BRIEF REPORTS

Brief Reports are accounts of completed research which do not warrant regular articles or the priority handling given to Rapid Communications; however, the same standards of scientific quality apply. (Addenda are included in Brief Reports.) A Brief Report may be no longer than four printed pages and must be accompanied by an abstract.

Observer dependence in quantum cosmology

Esteban Calzetta
Instituto de Astronomía y Física del Espacio, Casilla de Correos 67, Sucursal 28, 1428 Buenos Aires, Argentina and Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Ciudad Universitaria, 1428 Buenos Aires, Argentina

Alejandra Kandus
Instituto de Astronomía y Física del Espacio, Casilla de Correos 67, Sucursal 28, 1428 Buenos Aires, Argentina
(Received 15 October 1992)

The main thesis of this paper is that minisuperspace quantum cosmology reproduces the observer dependence of the vacuum found in quantum field theory in curved spacetime. We show that the vacua picked up by the Hartle-Hawking "no boundary" proposal in two different minisuperspaces sharing the same classical limit ($k=0$ and $k=1$ de Sitter minisuperspaces) are nonequivalent. A closer inspection suggests that the observer dependence is introduced through the choice of the Wick rotation. This hypothesis is supported by the analysis of the static de Sitter minisuperspace, where two different Euclidean sections are available, and where again two inequivalent vacua are obtained. These examples are not conclusive, but point to the possibility of a similar ambiguity even in full quantum cosmology.

PACS number(s): 04.60.+n, 98.80.Hw

I. INTRODUCTION

One of the fundamental issues in quantum field theory in curved space-time (QFTCST) is the determination of the vacuum state of the system under study. One criterion to pick up that state is for example the diagonalization of the Hamiltonian of the system. The starting point for its application is the selection of a foliation of the background space-time, or equivalently of a coordinate system [1] in which we will solve the field equations. Nevertheless, once that selection is done and the Hamiltonian diagonalization performed, the resulting vacuum state is generally not equivalent to the one obtained working with other possible foliations; under these circumstances, we talk about *observer-dependent quantum vacua* [1].

Another approach to obtaining the vacuum state of a system is given by the semiclassical limit of quantum cosmology (QC). It has two formulations, the Euclidean path integral approach and the canonical approach. In each of these we would expect that the resulting ground state wave function of the Universe will be independent of any choice of foliation, due to the fact that in this theory there is no *a priori* foliation of the space-time. Nevertheless, because of mathematical difficulties in both methods, we are compelled to work not with the infinite degrees of freedom of a realistic cosmological model, but only with a few, freezing out the others. These remaining degrees of freedom are called *minisuperspace.* The selection of a minisuperspace involves the selection of a foliation of the space-time. We are then led to the fact that a possible dependence on the minisuperspace selection could arise in the semiclassical limit of the minisuperspace approximation to the full quantum cosmology.

When working with the Euclidean path integral formalism, the ground state wave function for the considered system, let us say a field, is obtained considering Euclidean field configurations on a Euclidean background. In such backgrounds the concept of foliation loses the meaning that it has in Lorentzian manifolds, so we could think that the results obtained in this formalism, even when working within the minisuperspace approximation, would not depend on the selected minisuperspace. Nevertheless, this is not so. To apply this method, we start by Euclideanizing the metric by rotating the parameter that, with the signature convention of Ref. [2], plays the role of "time," performing $t\rightarrow -i\tau$. But the choice of such a time coordinate depends on the choice of an arbitrary coordinate system. As in general there are several such systems for each manifold, when we perform the mentioned rotation we will obtain different *Euclidean sections.* Even more, we can associate to a given parameter t more than one different Euclidean sections, as happens with the static coordinatization of de Sitter space-time or with Schwarzschild coordinates of a black-hole space-time: a periodically identified one that corresponds to freely falling observers, and a nonidentified one that corresponds to static observers. So if the *observer* is introduced through the selection of the

Euclideanization, then the results of any formulation of quantum cosmology which uses Euclidean methods, such as the Hartle-Hawking one, are potentially observer dependent.

Let us observe that there is no well-defined and covariant procedure to Euclideanize a generic Lorentzian manifold, nor to *Lorentzianize* a Euclidean one. There are examples where a Lorentzian manifold is obtained from a Euclidean one, such as de Sitter space-time [3]. That Euclidean manifold is referred to as a *true Euclidean section*. However it is not the case that we can build all possible space-times by a similar procedure. Although there is a covariant method to Euclideanize globally hyperbolic manifolds [4], it is not a general one, being valid only under restricted circumstances. Therefore, we cannot speak of a Euclidean section in general, but of a foliation or observer-dependent one. We believe that this is another ambiguity in the calculation of the wave function of the Universe, among others related to the use of Euclidean methods in cosmology [5,6].

In this Brief Report we study the dependence of the semiclassical limit of QC both on the choice of minisuperspace and on the Euclideanization procedure. To investigate the former issue, we compute the semiclassical wave function describing quantum fluctuations of the metric around a de Sitter configuration, applying the Hartle-Hawking [7] prescription first describing the de Sitter universe as a spatially closed Friedmann-Robertson-Walker model, and then as a spatially flat one. In each case, we perform a Wick rotation naturally related to the chosen minisuperspace. We conclude, nevertheless, that the semiclassical vacua found in each case are not equivalent.

The dependence of the semiclassical wave function on the choice of Euclidean section is further highlighted by considering the static foliation of the de Sitter background, where two Euclidean sections can be achieved from the same Lorentzian parameter t, by two inequivalent but physically meaningful Wick rotations; we obtain again observer-dependent graviton ground state wave functions.

The paper is organized as follows. We discuss the semiclassical wave function for metric perturbations of de Sitter space in spatially closed and spatially flat coordinates in Sec. II. In Sec. III, we perform a similar analysis in the static frame.

II. DEPENDENCE ON THE CHOICE OF MINISUPERSPACE

In this section, we shall demonstrate the dependence of semiclassical QC on the choice of minisuperspace, by showing that the Hartle-Hawking boundary condition actually picks up two inequivalent vacua for the graviton ground state in a de Sitter universe, depending on whether this is considered as spatially closed or as spatially flat.

de Sitter space-time is the solution of the Einstein equations with cosmological constant $\Lambda=3H^2$ and no stress-energy tensor. From all possible coordinatizations there are three most commonly used [3], namely the *spatially closed* one:

$$ds^2=-dt^2+H^2\cosh^2(Ht)[d\chi^2+\sin^2\chi\, d\Omega^2]\ ,$$

where $d\Omega^2=(d\theta^2+\sin^2\theta\, d\phi^2)$, $0\le\chi\le\pi$, $0\le\theta\le\pi$, $0\le\phi\le 2\pi$, $-\infty<t<+\infty$, which encloses the whole space-time; the *spatially flat* one:

$$ds^2=-dt^2+e^{2Ht}(dx^2+dy^2+dz^2)\ ,$$

where $-\infty<t<+\infty$, $-\infty<x,y,z<+\infty$, which encloses only one-half of the hyperboloid; and the *static* coordinates:

$$ds^2=-(1-H^2r^2)dt^2+\frac{dr^2}{(1-H^2r^2)}+r^2(d\Omega^2)\ ,$$

where $-\infty<t<+\infty$, $0<r<(1/H)$, $0<\theta<\pi$, $0<\phi<2\pi$, which does not cover all the hyperboloid and presents event horizons.

In the path integral formulation of quantum cosmology the idea is to extend the formalism of Euclidean path integrals in Minkowski space-time [8] to gravity. When matter fields are considered perturbatively, the action for them and for gravity decouples and the path integral can be written as the product of one for the background metric and another for the matter fields. The path integral for the background has been worked out first by Hartle and Hawking [7] and later by Halliwell and Louko [5].

In this Brief Report we calculate the wave function for three-metric perturbations δh_{ij} of a de Sitter universe. In the gauge $N^i=0$, $\dot N=0$, the path integral expression for the quantum amplitude $\Psi[\delta h_{ij}]$ associated to a given perturbation becomes [9,10]

$$\Psi[\delta h_{ij}]=\int dN(t_f-t_0)\int\mathcal{D}[\delta h_{ij}]\mathcal{D}[\pi^{ij}]e^{iS(g_{ij}^{\text{back}},N,\delta h_{ij},\pi^{ij})}\ , \tag{2.1}$$

where g_{ij}^{back} denotes the background three-metric. In order to obtain the ground state wave function, we must perform $t\to -i\tau$ and apply the chosen boundary condition.

Considering de Sitter space-time as an instance of a Friedmann-Robertson-Walker universe, we can follow Ford and Parker's treatment of perturbations on those backgrounds [11,12]. We will then consider three-metric perturbations that are transverse and traceless.

The equation for them is formally equivalent to that for a massless minimally coupled scalar field [11,12], and the action for the perturbation reads [11]

$$S=-\tfrac{1}{4}\int d^4x\sqrt{-\hat g}\,[h_{\mu\nu;\alpha}h^{\mu\nu;\alpha}-2\mathcal{R}_\rho^{(0)\mu}h_{\alpha\mu}^{\alpha\rho}-2\mathcal{R}^{(0)}_{\mu\rho\alpha\nu}h^{\mu\nu}h^{\rho\alpha}-\Lambda h_{\mu\nu}h^{\mu\nu}]\ . \tag{2.2}$$

When considering the perturbation as a superposition of modes, for the spatially flat case we can consider a superposition of plane waves of the form $\mathcal{G}_i^j(\mathbf{k},\mathbf{x})=\mathcal{A}_i^j(k)e^{i\mathbf{k}\cdot\mathbf{x}}+\text{H.c.}$, where $\mathcal{A}_i^j$ are symmetric tensors of unit norm, whose components are complex numbers and do not depend on the coordinates. For the spatially closed foliation we consider $\mathcal{G}_i^j$ as symmetric tensor spherical harmonics [13].

In the spatially closed coordinates, the path integral (2.1), replacing the mode decomposition in the action

(2.2), performing the Wick rotation $t\to(i/H)(\pi/2-\tau)$ and applying the Laplace method to evaluate the time integral, leads to the wave function Ψ_1 of Ref. [14]. The modes used to build this wave function are equivalent to those of Ref. [15], where it is shown that the ground state built with them is de Sitter invariant. So this wave function represents a state that carries the full de Sitter symmetry [16].

In the spatially flat coordinatization, a prescription of the form $t\to -i\tau$ turns the scale factor into a complex function. Nevertheless, as this is the usual way in which manifolds are Euclideanized, we will follow the treatment of Ref. [17], performing $t\to -i\tau$, $H\to iH$, the metric then becomes Euclidean but the scale factor, which in this case is $a(t)=e^{Ht}$, remains real. In Ref. [17] this criterion is employed to calculate the modes corresponding to a massive conformally coupled scalar field in de Sitter space-time, obtaining those associated to the Euclidean vacuum [18,19]. In view of this result we can consider this Euclideanization prescription as a good one for this minisuperspace.

Another thing that must be taken into account is the fact that flat coordinates only cover one-half of the full de Sitter manifold. This is of no importance because by transforming to conformal time $\eta=-(1/H)\exp[-H\tau]$ and letting it take values in the interval $-\infty<\eta<+\infty$, we cover the full manifold.

The path integral over phase space yields

$$\Psi\simeq\prod_k \mathcal{N}_k e^{-(1/4)a^3(\eta)\{\dot\phi_k^{(1)}(\eta)]/[\phi_k^{(1)}(\eta)]\}\phi_k^2(\eta)}, \qquad (2.3)$$

where the modes that satisfy the Euclidean field equation and the boundary condition, i.e., the ones that remain bounded when $\eta\to-\infty$, are [18]

$$\phi_k^{(1)}(\eta)=(ik\eta)^{3/2}[(ik\eta)^{-1/2}-i(ik\eta)^{-3/2}]e^{-i(ik\eta-\pi/4)} .$$

In this case the behavior of metric perturbations is the same as for a massless minimally coupled scalar field [11,12]. The range of mode solutions for the time-dependent equation is the same for the two systems. In de Sitter space-time this system has no vacuum states that are invariant with respect to the symmetry transformations of the space-time, as shown in Ref. [18]. The modes used to calculate the wave function (2.3) represent an idealized vacuum state with less symmetry than the one of the full de Sitter group [18,20] but with the highest possible symmetry for the chosen coordinatization of the background. In view of what we have just said, we can conclude that our wave function does not represent a de Sitter invariant state, and then the vacuum state obtained in these coordinates is not the same as the one obtained in the previous ones. Indeed, for a massless minimally coupled scalar field, the chosen boundary condition does not pick a de Sitter invariant state [19].

We want to stress here how, given a system, this boundary condition picks out different vacuum states when working in different Euclidean sections of the same manifold.

III. DEPENDENCE ON THE CHOICE OF EUCLIDEAN SECTION

In order to analyze the relationship between the Euclideanization procedure and the implementation of the Hartle-Hawking boundary condition, we are going to work with a minisuperspace that consists of three-metric perturbations $h_{\mu\nu}$ of the static de Sitter foliation, such that the line element reads

$$ds^2=-(1-H^2r^2)dt^2+\frac{dr^2}{(1-H^2r^2)}+r^2d\Omega_2^2 -2r^2\mathcal{C}(r,t)\sin\theta\, d\theta\, d\phi . \qquad (3.1)$$

It is easy, but somewhat lengthy, to show that $h_{\mu\nu}$ is a linear combination of odd waves according to the classification made by Regge and Wheeler [21].

The $(\theta\phi)$ Einstein equation to lowest order in the function $\mathcal{C}(r,t)$ reads

$$-\frac{r^2}{(1-H^2r^2)}\ddot{\mathcal{C}}+r(1-H^2r^2)\mathcal{C}'' +2r(1-2H^2r^2)\mathcal{C}'-2\mathcal{C}=0 \qquad (3.2)$$

where the dot means a time derivative and the prime a derivative with respect to r. Proposing as solution a mode decomposition of the form $\sum_\omega e^{i\omega t}\mathcal{C}_\omega(r)$, we obtain an equation for the spatial dependent part that can be put in the hypergeometric form. The solutions for the modes are

$$\mathcal{C}_\omega=e^{i\omega t}Hr(1-H^2r^2)^{(i\omega/2H)} \times F\left[\frac{1}{2}+\frac{i\omega}{2H};2+\frac{i\omega}{2H};\frac{5}{2};H^2r^2\right]+\text{c.c.} , \qquad (3.3)$$

where the hypergeometric function is expressed according to Ref. [22]. These solutions are orthonormalized by the relation [23]

$$\int_0^{1/H}dr\frac{r^2}{(1-H^2r^2)}\mathcal{C}_\omega\mathcal{C}^*_{\omega'} =\frac{r^2(1-H^2r^2)}{\omega^2-\omega'^2}[\mathcal{C}'_\omega\mathcal{C}^*_{\omega'}-\mathcal{C}_\omega\mathcal{C}'^*_{\omega'}] . \qquad (3.4)$$

The function $\mathcal{C}(r,t)$ is a minisuperspace variable, because the Einstein equation (3.2) coincides with the equation obtained by taking variations in the Einstein-Hilbert action, and the (00) Einstein equations also coincide with the Hamiltonian obtained by the usual Legendre transformation [24].

In order to write the path integral over all the Euclidean configurations, we must first work out the Euclidean action. For this purpose we replace a perturbation of the form $\int d\omega B_\omega(t)\mathcal{C}_\omega(r)$ in the Einstein-Hilbert action [2] and perform the Wick rotation $t\to -i\tau$, whereby we get the expected result

$$S_E=\int d\omega\int d\tau[\dot B_\omega^2(\tau)+\omega^2B_\omega^2(\tau)] .$$

The path integral over the modes and its momenta leads to the known result for a harmonic oscillator, describing the transition amplitude from an initial configuration $B_{\omega0}$

at Euclidean time τ_0, to the desired configuration $B_{\omega f}$ at Euclidean time τ_f.

To obtain the ground state wave function we must apply the boundary condition. Without loss of generality we can consider $\tau_f=0$ and $B_{\omega 0}=0\ \forall\omega$, then following steps of the previous section, we obtain a ground state wave function by taking the limit $\tau_0\rightarrow-\infty$. In doing so we can evaluate the remaining integral by the Laplace method obtaining

$$\Psi=\prod_\omega \mathcal{N}_\omega\left[\frac{\omega}{\pi\hbar}\right]^{1/2} e^{-(\omega/2\hbar)B^2_{\omega f}} . \tag{3.5}$$

This wave function is not, properly speaking, a Hartle-Hawking one, because we did not consider regular modes to evaluate it, i.e., modes defined in what is considered *the* Euclidean section of de Sitter space-time. In static coordinates, the Euclidean section of de Sitter space-time is obtained by the same rotation $t\rightarrow -i\tau$, but considering the Euclidean time τ as a periodic variable with period $2\pi/H$, where H^{-1} is the radius of the four-sphere. The configurations that are to be taken into account must be defined in a complete spacelike hypersurface; in our case this hypersurface consists of two disconnected parts. In order to build a regular configuration, we need to specify field amplitudes on the part of the hypersurface that is inside of one of the wedges, and field amplitudes on the part that corresponds to the other wedge. For simplicity we will consider that these two surfaces are separated in the Euclidean section by $\tau_f-\tau_0=\pi/H$.

All this is translated to the time integral by considering the integration over all possible Euclidean intervals between two subhypersurfaces located, say, in $\tau_0=0$ and in $\tau_f=\pi/H$, and field amplitudes $B_{\omega 0}$ on τ_0 and $B_{\omega f}$ on τ_f. As, again, our integral has a saddle point in the lower extreme of integration, we can apply the Laplace method, obtaining

$$\Psi\simeq\prod_\omega \mathcal{N}_\omega\left[\frac{\omega}{2\pi\hbar\sinh[\pi\omega/H]}\right]^{1/2} e^{-I_\omega/\hbar} . \tag{3.6}$$

Equation (3.6) is the Hartle-Hawking wave function. It was built taking into account the regularity of the Euclidean section and of the modes, by periodically identifying the Euclidean time. Applying the thermofields formalism with Euclidean path integrals [25], it can be shown that the vacuum state (3.6) is perceived by static observers as a thermal state with temperature $T=H/2\pi k_B$, where k_B is the Boltzmann constant.

We conclude that the path integral formalism leads to the vacuum perceived by static observers, when a non-periodically-identified Euclidean section is employed, and to the vacuum perceived by free falling observers (the Hartle-Hawking one), when the Euclidean section is periodically identified, to avoid a possible singularity. Since these vacua are not equivalent, we conclude that the Hartle-Hawking boundary condition does not pick up a unique vacuum but different ones, depending on the considered Euclidean section.

ACKNOWLEDGMENTS

We want to thank B. Allen, R. Ferraro, A. Higuchi, T. Jacobson, R. Laflamme, and L. Smolin for useful discussions. This work was partially supported by Consejo Nacional de Investigaciones Científicas y Técnicas de Argentina, by the Directorate General for Science, Research and Development of the Commission of the European Communities under Contract No. C11-0540-M(TT), by Universidad de Buenos Aires and by Fundación Antorchas.

[1] M. Castagnino and R. Ferraro, Phys. Rev. D **34**, 497 (1986).
[2] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).
[3] E. Schrödinger, *Expanding Universes* (Cambridge University Press, Cambridge, England, 1956).
[4] P. Candelas and D. J. Raine, Phys. Rev. D **15**, 1494 (1977).
[5] J. J. Halliwell and J. Louko, Phys. Rev. D **40**, 1868 (1989).
[6] G. W. Gibbons, S. W. Hawking, and M. J. Perry, Nucl. Phys. **B138**, 141 (1987); K. Schleich, Phys. Rev. D **36**, 2342 (1987); **39**, 2192 (1989).
[7] J. B. Hartle and S. W. Hawking, Phys. Rev. D **28**, 2960 (1983).
[8] S. Coleman, *Aspects of Symmetry* (Cambridge University Press, Cambridge, England, 1985).
[9] C. Teitelboim, Phys. Rev. D **25**, 3159 (1982).
[10] J. J. Halliwell, Phys. Rev. D **38**, 2468 (1988).
[11] L. H. Ford and L. Parker, Phys. Rev. D **16**, 1601 (1977).
[12] L. Parker and L. H. Ford, Phys. Rev. D **16**, 245 (1977).
[13] U. H. Gerlach and U. K. Sengupta, Phys. Rev. D **18**, 1773 (1978).
[14] S. Wada, Nucl. Phys. **B276**, 729 (1986).
[15] A. Higuchi, Class. Quantum Grav. **8**, 1961 (1991).
[16] F. Schwarz, in *Lectures in Theoretical Physics,* edited by A. O. Barut and W. E. Brittin (Colorado Associated University Press, Boulder, 1971), Vol. XIII.
[17] E. A. Calzetta and M. Castagnino, Phys. Rev. D **29**, 1602 (1984).
[18] B. Allen, Phys. Rev. D **32**, 3136 (1985).
[19] R. Laflamme, Phys. Lett. B **198**, 156 (1987).
[20] B. Allen and A. Folacci, Phys. Rev. D **35**, 3771 (1987).
[21] T. Regge and J. A. Wheeler, Phys. Rev. **108**, 1063 (1957).
[22] *Handbook of Mathematical Functions,* edited by M. Abramowitz and I. Stegun, Natl. Bur. Stand. (U.S.), Applied Mathematics Series No. 55, 10th ed. with corrections (U.S. GPO, Washington, DC, 1972).
[23] D. L. Kreider, R. G. Kuller, D. R. Ostberg, and F. W. Perkins, *An Introduction to Linear Analysis* (Addison-Wesley, Reading, MA, 1966).
[24] C. O. Lousto and F. D. Mazzitelli, Int. J. Mod. Phys. A **6**, 1017 (1991).
[25] R. Laflamme, Physica A **158**, 58 (1989).