

## Nonsingular black hole evaporation and “stable” remnants

D. A. Lowe\*

*Department of Physics, Princeton University, Princeton, New Jersey 08544*

M. O’Loughlin†

*Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08855-0849*

(Received 27 May 1993)

We examine the evaporation of two-dimensional black holes, the classical space-times of which are extended geometries, such as, for example, the two-dimensional section of the extremal Reissner-Nordström black hole. We provide evidence that the evaporation in two particular models proceeds to a stable end point. This should represent the generic behavior of a certain class of two-dimensional dilaton-gravity models. There are two distinct regimes depending on whether the back reaction is weak or strong in a certain sense. When the back reaction is weak, evaporation appears to proceed via an adiabatic evolution, whereas for strong back reaction, the decay proceeds in a somewhat surprising manner. Although information loss is inevitable in these models at the semiclassical level, it is rather benign, in that the information is stored in another asymptotic region.

PACS number(s): 04.60.+n, 97.60.Lf

### I. INTRODUCTION

One of the authors (M.O’L.) and Banks [1] had previously proposed that for a large class of modified scalar-gravity theories in which the classical geometries are all nonsingular, with a causal structure identical to that of Reissner-Nordström type, the Hawking evaporation to a final zero-temperature remnantlike object could be studied without singularity, as opposed to the original CGHS (Callan-Giddings-Harvey-Strominger) [2] models in which a now well-known singularity was found. Here we report on calculations that support this picture. When in-falling matter perturbs one of these extremal solutions, two apparent horizons form. As the evaporation takes place, these apparent horizons approach each other. We find two distinct regimes, depending on whether the back reaction is weak or strong in a certain sense. With weak back reaction, an adiabatic approximation gives a correct description, and the geometry appears to settle down to a stable remnant, with the apparent horizons meeting only after an *infinite* proper time. In the strong back-reaction regime, the apparent horizons meet after a *finite* proper time, and only after meeting do things appear to settle back down to the extremal solution. Black holes in these models therefore seem to evaporate in a completely nonsingular fashion, realizing the original objectives of CGHS. Information loss occurs at the semiclassical level, but only in a rather benign way.

In Sec. II we introduce the models of interest. We discuss in some detail the behavior near the double horizon of the extremal static semiclassical space-time in Sec. III. In Sec. IV we describe the adiabatic approximation [3] for the nonsingular models. Section V contains the re-

sults of our numerical analysis and Sec. VI is devoted to our conclusions and a discussion of their implications.

### II. THE MODELS

Consider a Lagrangian taken from the general class of two-dimensional renormalizable generally covariant field theories [4]:

$$\mathcal{L}_{\text{cl}} = \sqrt{-g} [D(\phi)R + G(\phi)(\nabla\phi)^2 + H(\phi)] . \quad (2.1)$$

We require that the potentials behave asymptotically like those of linear dilaton gravity [2]:

$$D(\phi) \rightarrow \frac{G(\phi)}{4} \rightarrow \frac{H(\phi)}{4} \rightarrow e^{-2\phi} , \quad (2.2)$$

as  $\phi \rightarrow -\infty$ .

The renormalization-group equations are hyperbolic on the two-dimensional target space of this model, and thus given a set of initial data one can consistently renormalize the model [5]. In the following, without loss of generality, we will restrict attention to the class of models satisfying  $G(\phi) = -2D'(\phi)$ . Other models may be obtained by a field redefinition of  $\phi$ . Performing a Brans-Dicke transformation on the metric  $\hat{g} = e^{-2\phi}g$  this Lagrangian may be rewritten in the simple form

$$\hat{\mathcal{L}}_{\text{cl}} = \sqrt{-\hat{g}} [D(\phi)\hat{R} + W(\phi)] , \quad (2.3)$$

where we have defined  $W(\phi) = e^{2\phi}H(\phi)$ . This form of the Lagrangian is convenient for finding the classical solutions as described in Ref. [1], in which reference the extended space-time geometries are discussed.<sup>1</sup>

All solutions are causally related to the two-dimensional  $r$ - $t$  section of the four-dimensional Reissner-

\*Electronic address: lowe@puhep1.princeton.edu

†Electronic address: ologhlin@bohr.uchicago.edu

<sup>1</sup>Other black-hole-like geometries present in generalized versions of (2.1) are introduced by Mann in Ref. [6].



anomaly [10]:

$$T_{\alpha}^{\alpha m} = 2T_{rv}^m + hT_{rr}^m = \frac{\kappa}{4}R . \quad (3.2)$$

The components of interest to us are the trace, and  $l^{\alpha l \beta} T_{\alpha \beta}$ , where  $l = (h/2, 1)$ .

The  $\phi$  and trace equations may be written as

$$\nabla^2 \phi = \frac{e^{-2\phi} [D'W + \kappa(W - \frac{1}{2}W')] - D''(D' + \kappa)(\nabla\phi)^2}{D'(D' + 2\kappa)} , \quad (3.3)$$

$$R = -\partial_r^2 h = \frac{-e^{-2\phi}(2W + W') + 2D''(\nabla\phi)^2}{(D' + 2\kappa)} , \quad (3.4)$$

where

$$\nabla^2 \phi = \partial_r(2\partial_v \phi + h \partial_r \phi) ,$$

and

$$(\nabla\phi)^2 = 2\partial_r \phi \partial_v \phi + h(\partial_r \phi)^2 .$$

The remaining local linear combination of the constraint equations is (conveniently the linear combination needed for matching across the shock wave)

$$\begin{aligned} l^{\alpha l \beta} T_{\alpha \beta} = & 2D'(\partial_v \phi + \frac{1}{2}h \partial_r \phi)^2 \\ & + [\frac{1}{2}h \partial_r + \partial_v - \frac{1}{2}(\partial_r h)](\frac{1}{2}h \partial_r + \partial_v)D \\ & + \frac{\kappa}{4} \left[ \frac{h}{2} \partial_r^2 h + \partial_v \partial_r h - \frac{1}{4}(\partial_r h)^2 \right] + t_r(r) , \end{aligned} \quad (3.5)$$

where  $t_r$  is determined by the initial state of the quantum vacuum, and for our collapse, and for static solutions with no net flux at infinity, we set  $t_r$  to zero in coordinates that are asymptotic to the linear dilaton vacuum.

For static solutions we set  $h = h(r)$  and  $\phi = \phi(r)$ . With linear dilaton asymptotics at  $\phi \rightarrow -\infty$ ,  $W \rightarrow 4$ , and  $D \rightarrow e^{-2\phi}$ , we find, to lowest order,<sup>2</sup>

$$\begin{aligned} h(r) &= 1 - 2Me^{-2r} , \\ \phi(r) &= -r . \end{aligned} \quad (3.6)$$

One can numerically integrate the static equations using the above as boundary conditions at  $r \rightarrow \infty$ .

Now let us consider the behavior of fields near the horizon,  $r = r_h$ , of the extremal solution. We know from the classical analysis in Ref. [1], that the classical extremal solution has to lowest order  $\phi = \phi_0$  and  $h = h_0 x^2$  near the horizon, where  $x = r - r_h$ . Let us look at the quantum corrections to these formulas:

$$\begin{aligned} \phi &= \phi_0 + \beta x^{1+\delta} , \\ h &= \alpha_1 x^2 + \alpha_2 x^{3+\eta} . \end{aligned} \quad (3.7)$$

<sup>2</sup>Certain choices of the potentials  $D$  and  $W$  will give an additional term in the asymptotics of  $h$  proportional to  $r \exp(-2r)$ .

Plugging this into the static equations we find (evaluating all functions at  $\phi = \phi_0$ )

$$\begin{aligned} D'W &= \kappa(W'/2 - W) , \\ \alpha_1 &= \frac{e^{-2\phi_0}(W + \frac{1}{2}W')}{D' + 2\kappa} \\ &= e^{-2\phi_0}W/\kappa , \\ \eta &= \delta , \\ (\delta + 1)(\delta + 2) &= \frac{D''W + D'W - \kappa(\frac{1}{2}W'' - W')}{D'(W + \frac{1}{2}W')} . \end{aligned} \quad (3.8)$$

$\beta$  and  $\alpha_2$  are related by

$$\beta = \frac{-\kappa\alpha_2(\delta + 2)}{2\alpha_1\delta D'} . \quad (3.9)$$

Notice that the solutions have two obvious extensions through  $\phi = \phi_0$  (see Ref. [3] for a similar discussion). To study perturbations by shock waves of matter we will restrict ourselves to the odd extension that is the smoother of the two continuations and is the one that is a deformation of the classical geometry:

$$\begin{aligned} \phi &= \phi_0 + \beta x|x|^\delta , \\ h &= \alpha_1 x^2 + \alpha_2 x|x|^{2+\delta} . \end{aligned} \quad (3.10)$$

This also means that immediately above the shock wave we will find two apparent horizons at  $\phi_+ < \phi_0$  and at  $\phi_- > \phi_0$ , with a geometry qualitatively the same as the positive mass classical solutions.

We can evaluate the parameters in the expansion near the horizon of the extremal solution and observe that the quantum vacuum near the horizon is indeed a small  $\kappa$  deformation of the classical vacuum. Let us fix

$$D = e^{-2\phi} - \gamma^2 e^{2\phi} \quad \text{and} \quad W = 4 - \mu^2 e^{4\phi} . \quad (3.11)$$

For small  $\kappa$  and large  $\mu$ , we find

$$\begin{aligned} e^{-2\phi_0} &= \frac{\mu}{2} + \frac{\kappa}{2} , \\ \alpha_1 = 4, \quad \delta &= \frac{2\kappa}{\mu}, \quad \frac{\beta}{\alpha_2} = \frac{1}{8} . \end{aligned} \quad (3.12)$$

For definiteness we set  $\beta = -1/\mu$  in the following.

To check that the quantum behavior near  $\phi = \phi_0$  that we have displayed is consistent with the linear dilaton at infinity we numerically integrated out from  $\phi = \phi_0$  to  $\phi = \pm\infty$ , and indeed have observed the linear dilaton vacuum for  $\phi \rightarrow -\infty$  and the large  $\phi$  classical behavior in the other asymptotic regime.

#### IV. THE ADIABATIC APPROXIMATION

In order to study the semiclassical stability of the static extremal solutions of the DW models, consider sending in a matter shock wave  $l^\mu l^\nu T_{\mu\nu}^f = 2M\delta(v)$ . We define

$$\Sigma = 2\partial_v \phi + h \partial_r \phi , \quad (4.1)$$

so that the future apparent horizon is the locus of  $\Sigma = 0$ .

The discontinuity in  $\Sigma$  across the shock is

$$\delta\Sigma = \frac{4M}{\sqrt{D'(D'+2\kappa)}}. \quad (4.2)$$

To obtain explicit expressions we use the potentials of Eq. (3.11) and make a large  $\mu$ , small  $\kappa$ , and small  $M$  expansion. Specifically, we have  $M/\mu \ll \kappa/\mu \ll 1$ . We may calculate the positions of the apparent horizons which turn out to be

$$r_{\pm} = r_h \pm \sqrt{\mu}(M/\mu)^{1/(2+\delta)}, \quad (4.3)$$

which is obtained by setting  $\Sigma + \delta\Sigma = 0$ . This implies that

$$\partial_r \Sigma(r_{\pm}) = \mp \frac{8}{\sqrt{\mu}}(M/\mu)^{(1+\delta)/(2+\delta)}. \quad (4.4)$$

The discontinuity in  $h$  across the shock is given by

$$\delta h = \frac{8M}{\kappa} \int dr \left[ -1 - \frac{D'}{\sqrt{D'(D'+2\kappa)}} \right]. \quad (4.5)$$

By inserting this expression into the constraint equation we find  $\partial_v \Sigma$  at the apparent horizons:

$$\partial_v \Sigma = -8M\kappa/\mu^2. \quad (4.6)$$

One may then try to make an adiabatic approximation to compute the relative positions of the horizons as a function of  $v$ , by assuming Eqs. (4.4) and (4.6) continue to hold for all  $v$  if things are changing slowly enough:

$$\partial_v \hat{r}^{\pm} = -\frac{\partial_v \Sigma}{\partial_r \Sigma} = \mp \frac{\kappa}{\mu}(\hat{r}^{\pm} - r_h), \quad (4.7)$$

which gives

$$\hat{r}^{\pm}(v) = r_h + r_0^{\pm} e^{-\kappa(v-v_0)/\mu}, \quad (4.8)$$

or expressing things in terms of an effective mass, which measures the difference between the actual mass and the extremal mass

$$M(v) = M_0 e^{-2\kappa(v-v_0)/\mu}. \quad (4.9)$$

The adiabatic approximation will break down when the energy flux due to Hawking radiation is comparable to  $M(v)$ , so will be valid as long as  $\kappa/\mu \ll 1$ .

These expressions are almost identical to ones obtained in Ref. [3] in the case of the spherically symmetric reduction of the Reissner-Nordström black hole, apart from slight changes in numerical coefficients. The RN case is obtained by picking different potentials  $D$  and  $W$ . In this case, the adiabatic approximation is valid as long as  $\kappa/Q^2 \ll 1$ , where  $Q$  is the charge of the extremal black hole.

## V. THE NUMERICAL COLLAPSE

When an extremal solution is perturbed by an incoming flux of matter the picture one expects is the following: above the shock wave two apparent horizons form on either side of the line  $\phi_0$ , and as the black hole evaporates these horizons approach each other, with the solution settling back down to the extremal solution at  $\mathcal{I}_R^+$ . In

this section we describe numerical solutions for a shock wave of in-falling matter impinging upon the extremal solution of the DW model described in Sec. III. We also consider the analogous calculation for the RN-type model. Our aim is to test whether the extremal black hole is stable, and in what manner the evaporation proceeds.

Another approach would be to do a linear stability analysis of the problem, where one might hope to be able to understand things analytically. In fact, it seems this is not so for the back-reaction-corrected equations. A numerical analysis of this problem is tractable, but is of comparable difficulty to numerically solving the full non-linear equations, so we choose to do the latter.

In the context of the CGHS model, numerical results have previously been obtained in [11–13]. Here we are solving equations of precisely the same form, but with somewhat different potentials. See Ref. [12] for a description of the numerical algorithm used in this paper, and also Appendixes A and B, where the gauge and coordinate choices, and equations of motion are stated. For numerical purposes it is convenient to work on a grid of null lines, so conformal gauge is appropriate. This should be borne in mind when making quantitative comparisons with the results of the previous section, where it was necessary to use light-cone gauge to get analytical results.

For the DW model of Eq. (3.11), with  $\kappa=10$ ,  $\mu=15$ ,  $\gamma=8$ , and shock mass  $M_s=1.5$ , the results are plotted in Fig. 2. Note that here  $\kappa/\mu > 1$ , so one is in the strong back-reaction regime and the adiabatic approximation of the previous section is not expected to hold. The integration is stopped near  $x^- = 0$  where the line  $\phi = \phi_0$  starts out. This means these calculations will hold for either of the extensions described in the previous section. The horizons meet at finite proper time, and the line  $\phi_0$  goes from being spacelike to timelike at this point. Note the equations of motion imply that  $\partial_+ \partial_- \phi = 0$  when an apparent horizon (where  $\partial_+ \phi = 0$ ) intersects  $\phi = \phi_0$ . The position of the apparent horizons  $\hat{x}^-(x^+)$  may be found by

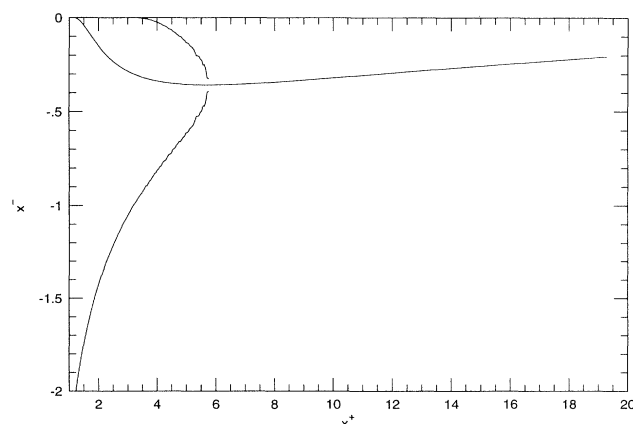


FIG. 2. Plot of motion of apparent horizons for DW model, in the strong back-reaction regime. The lower line is the outer apparent horizon, the middle line is  $\phi = \phi_0$ , and the upper line is the inner apparent horizon.

solving

$$\frac{\partial \hat{x}^-}{\partial x^+} = -\frac{\partial_+^2 \phi}{\partial_- \partial_+ \phi}, \quad (5.1)$$

which indicates that  $\partial \hat{x}^- / \partial x^+$  blows up as  $\partial_+ \partial_- \phi \rightarrow 0$ , as long as  $\partial_+^2 \phi$  remains finite. This is precisely what is happening at the meeting point of the apparent horizons, consistent with the numerical results. Also note that the wiggles in the path of the outer horizon are due to  $\partial_- \partial_+ \phi$  becoming very small in this region. This causes the error in the path of the line  $\partial_+ \phi = 0$  to be much larger than, for example, the error in the path of a line of constant  $\phi$ . The conclusive indication that the horizons meet comes from the observation that the line  $\phi = \phi_0$  becomes timelike. Numerical convergence has been checked by varying the stepsize, and varying the position of the initial surface.

Following a line of constant  $\phi$  as  $x^+$  become large, one finds the curvature approaches a constant which equals the initial curvature of the extremal solution, as shown in Fig. 3. This indicates that despite the fact that the apparent horizons have met, the solution still settles back down to the zero-temperature extremal state.

Lines of constant  $\phi$  appear to approach a null line  $x^- = x_0^-$ , which will become a global horizon, as shown in Fig. 4. It is difficult to tell from the numerics whether  $x_0^- \geq 0$ , or whether it is shifted out to more negative  $x^-$  by the in-falling matter. More sophisticated numerical calculations are needed to answer this question definitively.

Simulations were also performed for the weak back-reaction regime where  $\kappa/\mu \ll 1$ , when the adiabatic approximation is expected to hold. Here potential numerical errors were somewhat larger, but the results were found to be consistent with the adiabatic approximation. The critical line  $\phi = \phi_0$  remained spacelike, approaching the outer apparent horizon out to large values of  $x^+$ .

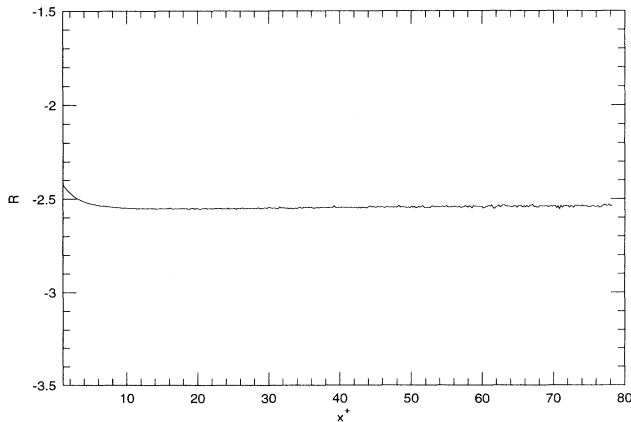


FIG. 3. Plot of the curvature along a line of constant  $\phi = -1.2$ , for the DW model, in the strong back-reaction regime. Here the range of  $x^+$  extends much further out. The curvature approaches a constant value which is the same as the initial curvature of the extremal solution.

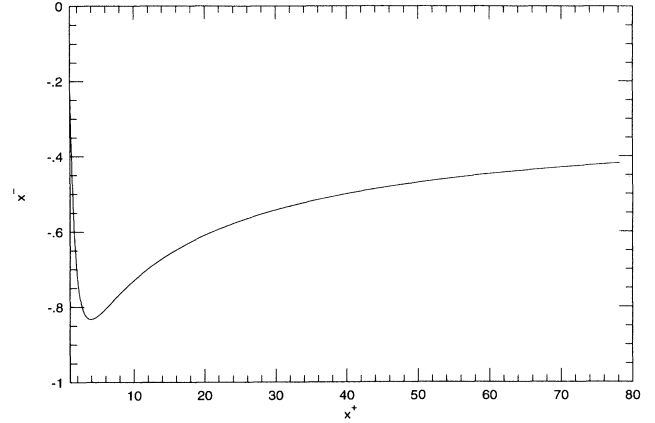


FIG. 4. Plot of the path of a line of constant  $\phi = -1.2$ , for the DW model, in the strong back-reaction regime. As  $x^+$  becomes large the line appears to asymptotically approach a null line.

Qualitatively similar results are obtained for the case of the RN model. In the strong back-reaction regime where  $\kappa/Q^2 > 1$ , with  $\kappa = 200$ ,  $Q^2 = 60$ , and shock mass  $M_s = 0.08$  (results are shown in Fig. 5). One difference here is that coordinates are chosen so the line  $\phi = \phi_0$  starts out at  $x^- = +\infty$ . Again the solution appears to settle back down to the extremal solution as  $x^+$  becomes large, after the apparent horizons have met. In the weak back-reaction regime ( $\kappa/Q^2 \ll 1$ ), results consistent with the adiabatic approximation were found, with the critical line remaining spacelike out to large values of  $x^+$ .

## VI. DISCUSSION AND CONCLUSIONS

The picture of the end point of the evaporation of these two-dimensional extremal black holes, which is consistent with the numerical calculations, is shown in Fig. 6. In the strong back-reaction regime, lines of constant  $\phi$  ap-

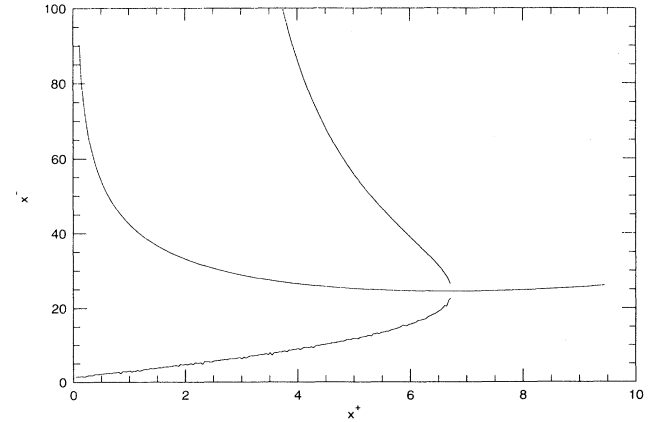


FIG. 5. Plot of motion of apparent horizons for RN model, in the strong back-reaction regime. The lower line is the outer apparent horizon, the middle line is  $\phi = \phi_0$ , and the upper line is the inner apparent horizon.

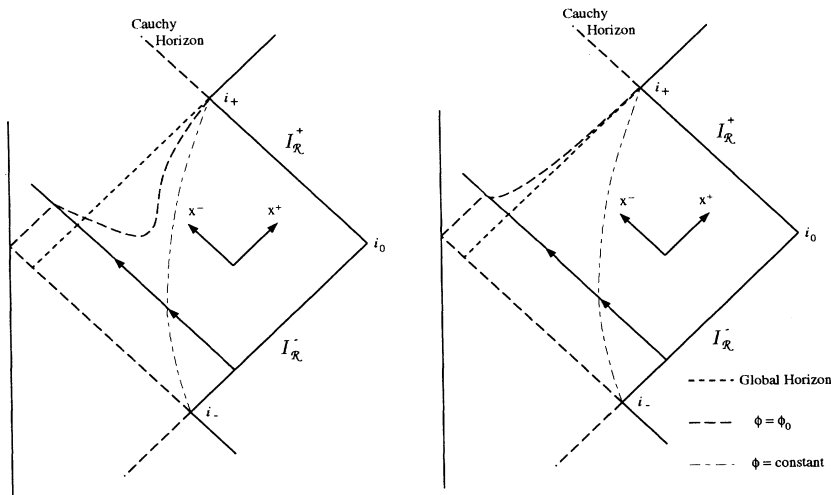


FIG. 6. Penrose diagrams showing the end points of the black-hole evaporation. The case of strong back reaction is shown on the left, the case of weak back reaction is shown on the right.

proach a global horizon, and the solution settles back down to the extremal one. After the apparent horizons meet the line  $\phi = \phi_0$  turns timelike and becomes asymptotically null as it approaches the global horizon. In the weak back-reaction regime the apparent horizons meet at  $\mathcal{I}_R^+$ , and the critical line  $\phi = \phi_0$  remains spacelike.

Our results indicate the presence of two qualitatively different regimes separated by some critical value of  $\kappa/\mu$  in the DW case, and  $\kappa/Q^2$  in the RN case. This should not be too surprising, since a similar phenomena occurs in the case of the damped harmonic oscillator, where there is a critical value of the damping separating two regimes of behavior. Although we have studied two particular models, we believe that this behavior will be generic to the class of models in which  $W(\phi)$  has a simple zero, which includes the nonsingular models discussed in Sec. II.

Because for large  $\kappa/\mu$  the apparent horizons collide, locally, the end point of the evaporation of these extremal solutions looks like an  $M^2 < Q^2$  static solution in the case of the RN model. One may wonder if two-dimensional quantum positive mass theorem prevents such an unexpected occurrence. However, positive mass theorems only give us information about the asymptotic structure of the space-time and certainly do not preclude the type of relaxation that we observe here. Classically, the DW and RN models obey positive mass theorems [14] with the mass defined on spacelike surfaces that become asymptotically null along  $\mathcal{I}_R^+$  and have their left boundary at the meeting of the two global horizons of the extremal solution below the shock wave. Unfortunately, the quantum positive mass theorems [15] do not appear to give any useful information in their present formulation due to the quantum corrections to the vacuum. Further, quantum positive mass theorems require a nonsingular spacelike surface that is asymptotically linear dilaton vacuum at both ends, see Refs. [15–17]. In our situation, this of course can never be the case, since the left boundary of our spacelike hypersurfaces meets the horizon of the extremal solution which looks distinctly not like the LDV.

These results appear to confirm the conjecture that

zero-temperature extremal (in the sense of RN) two-dimensional black holes are semiclassically stable. In the case of the DW models the evaporation of nonextremal black holes should proceed in a completely nonsingular way, realizing the original objectives of CGHS. Information loss is inevitable in these models at the semiclassical level,<sup>3</sup> since an in-falling flux of matter will always produce correlations with the region behind the global horizon, and hence inaccessible to an observer who finds themselves at  $i_+$  after an infinite proper time. The information loss is of a somewhat benign type, though. For observers outside the global horizon, the quantum mechanics that they participate in is unitary; indeed the spacetime before the Cauchy horizon can (by definition) be foliated by spacelike Cauchy hypersurfaces. The information that is no longer accessible to them is stored in a stable remnant, one of an infinitely degenerate set of possible final states. This infinity of states corresponds to all the possible field configurations on a spacelike slice through region III (Fig. 1). These remnants avoid the problems of overproduction in external fields and divergences in virtual loops by nature of their large internal geometry [18].

Let us note that the above discussion is based on semiclassical reasoning. It is still possible that when space-time is properly second quantized the information loss problem will be cured. Highly nonlocal quantum gravity effects may wind up giving the remnants a very long but finite lifetime. If this lifetime is of order the age of the Universe, experimenters making measurements over shorter times will still see an effective loss of information, which they would attribute to the existence of “stable” remnants.

The conjectured extremal state possesses a Cauchy horizon as shown in Fig. 1. We would like to comment on the sense in which this horizon is traversable<sup>4</sup> and the possibility of information loss for observers who traverse

<sup>3</sup>See Refs. [1,3,18] for related discussions.

<sup>4</sup>See Ref. [19] and references therein for earlier discussions.

it. We should first note that the Cauchy horizon in our model appears to be a double horizon and, as discussed in Ref. [9], has a softened divergence of the stress tensor as compared to the inner horizon of a nonextremal spacetime. It is also possible that the disturbance produced by the shock wave will separate the local horizon and the global Cauchy horizon in a manner similar to the mass inflation models of Poisson and Israel [20]. In mass inflation it was shown that an infinite tidal force appears along the Cauchy horizon, but the singularity is weak [21] in the sense that an observer can traverse it without getting stretched infinitely.

Let us assume then that a traversable Cauchy horizon is present. An observer who passes into the region above this Cauchy horizon faces a potential loss of unitarity associated with the lack of a global foliation of the spacetime by spacelike Cauchy hypersurfaces. The existence of a Cauchy horizon, its nature, if it existed, and the necessity of a unitarity restoring mechanism for those who cross it, are the subject of our ongoing investigations. The answers to these questions should shed light on an  $S$ -matrix description of quantum gravity in the presence of remnantlike objects.

#### ACKNOWLEDGMENTS

The research of D.L. was supported in part by DOE Grant No. DE-AC02-76WRO3072, NSF Grant No. PHY-9157482, and James S. McDonnell Foundation Grant No. 91-48. D.L. would like to thank C. Callan and A. Bilal for useful discussions. The research of M.O'L. was supported in part by the Department of Energy under Grant No. DE-FG05-90ER40559 and the National Science Foundation under Grant No. PHY89-04035. M.O'L. would like to thank T. Banks for innumerable enlightening conversations on the subject, and M. Douglas, S. Shenker, and A. Strominger for conversations. He would also like to thank the ITP at Santa Barbara for hospitality during part of this work. We thank A. Strominger and S. Trivedi for comments on a previous draft of this manuscript.

#### APPENDIX A: NUMERICAL METHOD FOR DW MODEL

The numerics are performed in the conformal gauge:

$$g_{++} = g_{--} = 0, \quad g_{+-} = g_{-+} = -\frac{1}{2}e^{2\rho}. \quad (\text{A1})$$

The linear dilation vacuum in the coordinates we choose is

$$\phi = \rho = -\frac{1}{2} \ln(-x^+ x^-) \quad (\text{A2})$$

while a solution with mass  $M$  looks like

$$\phi = \rho = -\frac{1}{2} \ln(M - x^+ x^-) \quad (\text{A3})$$

in the limit  $x^+ x^- \rightarrow -\infty$ . The equations to be solved are

$$\begin{aligned} \partial_+ \partial_- \rho &= \frac{-D'' + \phi \partial_- \phi - \frac{1}{4} e^{2\rho-2\phi} (W + W'/2)}{D' + 2\kappa}, \\ \partial_+ \partial_- \phi &= -\frac{D'' \partial_+ \phi \partial_- \phi + \frac{1}{4} e^{2\rho-2\phi} W + \kappa \partial_+ \partial_- \rho}{D'}. \end{aligned} \quad (\text{A4})$$

Here  $D = e^{-2\phi} - \gamma^2 e^{2\phi}$  and  $W = 4 - \mu^2 e^{4\phi}$ . The boundary conditions are that along  $x^+ = 1$  the solution correspond to the extremal solution discussed in Sec. III. Above the shock wave along  $\mathcal{I}^-$  the solution should agree with the classical shock solution. This amounts to setting

$$\partial_+ \phi = \partial_+ \rho \sim \frac{x^- + M}{2(M - x^+ x^- - Mx^+)} \quad (\text{A5})$$

as  $x^- \rightarrow -\infty$ . In practice, a large negative initial value of  $x^-$  is chosen.

#### APPENDIX B: NUMERICAL METHOD FOR THE RN MODEL

Here the numerics are also performed in conformal gauge (A1). Asymptotically, as  $x^+ - x^- \rightarrow \infty$  the solutions approach the vacuum

$$\phi = -\ln[\frac{1}{2}(x^+ - x^-)], \quad \rho = 0. \quad (\text{B1})$$

The equations to be solved are

$$\begin{aligned} \partial_+ \partial_- \rho &= \frac{\partial_+ \phi \partial_- \phi + \frac{1}{4} e^{2\rho} (e^{2\phi} - 2Q^2 e^{4\phi})}{1 - (\kappa/2) e^{2\phi}}, \\ \partial_+ \partial_- \phi &= \partial_+ \partial_- \rho + \partial_+ \phi \partial_- \phi + \frac{Q^2}{4} e^{2\rho+4\phi}. \end{aligned} \quad (\text{B2})$$

Here the boundary conditions are that along  $x^+ = 0$  the solution match onto the quantum corrected extremal solution [8], while above the shock wave along  $\mathcal{I}^-$  the solution agrees with a classical shock solution. This means

$$\partial_+ \phi \sim -\frac{1}{x^+ - x^-} + \frac{4M}{(x^+ - x^-)^2} \{1 - \ln[\frac{1}{2}(x^+ - x^-)]\}, \quad (\text{B3})$$

$$\partial_+ \rho \sim \frac{2M}{(x^+ - x^-)^2}$$

as  $x^- \rightarrow -\infty$ .

[1] T. Banks and M. O'Loughlin, Phys. Rev. D **48**, 698 (1993).  
[2] C. G. Callan, S. B. Giddings, J. A. Harvey, and A. Strominger, Phys. Rev. D **45**, R1005 (1992); J. Harvey and A. Strominger, "Quantum Aspects of Black Holes," Enrico Fermi Institute Report No. EFI-92-41, hep-th/9209055 (unpublished), and references cited therein.

[3] A. Strominger and S. Trivedi, Phys. Rev. D (to be published).  
[4] T. Banks and M. O'Loughlin, Nucl. Phys. **B362**, 649 (1991).  
[5] T. Banks and J. Lykken, Nucl. Phys. **B331**, 173 (1990).  
[6] R. Mann, Phys. Rev. D **47**, 4438 (1993).

- [7] S. Hawking, *Phys. Rev. Lett.* **69**, 406 (1992).
- [8] B. Birnir, S. Giddings, J. Harvey, and A. Strominger, *Phys. Rev. D* **46**, 638 (1992).
- [9] S. Trivedi, *Phys. Rev. D* **47**, 4233 (1993).
- [10] A. M. Polyakov, *Mod. Phys. Lett. A* **2**, 893 (1987).
- [11] S. W. Hawking and J. M. Stewart, *Nucl. Phys.* **B400**, 393 (1993).
- [12] D. A. Lowe, *Phys. Rev. D* **47**, 2446 (1993).
- [13] T. Piran and A. Strominger, *Phys. Rev. D* (to be published).
- [14] Y.-C. Park and A. Strominger, *Phys. Rev. D* **47**, 1569 (1993).
- [15] A. Bilal, *Phys. Rev. D* **48**, 1665 (1993).
- [16] A. Bilal and I. Kogan, *Phys. Rev. D* **47**, 5408 (1993).
- [17] S. P. de Alwis, "Two-dimensional quantum dilaton gravity and the positivity of energy," Report No. hep-th/9302144, COLO-HEP-309 (unpublished).
- [18] T. Banks, M. O'Loughlin, and A. Strominger, *Phys. Rev. D* **47**, 4776 (1993).
- [19] S. Chandrasekhar and J. Hartle, *Proc. R. Soc. London* **A284**, 301 (1982).
- [20] E. Poisson and W. Israel, *Phys. Rev. Lett.* **63**, 1663 (1989); *Phys. Lett. B* **233**, 74 (1989); *Phys. Rev. D* **41**, 1796 (1990).
- [21] A. Ori, *Phys. Rev. Lett.* **67**, 789 (1992).