Four-dimensional dynamically triangulated gravity coupled to matter

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We investigate the phase structure of four-dimensional quantum gravity coupled to Ising spins or Gaussian scalar fields by means of numerical simulations. The quantum gravity part is modeled by the summation over random simplicial manifolds, and the matter fields are located in the center of the four-simplices, which constitute the building blocks of the manifolds. We find that the coupling between spin and geometry is weak away from the critical point of the Ising model. At the critical point there is clear coupling, which qualitatively agrees with that of Gaussian fields coupled to gravity. In the case of pure gravity a transition between a phase with highly connected geometry and a phase with very "dilute" geometry has been observed earlier. The nature of this transition seems unaltered when matter fields are included. It was the hope that continuum physics could be extracted at the transition between the two types of geometries. The coupling to matter fields, at least in the form discussed in this paper, seems not to improve the scaling of the curvature at the transition point.

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I. INTRODUCTION

Last year a new regularized model of quantum gravity in four dimensions (4D) was introduced [1,2]. The path integral is approximated by a summation over randomly triangulated piecewise linear manifolds.[1] This model is a generalization of the one from two dimensions, which was very successful [4–7]. In 4D simplicial quantum gravity two different phases have been observed: one with a highly connected geometry and a large Hausdorff dimension and one with a low Hausdorff dimension. Based on numerical simulations it was suggested in [2] that the transition between the two types of geometries was of second order and that an interesting continuum limit might be extracted at the transition point.[2] This observation has been further corroborated in a sequence of papers [8–12].

One obstacle to the above-mentioned suggestion is that the numerical simulations performed so far seem to indicate that the average curvature, defined for a triangulated manifold by Eq. (3), does not scale to zero at the transition point. The average curvature does decrease (albeit slowly) with the volume of the simulated universes, and since it has so far only been possible to consider universes built out of less than 32 000 four-dimensional simplices, which corresponds to at most a 6^4 regular lattice, it is possible that the curvature scales to zero in the infinite volume limit. However, if this is not the case it prompts at least a reinterpretation of the meaning of the scaling limit, since naive scaling such as

$$\langle R_{\text{lattice}} \rangle = \langle R_{\text{cont}} \rangle a^2 \tag{1}$$

(where a is the lattice spacing) cannot be maintained. Of

[1]An older, related approach makes use of a fixed triangulation, but allows the variation of the length of the links. To the contrary, in the present approach one keeps the length of the links fixed, but varies the connectivity. We refer to [3] for a recent lucid review of the first approach, which we here will call "Regge gravity," while we will use the term "simplicial gravity" for the present approach.

[2]A similar transition has been observed in the computer simulations of 4D Regge gravity. The relation between the two approaches is not clarified yet. We would like both regularizations to represent the same continuum theory. However, there are indications that even in 2D gravity the theories might differ. According to [3] it seems as if the coupling of 2D Regge gravity to matter differs from that of simplicial 2D gravity coupled to matter, which, on the other hand, agrees with analytic continuum calculations. This observed difference is based on numerical simulations and maybe more extensive simulations will change the situation. With the present incomplete understanding of the relation between Regge gravity and simplicial gravity, it is clearly of importance to investigate both regularizations of quantum gravity.

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course, nobody presently knows how a quantum theory of Euclidean gravity will manifest itself and maybe a conventional scaling such as Eq. (1) is misleading. This is most likely the case if there exists a "topological" phase where $\langle g_{\mu\nu} \rangle = 0$, as sometimes conjectured. At this point we should mention a recent suggestion [13] of a different identification of the lattice results with continuum theory in which one considers the limit of the bare gravitational coupling constant going to infinity. This limit might in continuum language correspond to an infrared fixed point dominated by the quantum fluctuations of the conformal factor. The scaling relations derived in [13] agree at the qualitative level quite well with the numerical results, but they move the interesting region of continuum physics away from the transition in geometry and to a region in coupling constant space where (1) can be satisfied. We consider this suggestion as most interesting.

In this work, we will explore whether coupling of matter fields to gravity will change the phase structure of the theory and maybe even cure the problem with the scaling of the average curvature at the transition. In the best of all worlds one could even hope that the quest for a conventional scaling of gravity observables such as the average curvature would uniquely determine the matter content of the theory.[3] It is, of course, also of interest in itself to study matter fields coupled to dynamical random geometries.

The coupling of matter to two-dimensional gravity has revealed a rich and beautiful structure as long as the central charge of the field theory is less than or equal to one. This is summarized in the Knizhnik-Polyakov-Zamolodchikov (KPZ) formulas [14], but was first discovered in the simplicial gravity approach. As an example, when the Ising model is coupled to 2D simplicial gravity its phase transition changes from being second order to third order [15,16]. In addition the back reaction of matter changes the critical exponent γ of gravity *at* the critical point of the Ising model. Away from the critical point this exponent is unchanged.

Unfortunately the analytical methods of 2D have not yet been extended to higher dimensions. The coupling of the Ising model to 3D gravity was investigated by numerical simulations in [17–19]. The phase diagram was determined in [19] and the conclusion was that, although there was a clear coupling between gravity and the spins at the critical point of the spin system, this influence was not sufficiently strong to change the *first-order transition* observed in three dimensions [20,21] between the two phases of the geometrical system into a more interesting (from the point of view of continuum physics) second-order transition. In this respect the situation is better in 4D where the transition between the two phases of the geometrical system may already be of second order, as mentioned above.

The rest of this paper is organized as follows. In Sec. II, we define the model. In Sec. III, we discuss briefly the numerical method, while Sec. IV contains our numerical results. Finally, in Sec. V, we discuss the results obtained.

II. THE MODEL

Simplicial quantum gravity in 4D is described by the partition function (see, e.g., [2,10])

$$Z(\kappa_2,\kappa_4)=\sum_{T\in \mathcal{T}} e^{-\kappa_4 N_4+\kappa_2 N_2}\ , \qquad (2)$$

where the sum is over triangulations T in a suitable class of triangulations $\mathcal{T}$. The quantity N_4 denotes the number of four-simplices in the triangulation and N_2 the number of triangles. The coupling constant κ_2 is inversely proportional to the bare gravitational coupling constant, while κ_4 is related to the bare cosmological constant. The most important restriction to be imposed on $\mathcal{T}$ is that of a fixed topology. If we allow an unrestricted summation over all topologies in (2) the partition function is divergent [2]. In the following we will always restrict ourselves to consider manifolds with the topology of S^4.

$Z(\kappa_2,\kappa_4)$ is the grand canonical partition function. It is defined in a region $\kappa_4 \geq \kappa_4^c(\kappa_2)$ in the (κ_2,κ_4) coupling constant plane. The only way in which we can hope to obtain a continuum limit is by letting κ_4 approach $\kappa_4^c(\kappa_c)$ from above. This tentative continuum limit depends only on one coupling constant κ_2 and the transition between the two phases of 4D gravity mentioned above takes place at a critical value of κ_2, κ_2^c. It is often convenient to think about the canonical partition function where N_4 is kept fixed. Then κ_2 is the only coupling constant and the aspects of gravity which do not involve the fluctuation of the total volume of the Universe can be addressed in the limit of large N_4. For the geometrical system an observable which has our interest is the average curvature per volume, $\langle R \rangle$. The average curvature can for a simplicial manifold be defined by Regge calculus and in the case of equilateral simplexes one simply has

$$\langle R \rangle \propto (c_4 N_2/N_4 - 10)\ , \qquad (3)$$

where the constant c_4 is the number of four-simplices to which each triangle should belong if the manifold were flat. Furthermore one can by an appropriate interpretation of the Regge approach introduce the average of the squared curvature per volume by

$$\langle R^2 \rangle \propto \frac{\sum_{n_2} o(n_2)\{[c_4 - o(n_2)]/o(n_2)\}^2}{10 N_4}\ , \qquad (4)$$

where the sum is over triangles n_2 and $o(n_2)$ is the order of a given triangle, i.e., the number of four-simplices to which this triangle belongs. The correlator $\langle R^2 \rangle - \langle R \rangle^2$ will prove useful as an indicator of a change in geometry

One can now couple matter fields to simplicial quantum gravity. In the case of Ising spins the partition function will look like

[3]But we will, of course, not seriously pretend that the present stage of numerical simulations of quantum gravity is such that one could really determine the matter content.

$$Z(\beta,\kappa_2,\kappa_4)=\sum_{N_4} e^{-\kappa_4 N_4} \sum_{T\in\mathcal{T}(N_4)} \sum_{\{\sigma\}} e^{\kappa_2 N_2} \exp\left[\beta \sum_{\langle i,j\rangle} (\delta_{\sigma_i \sigma_j}-1)\right] . \tag{5}$$

In this formula $\mathcal{T}(N_4)$ signifies the subclass of $\mathcal{T}$ with volume N_4, $\sum_{\{\sigma\}}$ the summation over all spin configurations, while $\sum_{\langle i,j\rangle}$ stands for the summation over all neighboring pairs of four-simplices. As a function of β there might or might not be a phase transition for the spin system, depending on the value of κ_2 [assuming that $\kappa_4=\kappa_4^c(\kappa_2,\beta)$, where κ_4^c now depends on both κ_2 and β].

The coupling of scalar fields to simplicial quantum gravity is also straightforward. Here we will ignore the self-interaction of the scalar fields and direct coupling between the scalar fields and the curvature, and simply consider the partition function

$$Z(\kappa_2,\kappa_4)=\sum_{N_4} \sum_{T\in\mathcal{T}(N_4)} e^{\kappa_2 N_2-\kappa_4 N_4} \int \prod_{i,\alpha} \frac{d\phi_i^\alpha}{\sqrt{2\pi}} \prod_{\alpha=1}^{n_g} \delta\left[\sum_i \phi_i^\alpha\right] \exp\left[-\frac{1}{2}\sum_{\langle i,j\rangle,\alpha} (\phi_i^\alpha-\phi_j^\alpha)^2\right] . \tag{6}$$

Here i labels the four-simplices, α different components of the field ϕ, and n_g is the total number of independent Gaussian fields. There is no need for a coupling constant in front of the Gaussian action since it can always be absorbed in κ_4 by a rescaling of the ϕ's. Of course, the Gaussian action can, in principle, be integrated out explicitly, leaving us with an additional weight

$$(\mathrm{Det}C_T)^{-n_g/2} \tag{7}$$

for each triangulation T, where C_T is just the incidence matrix for the φ^5 graph which is dual to the triangulation T. In the case of Gaussian fields coupled to 2D gravity, this fact was used to determine qualitatively the phase diagram of noncritical strings as a function of the number of Gaussian systems, n_g [22–24]. In principle, one could try to do the same here. However, the class of allowed φ^5 graphs is not so easy to determine as in the case of 2D gravity. In the following we will rely on numerical simulations.

III. NUMERICAL METHODS

One annoying aspect of the above formalism is that we are forced to perform a grand canonical simulation where N_4 is not fixed. The reason is that we have no ergodic updating algorithm[4] which preserves the volume N_4. It is, however, possible to perform a grand canonical updating without violating ergodicity and still stay in the neighborhood of a prescribed value of N_4, which we will denote N_4(fix). The procedure involves fine-tuning of κ_4 to its critical value, $\kappa_4^c(\kappa_2,\beta)$. We refer to [10] for details.

In addition to the updating of the geometry, we also have to update the Ising spin system and the Gaussian systems. Let us first discuss the Ising spin system. In order to avoid critical slowing down close to the phase transition between the magnetized and the nonmagnetized phase the spin updating is performed by the single cluster variant of the Swendsen-Wang algorithm developed by Wolff [27]. The cluster updating algorithms have been successfully applied to the Ising model coupled to 2D gravity [28–31] and to the Ising model coupled to 3D gravity [19]. We update the spins once for every sweep, i.e., after N_4(fix) *accepted* updatings of the geometry.

In the simulations we have scanned the (κ_2,β) coupling constant plane by first fixing κ_2 and then varying β in the search for a critical value $\beta_c(\kappa_2)$ where the spin system undergoes a transition.[5] For values of κ_2 where we are well inside the phase with a highly connected geometry and a large Hausdorff dimension, 5000 sweeps are sufficient to achieve equilibrium for bulk quantities when the number of simplexes does not exceed $N_4=9000$. This is in agreement with the situation in pure gravity [2,10]. We have occasionally made longer runs in connection with the measurement of Binders cumulant [50 000 sweeps) and near critical points either in the spin or gravity coupling constant. It seems as if the situation is in all respects as in 2D and 3D gravity. In particular, the presence of the spins seems not to slow down the convergence of bulk geometries observables (in 2D it is known that spins speed it up). In this phase we have neither seen excessive signs of autocorrelations of spins (the longest of the order of 500 sweeps at the spin transition). This is in agreement with intuition since the connectivity of the system is large and the maximal distance between spins correspondingly small. The situation is somewhat different when we probe the phase where the geometry is elongated and where internal distances can be quite large. Without spin the convergence in geometry is slow in this phase and it is true also after coupling to spins.

The Gaussian fields are updated by a heat bath algorithm. There are two aspects of this updating. One type of updating is performed with a fixed background geometry and is standard. The other one is related to the Metropolis updating of the geometrical structure. Since there are slightly unconventional aspects connected with

[4]In 2D gravity we know how to perform a canonical updating, but even there the grand canonical updating is occasionally convenient to use [25,22,23,26].

[5]In order that the reader could appreciate the amount of work going into this please note that we have to fine tune κ_4 for each value of κ_2 and β.

the change of the fields, when the geometry is changed,[6] let us make a few comments. We will not go into details (which are trivial, but clumsy to write down explicitly), but rather sketch the main point: Consider a change in geometry where we take a four-simplex, remove the "interior," insert a vertex in the "empty" interior, and connect this vertex to the five vertices of the former four-simplex. With a proper identification of subsimplices we have by this procedure removed one four-simplex and created five new ones. This inverse "move" is one where we remove a vertex of order 5 and the associated five four-simplices and replace them by a single four-simplex. We must be careful to treat the Gaussian fields correctly in such moves. In the case where we insert a vertex we will have to introduce five new fields φ_i, $i=1,\ldots,5$. They will interact quadratically with each other, and each of them will interact with one field associated with a neighboring four-simplex untouched by the move. Let us denote these five fields ϕ_i, $i=1,\ldots,5$. In addition we have removed a field associated with the original four-simplex. We denote it by φ_0. It interacted with the five ϕ_i's. The correct probability distribution of the new five φ_i's is

$$dP_{\mathrm{new}}(\varphi_i)=C_{\mathrm{new}}(\phi_i)\prod_{i=1}^{5} d\varphi_i e^{-S_{\mathrm{new}}(\varphi_i,\phi_i)} , \tag{8}$$

where the additional part of the action S_{new} coming from added fields φ_i determined from (6) is

$$S_{\mathrm{new}}(\varphi_i)=\frac{1}{2}\sum_{i<j}(\varphi_i-\varphi_j)^2+\frac{1}{2}\sum_i(\varphi_i-\phi_i)^2 . \tag{9}$$

The factor $C(\phi_i)$ is a normalization factor, which contains the exponential of a quadratic form in the ϕ_i's and its all-over scale is fixed by the requirement that $\int dP_{\mathrm{new}}(\varphi_i)=1$. In a similar way the field φ_0 which was removed had a Gaussian probability distribution $dP_{\mathrm{old}}(\varphi_0)$, just with another action

$$S_{\mathrm{old}}(\varphi_0)=\frac{1}{2}\sum_i(\varphi_0-\phi_i)^2 \tag{10}$$

and an appropriate normalization factor $C_{\mathrm{old}}(\phi_i)$, which again contains the exponential of a Gaussian form in ϕ_i's. Assuming that the fields $\varphi_0,\ldots,\varphi_5$ are selected according to P_{new} and P_{old} it is easy to enlarge the condition for detailed balance for the change in geometry to include the additional change in field content.

The geometrical moves fall in three classes (see, e.g., [2] for details) of which we have described one above. A second class is one where two neighboring four-simplices are removed and replaced by three new ones having in common a link (a one-simplex), or the inverse move, where three four-simplices sharing a link are removed and replaced by two four-simplices being neighbors (i.e., sharing a three-simplex). Finally the third class of moves is "self-dual:" three four-simplices sharing a triangle (a two-simplex) are replaced by three others, sharing a different triangle. In all cases one can easily write down dP_{new} and dP_{old} as above and incorporate these probabilities in the requirement of detailed balance needed for performing the purely geometrical move.

The total updating is now organized in the following way. A sweep over the lattice with an updating of geometry and the above-described updating of field content is followed by a number of sweeps with the geometry fixed and ordinary heat bath updating of the Gaussian fields. The actual number of such heat bath updatings for each geometrical updating is chosen so that the fastest convergence to equilibrium is achieved. For one Gaussian field two heat bath updatings for each geometrical updating is usually sufficient as long as the geometry is highly connected. In the elongated phase up to 15 Gaussian updatings were needed. The number of necessary updatings per sweep increases with the number of Gaussian fields. For four Gaussian fields three updatings per sweep were needed in the highly connected phase of gravity.

IV. NUMERICAL RESULTS

A. Ising spins coupled to gravity

Pure 4D gravity has two phases and this fact is not changed by the coupling to a single Ising spin. In the phase where the geometry is highly connected the spin system has a phase transition. In Fig. 1, we show the absolute value of the magnetization:

$$|\sigma|=\frac{1}{N_4}\left|\sum_{i=1}^{N_4}\sigma_i\right| \tag{11}$$

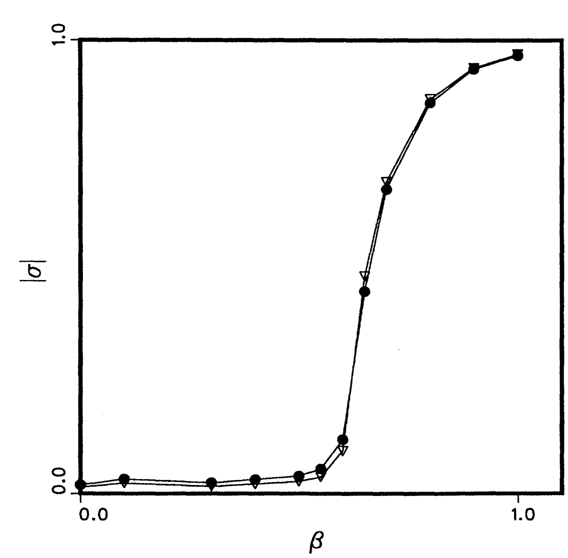

FIG. 1. The absolute value of the magnetization, as defined by (11), as a function of β for $\kappa_2=0.9$, i.e., in the phase with a highly connected geometry. The circles correspond to a volume $N_4=4000$, the triangles to $N_4=9000$.

[6]The same aspect is already present in the grand canonical algorithms used in 2D gravity, see, e.g., [25].

as a function of β for a value of κ_2 for which the geometrical system is highly connected. In Fig. 2 we show Binders cumulant defined by

$$B(\beta)=1-\frac{1}{3}\frac{\langle\sigma^4\rangle}{\langle\sigma^2\rangle^2} \tag{12}$$

and it is seen that the data are consistent with a transition which is second order or higher. We feel there is no reason to believe that the transition should be of higher than second order, since in this phase of the geometrical system the effective Hausdorff dimension is quite high which should favor mean-field results. In the phase with elongated geometry the situation is quite different. The magnetization curve well inside this phase is shown in Fig. 3. There is only a gradual crossover to $|\sigma|\approx 1$ for large β, and the crossover weakens (slightly) with increasing volume. This is in agreement with the measurements of the Hausdorff dimension d_H in this phase which seems to indicate that $d_H < 2$.

The phase diagram in the (κ_2,β) plane, as it appears for a system consisting of 9K simplexes, is shown in Fig. 4. It is in qualitative agreement with the phase diagram of 3D simplicial gravity coupled to Ising spins [19]. The shaded area reflects the uncertainty in the location of the transition line separating the two phases of the geometrical system. This uncertainty is due to a discrepancy between the results for κ_2^c arising when one uses different indicators for the change in geometry. One possible indicator is the Hausdorff dimension d_H, another one the correlator $\langle R^2\rangle-\langle R\rangle^2$. The left boundary of the shaded area results from determining κ_2^c as the value of κ_2 at the peak of $\langle R^2\rangle-\langle R\rangle^2$. The right boundary appears when κ_2^c is defined as the value of κ_2 for which there is a sudden change in Hausdorff dimension. While the left-hand boundary is relatively easy to determine (cf. Fig. 7), the right boundary is difficult to locate precisely due to large fluctuations in geometry and should only be taken as a rough estimate. The fact that the two boundaries do not coincide for the size of systems used here should be taken as a clear sign of finite-size effects. A related phenomenon is seen in the numerical studies of 2D gravity coupled to Ising spins, where the peak in the specific heat does not coincide with the peak in the susceptibility due to finite-size effects which seem to disappear only

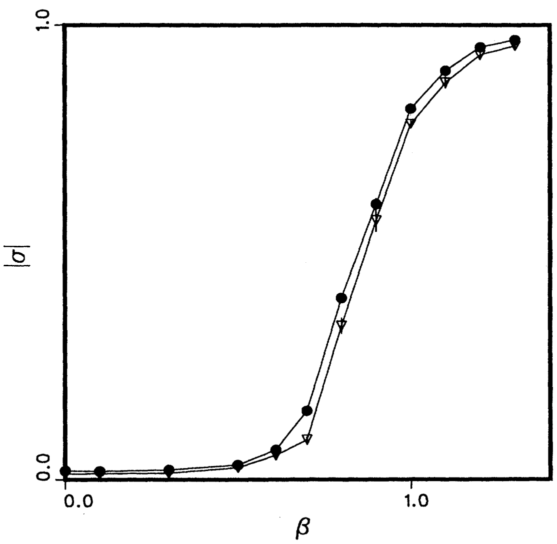

FIG. 3. The absolute value of the magnetization, as defined by (11), as a function of β for $\kappa_2=1.3$, i.e., in the phase with elongated geometry. The circles correspond to a volume $N_4=4000$, the triangles to $N_4=9000$.

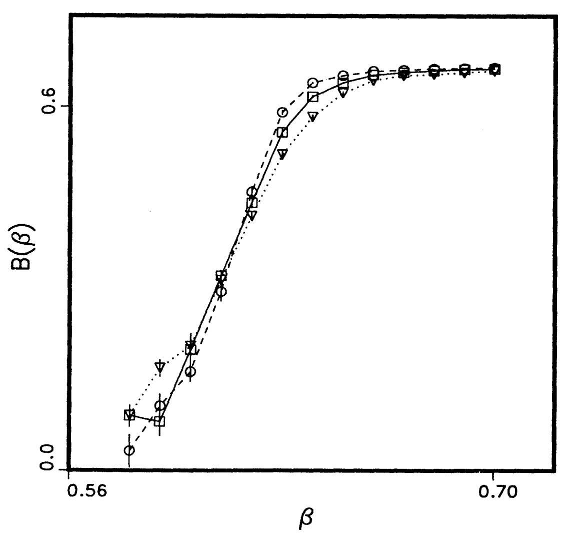

FIG. 2. Binder's cumulant (12) for $\kappa_2=0.9$ and three volumes: $N_4=4000$ (∇), $N_4=6000$ ($\square$), and $N_4=9000$ ($\bigcirc$). The shape corresponds to a transition of second or higher order and the point of intersection to β_c ($N_4=\infty$).

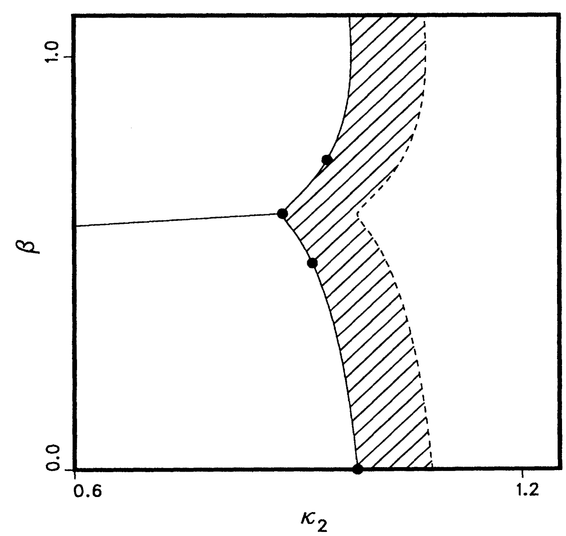

FIG. 4. The phase diagram in the (κ_2,β) plane as it appears when $N_4=9000$. As discussed in the text there are reasons to believe that part of the diagram is distorted by finite-size effects and that in the infinite volume the shaded region will be replaced by the dashed line.

very slowly when the size of the system is increased. The lines of the phase transition (treating the shaded area as a "line," which we expect it will be in the infinite volume limit) divide the coupling constant plane into three regions: The one to the right is characterized by no magnetization and elongated geometry, the lower left region is characterized by no magnetization and highly connected geometry, while the upper left corresponds to a magnetized phase and highly connected geometry. It is difficult to determine the exact position of the bifurcation point since we have here both a fluctuating geometry and large spin fluctuations. It is easy to understand that the transition line separating different geometries will approach the value of κ_2^c for pure gravity when $\beta \to \infty$ and $\beta \to 0$. In these limits the spin fluctuations decouple from gravity and the locations of the transition must agree with the one of pure 4D simplicial gravity.

In Fig. 5 we have shown the behavior of the average curvature of our manifolds when we fix κ_2 inside the highly connected phase fix and move vertically in the coupling constant plane varying β. The value of κ_2 is the same as in Figs. 1 and 2. The position of the peak in the average curvature exactly coincides with the value of β_c determined from the magnetization curve and the plot of Binders cumulant. This observation allows an easy and not so time-consuming determination of $\beta_c(\kappa_2)$. The transition line $\beta = \beta_c(\kappa_2)$ was determined using this idea. We note that this line shows little dependence on κ_2. The dependence of $\kappa_2^c(\beta)$ is more pronounced. The value of κ_2^c is smaller for the coupled system than for pure gravity. The shift in κ_2^c is largest when $\beta = \beta_c$ showing that the coupling between geometry and spins is indeed largest when the spin system is critical. This is in agreement with the intuition we have from the exactly solvable 2D Ising-gravity system. The transition line $\kappa_2 = \kappa_2^c(\beta)$ shows that effectively the spin system pushes geometry towards larger κ_2 values. The effect is strongest when β is close to $\beta_c(\kappa_2)$. On the other hand, we know that for large-κ_2 values the geometry is such that the system cannot be critical. This apparent contradiction seems to be generic for the interaction between gravity and matter of the kind considered here. This is highlighted in a recent paper on multiple spin systems coupled to 2D gravity [32]. In 2D the *back reaction* of the spin system on gravity is also largest close to criticality, but is such that it counteracts its own criticality by trying to deform the geometry into generic shapes where it cannot be critical (polymerlike geometries). It seems that we are observing a similar phenomenon here in 4D.

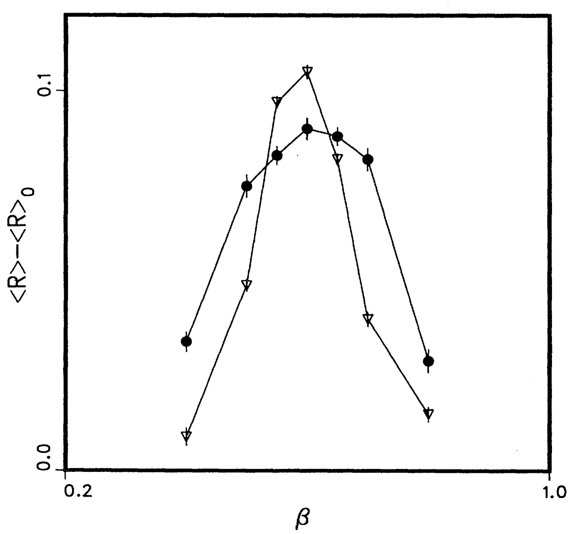

FIG. 5. The effect on the curvature $\langle R \rangle - \langle R \rangle_0$ [where $\langle R \rangle$ is defined by (3)] when we are in the phase with a large Hausdorff dimension and change β. The value of $\kappa_2 = 0.9$ and the circles correspond to $N_4 = 4000$ while the triangles correspond to $N_4 = 9000$. $\langle R \rangle_0$ denotes the average curvature in the case of pure gravity (it differs slightly for $N_4 = 4000$ and 9000 due to finite-size effects).

It is, of course, an interesting question whether the coupling between the spins and gravity changes the critical exponent of either of the systems as is the case in two dimensions. However, since the critical exponents of the pure 4D gravity system are yet not known and since it has proven quite difficult to extract by numerical methods the critical exponents of the Ising spins coupled to 2D gravity, we have chosen here the more modest approach to look at the influence of the spin system on bulk geometric quantities like the average curvature. As explained in the Introduction, this has special interest in relation to the scaling of gravity observables at the transition between geometries. We will return to this aspect after we have discussed briefly 4D gravity coupled to Gaussian fields.

B. Gaussian fields coupled to gravity

In the case of Gaussian fields we have, as explained above, no coupling constant to adjust. The fields will automatically be critical in the infinite volume limit. We have considered up to four Gaussian fields coupled simultaneously to gravity and for these systems we can make a statement similar to the one made for the Ising model: The two phases of geometry seem to survive the coupling to Gaussian matter. In Fig. 6, we have shown the expectation value $\langle \phi^2 \rangle$ of a single component of the Gaussian field as a function of κ_2. We see a change in $\langle \phi^2 \rangle$ linked to the change in geometry. The value of $\langle \phi^2 \rangle$ increases when we enter into the elongated phase. In fact, $\langle \phi^2 \rangle$ also has quite large fluctuations in this phase.

C. Behavior of gravity observables coupled to matter

In the computer simulations we can clearly see the back reaction of matter on the geometry for a given choice of coupling constants. It is less obvious, however, that this back reaction of matter leads to anything but trivial changes. Both for the coupling of Ising spins and Gaussian fields we still have two phases of the geometry: the highly connected one and the very elongated one. As mentioned in the Introduction, one could hope that the inclusion of matter would improve the scaling of the curvature at the transition. We have investigated this in the

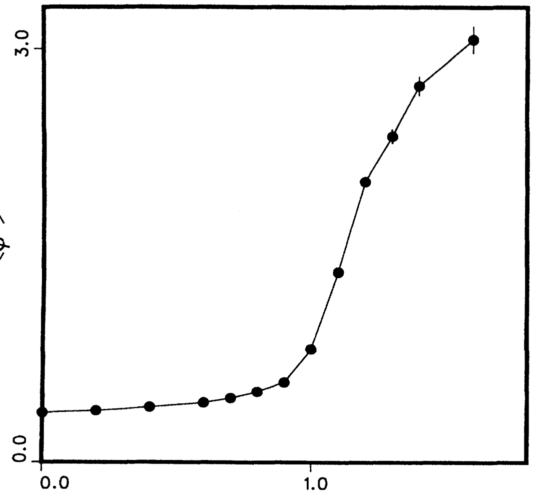

FIG. 6. The change in $\langle \phi^2 \rangle$ (a single component field) as a function of κ_2 for $N_4=4000$.

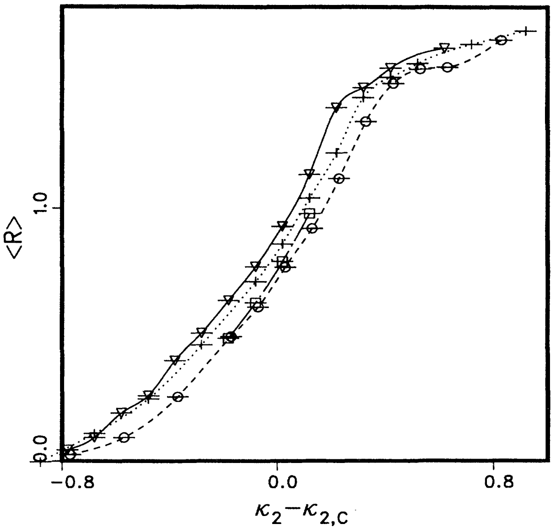

FIG. 8. $\langle R \rangle$ as a function of $\Delta\kappa_2$ for different matter content. Pure gravity (∇), gravity + Ising at β_c, (+), gravity + one Gaussian field ($\bigcirc$), and gravity + four Gaussian fields ($\Box$).

following way: As remarked above there are several indicators of the change in geometry. They result in slightly different values of κ_2^c. We have chosen to use the peak of $\langle R^2 \rangle - \langle R \rangle^2$ as an indicator of the transition, mainly because it is easier to identify than the change in Hausdorff dimension. The value of κ_2^c depends on the matter content, as can be seen from Fig. 7. In Fig. 8, we have plotted the average curvature as a function of the distance $\Delta\kappa_2$ from κ_2^c. It is seen that there is no improvement in the scaling behavior of $\langle R \rangle(\kappa_2^c)$ as a function of the matter content when we compare with the situation in

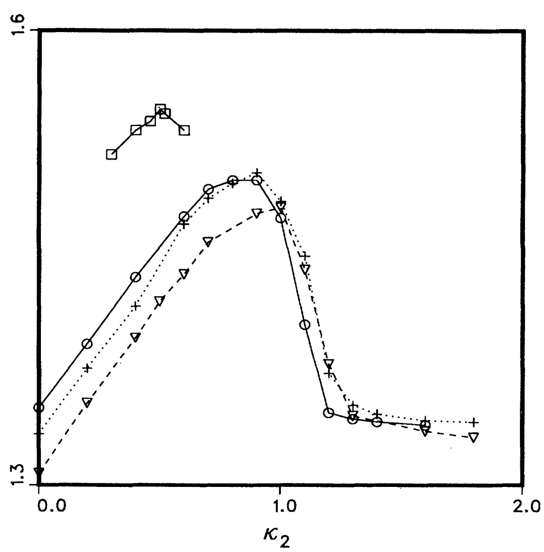

FIG. 7. $\langle R^2 \rangle - \langle R \rangle^2$ for a different matter field as a function of $\Delta\kappa_2$. Pure gravity (∇), gravity + Ising at β_c (+), gravity + one Gaussian field ($\bigcirc$), and gravity + four Gaussian fields ($\Box$). [The observables $\langle R \rangle$ and $\langle R^2 \rangle$ are defined in (3) and (4), respectively.]

pure gravity. In fact, the curves look remarkably insensitive to the inclusion of matter and one could at this point wonder whether the back reaction of matter has any effect on the geometry except to introduce an effective κ_2 which differs from the bare parameter. This is, of course, enough to explain the peak in the average curvature observed in Fig. 5 and it also provides us with an explanation why the peak is more narrow for a 9K system than for a 4K system. This is due to the fact that the change in average curvature across the phase transition is more sudden for the larger system. In Fig. 7, we have shown $\langle R^2 \rangle - \langle R \rangle^2$ for various matter fields coupled to gravity. We see that the peak grows with the number of Gaussian fields, indicating at least somewhat increased back reaction with the number of fields. Furthermore, we note that the larger the number of Gaussian fields is, the more κ_2^c is shifted towards smaller values. Hence systems with a large number of Gaussian fields favor elongated geometries. The same phenomenon is known from two dimensions where analytic considerations show that the path integral is dominated by elongated geometries when n_g is large. However, there is no indication that the presence of matter fields changes the nature of the phase transition of the geometrical system.

Let us comment here on a somewhat surprising feature of 4D simplicial gravity. As mentioned earlier the method of grand canonical simulation requires a fine-tuning of κ_4 to its critical value, κ_4^c. It appears that κ_4^c depends on κ_2 in a universal way. In Fig. 9, we have shown $\kappa_4^c(\kappa_2)$ for pure gravity, gravity coupled to Ising spins at $\beta=\beta_c$, and gravity coupled to one and four Gaussian fields, respectively. In Ref. [10], 4D simplicial gravity was simulated using the action

$$S=\kappa_4 N_4-\kappa_2 N_2+\frac{h}{c_4^2}\sum_{n_2} o(n_2)\left[\frac{c_4-o(n_2)}{o(n_2)}\right]^2 . \qquad (13)$$

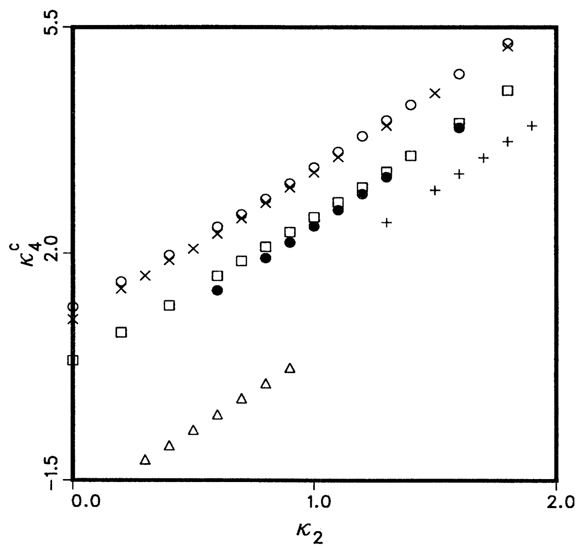

FIG. 9. κ_4^c as a function of κ_2 for different systems. Pure gravity (×), gravity + Ising at $\beta=\beta_c$ (○), gravity + one Gaussian field (□), gravity + four Gaussian fields (△), gravity with a higher derivative term for $h=10$ (●), and gravity with a higher derivative term for $h=20$ (+).

This corresponds to adding to the Einstein Hilbert action a typical higher derivative term [cf. Eq. (4)]. We have shown also $\kappa_4^c(\kappa_2)$ for this model when $h=10$ and 20. For all the systems studied $\kappa_4^c(\kappa_2)$ is a linear function with a slope of approximately 2.5.

V. DISCUSSION

It is clear that the numerical exploration of simplicial quantum gravity is still in its infancy. Finite-size effects are not under control and it would be most desirable to be able to simulate larger systems. In principle, it is possible and it *will* be possible in the future. The size of random lattices considered here, consisting of 9000 four-simplices, corresponds to a 4^4–5^4 regular lattice. But even on the small lattices used here one might reveal interesting aspects of the interaction between gravity and matter. Until now we have only considered the simplest matter systems, spins and Gaussian fields, but nothing prevents us from considering the coupling to, for instance, non-Abelian gauge fields. It is also, in principle, possible to define nonlocal observables such as spin-spin correlation functions as functions of geodesic distance (see, i.e., [19] for a discussion in the case of 3D gravity) and explore their quantum averages. In this paper we have not tried to extract any critical exponents of such observables since the experience from 3D is that it is not easy, and we decided in this first investigation to concentrate on bulk quantities.

The main result of the simulations is that coupling of matter to discretized gravity seems *not* to influence the geometry in a profound way. Of course, it is possible that critical indices change (as in the case of 2D gravity). Our measurements are still too poor to measure such subleading effects. As mentioned above, an interesting effect would be an improved scaling of the average curvature in the region where there is a transition in geometry. We have not seen any such effect. The tentative conclusion from these first numerized experiments is that matter fields (at least of the kind we have considered here) will not add very much to our attempts to understand the basic structure of four-dimensional quantum gravity.

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