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ARTICLES

Tensor-scalar cosmological models and their relaxation toward general relativity

Thibault Damour
Institut des Hautes Etudes Scientifiques, 91440 Bures sur Yvette, France
and Département d'Astrophysique Relativiste et de Cosmologie, Observatoire de Paris,
Centre National de la Recherche Scientifique, 92195 Meudon Cedex, France

Kenneth Nordtvedt
Institut des Hautes Etudes Scientifiques, 91440 Bures sur Yvette, France
and Physics Department, Montana State University, Bozeman, Montana 59717
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Cosmological models within general tensor-multiscalar theories of gravity are studied. By isolating an autonomous evolution equation for the scalar fields, one shows that the expansion of the Universe during the matter-dominated era tends to drive the scalar fields toward a minimum of the function $a(\varphi)$ describing their coupling to matter, i.e., toward a state where the tensor-scalar theory becomes indistinguishable from general relativity. The two main parameters determining the efficiency of this natural attractor mechanism toward general relativity are the redshift at the beginning of the matter era (or equivalently the present cosmological matter density) and the curvature of the coupling function $a(\varphi)$. Quantitative estimates for the present level of deviation from general relativity, as measured by the post-Newtonian parameters $\gamma - 1$, $\beta - 1$, and $\dot{G}/G$, are derived, which give greater significance to future improvements of solar-system gravitational tests. Another prediction of many tensor-scalar scenarios (whose consequences, particularly for the formation of structure in the Universe, remain to be studied in detail) is the existence of strong oscillations of the effective Newtonian coupling strength during the first few Hubble time scales of the matter era.

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I. INTRODUCTION

From the theoretical point of view, tensor-scalar theories of gravitation, in which gravity is mediated by one or several long-range scalar fields in addition to the usual tensor field present in Einstein's theory, are the most natural alternatives to general relativity. Indeed, most attempts at unifying gravity with the other interactions predict the existence of massless scalar fields with gravitational-strength couplings. The Jordan-Fierz-Brans-Dicke theory [1–3], which is the simplest tensor-scalar theory of gravitation, originated from taking seriously the scalar field arising in the Kaluza-Klein compactification of a fifth dimension. The more recent unification models based on supergravity or superstrings naturally associate long-range scalar partners to the usual tensor gravity of Einstein (see, e.g., [4]). An independent motivation for scalar fields is furnished by inflationary cosmology models which found in the framework of generic tensor-scalar theories of gravitation a technically natural (non-fine-tuned) way of terminating inflationary eras [5–8].

If we turn our attention to the experimental tests of the gravitational interaction [9, 10], the situation is two sided. On the one hand, the most precise gravitational tests (Hughes-Drever experiments and tests of the weak equivalence principle) are perfectly compatible with the existence of a scalar admixture to gravity, as long as the coupling of the scalars to matter is "metric," which means, in field theory language, a coupling to the trace of the energy-momentum tensor. [Actually, it seems that the only consistent field theories (without causality problems, negative-energy modes, etc.), satisfying exactly the weak equivalence principle, are the tensor-multiscalar metric theories; see Ref. [10].] On the other hand, the tests investigating relativistic effects in the solar system (notably the delay of radar signals passing near the Sun [11], and the bound on a possible violation of the strong equivalence principle in the Earth-Moon system [12, 13]) have set rather tight limits on the admixture of scalars to gravity. In field-theory language (see below), the solar-system experiments indicate that the maximum fractional contribution of scalars to the (Newtonian and post-Newtonian) gravitational interaction is

$$\alpha^2_{\text{solar system}} < 0.001 \quad (1\sigma \text{ confidence level}) . \qquad (1.1)$$

As one expects a fundamental tensor-scalar theory to exhibit a ratio of order unity between the couplings to matter of scalar and tensor fields, the limit (1.1) seems to argue against the existence of long-range scalars. We believe such a pessimistic conclusion is premature.

In this paper we study the general features of the cosmological evolution of tensor-scalar gravitational models and show that these models generically exhibit an attractive mechanism by which the observable predictions of tensor-scalar theories evolve toward those of Einstein's theory [14]. (Our main results were briefly presented in a previous communication [15].) The possibility of such a mechanism has been previously suggested in particular cases [16, 6–8], but without giving any general argument that general relativity is indeed a generic attractor of tensor-scalar theories. (As discussed below, the conclusion of Ref. [7] that the time evolution of the Jordan-Fierz-Brans-Dicke field Φ is always monotonic is actually incorrect.) Moreover, we give quantitative estimates of the efficiency of the relaxation toward general relativity. Because of the large but finite redshift factor separating us from the end of the radiation era, we find that the present observable total coupling strength of scalars α^2 is generically expected to be small [in accord with Eq. (1.1)] but not unmeasurably so. The present numerical value of α^2 is indeed found to be a number of rich cosmological significance, being very sensitive to the present total mass density.

From a technical point of view, the newest result of this paper will be to exhibit an *autonomous* evolution equation for the scalar fields, independent of the equations describing the evolution of the scale factor of the cosmological models we consider. This isolation of a scalar evolution equation (which is crucial to our demonstration of the generic "attractor" features of general relativity) was facilitated by our use of the framework in which the kinetic terms of the tensor and scalar fields do not mix ("Einstein conformal frame").

The plan of this paper is as follows. Section II presents the general formulation of tensor-scalar theories in the "Einstein" conformal frame. The system of equations describing cosmological models in tensor-multiscalar theories is given in Sec. III, ending with the autonomous evolution equation for the scalars. The implications of the latter equation in the simplest case of spatially flat cosmologies are explored in Sec. IV, while Sec. V discusses the observable consequences of generic tensor-scalar cosmological models.

II. TENSOR-SCALAR THEORIES IN THE EINSTEIN CONFORMAL FRAME

The simplest tensor-scalar theory, due to Jordan [1], Fierz [2], and Brans and Dicke [3], is usually formulated by using as the basic metric tensor the physical tensor $\tilde{g}_{\mu\nu}$ to which matter is universally coupled ("Jordan-Fierz" conformal frame). This Jordan-Fierz-Brans-Dicke (JFBD) theory contains only one free dimensionless parameter (denoted ζ by Jordan and Fierz, and ω by Brans and Dicke), and its predictions tend to those of general relativity when this parameter tends to infinity. Bergmann [17], Nordtvedt [18], and Wagoner [19] generalized the JFBD theory by considering the most general metric theory of gravity containing one tensor and one scalar. This general tensor-monoscalar theory can be described in the Jordan-Fierz frame by replacing the parameter ω by an arbitrary function of the JFBD field Φ, and by adding an arbitrary potential $V(\Phi)$ for the scalar. In this paper we explore the simplest class of exactly massless tensor-scalar theories, i.e., the case of the vanishing potential $V(\Phi) = 0$ [20]. The action describing a general massless tensor-monoscalar theory in the Jordan-Fierz frame reads

$$S = \frac{1}{16\pi}\int d^4x\sqrt{\tilde{g}}\left[\Phi\widetilde{R} - \frac{\omega(\Phi)}{\Phi}\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi\right] + S_m[\Psi_m, \tilde{g}_{\mu\nu}] \ , \tag{2.1}$$

where $\widetilde{R} \equiv \tilde{g}^{\mu\nu}\widetilde{R}_{\mu\nu}$ denotes the curvature scalar of the physical metric $\tilde{g}_{\mu\nu}$ [21]. The last term in Eq. (2.1) denotes the action of the matter, which is a functional of the matter variables, collectively denoted by Ψ_m, and of the metric $\tilde{g}_{\mu\nu}$, but which does not depend on the JFBD field Φ.

The advantage of the Jordan-Fierz frame is that the laws of evolution of the matter (i.e., nongravitational) fields take the same form as in general relativity. For instance, the energy-momentum conservation equation reads

$$\widetilde{\nabla}_\nu \widetilde{T}^{\mu\nu} = 0 \ , \tag{2.2}$$

where $\widetilde{\nabla}_\mu$ denotes the covariant derivative defined by the physical metric $\tilde{g}_{\mu\nu}$, and where

$$\widetilde{T}^{\mu\nu} \equiv \frac{2}{\sqrt{\tilde{g}}}\frac{\delta S_m[\Psi_m, \tilde{g}_{\mu\nu}]}{\delta \tilde{g}_{\mu\nu}} \tag{2.3}$$

denotes the physical stress-energy tensor of the matter. On the other hand, this frame has the disadvantage of featuring complicated evolution equations for the gravitational fields $\tilde{g}_{\mu\nu}$ and Φ:

$$\widetilde{R}_{\mu\nu} - \tfrac{1}{2}\widetilde{R}\tilde{g}_{\mu\nu} = 8\pi\Phi^{-1}\widetilde{T}_{\mu\nu} + \Phi^{-2}\omega(\Phi)\left[\partial_\mu\Phi\partial_\nu\Phi - \tfrac{1}{2}\tilde{g}_{\mu\nu}\tilde{g}^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi\right] + \Phi^{-1}[\widetilde{\nabla}_\mu\widetilde{\nabla}_\nu\Phi - \tilde{g}_{\mu\nu}\Box_{\tilde{g}}\Phi] \ , \tag{2.4a}$$

$$\Box_{\tilde{g}}\Phi = \frac{1}{2\omega(\Phi)+3}\left(8\pi\widetilde{T} - \frac{d\omega}{d\Phi}\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi\right) \ . \tag{2.4b}$$

In Eqs. (2.4a) and (2.4b) all index operations are performed with the metric $\tilde{g}_{\mu\nu}$, e.g., $\Box_{\tilde{g}} \equiv \tilde{g}^{\mu\nu}\widetilde{\nabla}_\mu\widetilde{\nabla}_\nu$, $\widetilde{T}_{\mu\nu} \equiv \tilde{g}_{\mu\alpha}\tilde{g}_{\nu\beta}\widetilde{T}^{\alpha\beta}$. The presence of second-order derivatives on the right-hand side of Eq. (2.4a) shows that the propagation modes of $\tilde{g}_{\mu\nu}$ and Φ are mixed together. [Physically this means that the $\tilde{g}_{\mu\nu}$ waves contain both helicity-2 and helicity-0 excitations, and mathematically this implies that the variables $(\tilde{g}_{\mu\nu}, \Phi)$ are inconvenient variables for formulating a Cauchy problem.] The other disadvantages of the formulation (2.4a) and (2.4b) are (i) the fact that the points in field space where $\omega(\Phi) = \infty$ enter the theory as mathematically singular boundary points, while we shall show that they are physically regular and (ii) the awkwardness of the action (2.1) as a starting point for generalizing the theory to the case where there are *several* scalar fields.

For many purposes tensor-scalar theories are better formulated by using as basic gravitational variables the pure-helicity propagation modes present in the theory ("Einstein" conformal frame). In this formulation, the generalization to the case where there are several scalar fields is quite straightforward [22]. Considering first the one-scalar case, the action [differing only by a surface term from the Jordan-Fierz action (2.1)] which describes a general (massless) tensor-scalar theory in the Einstein frame reads

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{g_*}[R_* - 2g_*^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi] + S_m[\Psi_m,\ A^2(\varphi)g^*_{\mu\nu}] \ . \tag{2.5}$$

Here G_* denotes a "bare" gravitational coupling constant, $g^*_{\mu\nu}$ the "Einstein" metric tensor conformally related to the physical ("Jordan-Fierz") one via

$$\tilde{g}_{\mu\nu} = A^2(\varphi)g^*_{\mu\nu} \ , \tag{2.6}$$

and $R_* \equiv g_*^{\mu\nu}R^*_{\mu\nu}$ the curvature scalar of $g^*_{\mu\nu}$. One should carefully distinguish the basic scalar field φ of the Einstein-frame formulation from the JFBD one, Φ [see Eq. (2.13a) below for the connection between the two]. Note that the kinetic terms in Eq. (2.5) have a universal structure and do not contain an arbitrary function [such as the $\omega(\Phi)$ of Eq. (2.1)]. In the approximation of linearized waves around a flat background $f^*_{\mu\nu} = \mathrm{diag}\,(-1,+1,+1,+1)$, $h^*_{\mu\nu} \equiv g^*_{\mu\nu} - f^*_{\mu\nu}$ represents a pure massless helicity-2 excitation decoupled from the massless helicity-0 one described by φ. It is only in the coupling to matter that an arbitrary function $A(\varphi)$ appears.

In field theory the kinetic terms play the basic role of defining a "metric" (i.e., a quadratic form) in field space. In that sense the variables (g_*, φ) of the Einstein frame are field analogues of "Cartesian coordinates" which reduce an intrinsically simple metric in field space to its simplest form [kinetic terms $\sim (\partial g_*)^2 + (\partial\varphi)^2$], by contrast with the variables $(\tilde{g}, \Phi)$ of the Jordan-Fierz frame which are analogues of "curved coordinates" in a flat space [kinetic terms $\sim \Phi(\partial\tilde{g})^2 + \omega(\Phi)(\partial\Phi)^2$]. As we find below the "curved coordinates" $(\tilde{g}, \Phi)$ introduce fictitious coordinate singularities in field space $[\omega(\Phi) = \infty]$ at points which are perfectly regular when described in the "Einstein" coordinates (g_*, φ).

The "coupling function" $A(\varphi)$, or more precisely its natural logarithm

$$a(\varphi) \equiv \ln A(\varphi) \tag{2.7}$$

plays a central role in discussing the observable consequences of tensor-scalar theories [22]. It is convenient to introduce a special notation for the first two gradients of the coupling function $a(\varphi)$: namely,

$$\alpha(\varphi) \equiv \frac{\partial a(\varphi)}{\partial\varphi} \equiv \frac{\partial \ln A}{\partial\varphi} \ , \tag{2.8a}$$

$$\kappa(\varphi) \equiv \frac{\partial \alpha(\varphi)}{\partial\varphi} = \frac{\partial^2 a}{\partial\varphi^2} \ . \tag{2.8b}$$

[Note that the quantity κ, which measures the curvature of the potential $a(\varphi)$, was denoted β (β_{ab} in the multiscalar case) in Ref. [22]. We change the notation here to avoid any confusion with the post-Newtonian parameter β.]

The original JFBD theory is defined by a linear coupling function: $a(\varphi) = \alpha\varphi$, i.e., $\alpha(\varphi) = \alpha = \text{const}$, and $\kappa(\varphi) = 0$ [with $\alpha^2 = 1/(2\omega + 3)$ in terms of the usual parameter ω, see Eq. (2.13b)].

The field equations in the Einstein conformal frame read

$$R^*_{\mu\nu} = 2\partial_\mu\varphi\partial_\nu\varphi + 8\pi G_*\left(T^*_{\mu\nu} - \tfrac{1}{2}T_* g^*_{\mu\nu}\right) \ , \tag{2.9a}$$

$$\Box_{g_*}\varphi = -4\pi G_*\alpha(\varphi)T_* \ , \tag{2.9b}$$

where

$$T_*^{\mu\nu} \equiv \frac{2}{\sqrt{g_*}}\frac{\delta S_m[\Psi_m, A^2 g^*_{\mu\nu}]}{\delta g^*_{\mu\nu}} \tag{2.10}$$

denotes the g^* stress-energy tensor. The tensorial operations appearing in Eqs. (2.9a) and (2.9b) are all performed by using the g^* metric: $T^*_{\mu\nu} \equiv g^*_{\mu\alpha}g^*_{\nu\beta}T_*^{\alpha\beta}$, $T_* \equiv g^*_{\mu\nu}T_*^{\mu\nu}$, $\Box_{g_*} \equiv g_*^{\mu\nu}\nabla^*_\mu\nabla^*_\nu$ where ∇^*_μ is the Levi-Civita connection of $g^*_{\mu\nu}$. It is clear from Eq. (2.9b) that the quantity $\alpha(\varphi)$, Eq. (2.8a), plays the role of the basic (field-dependent) coupling strength between the scalar field and matter. Its square α^2 appears in all quantities where a scalar interaction mediates between two material bodies (in the same way as e^2 appears in all the electromagnetic interactions).

The g^*-frame energy tensor (2.10) is related to the physical energy tensor through

$$T_*^{\mu\nu} = A^6\widetilde{T}^{\mu\nu} \ , \tag{2.11a}$$

$$T^\mu_{*\nu} = A^4\widetilde{T}^\mu_\nu \ . \tag{2.11b}$$

[Note that the indices in Eq. (2.11b) are moved by different metrics on each side.] In contrast with $\widetilde{T}^{\mu\nu}$ which satisfies the simple conservation law (2.2), $T_*^{\mu\nu}$ is acted upon by a scalar-gradient force:

$$\nabla^*_\nu T^\nu_{*\mu} = \alpha(\varphi)T_*\nabla^*_\mu\varphi \ . \tag{2.12}$$

[On the other hand it is clear from Eq. (2.9a) that the sum of $T^*_{\mu\nu}$ and the energy tensor of φ, $(4\pi G_*)^{-1}(\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g^*_{\mu\nu}(\nabla_*\varphi)^2)$, is conserved with respect to ∇_*.]

By taking into account the effect of the conformal transformation (2.6) on the action (2.5) one finds that the Einstein-frame variables are related to the Jordan-Fierz ones through

$$G_*A^2(\varphi) = \Phi^{-1} \ , \tag{2.13a}$$

$$\alpha^2(\varphi) = [2\omega(\Phi) + 3]^{-1} \ . \tag{2.13b}$$

It is straightforward to generalize the framework (2.5)–(2.12) to the case where there are several massless scalar fields, say $(\boldsymbol{\varphi}) = (\varphi^a)$ where $a = 1, \ldots, n$ labels the scalars [22]. The kinetic terms of the scalars become $-2g_*^{\mu\nu}\langle\partial_\mu\boldsymbol{\varphi}, \partial_\nu\boldsymbol{\varphi}\rangle$ where angular brackets denote a general field-dependent quadratic form (i.e., a metric) in the space of the scalar fields:

$$\langle d\boldsymbol{\varphi}, d\boldsymbol{\varphi}\rangle = \gamma_{ab}(\varphi^c)d\varphi^a d\varphi^b \tag{2.14}$$

[general nonlinear σ model]. Equation (2.6) still holds in the multiscalar case, with $A(\boldsymbol{\varphi}) \equiv \exp[a(\boldsymbol{\varphi})]$ an arbitrary function of the n scalar fields. The natural extensions of the definitions (2.8a) and (2.8b) give a vector $\alpha_a(\varphi) = D_a a(\varphi)$ and a tensor $\kappa_{ab}(\varphi) = D_a D_b a(\varphi)$ where D_a denotes the covariant derivative in the space of the scalars endowed with the metric (2.14). The right-hand side of Eq. (2.9a) now contains the contraction $2\langle\partial_\mu\boldsymbol{\varphi},\partial_\nu\boldsymbol{\varphi}\rangle$. Similarly the right-hand of Eq. (2.12) contains $\langle\boldsymbol{\alpha},\nabla^*_\mu\boldsymbol{\varphi}\rangle$ where $\boldsymbol{\alpha}\equiv\alpha^a\equiv\gamma^{ab}\alpha_b$, where γ^{ab} is the inverse of γ_{ab}. On the other hand Eq. (2.9b) is slightly modified by the appearance of a scalar-field-space Christoffel symbol $[\gamma^a_{bc}=\frac{1}{2}\gamma^{ad}(\partial\gamma_{cd}/\partial\varphi^b+\partial\gamma_{bd}/\partial\varphi^c-\partial\gamma_{bc}/\partial\varphi^d)]$

$$\Box_{g_*}\varphi^a+g_*^{\mu\nu}\gamma^a_{bc}(\varphi)\partial_\mu\varphi^b\partial_\nu\varphi^c=-4\pi G_*\alpha^a(\varphi)T_* \ . \tag{2.15}$$

If we consider the presently observable predictions of tensor-scalar theories in the quasistationary weak-field conditions (first post-Newtonian limit) of the solar system they can be completely described in terms of one dimensional coupling strength (the effective Newtonian "constant" $\widetilde{G}_0$) and two dimensionless post-Newtonian parameters γ and β [23–25]. (The most general "parametrized post-Newtonian" formalism is discussed in detail in Ref. [9]; for a streamlined formulation of the simple $\widetilde{G}-\gamma-\beta$ formalism see Sec. 3 of Ref. [22].)

The calculation of $\widetilde{G}$, γ, and β in terms of the general scalar functionals appearing in tensor-scalar theories can be performed either in the Jordan-Fierz frame [26, 18] or in the Einstein one [22]. The latter derivation exhibits more clearly the theoretical significance of these quantities, and generalizes straightforwardly to the multiscalar case. Its results are

$$\widetilde{G}_0=G_*[A^2(1+\alpha^2)]_{\varphi_0} \ , \tag{2.16a}$$

$$\gamma-1=-2\left[\frac{\alpha^2}{1+\alpha^2}\right]_{\varphi_0} \ , \tag{2.16b}$$

$$\beta-1=\frac{1}{2}\left[\frac{\alpha\kappa\alpha}{(1+\alpha^2)^2}\right]_{\varphi_0} \ . \tag{2.16c}$$

(The same formulas hold in the multiscalar case with the understanding that α^2 denotes $\langle\boldsymbol{\alpha},\boldsymbol{\alpha}\rangle\equiv\gamma_{ab}\alpha^a\alpha^b$, and $\alpha\kappa\alpha$ the matrix product $\alpha^a\kappa_{ab}\alpha^b$.) In Eqs. (2.16a)–(2.16c) the subscript φ_0 denotes the asymptotic ($r\to\infty$), cosmologically determined, values of the scalar fields around the considered local gravitating system (e.g., the solar system at the present cosmological epoch). The derivation of Eq. (2.16a) illustrates that in the $(1+\alpha^2)$ factor the 1 represents the exchange of spin-2 excitations, while the α^2 results from the exchange of spin-0 excitations (the A^2 factor comes from the rescaling of units between the Einstein and the Jordan-Fierz frames). One sees from Eq. (2.16b) that the parameter $\gamma-1$ gives a direct measure of the amount of scalar admixture in gravity. The 1σ limit $|\gamma-1|<2\times10^{-3}$ obtained from the analysis of the Shapiro time delay in the Viking data [11] gives the upper bound (1.1) quoted in the Introduction. The Lunar Laser Ranging test of the strong equivalence principle yields on the other hand a direct bound on the combination

$$\eta\equiv4\beta-\gamma-3=2\frac{\alpha^2}{1+\alpha^2}\left[1+\frac{\kappa}{1+\alpha^2}\right] \ , \tag{2.17}$$

namely, $|\eta|<5\times10^{-3}(1\sigma)$ [12, 13]. All the other completed solar-system tests give weaker limits on tensor-scalar theories. (See below for the limits coming from the time-variation of $\widetilde{G}$.)

Summarizing: all empirical knowledge of gravity in the solar system is compatible with a generic tensor-scalar theory of gravity if it satisfies (1σ level)

$$\alpha^2_{\text{solar system}}<0.001 \ , \tag{2.18a}$$

$$[(1+\kappa)\alpha^2]_{\text{solar system}}<0.0025 \ . \tag{2.18b}$$

Given the first inequality (2.18a), the second one seems theoretically natural in the sense that one might *a priori* expect the theory parameter κ [defined by Eq. (2.8b)] to be of order unity. The fundamental question is to understand why the coupling strength α [Eq. (2.8a)] is found to be so small when measured in the solar system at this epoch. If one assumes that the underlying theory of gravity is the original Jordan-Fierz-Brans-Dicke one ($\alpha=$ const, $\kappa=0$) the limit (2.18a) casts doubt on the plausibility of the theory. The situation turns out to be entirely different if one considers general tensor-scalar theories.

III. TENSOR-MULTISCALAR COSMOLOGIES

We consider homogeneous cosmological models with perfect-fluid matter distributions. Let k ($=+1,0$, or -1) denote the sign of the spatial curvature and

$$d\ell^2=\frac{dr^2}{1-kr^2}+r^2(d\theta^2+\sin^2\theta d\varphi^2) \tag{3.1}$$

the metric on a three-space of constant curvature k.

Homogeneous cosmological spacetimes can be represented in either the Einstein frame

$$ds_*^2=-dt_*^2+R_*^2(t_*)d\ell^2 \ , \tag{3.2a}$$

or in the Jordan-Fierz frame

$$d\tilde{s}^2=-d\tilde{t}^{\,2}+\widetilde{R}^2(\tilde{t})d\ell^2 \ . \tag{3.2b}$$

In either case, the set of scalar fields depends only on time, e.g., $\boldsymbol{\varphi}=\boldsymbol{\varphi}(t_*)$ in the g_* frame.

The matter distribution admits a perfect-fluid representation in either conformal frame:

$$T_*^{\mu\nu}=(\rho_*+p_*)u_*^\mu u_*^\nu+p_*g_*^{\mu\nu} \ , \tag{3.3a}$$

$$\widetilde{T}^{\mu\nu}=(\tilde{\rho}+\tilde{p})\tilde{u}^\mu\tilde{u}^\nu+\tilde{p}\tilde{g}^{\mu\nu} \ , \tag{3.3b}$$

with $g^*_{\mu\nu}u_*^\mu u_*^\nu=-1=\tilde{g}_{\mu\nu}\tilde{u}^\mu\tilde{u}^\nu$. From Eq. (2.11b) one has

$$\rho_*=A^4\tilde{\rho} \ , \tag{3.4a}$$

$$p_*=A^4\tilde{p} \ . \tag{3.4b}$$

For reasons explained above it is more convenient to work with the Einstein-frame field equations. The Jordan-Fierz variables $\tilde{t}$ (physical cosmic time) and $\widetilde{R}$ (physical cosmic scale factor) are then obtained by using $d\tilde{s}^2=A^2(\boldsymbol{\varphi})ds_*^2$, Eq. (2.6), i.e.,

$$d\tilde{t}=A[\boldsymbol{\varphi}(t_*)]dt_* \ , \tag{3.5a}$$

$$\widetilde{R}(\tilde{t})=A[\boldsymbol{\varphi}(t_*)]R_*(t_*) \ . \tag{3.5b}$$

The Einstein equations (2.9a) (considered in the general multiscalar case) give

$$-3\frac{\ddot{R}_*}{R_*} = 4\pi G_*(\rho_* + 3p_*) + 2(\dot{\varphi})^2 \ , \tag{3.6a}$$

$$3\left(\frac{\dot{R}_*}{R_*}\right)^2 + 3\frac{k}{R_*^2} = 8\pi G_*\rho_* + (\dot{\varphi})^2 \ , \tag{3.6b}$$

while the scalar field equation (2.9b) yields

$$\ddot{\varphi}^a + \gamma^a_{bc}(\varphi)\dot{\varphi}^b\dot{\varphi}^c + 3\frac{\dot{R}_*}{R_*}\dot{\varphi}^a = -4\pi G_*\alpha^a(\varphi)(\rho_* - 3p_*) \ . \tag{3.6c}$$

The overdots denote Einstein-time derivatives, d/dt_*, $(\dot{\varphi})^2$ is the square with respect to the σ-model metric (2.14), i.e., $\langle\dot{\varphi},\dot{\varphi}\rangle \equiv \gamma_{ab}(\varphi)\dot{\varphi}^a\dot{\varphi}^b$, while $\alpha^a \equiv \gamma^{ab}\alpha_b$. We recall that $\gamma^a_{bc}(\varphi)$ denote the σ-model Christoffel symbols [which are absent in the one-scalar case, and in the simplest multiscalar case where $\gamma_{ab}(\varphi) = \delta_{ab}$]. As usual the field equations (3.6a)–(3.6c) are not independent, being related by a Bianchi identity, or equivalently by an energy-conservation law which reads, in the Einstein-frame [time component of Eq. (2.12)],

$$d(\rho_* R_*^3) + p_* dR_*^3 = (\rho_* - 3p_*)R_*^3 da(\varphi) \ . \tag{3.7}$$

Since all quantities in Eq. (3.7) depend only on one (time) variable, it was convenient to write this equation in differential form by multiplying both sides by dt_*. To complete the set of dynamical equations one needs an equation of state for the cosmic matter. Because of the universal coupling of matter to the physical metric $\tilde{g}_{\mu\nu}$ this equation of state will hold (as a scalar-field independent relation) only between the physical matter variables:

$$\tilde{p} = f(\tilde{\rho}) \ . \tag{3.8}$$

In the form given above the time evolutions of the scale factor $R_*(t_*)$ and the scalar fields $\varphi(t_*)$ are intimately coupled. However, we found that the two evolutions can be conveniently unmixed by introducing a new evolution parameter [27], namely,

$$p \equiv \ln R_* + \text{ const}, \tag{3.9a}$$

such that

$$dp = H_* dt_* \ , \tag{3.9b}$$

where $H_* \equiv \dot{R}_*/R_*$ is the Einstein-frame Hubble parameter. (One should take care to distinguish the evolution parameter p from the fluid pressures p_* and $\tilde{p}$.) Using

$$\dot{\varphi}^a = H_*\frac{d\varphi^a}{dp} \ , \tag{3.10a}$$

$$\ddot{\varphi}^a = H_*^2\frac{d^2\varphi^a}{dp^2} + \dot{H}_*\frac{d\varphi^a}{dp} \ , \tag{3.10b}$$

$$\frac{\ddot{R}_*}{R_*} = \dot{H}_* + H_*^2 \ , \tag{3.10c}$$

we obtain from the field equations (3.6a)–(3.6c) an evolution equation for the scalars as function of the "p time" (3.9a) and (3.9b) whose coefficients depend only on the ratios p_*/ρ_* and $k/(R_*^2\rho_*)$, and one can use Eqs. (3.4a) and (3.4b), (3.7), and (3.8) to express these ratios as functions of the parameter p of Eqs. (3.9a) and (3.9b). To replace the above system of coupled cosmological evolution equations by one decoupled equation we first integrate the physical energy-conservation law

$$d[\tilde{\rho}\widetilde{R}^3] = -\tilde{p}(\tilde{\rho})d\widetilde{R}^3 \tag{3.11}$$

[equivalent to Eq. (3.7)] to obtain $\tilde{\rho}$ as a function of $\widetilde{R}$:

$$\tilde{\rho} = F(\widetilde{R}) \ . \tag{3.12a}$$

For instance, if the equation of state has the simple form $\tilde{p} = \lambda_0\tilde{\rho}$ with a constant λ_0, the integration of Eq. (3.11) yields

$$\tilde{\rho} = C\,\widetilde{R}^{-3(1+\lambda_0)} \ , \tag{3.12b}$$

with C being a constant of integration. Then, using Eq. (3.5b) and the definition $R_* \equiv \exp(p - \text{const})$ from Eqs. (3.9a) and (3.9b) we express $\widetilde{R}$ in terms of p and φ

$$\widetilde{R} = \exp[p + a(\varphi) - \text{ const}] \ . \tag{3.13}$$

From Eqs. (3.8), (3.12a), and (3.13), we have $\tilde{\rho}$ and $\tilde{p}$ as functions of p and φ, and consequently the functions

$$\lambda(p,\varphi) \equiv \frac{\tilde{p}}{\tilde{\rho}} \ , \tag{3.14a}$$

$$\epsilon(p,\varphi) \equiv \frac{3k}{8\pi G_*\tilde{\rho}A^4(\varphi)R_*^2} \ , \tag{3.14b}$$

which enter the decoupled equation for the p evolution of the scalars: namely,

$$\frac{2(1-\epsilon)}{3-(\varphi')^2}\frac{D}{dp}\left(\frac{d\varphi}{dp}\right) + \left(1 - \lambda - \frac{4}{3}\epsilon\right)\frac{d\varphi}{dp} = -(1-3\lambda)\boldsymbol{\alpha}(\varphi) \ . \tag{3.15}$$

In Eq. (3.15) $(\varphi')^2$ denotes $\gamma_{ab}(\varphi)\varphi'^a\varphi'^b$ with $\varphi'^a \equiv d\varphi^a/dp$, and D/dp denotes the *covariant* derivative of the scalar field "velocity vector" $\varphi' \equiv d\varphi/dp$ [i.e., $\varphi''^a + \gamma^a_{bc}(\varphi)\varphi'^b\varphi'^c$ as in Eq. (3.6c)]. The key feature of Eq. (3.15), thanks to the functions of Eqs. (3.14a) and (3.14b), is that this p-time evolution equation gives an ordinary second-order differential equation for $\varphi(p)$, independent of the evolution of the cosmic scale factor.

After integration of the decoupled scalar equation (3.15) we can produce from Eqs. (3.9a) and (3.9b), (3.12a), (3.12b), and (3.13) the p-evolution of R_*, $\widetilde{R}$, and the matter variables. All that remains to determine is the link between the p-time-scale and the Einstein time t_*, or better the physical proper time $\tilde{t}$. This is done as a final step by a simple quadrature. Indeed, we obtain, from Eq. (3.6b),

$$\frac{d\tilde{t}}{dp} = \left[\frac{8\pi G_*\tilde{\rho}(1-\epsilon)A^2}{3-\varphi'^2}\right]^{-1/2} \ , \tag{3.16}$$

the right-hand side of which is at this point a known function of p. In the following sections we discuss the general features of the solutions of Eq. (3.15), starting with the simplest case in which the cosmic three-space is flat ($k = 0$).

IV. EVOLUTION OF SCALAR FIELDS IN SPATIALLY FLAT COSMOLOGIES

It is well known that the inflationary scenario creates a postinflation universe of negligible spatial curvature. We first consider the spatially flat case ($k = 0$) of the scalar field evolution, and then later consider the important differences that arise if the universe is "open" ($k = -1, \Omega < 1$) as is in fact suggested by present observational data. There are some exact properties of the scalar-field evolution equation (3.15) in the $k = 0$ case which are worth discussing.

A. Mechanical analogue of the scalar-field evolution equation

With $k = 0$, the quantity ϵ, Eq. (3.14b), vanishes and Eq. (3.15) simplifies to $[\boldsymbol{\varphi}' \equiv d\boldsymbol{\varphi}/dp]$

$$\frac{2}{3-\boldsymbol{\varphi}'^2}(\boldsymbol{\varphi}'')_{\rm cov} + (1-\lambda)\boldsymbol{\varphi}' = -(1-3\lambda)\boldsymbol{\alpha}(\boldsymbol{\varphi}) \ , \qquad (4.1)$$

where $\lambda(p,\boldsymbol{\varphi}) \equiv \tilde{p}/\tilde{\rho}$ as in Eq. (3.14a).

It is useful to think of $\boldsymbol{\varphi} = (\varphi^a)$ as representing the position of a "particlelike dynamical variable" living in an n-dimensional Riemannian manifold endowed with the metric (2.14) ("σ manifold"), and of p as simply being the "time": $\boldsymbol{\varphi}' \equiv (d\varphi^a/dp)$ and $(\boldsymbol{\varphi}'')_{\rm cov} \equiv [d\varphi'^a/dp + \gamma^a_{bc}(\boldsymbol{\varphi})\varphi'^b\varphi'^c]$ are the velocity and acceleration, respectively, of the "particle." The right-hand side of Eq. (4.1) represents a "force term" proportional to (minus) the gradient of a potential:

$$-\alpha^a(\boldsymbol{\varphi}) \equiv -\gamma^{ab}\frac{\partial a(\boldsymbol{\varphi})}{\partial\varphi^b} \ . \qquad (4.2)$$

As Eq. (3.6b) can be expressed in the $k = 0$ case as

$$H_*^2(3-\boldsymbol{\varphi}'^2) = 8\pi G_*\rho_* = 8\pi G_* A^4\tilde{\rho} \ ,$$

the local positivity of the energy density implies that the velocity $\boldsymbol{\varphi}'$ is constrained by the inequality

$$(\boldsymbol{\varphi}')^2 < 3 \ . \qquad (4.3)$$

Equation (4.1) can be thought of as describing a kind of "relativistic dynamics" for a particle moving in a curved space, with a limiting speed equal to $\sqrt{3}$, a velocity-dependent mass term

$$m(\boldsymbol{\varphi}') = \frac{2}{3-\boldsymbol{\varphi}'^2} \qquad (4.4)$$

(diverging to infinity when $|\boldsymbol{\varphi}'| \to \sqrt{3}$), a friction term proportional to the velocity and a force term proportional to the gradient of the potential $a(\boldsymbol{\varphi}) \equiv \ln A(\boldsymbol{\varphi})$. Note that it seems physically reasonable to assume that the physical energy density $\tilde{\rho}$ is positive and that the ratio $\lambda \equiv \tilde{p}/\tilde{\rho}$ is between -1 and $+1$ (dominant energy conditions; they are satisfied in all the cosmological eras that are usually considered, including inflation). This implies that the second term in Eq. (4.1) always represents *damping* rather than antidamping. The sign of the coefficient $(1-3\lambda)$ in front of the force term is strictly positive if we consider an inflationary era ($\lambda = -1$), or a matter-dominated one ($\lambda = 0$), and nearly vanishes during a radiation-dominated era ($\lambda \simeq 1/3$). If we were to consider an era where $\lambda > 1/3$ (for instance, with $\lambda \approx 1$, i.e., $\tilde{p} \approx +\tilde{\rho}$ as suggested long ago [28]) the sign of the force term would change. The particle would then tend to go up the potential $a(\boldsymbol{\varphi})$ rather than down it. Keeping in mind this nonstandard possibility (to which we shall return later) we first consider the standard cases $\lambda = -1$, $1/3$, and 0 and the transitional cases between them.

We qualitatively describe the general features of the p-time evolution of the scalar fields as follows. The coupling function $a(\boldsymbol{\varphi}) \equiv \ln A(\boldsymbol{\varphi})$ defines a potential on the Riemannian manifold in which the scalar fields live (σ manifold). Then, except during the radiation era during which the particle $\boldsymbol{\varphi}$ does not feel the potential $a(\boldsymbol{\varphi})$ but has a damped inertial motion, the particle $\boldsymbol{\varphi}$ moves under the combined influence of the gradient of the potential $a(\boldsymbol{\varphi})$ and a damping linear in the velocity $\boldsymbol{\varphi}'$. We therefore expect (in the long term) the particle to end up being caught near a minimum of $a(\boldsymbol{\varphi})$ [and therefore near where the gradient $\boldsymbol{\alpha}(\boldsymbol{\varphi}) = \nabla_{\boldsymbol{\varphi}} a(\boldsymbol{\varphi})$ vanishes], if such points exist. Remembering from Sec. II that the magnitude of $\boldsymbol{\alpha}(\boldsymbol{\varphi}_0)$ [$\boldsymbol{\varphi}_0$ is the present cosmological value of $\boldsymbol{\varphi}$] measures the total admixture of the scalars in the gravitational interaction, i.e., the deviation from general relativity, we see that Eq. (4.1) [and more generally (3.15)] naturally suggests an attractor mechanism towards general relativity. Before studying in quantitative detail the efficiency of this attractor mechanism it is useful to discuss some exact properties of the evolution equation (4.1).

B. Energy evolution equation, Lagrangian, and momentum

Let us define the kinetic energy of the $\boldsymbol{\varphi}$ particle as $-\ln(1-\boldsymbol{\varphi}'^2/3)$ and its potential energy as $(1-3\lambda)a(\varphi)$, and consider the total energy

$$E(p,\boldsymbol{\varphi},\boldsymbol{\varphi}') \equiv -\ln(1-\boldsymbol{\varphi}'^2/3) + [1-3\lambda(p,\boldsymbol{\varphi})]a(\varphi) \ . \qquad (4.5)$$

Note that the kinetic energy is positive and tends to infinity when the velocity tends to its limiting value (4.3). Taking the p-time (covariant) derivative of Eq. (4.5) and using Eq. (4.1) yields the total time derivative

$$\frac{d}{dp}E(p) = -(1-\lambda)\boldsymbol{\varphi}'^2 - 3a(\varphi)\frac{d}{dp}\lambda(p) \ . \qquad (4.6)$$

The first term on the right-hand side of Eq. (4.6) can be thought of as the energy loss due to the damping term in (4.1), while the last is understood as the effect of an explicit time dependence $\lambda(p) \equiv \lambda(p,\boldsymbol{\varphi}(p))$ in the energy functional (4.5). When approximating the cosmological evolution as a sequence of eras where λ is constant (successively -1, $1/3$, and 0) one can neglect the last term in Eq. (4.6) and conclude that $E(p)$ is a monotonically decreasing function of time.

In the simple case where there is only one scalar field Eq. (4.1) reads

$$\frac{2}{3-\varphi'^2}\varphi'' + (1-3\lambda)\frac{\partial a(\varphi)}{\partial\varphi} = -(1-\lambda)\varphi' \ . \qquad (4.7)$$

For simplicity we consider that λ is either a constant or a known function of p: $\lambda(p)$ (without explicit dependence on the position variable φ). It is then possible to define a Lagrangian which reproduces the conservative terms in the equation of motion (4.7), i.e., the left-hand side at the exclusion of the friction term on the right-hand side. With

$$\begin{aligned} L(\varphi,\varphi',p) = &\left(1+\frac{\varphi'}{\sqrt{3}}\right)\ln\left(1+\frac{\varphi'}{\sqrt{3}}\right) \\ &+\left(1-\frac{\varphi'}{\sqrt{3}}\right)\ln\left(1-\frac{\varphi'}{\sqrt{3}}\right) \\ &-(1-3\lambda(p))a(\varphi) \ , \end{aligned} \qquad (4.8)$$

the associated "linear momentum" of the φ particle is

$$\pi \equiv \frac{\partial L}{\partial\varphi'} = \frac{1}{\sqrt{3}}\ln\left(\frac{1+\varphi'/\sqrt{3}}{1-\varphi'/\sqrt{3}}\right) \ , \qquad (4.9)$$

and

$$\frac{d\pi}{dp} = m(\varphi')\varphi'' \equiv \frac{2}{3-\varphi'^2}\varphi'' \ , \qquad (4.10)$$

so that Eq. (4.7) can be rewritten as

$$\frac{d}{dp}\frac{\partial L}{\partial\varphi'} - \frac{\partial L}{\partial\varphi} = -(1-\lambda)\varphi' \ . \qquad (4.11)$$

And the energy function canonically associated with the Lagrangian (4.8) reproduces the definition (4.5) above,

$$E = \varphi'\frac{\partial L}{\partial\varphi'} - L \ , \qquad (4.12)$$

from which it is straightforward to again derive Eq. (4.6). One should also note that the expansion of the kinetic terms of the Lagrangian (4.8) contain only even powers of the velocity,

$$\begin{aligned} K^{\rm kinetic}(\varphi') = 2\Bigg[&\frac{1}{1\times 2}\left(\frac{\varphi'}{\sqrt{3}}\right)^2 + \frac{1}{3\times 4}\left(\frac{\varphi'}{\sqrt{3}}\right)^4 \\ &+\frac{1}{5\times 6}\left(\frac{\varphi'}{\sqrt{3}}\right)^6 + \cdots\Bigg] \ . \end{aligned} \qquad (4.13)$$

However the replacement $\varphi'^2 \to \boldsymbol{\varphi}'^2$ in Eq. (4.13) does not define a useful Lagrangian in the multiscalar case.

C. Scalar-field evolution during the radiation-dominated era

A "radiation-dominated era" means a period of cosmological expansion during which the equation of state of matter is well approximated by $\tilde{p} \simeq \tilde{\rho}/3$, i.e., $\lambda \simeq 1/3$ in Eq. (4.1). In first approximation this suppresses the force term in Eq. (4.1) and leaves

$$\frac{2}{3-\boldsymbol{\varphi}'^2}(\boldsymbol{\varphi}'')_{\rm cov} + \frac{2}{3}\boldsymbol{\varphi}' = 0 \ . \qquad (4.14)$$

It is clear from Eq. (4.14) that in the general case of a curved σ manifold, the trajectory of $\boldsymbol{\varphi}$ will be a (segment of a) geodesic of the metric (2.14). The motion along this geodesic will be damped. To show that the damping is strong enough to confine $\boldsymbol{\varphi}$ to move only a finite amount as $p\to\infty$, it is sufficient to consider the one-scalar case in which the exact solution of Eq. (4.14) is

$$\varphi(p) = \varphi_\infty - \sqrt{3}\ln[Ke^{-p} + (1+K^2e^{-2p})^{1/2}] \ , \qquad (4.15)$$

the constant K being the following function of the initial velocity $[\varphi_0' = \varphi'(p=0)]$:

$$K = \frac{\varphi_0'/\sqrt{3}}{\sqrt{1-\varphi_0'^2/3}} \ . \qquad (4.16)$$

The total displacement of the particle between $p=0$ and $p=+\infty$, under the influence of inertia and friction, is therefore the function of the initial velocity

$$\Delta\varphi \equiv \varphi_\infty - \varphi_0 = \frac{1}{2}\sqrt{3}\ \ln\frac{1+\varphi_0'/\sqrt{3}}{1-\varphi_0'/\sqrt{3}} \ . \qquad (4.17)$$

[The total shift (4.17) is also easily obtained by integrating Eq. (4.11) between $p=0$ and $p=+\infty$.] In the nonrelativistic limit $\varphi_0' \ll \sqrt{3}$ one has simply

$$\Delta\varphi \approx \varphi_0' \ , \qquad (4.18)$$

as is easily obtained from the nonrelativistic limit of Eq. (4.14), namely, $\varphi'' + \varphi' = 0$. In other words, except in a case where, on coming out say of inflation, the particle φ enters the radiation era with an ultrarelativistic velocity [large momentum π, Eq. (4.9)] it will exponentially come to rest and move forward only by an amount of order unity.

The previous behavior results by setting equal to zero the coefficient $1-3\lambda = 1-3\tilde{p}/\tilde{\rho}$ which multiplies the potential force. This coefficient, however, though small, is never quite zero. Each time the universe, as it cools down during its expansion, passes through the threshold $k\widetilde{T}_i \sim \tilde{m}_i c^2$ for the participation in the total relativistic gas of a particular species of particles and antiparticles of mass $\tilde{m}_i$, the quantity $1-3\tilde{p}/\tilde{\rho}$ rises up to a value of order $(\tilde{m}_i/k\widetilde{T})^2$ and generates a kick on the $\boldsymbol{\varphi}$ particle, which (because of the ever present damping) causes $\boldsymbol{\varphi}$ to move by a finite amount in the direction of $-\boldsymbol{\alpha}$. Integrating Eq. (4.1) over p through such a threshold kick one obtains, for the total displacement of $\boldsymbol{\varphi}$,

$$\Delta^{i\text{th kick}}\boldsymbol{\varphi} = -\frac{3}{2}\int dp\,\frac{\tilde{\rho}_i - 3\tilde{p}_i}{\tilde{\rho}^{\rm tot}}\,\boldsymbol{\alpha}(\varphi) \ . \qquad (4.19)$$

The total energy density $\tilde{\rho}^{\rm tot}$ is approximately (in units where $\hbar = c = k = 1$)

$$\tilde{\rho}^{\rm tot} = g_*(\widetilde{T})\,\frac{\pi^2}{30}\,\widetilde{T}^4 \ ,$$

where

$$g_*(\widetilde{T}) = \sum_{\rm Bose} g_i^B (T_i/\widetilde{T})^4 + \left(\frac{7}{8}\right)\sum_{\rm Fermi} g_i^F (T_i/\widetilde{T})^4$$

is the effective number of relativistic degrees of freedom, while the contribution of the species of fermions of mass $\tilde{m}_i$ to $\tilde{\rho} - 3\tilde{p}$ is

$$\tilde{\rho}_i(\widetilde{T}) - 3\tilde{p}_i(\widetilde{T}) = \frac{g_i \tilde{m}_i^2}{2\pi^2} \int_0^\infty \frac{dq\, q^2}{\widetilde{E}_i[1 + \exp(\widetilde{E}_i/\widetilde{T})]} ,$$

with $\widetilde{E}_i = \sqrt{q^2 + \tilde{m}_i^2}$. The integration over p in Eq. (4.19) can be analytically performed if $\boldsymbol{\alpha}(\varphi)$ is approximated by a constant and the integration over p is replaced by an integration over $\tilde{\beta} = 1/\widetilde{T}$ via the relation $dp = d \ln R_* \approx d \ln \widetilde{R} = d\tilde{\beta}/\tilde{\beta}$ [using the near constancy of $A(\boldsymbol{\varphi})$ during the radiation era]. These integrals yield

$$\Delta^{i\text{th kick}} \boldsymbol{\varphi} \approx -k_i \langle \boldsymbol{\alpha} \rangle^{\text{kick}} , \tag{4.20a}$$

$$k_i = \frac{1}{2} \frac{7 g_i/8}{g_*(\tilde{m}_i^+)} , \tag{4.20b}$$

where $\langle \boldsymbol{\alpha} \rangle^{\text{kick}}$ is some average value of $\boldsymbol{\alpha}$ during the kick, and where $g_*(\tilde{m}_i^+)$ is the effective number of relativistic degrees of freedom just before the ith threshold (i.e., for $\widetilde{T} > \tilde{m}_i$), which includes the contribution $7g_i/8$ from the i-type particle.

The result (4.20a) and (4.20b) indicates that after its initial velocity when entering the radiation era is damped out, the φ particle climbs down the potential $a(\boldsymbol{\varphi})$ by steps each time the temperature of the universe crosses a threshold $k\widetilde{T} = \tilde{m}_i c^2$. In the standard big-bang scenario (with three light neutrinos) one has $g_* = 10.75$ before the freeze-out of the e^+e^- pairs ($g_e = 4$; $\widetilde{T}_e \sim 5 \times 10^9$ K) and $g_* = 14.25$ before that of the $\mu^+\mu^-$ pairs ($g_\mu = 4$; $\widetilde{T}_\mu \sim 10^{12}$ K) so that the corresponding "kick" coefficients in Eq. (4.20a) are

$$k_e = 0.163, \quad k_\mu = 0.123 . \tag{4.21}$$

Above the temperature corresponding to the $\mu^+\mu^-$ threshold one enters the less understood hadron era. If one formally estimates the integrated effect of all the thresholds between a temperature above a few hundred GeV (where g_* is of order 100) down to the last e^+e^- threshold by summing the k_i's of Eq. (4.20b) over all the fundamental fermionic g_i's one gets what is probably an upper bound on the total kick coefficient of $\sum_i k_i \sim \frac{1}{2} \ln(100/10) = 1.15$.

In summary, after the inertial displacement (4.17), (4.18) upon entering the radiation era the $\boldsymbol{\varphi}$ particle probably climbs down the potential $a(\boldsymbol{\varphi})$ [Eq. (4.20a)] by only a modest total amount $0.286 < \Sigma k_i \lesssim 1$. If upon entering the radiation era the slope $|\boldsymbol{\alpha}|$ of the potential function was of order unity (i.e., order 1 differences from general relativity; as is suggested by inflationary scenarios) it will generically be still so upon exiting the radiation era [excluding very rapidly varying or fine-tuned shapes of $a(\boldsymbol{\varphi})$]. One must therefore examine the matter era to find whether tensor-scalar theories naturally tend to a general-relativisticlike state.

For analytical simplicity we approximate, in the text, the transition between radiation domination ($\lambda \approx 1/3$) and matter domination ($\lambda \ll 1$) as being a sharp one. The effect of the gradual transition between the two eras has been however duly taken into account in the numerical calculations that we quote below, with the consequence of augmenting the efficiency of the relaxation toward general relativity.

D. Scalar-field evolution during the matter-dominated era

In the matter-dominated era ($\lambda = \tilde{p}/\tilde{\rho} \ll 1$) the evolution equation (4.1) reads

$$m(\boldsymbol{\varphi}')(\boldsymbol{\varphi}'')_{\text{cov}} + \boldsymbol{\varphi}' = -\boldsymbol{\alpha}(\boldsymbol{\varphi}) , \tag{4.22}$$

with a velocity-dependent mass

$$m(\boldsymbol{\varphi}') = 2/(3 - \boldsymbol{\varphi}'^2) . \tag{4.23}$$

The analysis of the previous subsection shows that the φ particle has damped away any preradiation-era velocity, so that it starts the matter era with an essentially zero initial velocity. The scalar-field evolution during the matter era is then an analogue to the dynamics of a released particle, of mass (4.23), subjected to the external potential $a(\boldsymbol{\varphi})$, and a velocity damping force with coefficient equal to one. The resulting evolution of $\boldsymbol{\varphi}$ is qualitatively clear. The $\boldsymbol{\varphi}$ particle initially tends to fall along the steepest slope of $a(\boldsymbol{\varphi})$. If the magnitude of the slope $\alpha \equiv |\boldsymbol{\alpha}|$ is of order unity it initially accelerates so as to acquire (on a p time scale of order 1) a relativistic velocity $|\boldsymbol{\varphi}'| \lesssim \sqrt{3}$. In the case where the slope is everywhere greater than $\sqrt{3}$ along its trajectory, the particle will tend to go straight (in the sense of the geodesics of the σ manifold) with a velocity constantly near the limiting speed $\sqrt{3}$ [inertia-dominated Eq. (4.22), with $m(\boldsymbol{\varphi}')$ large and $(\boldsymbol{\varphi}'')_{\text{cov}}$ small]. The picture is entirely different if the slope is (almost) everywhere smaller than $\sqrt{3}$. In this case, all three terms in Eq. (4.22) will generically soon become comparable, and with the influence of the damping the velocity tends to remain nonrelativistic, allowing the further simplification of Eq. (4.22) by replacing the mass by its nonrelativistic limit $m_0 = 2/3$. The long-term behavior of the particle then depends dominantly on the shape of the potential $a(\boldsymbol{\varphi})$. In the multidimensional case, the particle might undergo a complicated evolution, starting with oscillations up and down the ridges around gentle valleys, continuing (after the friction has damped the transverse oscillations) with a slower fall down a valley, down to a point where the particle gets trapped in a hollow. The generic long-term behavior emerges: the particle ends up near a local minimum of $a(\boldsymbol{\varphi})$ (or a minimum at infinity if the potential flattens out as $|\boldsymbol{\varphi}| \to \infty$). To make this more quantitative, assumptions are needed about the global shape of $a(\boldsymbol{\varphi})$. We consider the simplest cases: (a) a local minimum of parabolic shape, and (b) a power-law approach to a minimum at infinity in field space. We also consider the case of a single scalar field. This is not a big restriction because in the case, e.g., of the parabolic minimum one can separate the n-dimensional solution of Eq. (4.22) in n, nearly independent, normal mode oscillations (they become independent in the late stages where the velocities and displacements are small so that one can neglect the velocity dependence of the mass, and the Christoffel symbols hidden in the "cov" suffix).

Consider a one-dimensional parabolic potential

$$a(\varphi) = \tfrac{1}{2}\,\kappa\,\varphi^2 \,, \tag{4.24}$$

where we have normalized $a(\varphi)$ so that its minimum value is zero (i.e., $A_{\min} = 1$). The parameter κ entering Eq. (4.24) is the curvature (2.8b) calculated at the bottom of the potential. [In the parabolic approximation $\kappa(\varphi) = \kappa$ independently of φ, therefore the parameter κ of Eq. (4.24) is the same one that enters the post-Newtonian parameter β, Eq. (2.16c).] Neglecting the velocity dependence of the mass so that $m \approx 2/3$ in Eq. (4.22) (numerical calculations show that this is a very good approximation) we write down the solution for the motion of a φ particle released without velocity at an initial position φ_R [where the label R stands for (end of) radiation era]:

$$\varphi(p) = a^+ e^{\lambda^+ p} + a^- e^{\lambda^- p} \,, \tag{4.25}$$

where

$$a^\pm = \frac{1}{2}\left(1 \pm \frac{1}{r}\right)\varphi_R \,, \tag{4.26a}$$

$$\lambda^\pm = \tfrac{3}{4}\,(-1 \pm r) \,, \tag{4.26b}$$

$$r \equiv \left(1 - \tfrac{8}{3}\kappa\right)^{1/2} \,, \tag{4.26c}$$

when $\kappa \neq 3/8$, and

$$\varphi(p) = \varphi_R\left(1 + \tfrac{3}{4}p\right) e^{-\frac{3}{4}p} \,, \tag{4.27}$$

for $\kappa = 3/8$. [Here and in the following we choose the arbitrary constant in the definition (3.9a) of the evolution parameter p so that $p = 0$ corresponds to the time of transition between radiation and matter domination, i.e., $\varphi(0) = \varphi_R$, $\varphi'(0) = 0$.]

For $0 < \kappa < 3/8$, the square root r in Eq. (4.28) is real, the motion is monotonic, and is a linear combination of decreasing exponentials with positive coefficients (the traditional overdamped case). For large p time the least-decreasing exponential (λ^+) dominates and

$$\varphi(p) \approx \frac{1}{2}\left(1 + \frac{1}{r}\right)\varphi_R \exp\left[-\frac{3}{4}(1-r)p\right] \,. \tag{4.28}$$

In the limit where κ is appreciably smaller than 3/8 the result (4.28) simplifies to

$$\varphi(p) \approx \varphi_R\, e^{-\kappa p} \,. \tag{4.29}$$

Equation (4.29), being valid for small values of the curvature κ of the potential, admits a generalization to very general shapes of the potential $a(\varphi)$. Indeed, if the curvature $\kappa(\varphi) = \partial^2 a/\partial\varphi^2$ is much smaller than one, the general solution of Eq. (4.22) becomes *damping dominated* on a p time scale of order unity, leaving a motion approximately described by the first-order (Aristotelian) equation of motion

$$\boldsymbol{\varphi}' = -\boldsymbol{\alpha}(\boldsymbol{\varphi}) \qquad \text{(friction dominated)} \,. \tag{4.30}$$

[The consistency condition that the solution of Eq. (4.30) is a good approximation to Eq. (4.22) is that $\kappa_{ab} = D_a D_b a(\varphi) \ll 1$.]

For the cases where $\kappa > 3/8$, the square root (4.26c) is imaginary and the motion (4.25) is a damped oscillatory one. Denoting

$$\omega \equiv \tfrac{3}{4}\left(\tfrac{8}{3}\kappa - 1\right)^{1/2} \,, \tag{4.31a}$$

$$\theta_R \equiv \arctan\left[\left(\tfrac{8}{3}\kappa - 1\right)^{1/2}\right] \,, \tag{4.31b}$$

the solution (4.25) is

$$\varphi(p) = \varphi_R\left(1 - \frac{3}{8\kappa}\right)^{-1/2} e^{-\frac{3}{4}p}\sin(\omega p + \theta_R) \,. \tag{4.32}$$

When $\kappa \to 3/8$, Eq. (4.32) tends to the critically damped solution (4.27).

It is worth commenting on the fact that the existence of damped-oscillatory approaches to a point where α vanishes, while evident when using the variables $[g^*_{\mu\nu}, \varphi, A(\varphi)]$ can be missed when working with the variables $[\tilde{g}_{\mu\nu}, \Phi, \omega(\Phi)]$ (as actually happened in Ref. [7] which considered the case (translated in $g^* - \varphi$ language) $A(\varphi) = \cosh\sqrt{\kappa}\varphi$ with $\kappa > 0$). Indeed, what is a regular local minimum in the former language takes the form of a mathematical singularity $\omega = \infty$ in the latter one. As said in Sec. II above, the latter singularity must be thought of as being just a "coordinate singularity" in field space.

Now consider the case where the potential $a(\varphi)$ flattens out in a power-law fashion to a minimum at $\varphi = +\infty$:

$$a(\varphi) = a_R(\varphi/\varphi_R)^{-n} \,. \tag{4.33}$$

[Again we normalized to zero the minimum value of $a(\varphi)$.] Note that the initial ($\varphi = \varphi_R$) slope of the potential is

$$\alpha_R = -n\, a_R/\varphi_R \,. \tag{4.34}$$

Since the curvature $\kappa(\varphi)$ tends to zero as $\varphi^{-(n+2)}$ we can here approximately solve for the motion along the power-law slope (4.33) by using the friction-dominated approximation (4.30). This yields

$$\varphi(p) \approx \varphi_R\left[1 + \frac{n+2}{n}\frac{\alpha_R^2}{a_R}\,p\right]^{\frac{1}{n+2}} \,. \tag{4.35}$$

In summary, depending on the shape of the potential, we find three main types of approach to a minimum of $a(\varphi)$: (i) a damped-oscillatory motion around and toward a generic local minimum with curvature $\kappa > 3/8$; (ii) an overdamped, monotic approach to a weakly curved local minimum ($\kappa < 3/8$); and (iii) some power-law approach to a generic minimum at infinity.

The observable consequences of these various scenarios are discussed in the next section.

V. OBSERVABLE CONSEQUENCES OF TENSOR-SCALAR COSMOLOGIES

A. Inflation, the present value of $\tilde{\Omega}$, and nucleosynthesis

In Sec. III we presented the evolution equations for the universe scale factor in the Einstein frame because of its mathematical convenience. However, the physical quantities which are directly accessible from nongravitational measurements (using, e.g., standard rods, comparisons of electromagnetic wavelengths, clocks based on atomic transitions or nuclear decay) are all obtained in the Jordan-Fierz frame. This is as well the case for the (direct or indirect) measurements of the present expansion rate of the Universe $\tilde{H} = (d\tilde{R}/d\tilde{t})/\tilde{R}$, of the total matter density $\tilde{\rho}$, and of the age of the Universe $\tilde{t}$. Rewriting Eq. (3.6b) in terms of Jordan-Fierz quantities and the solution $\varphi(p)$ of the decoupled scalar evolution equation (3.15), we get

$$\frac{1-\varphi'^2/3}{(1+\alpha\cdot\varphi')^2}\tilde{H}^2 = \frac{8\pi\tilde{G}\tilde{\rho}}{3(1+\alpha^2)} - \frac{k}{\tilde{R}^2} . \quad (5.1)$$

Here we used a simplified notation for the scalar product of the σ-manifold vectors α and φ' as defined by Eq. (2.14), e.g., $\varphi'^2 \equiv \varphi'\cdot\varphi' \equiv \langle\varphi',\varphi'\rangle$, and $\tilde{G}$ denotes the value of Newton's coupling strength at the corresponding cosmological epoch, i.e.,

$$\tilde{G}(\varphi) = G_* A^2(\varphi)[1+\alpha^2(\varphi)] . \quad (5.2)$$

We define the dimensionless measure of the cosmological matter density $\tilde{\Omega}$ in the usual way:

$$\tilde{\Omega} \equiv \frac{8\pi\tilde{G}\tilde{\rho}}{3\tilde{H}^2} . \quad (5.3)$$

Equation (5.1) then becomes

$$\tilde{\Omega} = \frac{(1+\alpha^2)(1-\varphi'^2/3)}{(1+\alpha\cdot\varphi')^2} + \frac{(1+\alpha^2)k}{\tilde{H}^2\tilde{R}^2} . \quad (5.4)$$

In the case where $k = 0$ (as generically predicted by inflation) the usual consequence $\Omega^{\rm GR} = 1$ gets, as is well known from previous discussions in the Jordan-Fierz frame, modified by scalar-field contributions. Using the experimental limits on the present magnitude of α, Eq. (2.18a), and neglecting the corresponding terms in Eq. (5.4) we get at the present cosmological epoch

$$\tilde{\Omega}_0^{(k=0)} = \left[\frac{(1+\alpha^2)(1-\varphi'^2/3)}{(1+\alpha\cdot\varphi')^2}\right]_0 \approx 1 - \frac{\varphi_0'^2}{3} . \quad (5.5)$$

A scalar contribution to gravity could *a priori* help to reconcile inflationary scenarios with the observational data which tend to favor $\tilde{\Omega} < 1$ (even when assuming the presence of substantial nonluminous matter in galaxies and clusters of galaxies; see, e.g., Ref. [29]). The maximum possible present value of φ'^2 is constrained, however, by the energy evolution equation (4.6). Applying the latter equation during the matter era ($\lambda = 0$), we find that the total energy $E = -\ln(1-\varphi'^2/3) + a(\varphi)$ must have decreased between the radiation era and now, yielding the strict inequality

$$1 - \frac{\varphi_0'^2}{3} > \exp(a_0 - a_R) = \frac{A_0}{A_R} , \quad (5.6)$$

where the subscript R denotes the value of a quantity at the end of the radiation era. The right-hand side of the inequality (5.6) can be connected to the "speed-up factor" during nucleosynthesis, which is defined as $\xi_{\rm nucleo} \equiv \tilde{H}/\tilde{H}^{(\rm standard)}$, where $\tilde{H}$ is the actual value of the cosmological rate of expansion during nucleosynthesis, and $\tilde{H}^{(\rm standard)}$ is the value predicted by the standard big-bang model [i.e., general relativity with 3 light neutrino families; $\tilde{H}^2_{(\rm standard)} = (8\pi/3)\tilde{G}_{\rm Newton}\tilde{\rho}_{\rm rad}$ with $\tilde{\rho}_{\rm rad} = \frac{43}{8}a\tilde{T}^4$]. From Eq. (5.1) (with $\varphi' = 0$ and the neglect of $k/\tilde{R}^2$, as is appropriate to the radiation era) we obtain $\tilde{H}^2 = (8\pi/3)G_* A_R^2\tilde{\rho}_{\rm rad}$, and therefore

$$\left(\frac{\tilde{H}}{\tilde{H}^{\rm standard}}\right)_{\rm nucleosynthesis} \equiv \xi_{\rm nucleo} = \left(\frac{G_* A_R^2}{\tilde{G}_{\rm Newton}}\right)^{1/2} = \frac{1}{(1+\alpha_0^2)^{1/2}}\frac{A_R}{A_0} . \quad (5.7)$$

Like φ itself, $A(\varphi) \simeq A_R$ is essentially constant throughout the radiation era. Finally there results the rigorous inequality

$$1 - \frac{\varphi_0'^2}{3} > \frac{1}{(1+\alpha_0^2)^{1/2}\xi_{\rm nucleo}} , \quad (5.8)$$

which is an upper bound to the present value of the "scalar velocity" $|d\varphi/dp|$ in terms of the speed-up factor during nucleosynthesis. Combining Eqs. (5.5) and (5.8) (neglecting α_0^2 and $\alpha_0\cdot\varphi_0'$) yields

$$\frac{1}{\xi_{\rm nucleo}} \le \tilde{\Omega}_0^{(k=0)} \le 1 . \quad (5.9)$$

Although standard big-bang nucleosynthesis ($\xi_{\rm nucleo} = 1$) seems to be compatible with the observed abundances of light elements, it has been repeatedly emphasized that there are many uncertainties in both the theoretical predictions (because of the possible effects of inhomogeneities created during the quark-hadron transition [30]) and in the comparison with observational data (because of the necessity to extrapolate to zero metallicity [31]). It would be interesting to reexamine critically these uncertainties to assert whether a value of $\xi_{\rm nucleo}$ as large as, say, 5 (which would render $k = 0$ compatible with $\tilde{\Omega} = 0.2$) is or is not definitely excluded.

Tensor-scalar gravity could therefore have an impact on the question of dark matter. However, it must be admitted that a scenario where the present slope $\alpha(\varphi_0)$ is small but the present value of the scalar velocity $|\varphi_0'|$ is close to $\sqrt{3}$ is a rather fine-tuned possibility among generic scalar evolutions. In the following we consider only the more generic evolutions in which the scalar field now approaches a minimum of $a(\varphi)$ with a velocity φ_0' already damped to a small value of order α_0. In that case, the prediction (5.5) is essentially the standard one:

$$|\tilde{\Omega}_0^{(k=0)} - 1| = O(\alpha_0^2) . \quad (5.10)$$

B. Theoretical expectations for the present values of γ, β and $\dot{\tilde{G}}/\tilde{G}$ in the spatially flat, parabolic-attractor case

The analysis of Sec. IV has shown that the cosmological evolution of scalar fields tends to drive them toward a minimum of $a(\boldsymbol{\varphi})$, i.e., toward small values of the slope $\boldsymbol{\alpha}(\varphi) = \nabla_{\boldsymbol{\varphi}} a(\boldsymbol{\varphi})$. First consider the simplest scenario: a single scalar field attracted by a local minimum of $a(\varphi)$, approximated by a parabola with curvature κ. Let α_R be the value of the slope at the end of the radiation era, and p_0 be the amount of elapsed p time since the end of the radiation era. From Sec. IV we have the present values of the slope, $\alpha_0 \equiv \alpha(p_0)$, and of the scalar velocity $\varphi'_0 \equiv (d\varphi/dp)_{p_0}$, for the three cases $\kappa > 3/8$, $= 3/8$, or $< 3/8$.

For $\kappa > 3/8$,

$$\alpha_0 = \alpha_R \left(1 - \frac{3}{8\kappa}\right)^{-1/2} e^{-\frac{3}{4}p_0} \sin\theta_0 \ , \tag{5.11a}$$

$$\varphi'_0 = \kappa^{-1}[\omega \cot\theta_0 - 3/4]\alpha_0 \ , \tag{5.11b}$$

with $\theta_0 \equiv \omega p_0 + \theta_R$ and ω and θ_R as defined in Eqs. (4.31a) and (4.31b) above.

For $\kappa = 3/8$,

$$\alpha_0 = \alpha_R \left(1 + \tfrac{3}{4}p_0\right) e^{-\frac{3}{4}p_0} \ , \tag{5.12a}$$

$$\varphi'_0 = -\tfrac{3}{2}p_0 \left(1 + \tfrac{3}{4}p_0\right)^{-1} \alpha_0 \ , \tag{5.12b}$$

and for $0 < \kappa < 3/8$

$$\alpha_0 = \alpha_R \frac{1+r}{2r} e^{-\frac{3}{4}(1-r)p_0} \ , \tag{5.13a}$$

$$\varphi'_0 = -\frac{3}{4}\frac{1-r}{\kappa}\alpha_0 \ , \tag{5.13b}$$

with $r \equiv (1 - 8\kappa/3)^{1/2}$. In the case where $\kappa \ll 3/8$ the latter relations simplify to

$$\alpha_0 \approx \alpha_R \, e^{-\kappa p_0} \ , \tag{5.14a}$$

$$\varphi'_0 \approx -\alpha_0 \ . \tag{5.14b}$$

These small but nonzero values of α_0 and φ'_0 translate into small but nonzero values for the post-Newtonian parameters $\gamma - 1$ and $\beta - 1$ of gravitational physics, and additionally the present coupling strength of gravity still slowly changes with time. By differentiating Eq. (5.2) with respect to the physical time $\tilde{t}$, and using the link

$$\frac{dp}{d\tilde{t}} = \frac{\tilde{H}}{1 + \boldsymbol{\alpha}\cdot\boldsymbol{\varphi}'} \tag{5.15}$$

from Eq. (3.13), one gets (in the one-scalar case)

$$\frac{d\tilde{G}/d\tilde{t}}{\tilde{H}\tilde{G}} = 2\left(1 + \frac{\kappa}{1+\alpha^2}\right)\frac{\alpha\varphi'}{1+\alpha\varphi'} \ . \tag{5.16}$$

For application of Eq. (5.16) to the present epoch the fractional corrections of order α_0^2 can be neglected, and

$$(\dot{\tilde{G}}/\tilde{H}\tilde{G})_0 \simeq 2(1+\kappa)\alpha_0\varphi'_0 \ . \tag{5.17a}$$

Under the same approximation

$$1 - \gamma \simeq 2\,\alpha_0^2 \ , \tag{5.17b}$$

$$\beta - 1 \simeq \tfrac{1}{2}\,\kappa\alpha_0^2 \ . \tag{5.17c}$$

Note that the values of $(1-\gamma)$ and $(\beta - 1)$ are positive under these conditions, while the sign of $\dot{\tilde{G}}$ depends upon the relative sign of α_0 and φ'_0. In the friction-dominated case (5.13a) and (5.13b) [and (5.14a) and (5.14b)] one has $\dot{\tilde{G}} < 0$, while in the oscillatory case $\dot{\tilde{G}}$ can have either sign depending upon the phase θ_0 of the oscillation. It is also interesting to note that if one takes into account only the existing experimental upper bound on α_0^2, and the theoretical upper bound (5.8) on φ'_0, one gets the following rigorous upper bound on the magnitude of $\dot{\tilde{G}}/\tilde{G}$:

$$(|\dot{\tilde{G}}|/\tilde{H}\tilde{G})_0 < 2\sqrt{3}\,|1+\kappa||\alpha_0|(1 - \xi_{\rm nucleo}^{-1})^{1/2} \ , \tag{5.18a}$$

i.e.,

$$(|\dot{\tilde{G}}|/\tilde{G})_0 < 0.84|1+\kappa|(1-\xi_{\rm nucleo}^{-1})^{1/2}\tilde{h}_{75}\times 10^{-11}\ {\rm yr}^{-1} \ , \tag{5.18b}$$

where $\tilde{h}_{75} = \tilde{H}/75\ {\rm km\,sec^{-1}\,Mpc^{-1}}$. However, as we said above, the upper bounds (5.18a) and (5.18b) will be attained only in some fine-tuned scenarios. In generic cases one expects φ'_0 to be of order of α_0 (except if $\theta_0 \approx 0$ modulo π, in the oscillatory case). Therefore, one expects $(\dot{\tilde{G}}/\tilde{H}\tilde{G})_0$ to be of order $2(1+\kappa)\alpha_0^2 \simeq 4\beta - \gamma - 3$.

To convert Eqs. (5.11a) and (5.11b)–(5.14a) and (5.14b) into more explicit estimates of present-day deviations from general relativity we need (i) an assumption about α_R and (ii) an estimate for the value of p_0.

In line with the working assumption of this paper that gravity is described by a tensor-scalar theory, with a coupling function $a(\varphi)$ involving only dimensionless numbers of order unity, we assume that the (nearly constant) value of α during the radiation era is of order unity:

$$\alpha_R \sim 1 \ . \tag{5.19}$$

If inflation did not occur, the assumption (5.19) is natural for the state coming out of the primordial Planck era and having had the scalar field evolution frozen during the subsequent radiation era. On the other hand, if inflation did occur, it has been argued that the slope upon entering radiation era must be *greater* than some lower-bound $\alpha_R > \alpha_{\rm low}$ if inflation is to terminate adequately [32, 33] (e.g., the conclusion of Ref. [32] that the true upper bound on ω is probably even lower than 18 translates into $\alpha_{\rm low} > 0.16$). If this constraint from inflation is to be met in a non-fine-tuned way, we expect (5.19) to hold. Note however that the scalar field evolution during the inflation era [Eq. (4.1) with $\lambda = -1$] also exhibits a generic tendency to be trapped near a minimum of $a(\varphi)$. This may mean that inflation requires special shapes of $a(\varphi)$ and special initial conditions to terminate with $\alpha > \alpha_{\rm low}$.

The value of p_0 is obtained by combining Eq. (3.13) with the redshift value $Z \equiv \tilde{R}_0/\tilde{R}_R$ at the end of the radiation era (defined as the equivalence between the radiation and matter energy densities). From $\tilde{\rho}_0^{\rm rad} = 1.6813\, a\, \widetilde{T}^4$ (taking into account three light neutrinos) and $\widetilde{T} = 2.735K$ [34] we get $\tilde{\rho}_0^{\rm rad} = 7.918 \times 10^{-34}$ g cm^{-3} and hence

$$Z = \frac{\tilde{\rho}_0^{\rm matter}}{\tilde{\rho}_0^{\rm rad}} \approx 13350 \times \widetilde{\Omega}_{75} \ , \tag{5.20}$$

where we have introduced the following measure of the present matter density:

$$\widetilde{\Omega}_{75} \equiv \frac{8\pi \widetilde{G}_0 \tilde{\rho}_0^{\rm matter}}{3(75\,{\rm km\,sec^{-1}Mpc^{-1}})^2} = \frac{\tilde{\rho}_0^{\rm matter}}{1.0568 \times 10^{-29}\ {\rm g\,cm^{-3}}} . \tag{5.21}$$

[Note that the quantity 75 km sec^{-1}Mpc^{-1} is used only to define a convenient unit for $\tilde{\rho}_0^{\rm matter}$, and does not presume anything about the actual value of $\widetilde{H}_0$]. In terms of these definitions one has

$$p_0 = \ln Z + a_R - a_0 = 9.50 + \ln \widetilde{\Omega}_{75} + a_R - a_0 \ . \tag{5.22}$$

In the parabolic approximation the last term in Eq. (5.22) is approximately $a_R - a_{\rm minimum} = \frac{1}{2}\kappa\varphi_R^2 = \alpha_R^2/2\kappa \approx 0.5$ if $\alpha_R \approx 1$ and $\kappa \approx 1$. Using $\widetilde{\Omega}_0 \equiv \widetilde{\Omega}_{75} \times \tilde{h}_{75}^{-2}$, the theoretical value (5.10) for $\widetilde{\Omega}_0$ in the spatially flat case, and the observational limits $0.67 < \tilde{h}_{75} < 1.33$, we get $p_0 \approx 10$. Using this value of p_0 in Eqs. (5.11a), (5.11b)–(5.14a), (5.14b), and Eq. (5.17b) we obtain, in the spatially flat case, the level at which it is likely to observe deviations from general relativity in solar-system experiments.

For $\kappa > 3/8$ (with $\theta_0 \equiv \omega p_0 + \theta_R$)

$$1 - \gamma \simeq \frac{2\alpha_R^2}{1 - 3/(8\kappa)} \sin^2\theta_0 e^{-\frac{3}{2}p_0} \sim 3 \times 10^{-7} \ , \tag{5.23a}$$

when considering values of κ sufficiently above 3/8, and using Eq. (5.19) and $\sin^2\theta_0 \sim 1/2$.

For $\kappa = 3/8$, using Eq. (5.19),

$$1 - \gamma \simeq 2\alpha_R^2 \left(1 + \tfrac{3}{4}p_0\right)^2 e^{-\frac{3}{2}p_0} \sim 4 \times 10^{-5} \ , \tag{5.23b}$$

while for κ sufficiently below 3/8,

$$1 - \gamma \simeq 2\alpha_R^2 e^{-2\kappa p_0} \gtrsim 4 \times 10^{-5} \ . \tag{5.23c}$$

The dependence of the expected value of $1 - \gamma$ upon the curvature κ is illustrated in Fig. 1 (lower curve). Figure 1 was drawn by using direct numerical integrations of Eq. (3.15), starting from $\alpha_R^2 = 1$ deep into the radiation era, and taking into account the progressive change between radiation domination and matter domination. Note that this yields present values for $1 - \gamma$ which are slightly but systematically smaller than the analytical estimates given in the text. From Eq. (5.17c), the corresponding values of $\beta - 1$ are obtained by multiplying the results (5.23a)–(5.23c) by $(\kappa/4)$ which, under the assumption of theoretical naturalness, would be $\lesssim 1$. The expected size of $\dot{\widetilde{G}}/\widetilde{H}\widetilde{G}$, which is of order $2(1+\kappa)\alpha_0^2 \simeq 4\beta - \gamma - 3$, represents a difficult challenge for solar-system experiments (even if one can get centimeter-level ranging data to the inner planets [35]).

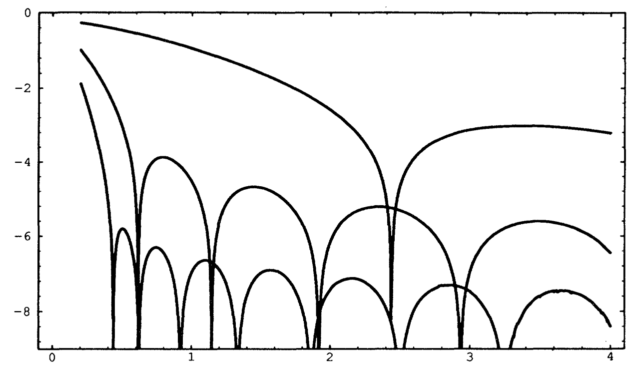

FIG. 1. Present value of $\log_{10}(1 - \gamma)$ as a function of the curvature κ of a parabolic minimum, $a(\varphi) = \kappa\varphi^2/2$. The upper curve corresponds to $\widetilde{\Omega}_{75} = 0.01$, the intermediate one to $\widetilde{\Omega}_{75} = 0.1$ and the lower one to $\widetilde{\Omega}_{75} = 1$. The starting value of α^2 deep into the radiation era is taken to be $\alpha_R^2 = 1$.

Staying within the $k = 0$ framework, but considering scalar field evolutions toward a minimum at infinity in field space, Eq. (4.35), Eqs. (5.11a) and (5.11b) are replaced by

$$\alpha_0 = \alpha_R \left[1 + \frac{n+2}{n}\frac{\alpha_R^2}{a_R} p_0\right]^{-\frac{n+1}{n+2}} \ , \tag{5.24a}$$

$$\varphi_0' = -\alpha_0 \ . \tag{5.24b}$$

With the assumption (5.19) (and $a_R \sim 1$) one estimates a present value of $1 - \gamma$ of order

$$1 - \gamma \sim p_0^{-2\frac{n+1}{n+2}} \sim 10^{-x} \ , \tag{5.25}$$

with an exponent $x = 2(n+1)/(n+2)$ somewhere between 1 and 2 [depending on the value of the exponent $n \geq 0$ in the power-law fall-off (4.33)]. A power-law (or exponential) attractor at infinity is a rather inefficient one. In view of the observed upper bound $|\gamma - 1| < 2 \times 10^{-3}$, a scenario involving a local minimum as an attractor seems more compatible with the idea of leaving the radiation era in a scalar field state which differs significantly from general relativity [Eq. (5.19)].

C. Post-Newtonian measurements as probes of the cosmological density of matter

We now consider the cosmological case of $k < 0$, which is, if one goes beyond the inflationary paradigm, of direct physical interest since most observational data suggests subcritical values for the cosmological matter density. The scalar evolution equation with $k \neq 0$ is given by Eq. (3.15), and contains the new quantity ϵ, given by Eq. (3.14b) or

$$\epsilon = \frac{3k(1+\alpha^2)}{8\pi \widetilde{G}\tilde{\rho}\widetilde{R}^2} \ . \tag{5.26}$$

As we are now very near a general relativistic situation the present value of ϵ can be approximated by

$$\epsilon_0 = 1 - \widetilde{\Omega}_0^{-1} , \qquad (5.27)$$

where $\widetilde{\Omega}$ is defined in the usual way, Eq. (5.3). From Eq. (5.26) (with $\tilde{\rho} \propto \widetilde{R}^{-3}$) $\epsilon(p)$ approximately increases as $\widetilde{R} = \exp[p+a(\varphi(p))]$. It can therefore bring noticeable modifications to the coefficients of the scalar equation only during the last few units of p time before the present cosmological epoch. In the limit where $\epsilon_0 \gg 1$, Eq. (3.15) yields (in the one-scalar case)

$$\varphi'' + 2\varphi' \simeq 0 , \qquad (5.28)$$

near the present epoch. This means that a negative spatial curvature ultimately quenches any oscillatory behavior, and accelerates the rate of approach toward a local minimum.

But, a negative value of k implies a lower elapsed total p time since the end of the radiation era. From Eq. (5.22) one gets [with the usual assumptions (5.19) and $\kappa \sim 1$]

$$p_0 \approx 10 + \ln \widetilde{\Omega}_{75} , \qquad (5.29)$$

where one should recall that $\widetilde{\Omega}_{75}$, Eq. (5.21), is a convenient dimensionless measure of the cosmological matter density (contrary to the standard $\widetilde{\Omega}_0 \equiv \widetilde{\Omega}_{75} \times \tilde{h}_{75}^{-2}$, $\widetilde{\Omega}_{75}$ does not depend on the value of $\widetilde{H}_0$). Numerical computation of the solution of Eq. (3.15) for various values of $\widetilde{\Omega}_{75}$ shows that the diminution of p_0, Eq. (5.29), is the dominant effect in controlling the efficiency of approach toward general relativity. The direct effects of a nonzero ϵ [with the limiting equation (5.28) when ϵ_0 is large] become numerically important only around and below $\widetilde{\Omega}_{75} \sim 0.01$. [For such small values of $\widetilde{\Omega}$ one finds that the approach towards general relativity is too slow to be compatible with the present observational limits on α_0, assuming (5.19).] For larger values of $\widetilde{\Omega}_{75}$ ($0.05 \leq \widetilde{\Omega}_{75} \leq 1$) one finds

$$1 - \gamma \simeq 2\alpha_0^2 \sim 2\alpha_R^2 e^{-\frac{3}{4}\frac{\alpha_R^2}{\kappa}} Z^{-3/2} \qquad (5.30)$$

which yields numerically, under the assumptions $\kappa \sim 1 \sim \alpha_R$,

$$1 - \gamma \sim 2 \times \left(\frac{\widetilde{\Omega}_{75}}{0.1}\right)^{-3/2} \times 10^{-5} , \qquad (5.31a)$$

or equivalently

$$1 - \gamma \sim 2 \times \left(\frac{\tilde{\rho}_0^{\rm matter}}{10^{-30}\ {\rm g\,cm^{-3}}}\right)^{-3/2} \times 10^{-5} . \qquad (5.31b)$$

These rough analytical estimates should be compared with the results of a direct numerical integration shown in Fig. 1.

Much of the present cosmological data is compatible with $\widetilde{\Omega}_{75} \sim 0.1$, i.e., $\tilde{\rho}_0^{\rm matter} \sim 10^{-30}\ {\rm g\,cm^{-3}}$ (and $k = -1$). If this is indeed the case, the level $1-\gamma \sim 10^{-5}$ would be a *lower bound* for the present value of $1 - \gamma$ (excluding the exceptional values of κ for which $1 - \gamma$ vanishes; see Fig. 1). Indeed, Eq. (5.30) was obtained for the case of a significantly curved ($\kappa > 3/8$) local minimum of the scalar coupling function $a(\varphi)$. The results of the previous section [which are applicable to the initial damped approach to the minimum, when $\epsilon(p)$ is still small] show that the other cases are less efficient attractors toward $\gamma = 1$.

VI. CONCLUSIONS

The results obtained in this paper suggest a new interpretation of the present tests of general relativity. The aquisition in 1979 [11] of a very tight bound ($\omega > 500$, $\alpha^2 < 0.001$) on the possible admixture of a scalar component to gravity, later confirmed by other high-precision tests, has been generally interpreted as reducing the likelihood that massless gravitational scalar fields exist. Our finding that tensor-scalar theories of gravity generically contain a natural attractor mechanism tending to drive the world toward a state close to a pure general relativistic one render this conclusion premature. The important point is that this idea is observationally testable. Assuming that during the radiation era (during which the evolution of the scalar fields is nearly frozen) the tensor-scalar theory was order-of-unity away from a quasigeneral-relativistic state, Eq. (5.19), we found that there exists a *lower bound* to the presently observable deviations from general relativity in the post-Newtonian regime. This lower bound is $1 - \gamma \sim 3 \times 10^{-7}$ and corresponds to the case of a spatially flat cosmological model and a parabolic local minimum (with curvature $\kappa > 3/8$) in the coupling function $a(\varphi)$ of the scalar. All other cases (negative curvature cosmology, local minimum with $\kappa \leq 3/8$, or a minimum at infinity in field space) yield higher values for $1 - \gamma$. In particular, a local minimum with $\kappa \leq 3/8$ yields $1 - \gamma \gtrsim 4 \times 10^{-5}$, and a negatively curved universe with $\tilde{\rho}_0^{\rm matter} < 10^{-30}\ {\rm g\,cm^{-3}}$ ($\widetilde{\Omega}_{75} < 0.1$) yields $1-\gamma \geq 2\times 10^{-5}$. Note also that more complicated shapes for $a(\varphi)$, and/or the existence of several scalar fields, are generically expected to delay the approach to a minimum, thereby leading to higher values for $1 - \gamma$. [Indeed, in more complicated cases only a fraction of the redshift (5.20) is effectively used to relax toward a minimum.]

These numerical estimates should hopefully provide new, strong motivations for experiments which push beyond the present empirical upper bound on γ ($|\gamma - 1| < 2 \times 10^{-3}$; 1σ level). In particular, it is to be noted that GPB [36], POINTS [37], TROLL [38], and a proposed Mercury Relativity Satellite [35] all plan to probe the level $|\gamma - 1| \sim 10^{-5}$–$10^{-6}$. In addition to experiments which probe the post-Newtonian parameter $\gamma - 1$, any experiment pushing by 2 orders of magnitude the present upper bounds on $\beta - 1$ ($|\beta - 1| < 2 \times 10^{-3}$; 1σ level) or $\eta \equiv 4\beta - \gamma - 3$ ($|\eta| < 5 \times 10^{-3}$; 1σ level) would also be of great interest. Indeed in many of the attractor scenarios considered here the ratio $(\beta - 1)/(1 - \gamma) \approx \kappa$ is expected to be (positive and) of order unity. (An important exception would be the approach to a minimum at infinity where one would expect relatively large values for $1 - \gamma$, but much smaller ones for $\beta - 1$ because $\kappa \to 0$ at such a minimum.) The scientific implications of a nonzero result for any post-Newtonian parameter at these discussed levels would be enormous: it would signal the existence of a new interaction, and would potentially give us an indirect handle on the average mass density in the universe, Eqs. (5.31a) and (5.31b).

In addition to hopefully providing a new stimulus for improving weak-field tests of relativistic gravity the present results suggest the need to revisit various cosmological issues within a larger framework. As mentioned in Sec. V A it would be important to assess the extent to which present data on the abundances of light elements, when keeping one's mind open to systematic uncertainties in the interpretation of data and in the predictions of inhomogeneous nucleosynthesis, do constrain the value of the coupling function $a_R = a(\varphi_R)$ during the radiation era.

An interesting prediction of many of our scenarios is the damped oscillatory behavior of the effective Newtonian constant $\widetilde{G}$ during the matter era. It would be worthwhile to study the possible consequences of these initially strong oscillations ($\Delta\widetilde{G}/\widetilde{G}$ of order unity over the first few Hubble time scales of the matter era) on the formation of structure in the Universe. Could, for example, such strong oscillations in $\widetilde{G}$ provide, via a parametric amplification effect in the relativistic Jeans equation, a mechanism for accelerating the rate of formation of structures? On the other hand, one should note that, contrary to some recent investigations where the period of cosmological oscillations of $\widetilde{G}$ was fixed in an *ad hoc* way by giving a very small but finite mass to the scalar field [39], in our case that period is predicted to be of order unity when time is measured in Hubble units, $\widetilde{H}^{-1}$. More precisely, from Eq. (4.32) one gets (when $k = 0$ and $\kappa > 3/8$)

$$\widetilde{G}(p) \approx G_* \left[1 + \frac{\kappa+1}{\kappa - 3/8}\alpha_R^2 e^{-\frac{3}{2}p}\sin^2(\omega p + \theta_R)\right] , \quad (6.1)$$

where $\omega = \frac{3}{4}(8\kappa/3 - 1)^{1/2}$. During the final oscillations before the present epoch one has approximately $p \approx \ln\widetilde{R} + \text{const} = -\ln(1+z) + \text{const}$, with z being the usual redshift. Equation (6.1) therefore represents a damped oscillation which is periodic in $\ln(1+z)$ with period

$$P = \frac{\pi}{\omega} = \frac{4\pi}{3}\left(\frac{8\kappa}{3} - 1\right)^{-1/2} . \quad (6.2)$$

Although it is tempting to associate these oscillations with the periodicities in $\ln(1+z)$ that have been claimed to exist in quasar or galaxy observations [40,41] they have probably nothing to do with them, because the periods given by Eq. (6.2) are too large ($P > 0.8$ if $\kappa < 10$), and their amplitude $\sim \alpha_R^2\exp(-3p_0/2) \sim (1-\gamma)$ too small even considering the strong dependence $\propto \widetilde{G}^7$ of stellar luminosities on the value of $\widetilde{G}$.

Note that during the entire history of our Galaxy, i.e., for the last 6 or 7 billion years, Eq. (6.1) indicates that $\widetilde{G}$ had already been driven extremely close to its present value. Contrary to the initial hope of Jordan, generic tensor-scalar theories seem to have negligible impact on geophysics, astronomy or galactic astrophysics.

In this paper we have concentrated on estimating the coupling strength α_0^2 which measures the present weak-field deviation from general relativity. Recently one uncovered the existence of nonperturbative strong-gravitational-field effects in tensor-scalar theories [42] allowing, in certain cases, for order unity deviations in neutron star models in spite of a very small α_0^2. A necessary condition for such nonperturbative effects to take place is that $\kappa = \partial^2 a/\partial\varphi_0^2$ be negative. However, the cosmological attractor scenario investigated here tends to drive the world to a state where κ is positive [*minimum* of $a(\varphi)$]. In such a case, the results of Ref. [42] are, on the contrary, that strong-field effects further quench the level of deviation from general relativity. Let us finally mention that the existence of a long-range scalar field could have important consequences on gravitational waves. It would be interesting to study the impact of tensor-scalar gravity on possible cosmological backgrounds of gravitational waves (particularly on the stochastic scalar gravitational waves generated during inflation) and on the efficiency of the earliest gravitational collapses in the Universe's evolution as scalar gravitational radiation emitters [43].

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