PHYSICAL REVIEW D VOLUME 48, NUMBER 1 1 JULY 1993

Excited heavy mesons

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(Received 8 February 1993)

We discuss an effective action for heavy excited mesons interacting with light pseudoscalars and vector mesons that combines both chiral and heavy-quark symmetry. We estimate the axial-vector couplings, masses, and mixed axial-vector couplings of the excited mesons to order one in the heavy-quark mass. The general issue of reparametrization invariance in heavy-quark effective theories is also discussed.

PACS number(s): 14.40.Jz, 11.30.Hv, 11.30.Rd

Recently, we have suggested an approximate bosonization scheme for QCD-motivated models involving heavy and light mesons [1]. The construction underlines the necessity of including the chiral partners of the heavy mesons, allows for the introduction of vector mesons, and provides an estimation of the masses and couplings to order one in the heavy-quark mass. The phenomenological implications of the effective action formulation to heavy hadron dynamics has been the subject of much discussion recently [2].

In the present paper, we wish to extend our analysis to the excited states of heavy mesons, and derive an explicit action for the $1^{\pm}$, $2^{\pm}$ heavy mesons. The interest in these states is motivated by the experimental identification of the 1^+, 2^+ with the $D_1(2420)$ and $D_2^*(2460)$ for the charmed particles. The spin-parity assignment for the light quark is $s_l^{P_l}=\frac{3}{2}^{\pm}$. A similar construction based solely on symmetry arguments has been recently discussed in [3,4]. We also address the issue of reparametrization invariance [5] in the effective action formulation, and show that while the invariance is manifest in the nonlocal formulation, it requires further amendments in the local approximation.

If the mass of the heavy quark is infinitely large, then the heavy-quark momentum is large and conserved: $P_\mu=m_Q v_\mu$. In this case, it is convenient to rewrite the heavy fields $Q(x)$ as [6]

$$Q(x)=\frac{1+\not{v}}{2}e^{-im_Q v\cdot x}Q_v^+(x)+\frac{1-\not{v}}{2}e^{im_Q v\cdot x}Q_v^-(x)\ , \qquad (1)$$

as suggested by the Foldy-Wouthuysen transformation. In terms of (1) the QCD action reads

$$S=-\int d^4x\frac{1}{4g^2}\mathrm{Tr}(G^2)+\int d^4x\left[\bar{q}(i\not{\partial}-m_q)q+\sum_v[\bar{Q}_v(i\not{v}v\cdot\partial)Q_v+\mathrm{Tr}(j\cdot G)]\right] \qquad (2)$$

to leading order in the heavy-quark mass. The colored current j is given by

$$j_\mu^a=\bar{q}\gamma_\mu T^a q+\bar{Q}_v\not{v}v_\mu T^a Q_v\ . \qquad (3)$$

To bosonize the QCD action (2), we define $\psi=(q,Q_v)$ and insert unity,

$$1=\int dM\,dN\exp\left[\int d^4x_1d^4x_2\mathrm{Tr}(M(x_1,x_2)[N(x_2,x_1)-\psi(x_2)\bar{\psi}(x_1)])\right]\ , \qquad (4)$$

in the QCD functional integral. Integrating over the gluons and the auxiliary field N yields the effective action for the bilocal field $M(x,y)$:

$$S_F(M)=\sum_v\int d^4x\,d^4y\,\bar{\psi}(x)[1_{xy}1_2i\not{\partial}+1_{xy}1_3i\not{v}v\cdot\partial+M(x,y)]\psi(y)+G(M)\ . \qquad (5)$$

The potential term $G(M)$ is the Legendre transform of the integrated gluonic action after proper gauge fixing. This term is nonlocal in M and invariant under *local* chiral and heavy-quark symmetry. It *does not* involve derivative terms acting on M. Generically,

$$G(M)=G(0)+\frac{a^2}{2}\int\mathrm{Tr}(M^\dagger M)+\cdots\ , \qquad (6)$$

where the a's are unknown dimensionful coefficients of order $\sqrt{N_c}$. The form of the potential determines the spontaneous breaking of chiral symmetry in the light sector. Here it will be assumed.

The mesonic quantum numbers follow from the spin-isospin assignment carried by M. In the light-light case, the assignment is standard, and will not be repeated here [7–10]. In the heavy-light case a Taylor expansion of the light

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quark around the standing heavy quark gives ($y=x+z$)

$$M^{ij}(x,y)=\sum_{n=0}^{\infty} F_n(z)M^{ij}_{\mu_1\cdots\mu_n}(x)z^{\mu_1}\cdots z^{\mu_n} , \tag{7}$$

where the F_n's are pertinent form factors[1] for the light quark that go to zero as z^2 goes to ∞. The index i refers to the heavy-quark spin, and the indices $[j,\mu_1,\ldots,\mu_n]$ refer to the light-quark spin-orbital wave function.

If we recall that the M are Legendre transforms of $Q\bar{q}$, then the heavy $1^+,2^+$ mesons are triggered by M^{ij}_{μ}, where the light quark carries spin $\frac{1}{2}$ in a P orbital. The light quark can be either in a $\frac{1}{2}^+$ or in a $\frac{3}{2}^+$ state (taking into account the intrinsic parity). The 1^+, 2^+ multiplet corresponds to the light quark being in the $\frac{3}{2}^+$ state. This implies that the vector source M^{ij}_{μ} for the 1^+, 2^+ obeys the Rarita-Schwinger condition, along with the usual transversality condition

$$M_{\mu}\gamma^{\mu}=0 , \quad v^{\mu}M_{\mu}=0 , \tag{8}$$

respectively. The chiral partners 1^-, 2^- correspond to a light quark with spin $\frac{1}{2}$ in a D orbital coupled to a $\frac{3}{2}^-$ state. These states follow from $M^{i}_{\mu\nu}(\gamma^{\nu}+v^{\nu})/\sqrt{3}$ by reduction, and can be identified with the $\frac{3}{2}^-$ components of M^{i}_{μ} after redefinition. With this in mind, the general decomposition of the vector source is

$$M_{\mu}=M_{\alpha}{}^{S}[g^{\alpha}_{\mu}-\tfrac{1}{3}\gamma_{\mu}(\gamma^{\alpha}+v^{\alpha})]+M_{\alpha}{}^{P}[g^{\alpha}_{\mu}-\tfrac{1}{3}\gamma_{\mu}(\gamma^{\alpha}+v^{\alpha})]\gamma^5+\gamma^{\nu}M^{V}_{\nu\mu}+\gamma^{\nu}\gamma^5 M^{A}_{\nu\mu} , \tag{9}$$

where $M^{V,A}_{\nu\mu}$ are traceless symmetric rank-2 tensors. The tensor part in (9) drops because of chiral symmetry (light sector). This argument is generic and extends to the higher spin states as discussed by [11] in the context of the nonrelativistic quark model. For our purposes, it is more convenient to work in the chiral basis. For that define instead

$$\hat{T}^{\mu}_{\pm}=\frac{1+\not{v}}{2}\{R^{*,\mu\nu}_{\pm}\gamma_{\nu}-\left[\tfrac{3}{2}\right]^{1/2}\{R^{\nu}_{\pm}[g^{\mu}_{\nu}-\tfrac{1}{3}\gamma_{\nu}(\gamma^{\mu}+v^{\mu})]\gamma^5\}\gamma_5^{\pm} , \tag{10}$$

which are linear combinations of (9). If we denote by

$$\hat{H}_{\pm}=\frac{1+\not{v}}{2}(\gamma^{\mu}\hat{P}^{*}_{\mu,\pm}+i\gamma_5\hat{P}_{\pm})\gamma_5^{\mp}+\text{H.c.} \tag{11}$$

the pseudoscalar and vector *bare* heavy mesons with specific light chirality, then using Eqs. (7)–(9) and integrating over z yields

$$S_F(M)=\sum_{v}\int\bar{\psi}[1_2 i\not{\partial}+1_3 i\not{v}v\cdot\partial+1_2(\hat{\not{L}}\gamma_5^{+}+\hat{\not{R}}\gamma_5^{-})-1_2(M\gamma_5^{+}+M^{\dagger}\gamma_5^{-})+\hat{H}_{+}+\hat{H}_{-}$$
$$+\tfrac{1}{2}(\hat{T}^{\mu}_{+}\vec{\partial}_{\mu}+\overleftarrow{\partial}_{\mu}\hat{T}^{\dagger,\mu}_{+})+\tfrac{1}{2}(\hat{T}^{\mu}_{-}\vec{\partial}_{\mu}+\overleftarrow{\partial}_{\mu}\hat{T}^{\dagger,\mu}_{-})]\psi+G(M) . \tag{12}$$

Here $\hat{L}_{\mu}$ and $\hat{R}_{\mu}$ are the bare light vector fields valued in $U(2)_L$ and $U(2)_R$ respectively, $1_2=\text{diag}(1,1,0)$, $1_3=\text{diag}(0,0,1)$ are the projectors onto the light and heavy sectors respectively, and $\gamma_5^{\pm}\equiv\frac{1}{2}(1\pm\gamma_5)$. The quark field ψ in (12) stands for $\psi=(q;Q_v)$. The $\hat{P}$'s and the $\hat{R}$'s are off-diagonal in flavor space, so, e.g., $P=P_aT^a$, with $a=1,2$ and $T^1=(\lambda_4-i\lambda_5)/2$, $T^2=(\lambda_6-i\lambda_7)/2$. The generalization of the above construction to the case of three light flavors and one heavy flavor is straightforward.

The action (12) enjoys both heavy-quark spin symmetry, denoted by $SU(2)_Q$, and rigid chiral symmetry $U(2)_L\times U(2)_R$. We have set the light quark masses to zero for convenience. The potential part (6) breaks spontaneously chiral $U(2)_L\times U(2)_R$ to $SU(2)_V$ with the appearance of three Goldstone bosons [the chiral anomaly taking care of the $U(1)_A$] around a chiral symmetric condensate. With this in mind, we will decompose the 2×2 complex matrix M as follows (a pertinent choice of gauge to avoid doubling of the Goldstone bosons will be $\xi_L^{+}=\xi_R=e^{i\pi/2f_\pi}$):

$$M=\xi_L^{\dagger}\Sigma\xi_R , \tag{13}$$

with the ξ's as elements in the coset $SU(2)_L\times SU(2)_R/SU(2)_V$. In the vacuum, Σ is diagonal. As a result, the "bosonized" QCD action is, in addition, invariant under local $SU(2)_V$ symmetry: $\xi_L\to h(x)\xi_L g_L^{\dagger}$, $\xi_R\to h(x)\xi_R g_R^{\dagger}$ and $\Sigma\to h(x)\Sigma h(x)^{\dagger}$.

The constituent (dressed) quark field χ relates to the bare quark field ψ through $\chi_{L,R}=(\xi_{L,R}q_{L,R};Q_v)$. In terms of the constituent field, the "bosonized" action reads, after integrating over the constituent quarks,

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[1]We are still using a covariant Taylor expansion in the source. Noncovariant expansions are also possible; they will amount to defining different form factors. Since we will ignore the finite size effects of the source below (pointlike approximation), this distinction is not important.

$$S(M)=N_c \sum_v \mathrm{Tr}\ln[\mathbf{1}_2(i\nabla\!\!\!/_L-\Sigma)\gamma_5^- +\mathbf{1}_2(i\nabla\!\!\!/_R-\Sigma)\gamma_5^+ +\mathbf{1}_3 i v\!\!\!/ v\cdot\partial+H+\bar{H}+G+\bar{G}$$
$$+(T^\mu+S^\mu)(\vec{\partial}_\mu+\xi_5\partial_\mu\xi_5^\dagger)+(\overleftarrow{\partial}_\mu+\xi_5\partial_\mu\xi_5^\dagger)(\bar{T}^\mu+\bar{S}^\mu)]+G(M)\ . \tag{14}$$

It is invariant under local $SU(2)_V$ symmetry and global $SU(2)_Q$ symmetry. Invariance under reparametrization of the fields will be discussed below. The dressed light fields are defined by

$$L_\mu=\xi_L\hat{L}_\mu\xi_L^\dagger+i\xi_L\partial_\mu\xi_L^\dagger\ ,\quad R_\mu=\xi_R\hat{R}_\mu\xi_R^\dagger+i\xi_R\partial_\mu\xi_R^\dagger\ , \tag{15}$$

and also

$$\xi_5=\frac{1+\gamma_5}{2}\xi_R+\frac{1-\gamma_5}{2}\xi_L\ . \tag{16}$$

The dressed heavy fields are defined as

$$H=\frac{1+v\!\!\!/}{2}(\gamma^\mu P_\mu^*+i\gamma_5 P)=\frac{1+v\!\!\!/}{2}[\gamma^\mu(P_{\mu,+}^*\xi_R^\dagger+P_{\mu,-}^*\xi_L^\dagger)+i\gamma_5(P_+\xi_R^\dagger+P_-\xi_L^\dagger)]\ ,$$
$$G=\frac{1+v\!\!\!/}{2}(\gamma^\mu\gamma_5 Q_\mu^*+Q)=\frac{1+v\!\!\!/}{2}[\gamma^\mu\gamma_5(P_{\mu,+}^*\xi_R^\dagger-P_{\mu,-}^*\xi_L^\dagger)+(P_+\xi_R^\dagger-P_-\xi_L^\dagger)] \tag{17}$$

and

$$T^\mu=\frac{1+v\!\!\!/}{2}\{R^{*\mu\nu}\gamma_\nu-(\tfrac{3}{2})^{1/2}R^{\nu}\gamma_5[g_\nu^\mu-\tfrac{1}{3}\gamma_\nu(\gamma^\mu-v^\mu)]\}$$
$$=\frac{1+v\!\!\!/}{2}\{(R_+^{*\mu\nu}\xi_R^\dagger+R_-^{*\mu\nu}\xi_L^\dagger)\gamma_\nu-(\tfrac{3}{2})^{1/2}\gamma_5(R_+^{\nu}\xi_R^\dagger+R_-^{\nu}\xi_L^\dagger)[g_\nu^\mu-\tfrac{1}{3}\gamma_\nu(\gamma^\mu-v^\mu)]\}\ ,$$
$$S^\mu=\frac{1+v\!\!\!/}{2}\{S^{*\mu\nu}\gamma_\nu\gamma_5-(\tfrac{3}{2})^{1/2}S^{\nu}[g_\nu^\mu-\tfrac{1}{3}\gamma_\nu(\gamma^\mu+v^\mu)]\}$$
$$=\frac{1+v\!\!\!/}{2}\{(S_+^{*\mu\nu}\xi_R^\dagger+S_-^{*\mu\nu}\xi_L^\dagger)\gamma_\nu\gamma_5-(\tfrac{3}{2})^{1/2}(S_+^{\nu}\xi_R^\dagger+S_-^{\nu}\xi_L^\dagger)[g_\nu^\mu-\tfrac{1}{3}\gamma_\nu(\gamma^\mu+v^\mu)]\}\ . \tag{18}$$

The H and G fields refer to (D,D^*) and their chiral partners $(\tilde{D},\tilde{D}^*)$, while the T and S fields refer to (D_1,D_2^*) and their chiral partners $(\tilde{D}_1,\tilde{D}_2^*)$. The covariant L, R derivatives in (14) are $\nabla_L=\partial-iL$ and $\nabla_R=\partial-iR$. $\bar{T}$ and $\bar{S}$ are related, respectively, to T and S through

$$\bar{T}=\gamma^0T^\dagger\gamma^0\ ,\quad \bar{S}=\gamma^0S^+\gamma^0\ , \tag{19}$$

and similarly for $\bar{H}$ and $\bar{G}$. The light-light- and heavy-light-quark dynamics follows from the dressed action (14) through a derivative expansion. The cutoff Λ_c reflects on the chiral symmetry-breaking scale [12]. Our expansion of (14) will be understood in the sense of $m_Q/\Lambda_c\to\infty$. The effective action for H, G has been discussed in [1], and will not be repeated here. For the excited states, to second order in $t_\mu=T_\mu+S_\mu$ we have

$$S_t=-N_c\mathrm{Tr}(+\Delta_h t\cdot\vec{D}\Delta_l\overleftarrow{D}\cdot\bar{t}+\Delta_h t\cdot\vec{D}\Delta_l\gamma_5\mathcal{A}\cdot\bar{t}+\Delta_h t\cdot\mathcal{A}\gamma_5\Delta\overleftarrow{D}\cdot\bar{t}-\Delta_h t\cdot\vec{D}\Delta_l(V\!\!\!/+A\!\!\!/\gamma_5)\Delta_l\overleftarrow{D}\cdot t)+\cdots\ , \tag{20}$$

where the functional trace includes tracing over space, flavor, and spin indices. We have denoted by $\Delta_l=(i\partial\!\!\!/-\Sigma)^{-1}$, $\Delta_h=(iv\!\!\!/ v\cdot\partial)^{-1}$, $D=\partial+\mathcal{V}$, and

$$\xi_5\partial_\mu\xi_5^\dagger=\mathcal{V}_\mu+\gamma_5\mathcal{A}_\mu\ . \tag{21}$$

The ellipsis in (20) stands for higher insertions of vectors and axial vectors. Note that to the order considered, there are cross terms between the T's and the S's generated by the axial current.

After carrying out the trace over space in (20) and renormalizing the heavy-quark fields, $T\to T/\sqrt{Z_T}$ and $S\to S/\sqrt{Z_S}$, we obtain, to leading order in the gradient expansion for the T's ($s_l^{P_l}=\frac{3}{2}^{\pm}$),

$$\mathcal{L}_v^T=\frac{i}{2}\mathrm{Tr}(\bar{T}^\mu\partial_\alpha T^\mu-\partial_\alpha\bar{T}^\mu T^\mu)v_\alpha-\mathrm{Tr}(\bar{T}^\mu V_\alpha T^\mu)v_\alpha-g_T\mathrm{Tr}(T^\mu A\!\!\!/\gamma_5\bar{T}^\mu)+m_T\mathrm{Tr}(\bar{T}^\mu T_\mu)\ , \tag{22}$$

and similarly for the S's ($s_l^{P_l}=\frac{3}{2}^-$):

$$\mathcal{L}_v^S=\frac{i}{2}\mathrm{Tr}(\bar{S}^\mu\partial_\alpha S^\mu-\partial_\alpha\bar{S}^\mu S^\mu)v_\alpha-\mathrm{Tr}(\bar{S}^\mu V_\alpha S^\mu)v_\alpha-g_s\mathrm{Tr}(S^\mu A\!\!\!/\gamma_5\bar{S}^\mu)+m_S\mathrm{Tr}(\bar{S}^\mu S_\mu)\ . \tag{23}$$

The parameters in (22) and (23) are given by ($P_l=\pm$ is the parity of the light part of T, S)

$$Z_{P_l}=N_c\int_0^{\Lambda_c}\frac{d^4Q}{(2\pi)^4}\frac{1}{3}\frac{Q^2}{Q^2-\Sigma^2+i\epsilon}\left[\frac{1}{v\cdot Q+i\epsilon}+\frac{3}{2}P_l\frac{\Sigma}{Q^2-\Sigma^2+i\epsilon}\right],$$

$$g_{P_l}=\frac{N_c}{Z_{P_l}}\int_0^{\Lambda_c}\frac{d^4Q}{(2\pi)^4}\frac{1}{3}\frac{Q^2}{(Q^2-\Sigma^2+i\epsilon)^2}\left[\Sigma^2-\frac{3}{5}Q^2\right], \tag{24}$$

$$m_{P_l}=\frac{N_c}{Z_{P_l}}\int_0^{\Lambda_c}\frac{d^4Q}{(2\pi)^4}\frac{1}{3}\frac{q^2}{Q^2-\Sigma^2+i\epsilon}\left[P_l\frac{\Sigma}{v\cdot Q+i\epsilon}-\frac{3}{4}\right].$$

Note that m_T and m_S are of order m_Q^0. The mass difference between the chiral partners T and S is of order $O(\Sigma)$. The behavior of the coupling constants as a function of the ratio $x=\Sigma/\Lambda_c$ is shown in Fig. 1. For completeness, we have also shown the values of the couplings for the $s_l^{P_l}=\frac{1}{2}^{\pm}$ mesons [1]. In the limit $x\rightarrow 0$ the couplings reduce to $g_H(0)=g_G(0)=\frac{1}{3}$, $g_T(0)=g_S(0)=-\frac{3}{5}$. The only available other theoretical estimates come from the nonrelativistic quark model with $g_H=g_A$, $g_G=g_A/3$, $g_T=g_A$ [3], and $g_A=0.75$ [13]. While the coupling constants for the G, S mesons are sensitive to the value Σ of the constituent mass, the couplings for the H, T mesons are not. Note that the H coupling is within the empirical bound $g_H^2\leq 0.58$ [14]. At $x\sim 0.25$ the ratios g_S^2/g_T^2 and g_G^2/g_H^2 are an order of magnitude larger than the corresponding ratios at $x=0$. This suggests that the width of the G and S mesons may be larger than the width of their chiral partners H and T. This result is consistent with the fact that the observed heavy mesons D, D^*, $D_1(2400)$, $D_2^*(2460)$ correspond solely to the H and T assignments.

The interaction between the T, S's and the H, G's is given by

$$S_I=-N_c\,\mathrm{Tr}[\Delta_h(t\cdot\vec{D}+t\cdot\mathcal{A}\gamma_5)\Delta_l(\overline{H}+\overline{G})+\Delta_h(H+G)\Delta_l(\overleftarrow{D}\cdot t+\gamma_5\mathcal{A}\cdot\overline{t})$$
$$+\Delta_h(t\cdot\vec{D}+t\cdot\mathcal{A}\gamma_5)\Delta_l(\mathcal{V}+\mathcal{A}\gamma_5)\Delta_l(\overline{H}+\overline{G})+\Delta_h(H+G)\Delta_l(\mathcal{V}+\mathcal{A}\gamma_5)\Delta_l(\overleftarrow{D}\cdot\overline{t}+\gamma_5\mathcal{A}\cdot\overline{t})]\,. \tag{25}$$

The derivative expansion of the above nonlocal effective action leads to a local effective action involving terms with one or two derivatives, as already suggested by [3,4], as well as terms of the form $\mathrm{Tr}\overline{H}iD_\mu T^\mu$ which are not reparametrization invariant [5]. The resulting local effective action, while invariant under chiral and spin symmetry, *appears* to violate reparametrization invariance in which $v\rightarrow w=v-\hat{q}/m_Q$ with the heavy fields transforming as (dropping terms of order $1/m_Q$)

$$H_v(x),G_v(x)\rightarrow e^{i\hat{q}\cdot x}H_w(x),G_w(x)\,,\quad T_v^\mu,S_v^\mu\rightarrow e^{i\hat{q}\cdot x}T_w^\mu,S_w^\mu\,. \tag{26}$$

The derivative expansion, while consistent with chiral and heavy-quark symmetry, is at odds with reparametrization invariance. Notice, however, that the full nonlocal effective action (25) is reparametrization invariant. To see this, consider, for instance, one of the terms in (25) (showing explicitly the space traces and the dependence on the heavy-quark velocity):

$$S_I^0=-N_c\sum_v\int dx\,dy\,\mathrm{Tr}(H_v(x)\Delta_l(x,y)\overleftarrow{D}\cdot\overline{T}_v(y)\Delta_{v,h}(y,x))\,. \tag{27}$$

Under the transformations (26), the above term is invariant if we note that

$$\Delta_{v,h}(x,y)\rightarrow e^{-i\hat{q}\cdot x}\Delta_{w,h}(x,y)e^{i\hat{q}\cdot y}\,. \tag{28}$$

This is generic of all the terms in (25). This point can be further appreciated if we were to recast (27) in a mixed momentum-space representation:

$$S_I^0=\sum_v\int dx\int^{\Lambda}dQ\,\mathrm{Tr}\left[H_v(x)\frac{1}{\not{Q}+i\not{\partial}-\Sigma}(Q+i\partial)\cdot\overline{T}_v(x)\frac{-i\not{v}}{v\cdot Q+\epsilon}\right.$$
$$\left.+i\,\mathrm{Tr}\left[H_v(x)\mathcal{V}\frac{1}{\not{Q}+i\not{\partial}-\Sigma}\overline{T}_v(x)\frac{i\not{v}}{v\cdot Q+\epsilon}\right]\right.. \tag{29}$$

Clearly, under (26), $Q\rightarrow Q+\hat{q}$ with the exception of the heavy-quark propagator, which is left unchanged. This transformation mixes up infinitely many terms in the derivative expansion, the sum of which is known in this case. In the resummed form, the shift in Q can be reabsorbed in the Q integration, the result being

$$S_I^0=\sum_w\int dx\int^{\Lambda+\hat{q}}dQ\,\mathrm{Tr}\left[H_w(x)\frac{1}{\not{Q}+i\not{\partial}-\Sigma}(Q+i\partial+\mathcal{V})\cdot\overline{T}_w(x)\frac{-i\not{w}}{w\cdot(Q-\hat{q})+\epsilon}\right]$$
$$+i\,\mathrm{Tr}\left[H_w(x)\mathcal{V}\frac{1}{\not{Q}+i\not{\partial}-\Sigma}\overline{T}_w(x)\frac{i\not{w}}{w\cdot(Q-\hat{q})+\epsilon}\right]. \tag{30}$$

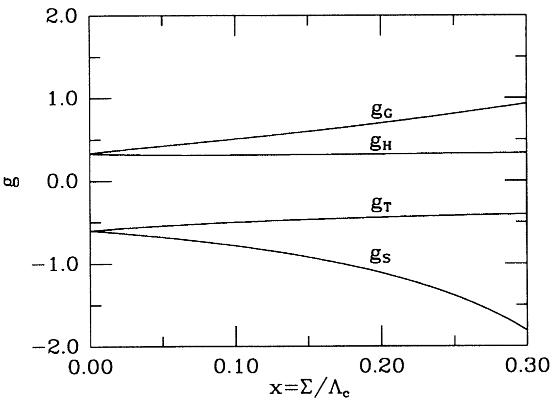

FIG. 1. The behavior of the coupling constants g_H $(s_l^{P_l}=\frac{1}{2}^-)$, g_G $(s_l^{P_l}=\frac{1}{2}^+)$, g_T $(s_l^{P_l}=\frac{3}{2}^+)$, and g_S $(s_l^{P_l}=\frac{3}{2}^-)$ as a function of the dimensionless ratio Σ/Λ_c, where Σ is the constituent mass of the light quark and Λ_c the scale for chiral symmetry breaking.

The new term in the heavy-quark propagator $w\cdot\hat{q}=-\hat{q}^2/2M$ is subleading in $1/m_Q$. Since after renormalization our results are cutoff independent to order m_Q^0, this effect can be ignored. Note that under $v\to w$ the range of Q is effectively shifted $\Lambda\to\Lambda+\hat{q}$, hence $S_I^0\to S_I^0$.

In short, the reparametrization invariance, while manifest at the nonlocal level, is highly nontrivial in the local version of the effective action. In our case, the breaking occurs in infinitely many terms of different order in the derivative expansion. Given the structure of the fermion determinant, this breaking is resummable to a subleading effect. Note that this argument is independent of the structure of the potential $G(M)$ which does not involve derivatives of the effective fields [cf. Eq. (6)]. Our conclusion is therefore generic. In a way, our problem is reminiscent of the chiral invariance of the Wess-Zumino term in Witten form. Chiral invariance is manifest in the nonlocal form, but broken in the (expanded) local form. The expanded form is more useful for practical calculations.

At this stage there are various courses of action. One might just stick to the nonlocal form with all symmetries manifest but all the inherent complications of the nonlocal formulation. Alternatively, one can try to find variants of the naive derivative expansion that may preserve reparametrization invariance locally, much like the general scheme advocated by Luke and Manohar [5]. So far we have not succeeded. If we were only to consider processes involving on-shell heavy mesons (as, in fact, discussed in [4]), then the derivative terms discussed above vanish to order one in the heavy-quark mass. In this case the derivative expansion is consistent with all the symmetries of the original action, and can be used to estimate the various couplings needed in the evaluation of the covariant decay amplitudes for the one-pion P-wave to S-wave transitions. Finally, we can choose to work with a fixed velocity v (such as gauge fixing) and expand the nonlocal effective action, giving up manifest reparametrization invariance. To what extent this expansion is justified can be addressed empirically only.

To summarize, we have constructed an effective action for the excited states of the heavy mesons, using an approximate bosonization scheme to QCD-inspired models, that combines both chiral and heavy-quark symmetry. The effective action is nonlocal in the mesonic fields with manifest invariance under reparametrization of the velocity of the heavy-quark field. Using a derivative expansion, we have estimated the various couplings and masses to order one in the heavy-quark mass, and compared these results with the nonrelativistic quark model estimation. Our results are consistent with the empirical measurements. The specific issue of reparametrization invariance in the local approximation requires care in assessing the properties of the heavy mesons off shell. Our construction is readily amenable to the analysis of the excited states with higher spin. It would also be interesting to see how the present local effective action compares with the nonlocal version in the Bethe-Salpeter form.

This work has been supported in part by DOE Grant No. DE-FG02-88ER40388 and by KBN Grant No. PB 2675/2.

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