PHYSICAL REVIEW D VOLUME 48, NUMBER 7 1 OCTOBER 1993

Chiral theory calculation of $\eta \rightarrow \pi^+ \pi^- e^+ e^-$

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(Received 14 April 1993)

We analyze the decay $\eta \rightarrow \pi^+\pi^- e^+ e^-$ using a recently developed chiral model which incorporates vector mesons. We find a partial width of 0.38 eV, and also obtain distributions in terms of the $\pi^+\pi^-$ and e^+e^- invariant masses. We anticipate the opportunity to test our results against future experimental data from the η factories.

PACS number(s): 13.20.Jf, 12.40.Vv, 13.40.Hq, 14.40.Aq

In a recent paper [1] we analyzed the decay $\eta \rightarrow \pi^+\pi^-\gamma$ using a model which incorporates vector mesons in the chiral Lagrangian. There has been much work over the years on effective chiral Lagrangians that include the Wess-Zumino non-Abelian anomaly. In particular, there have been two fundamental approaches in incorporating vector mesons into these theories. One, by Schecter and co-workers [2], is based on treating the spin-one mesons as massive Yang-Mills bosons. In the other [3], the vector mesons are identified as the dynamical gauge bosons of a hidden local symmetry in the nonlinear chiral Lagrangian. Aside from the numerical value for the parameters, the general structures of both approaches have been demonstrated to be equivalent, and radiative processes such as this one can be described in a unique way with either model. In Ref. [1] we used the second model to calculate the rate for the decay $\eta \rightarrow \pi^+\pi^-\gamma$. The resulting amplitude yields a decay width and photon energy spectrum which are in excellent agreement with the data obtained by a high-statistics experimental study [4], in contrast with earlier unsuccessful attempts based on vector-meson dominance.

In this Brief Report we extend the calculation to the decay $\eta \rightarrow \pi^+\pi^- e^+ e^-$. This process presents a more stringent test of the theory, since it involves photons off shell. It is thus a greater challenge to the chiral models and, in particular, provides the opportunity to extend the study of the role played by the vector mesons. On the experimental side, the only data that exist consist of a single event [5]. However, with the development of high-intensity beams, such as the tagged-η beam at Saturne [6], it should be possible to carry out a high-statistics experimental analysis. Since a match between chiral models and experimental results still eludes us in some areas (for example [7], in the decay $\eta \rightarrow \pi\gamma\gamma$), we hope that this calculation will generate interest in a future experimental measurement.

We begin by writing down the amplitude for $\eta \rightarrow \pi^+\pi^-\gamma$. The three graphs that contribute to this process are shown in Fig. 1 of Ref. [1]. From Eq. (21) we obtain the amplitude

$$A(\eta \rightarrow \pi^+\pi^-\gamma) = M \epsilon^{\mu\nu\alpha\beta} p^+_\mu p^-_\nu k_\alpha \epsilon_\beta \times \left[\frac{1}{2} - \frac{3}{2} \frac{m_\rho^2}{m_\rho^2 - (p^+ + p^-)^2} \right], \tag{1}$$

$$M = \frac{ie}{4\pi^2 f_\pi^2} \left[\frac{1}{\sqrt{3} f_8} \cos\theta - \sqrt{2/3} \frac{1}{f_0} \sin\theta \right].$$

Here $f_\pi = 93$ MeV, and we used the mixing prescription

$$|\eta\rangle = \cos\theta |\eta^8\rangle - \sin\theta |\eta^0\rangle,$$
$$|\eta'\rangle = \sin\theta |\eta^8\rangle + \cos\theta |n^0\rangle, \tag{2}$$

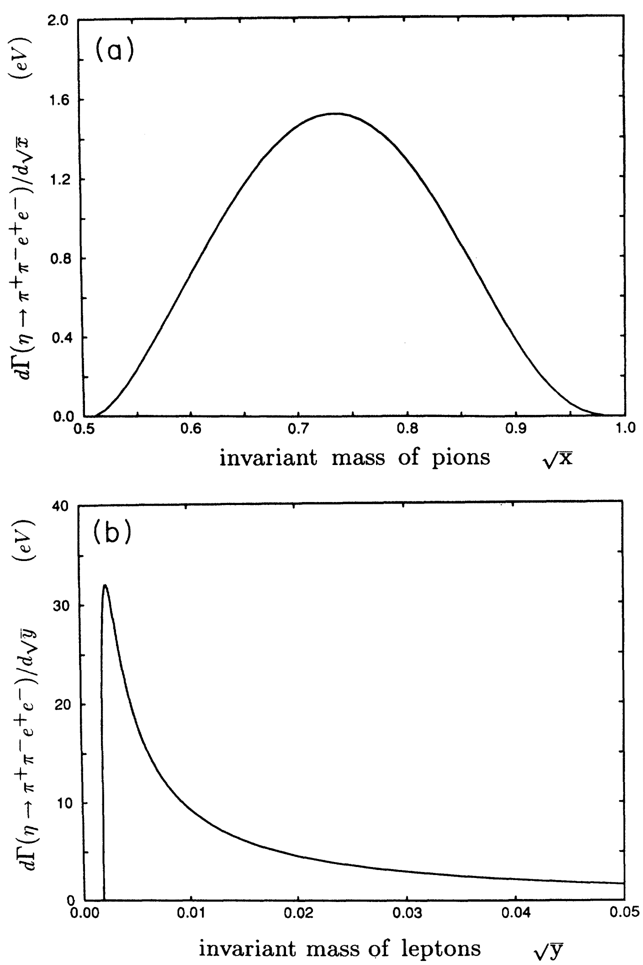

FIG. 1. Differential spectra for $\eta \rightarrow \pi^+\pi^- e^+ e^-$. The pion and lepton normalized invariant masses are defined by $x = (p^+ + p^-)^2/m_\eta^2$ and $y = (k^+ + k^-)^2/m_\eta^2$.

with $\theta=-20°$ being the most recent value for the mixing angle [8]. The pion momenta are $p^{\pm}$ and ϵ_{β} is the photon polarization. The physical values $f_8=1.25f_{\pi}$ and $f_0=1.04f_{\pi}$, which parametrize SU(3) breaking, were taken from Donoghue, Holstein, and Lin [9]. The amplitude of Eq. (1) can now be modified to describe the process $\eta\rightarrow\pi^{+}\pi^{-}e^{+}e^{-}$. We take the photon off shell and replace ϵ^{μ} by $(e/k^2)\bar{u}(k^{-})\gamma^{\mu}v(k^{+})$, where $k^{\pm}$ are the electron momenta. Although it would now be possible to have transitions into final states with $(\pi^{+}\pi^{-})_{J=0}$, which are forbidden by angular momentum conservation for $k^2=0$, these channels would not contribute significantly and can be ignored. With this modification, the amplitude becomes

$$A(\eta\rightarrow\pi^{+}\pi^{-}e^{+}e^{-}) = M\epsilon^{\mu\nu\alpha\beta}p_{u}^{+}p_{\nu}^{-}k_{\alpha}\left[\frac{e}{k^2}\bar{u}(k^{-})\gamma_{\beta}v(k^{+})\right] \times\left[\frac{1}{2}-\frac{3}{2}\frac{m_{\rho}^2}{m_{\rho}^2-p^2}\frac{m_{\rho}^2}{m_{\rho}^2-k^2}\right], \tag{3}$$

where now $k=k^{+}+k^{-}$ and $p=p^{+}+p^{-}$.

Using this amplitude we have calculated the differential decay rate of $\eta\rightarrow\pi^{+}\pi^{-}e^{+}e^{-}$ in terms of the variables $x=(p^{+}+p^{-})^2/m_{\eta}^2$, the normalized mass of the pions, and $y=(k^{+}+k^{-})^2/m_{\eta}^2$, the normalized mass of electrons. The notation is similar to that used by Sehgal and Wanninger [10] in an analysis of $K_L\rightarrow\pi^{+}\pi^{-}e^{+}e^{-}$. After considerable manipulation we obtained the result

$$\frac{d^2\Gamma}{dx\,dy}=\frac{e^2M^2m_{\eta}^7}{18(4\pi)^5}\left[\frac{1}{2}-\frac{3}{2}\frac{m_{\rho}^4}{(m_{\rho}^2-xm_{\eta}^2)(m_{\rho}^2-ym_{\eta}^2)}\right]^2 \times\frac{\lambda^{3/2}(1,x,y)\lambda^{1/2}(y,\nu^2,\nu^2)\lambda^{3/2}(x,\mu^2,\mu^2)}{x^2y^2} \times\left[\frac{1}{4}+\frac{\nu^2}{2y}\right], \tag{4}$$

where $\lambda(x,y,z)=x^2+y^2+z^2-2xy-2xz-2yz$, $\mu=m_{\pi}/m_{\eta}$, and $\nu=m_e/m_{\eta}$. In contrast to Ref. [10] we did not neglect the electron mass, which yields a small but not insignificant contribution.

The individual distributions can be computed by integrating Eq. (4) with respect to x or y, and the results are shown in Fig. 1. The decay width obtained for $\eta\rightarrow\pi^{+}\pi^{-}e^{+}e^{-}$ is

$$\Gamma(\eta\rightarrow\pi^{+}\pi^{-}e^{+}e^{-})=0.38\text{ eV}. \tag{5}$$

This partial width corresponds to a branching ratio $(0.32\pm0.03)\times10^{-3}$. The only data that exist [5] consist of a single event. Based on this event, the experimental branching ratio is $1.3^{+1.3}_{-0.8}\times10^{-3}$. Clearly, a substantial increase in statistics is required to establish a reliable experimental rate and the shape of the decay spectrum.

Decays of this type provide very useful information in our understanding of low-energy QCD, and additional experimental effort would be extremely helpful. We would welcome a precision measurement in future experiments at the η factories.

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

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