Regge trajectories from the two-body, bound-state Thompson equation using a quark-confining interaction in momentum space

David E. Kahana
Physics Department, Kent State University, Kent, Ohio 44242

Khin Maung Maung
Physics Department, Hampton University, Hampton, Virginia 23668

John W. Norbury
Physics Department, University of Wisconsin, La Crosse, Wisconsin 54601
(Received 22 March 1993)

Solutions of two-body, bound-state equations have recently been developed for quark-antiquark bound-state pairs. These solutions use a confining potential in momentum space as input into three-dimensional reductions of the Bethe-Salpeter equation using special subtraction procedures. Regge trajectories are calculated for the Schrödinger equation which display the well-known unphysical behavior where lower mass trajectories overlap higher mass ones and also display nonlinearity. Both of these features contradict experiment. Regge trajectories obtained from the Thompson equation, which is relativistic in origin, avoid both of these problems.

PACS number(s): 11.10.St, 11.10.Qr, 12.40.Gg

Many mesons are made of equal mass quarks which are light and therefore move at an appreciable fraction of the speed of light. One-body relativistic equations, such as the Dirac or Klein-Gordon equations, or the two-body nonrelativistic Schrödinger equation do not incorporate the full kinematic or dynamic features of all mesons. Thus there has been considerable interest [1–7] recently in solving two-body, relativistic, and semirelativistic, bound-state equations with realistic quark-confining interactions. The equations which are most convenient to solve are three-dimensional reductions of the Bethe-Salpeter equation [8,9].

Recently we presented methods [1–3] for solving these three-dimensional equations in momentum space using a momentum space confining interaction (which is relativistic in origin) which reduces to the linear confining potential discovered in studies of lattice quantum chromodynamics [10]; namely,

$$V(r)=kr\ . \tag{1}$$

The equation that we shall solve in the present work is the Thompson equation. This is a particular three-dimensional reduction of the Bethe-Salpeter equation where one particle is kept on shell, but retardation is neglected. As such the Thompson equation is just the Gross equation [3,8] neglecting retardation. The Thompson equation reads [1,3,8,11]

$$(2E_p-W)\phi(\mathbf{p})=-\int d\mathbf{p}'V(\mathbf{q})\phi(\mathbf{p}') \tag{2}$$

where the total energy of the state is $W=2m+E$ and E is the binding energy and $E_p=\sqrt{p^2+m^2}$. Also $\mathbf{q}\equiv\mathbf{p}'-\mathbf{p}$ and $\phi(\mathbf{p})$ is a Schrödinger-like wave function. The kernel is given by [1,5]

$$V(\mathbf{q})=\frac{k}{2\pi^2}\lim_{\eta\to 0}\frac{\partial^2}{\partial\eta^2}\left[\frac{1}{\mathbf{q}^2+\eta^2}\right] \tag{3}$$

which is just the Fourier transform of (1). Several authors [7,12] solve the Bethe-Salpeter equation or a three-dimensional reduction by keeping η finite. However, keeping η finite will not correspond to an infinitely high linear confining potential, but to a potential barrier with a finite height and will introduce continuum solutions together with the solutions of physical interest. Zhu *et al.* [7] have recently addressed this problem and they also found that the solutions are extremely sensitive to the value of this parameter η [7]. Therefore it is important to employ calculational methods which will allow us to take the $\lim_{\eta\to 0}$ explicitly. In previous work [1,2] we have shown how to take the exact limit in Eq. (3). This has also been discussed by Gross and Milana [3] and Spence and Vary [5]. In taking this correct limit, however, one obtains singularities in momentum space which, after very careful manipulation, we have shown how these can be removed exactly [1,2]. This then enables momentum space equations to be solved [1,2] which give the correct nonrelativistic limit (1) for the confining interaction.

Previous work [1,2] has concentrated primarily on developing the mathematical techniques necessary to solve the singular equations. Some test results were presented for the purpose of verifying that the solution did indeed display the correct physical behavior, agreed with some results of other authors and also coordinate space calculations, and displayed the correct nonrelativistic limits. The main results obtained were a set of equations which had the singularities exactly removed [1].

In the present paper we wish to report some new results concerning the Regge trajectories [13,14] of the quark bound states. A Regge trajectory is simply a plot of the mass (squared) versus the angular momentum of a set of energy levels. As such it displays the global behavior of the energy level spectrum and provides a very stringent test for theoretical models. From the experimental results concerning these Regge trajectories it is well known that the trajectories are linear [13–15].

Both the Schrödinger equation and the Thomson equation (2) are solved with the momentum space techniques described in Refs. [1,2]. (Of course we have checked that the Schrödinger results agree with coordinate space calculations.) Regge trajectories are calculated for both equations using the techniques for exactly removing the momentum space singularities [1,2]. The present paper represents the first ever calculation of Regge trajectories from the Thompson equation for the problem of a quark bound-state pair interacting through the confining potential deduced from lattice gauge theories [10].

The trajectories are plotted in the accompanying figure. It is immediately apparent that the nonrelativistic trajectories display the well-known [13] unphysical behavior in that the lower mass trajectories (quark mass of 0.3 GeV) overlap the higher mass trajectories (quark mass of 1.5 GeV). This overlap means that mesons composed of light quarks are heavier than mesons made of heavy quarks. Another problem with the nonrelativistic trajectories is that they are *curved*, yet it has been known experimentally [13–15] for many years that they are *linear*. It is immediately apparent from the figure that the Thompson equation avoids both of these problems. The trajectories are all linear and they diverge for unequal masses.

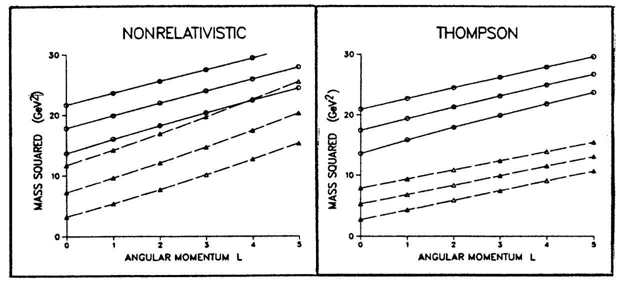

FIG. 1. Regge trajectories for Nonrelativistic and Thompson equations with $k = 0.2$ GeV^2. The solid and dashed lines are for quark masses of 1.5 and 0.3 GeV, respectively.

In summary, we present the first calculations of Regge trajectories for quark bound states from the two-body, relativistic Thompson equation using a confining interaction that agrees with the coordinate space linear confining potential. The Schrödinger calculations predict overlapping and curving trajectories, in contradiction to experiment [15]. The Thompson equation predicts linear and diverging trajectories.

We are very grateful to Professor Franz Gross for providing many useful insights and suggestions over the years. We would also like to thank Professor Warren Buck for his support and the CEBAF theory group for its generous hospitality. K.M.M. was supported by NSF Grant No. HRD-9154080 and J.W.N. was supported in part by NASA Grant No. NAG-1-1457.

[1] K. Maung Maung, D. E. Kahana, and J. W. Norbury, Phys. Rev. D **47**, 1182 (1993).
[2] J. W. Norbury, D. E. Kahana, and K. Maung Maung, Can. J. Phys. **70**, 86 (1992).
[3] F. Gross and J. Milana, Phys. Rev. D **43**, 2401 (1991); **45**, 969 (1992).
[4] P. C. Tiemeijer and J. A. Tjon, Phys. Lett. B **277**, 38 (1992).
[5] J. R. Spence and J. P. Vary, Phys. Rev. D **35**, 2191 (1987).
[6] L. Durand, B. Durand, and J. B. Whitenton, Phys. Rev. D **28**, 607 (1983).
[7] X. Q. Zhu, F. C. Khanna, R. Gourishankar, and R. Teshima, Phys. Rev. D **47**, 1155 (1993).
[8] R. M. Woloshyn and A. D. Jackson, Nucl. Phys. **B64**, 269 (1973).
[9] H. A. Bethe and E. E. Salpeter, Phys. Rev. **82**, 309 (1951); **84**, 1232 (1951).
[10] J. D. Stack, Phys. Rev. D **27**, 412 (1983); H. Q. Ding, *ibid.* **44**, 2200 (1991); A. Hasenfratz and P. Hasenfratz, Annu. Rev. Nucl. Part. Sci. **35**, 559 (1985).
[11] R. H. Thompson, Phys. Rev. D **1**, 110 (1970).
[12] A. B. Henriques, B. H. Kellett, and R. G. Moorehouse, Phys. Lett. **64B**, 85 (1976).
[13] W. Lucha, F. Schoberl, and D. Gromes, Phys. Rep. **200**, 127 (1991).
[14] W. Lucha and F. F. Schoberl, Z. Phys. C **53**, 83 (1992); W. Lucha, H. Rupprecht, and F. F. Schoberl, Phys. Rev. D **46**, 1088 (1992); W. Lucha, H. Rupprecht, and F. F. Schoberl, Phys. Lett. B **261**, 504 (1991); A. Martin, Z. Phys. C **32**, 359 (1986); S. Godfrey, Phys. Rev. D **31**, 2375 (1985).
[15] Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992).