$\pi^0 
ightarrow e^+e^- ext{ and } \eta 
ightarrow \mu^+\mu^- ext{ decays reexamined}$ 

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The rare  $\pi^0 \to e^+e^-$  and  $\eta \to \mu^+\mu^-$  decays are calculated in different schemes, which are seen to be essentially equivalent to and produce the same results as conventional vector-meson dominance. We obtain the theoretical predictions  $B(\pi^0 \rightarrow e^+e^-) = (6.41 \pm 0.19) \times 10^{-8}$  and  $B(\eta \rightarrow \mu^+ \mu^-) = (1.14^{+0.07}_{-0.03}) \times 10^{-5}$ , where  $B(P \rightarrow l^+ l^-) = \Gamma(P \rightarrow l^+ l^-) / \Gamma(P \rightarrow \gamma \gamma)$ , in reasonable agreement with recent experimental data.

PACS number(s): 13.20.Cz, 13.20.Jf

New experimental data on the rare  $\eta \rightarrow \mu^+ \mu^-$  and  $\pi^0 \rightarrow e^+ e^-$  decays have been obtained very recently by different groups, and more accurate results are expected in the near future. On the one hand, the  $\eta$  tagging facility at Saturne reports [1] a branching ratio  $B(\eta \rightarrow \mu^+ \mu^-) =$  $\Gamma(\eta \to \mu^+ \mu^-) / \Gamma(\eta \to \text{all}) = (5.6^{+0.6}_{-0.7} \pm 0.5) \times 10^{-6}$ , to be compared to the old result [2]  $B(\eta \to \mu^+ \mu^-) = (6.5 \pm$  $2.1) \times 10^{-6}$ . Normalizing the new Saturne's measurement to  $\eta \to \gamma \gamma$  one gets

$$B(\eta \to \mu^+ \mu^-) \equiv \frac{\Gamma(\eta \to \mu^+ \mu^-)}{\Gamma(\eta \to \gamma \gamma)} = (1.4 \pm 0.2) \times 10^{-5} ,$$
(1)

where the branching ratio [3]  $B(\eta \rightarrow \gamma \gamma) = 0.389 \pm$ 0.005 has been used. On the other hand, the  $\pi^0 \rightarrow$  $e^+e^-$  branching ratio has been recently measured at Brookhaven [4] and at Fermilab [5] with the results  $(6.9 \pm 2.4) \times 10^{-8}$  and  $(7.8 \pm 3.1) \times 10^{-8}$ , respectively. Averaging these data and using [3]  $B(\pi^0 \to \gamma\gamma) = 0.988$ one similarly has

$$B(\pi^{0} \to e^{+}e^{-}) \equiv \frac{\Gamma(\pi^{0} \to e^{+}e^{-})}{\Gamma(\pi^{0} \to \gamma\gamma)} = (7.3 \pm 1.9) \times 10^{-8} .$$
(2)

The "reduced" ratios (1,2) can be expressed in terms of a dimensionless "reduced" amplitude  $R(P \rightarrow l^+ l^-) \equiv$ R, normalized to the intermediate  $P \rightarrow \gamma \gamma$  amplitude, leading to

$$B(P \to l^+ l^-) = 2\beta \left(\frac{\alpha}{\pi} \frac{m_l}{m_P}\right)^2 |R(P \to l^+ l^-)|^2, \quad (3)$$

where  $\beta = \sqrt{1 - 4m_l^2/m_P^2}$ . The on-shell  $\gamma\gamma$  intermediate state generates the model-independent imaginary part of R:

$$\operatorname{Im} R(P \to l^+ l^-) = \frac{\pi}{2\beta} \ln \frac{1-\beta}{1+\beta} .$$
(4)

The unitary bound on  $B, B \ge B^{\text{unit}}$ , is then obtained by setting  $\operatorname{Re} R = 0$  in (3). It takes the values

$$B^{\text{unit}}(\eta \to \mu^+ \mu^-) = 1.11 \times 10^{-5}, \tag{5}$$

$$B^{\text{unit}}(\pi^0 \to e^+ e^-) = 4.75 \times 10^{-8}$$
 (6)

0556-2821/93/48(7)/3388(4)/\$06.00

In units of  $B^{\text{unit}}$ , the Saturne result (1) and the average (2) are

$$B(\eta \to \mu^+ \mu^-)/B^{\text{unit}} = 1.3 \pm 0.2,$$
 (7)

$$B(\pi^0 \to e^+ e^-)/B^{\text{unit}} = 1.54 \pm 0.40$$
 . (8)

The values on the right-hand side (RHS) of Eqs. (7) and (8), which correspond to  $1 + (\text{Re}R/\text{Im}R)^2$ , can be used to extract  $\operatorname{Re} R$  from experiment:

$$\operatorname{Re} R(\eta \to \mu^+ \mu^-) = \pm \left(3.0^{+0.9}_{-1.2}\right),\tag{9}$$

$$\operatorname{Re} R(\pi^0 \to e^+ e^-) = \pm \left(12.9^{+4.0}_{-6.5}\right) \ .$$
 (10)

We will see below that we are able to choose a sign for  $\operatorname{Re}R$  from theoretical considerations. While the imaginary part of R is finite, model independent, and dominant, the real part of R contains an *a priori* divergent  $\gamma\gamma$  loop, depends on the hadronic physics governing the  $P \rightarrow \gamma^* \gamma^*$  transition (with off-shell photons), and, according to Eqs. (7) and (8), amounts only to a fraction of Im R.

Calculations of  $\operatorname{Re} R$  have been performed by many authors in, essentially, two different contexts. One consists in using vector-meson dominance (VMD) ideas [6], thus introducing the corresponding VMD form factor to regularize the photon-photon loop. Hadronic couplings cancel precisely in the "reduced" amplitude R, which turns out to depend essentially only on the vector-meson mass  $M_V$  in the form factor. Alternatively, one can rely on (constituent) quark model ideas [7] to regularize the  $P \rightarrow \gamma^* \gamma^*$  vertex thus obtaining a finite and reasonable value for ReR. A recent paper by Margolis *et al.* [8] confirms the validity (as well as some degree of model independency) of this approach. In both contexts, one obtains rather stable results which are in reasonable agreement with the above data. The accuracy and reliability of these methods can obviously be improved when used to compute differences of two  $\operatorname{Re} R$ 's rather than  $\operatorname{Re} R$ 's themselves, as shown by the authors [9] a decade ago. The recent and partly related paper by Savage, Luke, and Wise [10], as well as the publication of new experimental

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results, have prompted us to reconsider the situation.

Assuming the dominance of the two photon contribution, the reduced amplitude  $R(q^2) = R(P \rightarrow l^+l^-)$  can be written as (see Ref. [11], where details can be found)

$$R(q^{2}) = \frac{2i}{\pi^{2}q^{2}} \int d^{4}k \frac{q^{2}k^{2} - (q \cdot k)^{2}}{k^{2}(q - k)^{2}[(p - k)^{2} - m_{l}^{2}]} F(k^{2}, k'^{2}),$$
(11)

where  $q^2 = m_P^2$ ,  $p^2 = m_l^2$ , and k' = q - k, and F is a generic and model-dependent form factor, with F(0,0) = 1 for on-shell photons. The simplest and more transparent way to fix F is by invoking conventional vector-meson dominance (VMD) ideas. This essentially implies neglecting direct  $P\gamma\gamma$  and (single V)  $VP\gamma$  vertices, thus assuming the full dominance of the (double V) chain  $P \to VV \to \gamma\gamma$ . The form factor in this case is

$$F = F_{VV} = \frac{M_V^2}{M_V^2 - k^2} \frac{M_V^2}{M_V^2 - k'^2}.$$
 (12)

Let us present the theoretical predictions for the real part of the amplitude in this naive and conventional VMD model. Taking  $M_V = M_{\rho,\omega} = 0.77 \pm 0.10$  GeV (this error will be justified later), one gets

$$\operatorname{Re} R_{\rho,\omega}(\pi^0 \to e^+ e^-) = 10.4 \pm 0.6.$$
(13)

Quite independently, one also obtains  $\text{Im}\,R(\pi^0 \to e^+e^-) = -17.5$  and then the ratio

$$B(\pi^0 \to e^+e^-) = (1.35 \pm 0.04) B^{\text{unit}} = (6.41 \pm 0.19) \times 10^{-8},$$
(14)

in good agreement with the recent data (2) (but in sharp contrast with the old data [12] available when similar VMD estimates were commonly performed). The  $\eta \rightarrow \mu^+\mu^-$  decay can be similarly analyzed. In the SU(3)-symmetric limit, i.e., using  $M_V = M_{\rho,\omega}$  in (12) and ignoring  $M_{\phi} > M_{\rho,\omega}$ , we get

$$\operatorname{Re} R_{\rho,\omega}(\eta \to \mu^+ \mu^-) = -1.3^{+0.7}_{-0.5} , \qquad (15)$$

whereas  $\text{Im} R(\eta \to \mu^+ \mu^-) = -5.47$ . The predicted ratio is then

$$B(\eta \to \mu^+ \mu^-) = (1.06^{+0.06}_{-0.05}) B^{\text{unit}} = (1.18^{+0.08}_{-0.06}) \times 10^{-5}$$
, (16)

in reasonable agreement with the corresponding experimental value (1). To calculate  $B(\eta \to \mu^+\mu^-)$  in the more realistic case of SU(3) breaking we use the  $\eta$ - $\eta'$  mixing angle  $\theta_P = -19.5^{\circ}$  [13], corresponding to an  $\eta$  quark content  $\eta = (u\bar{u} + d\bar{d} - s\bar{s})/\sqrt{3}$ , and introduce  $M_{\phi} > M_{\rho,\omega}$ . This easily leads to

$$\operatorname{Re} R(\eta \to \mu^+ \mu^-) = \frac{5}{4} \operatorname{Re} R_{\rho,\omega}(\eta \to \mu^+ \mu^-) -\frac{1}{4} \operatorname{Re} R_{\phi}(\eta \to \mu^+ \mu^-) = -1.0^{+0.9}_{-0.6}, B(\eta \to \mu^+ \mu^-) = (1.03^{+0.06}_{-0.03}) B^{\text{unit}} = (1.14^{+0.07}_{-0.03}) \times 10^{-5},$$
(17)

marginally consistent with the data. For completeness, we also quote the corresponding results for the  $\eta \rightarrow e^+e^-$  decay amplitude: Re  $R(\eta \rightarrow e^+e^-) = 31.3 \pm 2.0$ , Im  $R(\eta \rightarrow e^+e^-) = -21.9$ , and  $B(\eta \rightarrow e^+e^-) = (3.04 \pm 0.26)B^{\text{unit}} = (1.37 \pm 0.12) \times 10^{-8}$ .

Two comments about our results are in order. First, the reasonable agreement we get allows us to solve the sign ambiguity when extracting the real part of the amplitude from experiment in (9,10): We have to choose the positive value for  $\operatorname{Re} R(\pi^0 \to e^+e^-)$  and the negative value for  $\operatorname{Re} R(\eta \to \mu^+\mu^-)$ . The discarded values are 4 and 3 experimental standard deviations away from the theoretical results. Second, as was noticed by the authors in Ref. [9], some of the uncertainties related to hadronic scales or cutoffs disappear when considering differences of two ReR, as  $\operatorname{Re}_{\pi ee} - \operatorname{Re}_{\eta \mu \mu}$ . From (13) and (17) we get the numerical result

$$\operatorname{Re}R_{\pi ee}(m_{\pi}^2) - \operatorname{Re}R_{\eta\mu\mu}(m_{\eta}^2) = +11.4 \pm 0.4, \qquad (18)$$

where the smallness of the error comes from a large cancellation of the uncertainties in  $M_V$  taking place because of the difference on the LHS.<sup>1</sup> Equation (18) is fully compatible with the experimental value

$$\operatorname{Re}R_{\pi ee}(m_{\pi}^{2}) - \operatorname{Re}R_{\eta\mu\mu}(m_{\eta}^{2}) = +16^{+5}_{-7},$$
(19)

which is deduced from (9) and (10) when solving the sign ambiguities according to our analysis, and adding the errors linearly.

We are well aware that the above VMD calculations could be (and essentially were) performed many years ago. One may argue that VMD is certainly a successful phenomenological scheme but old fashioned and lacking of solid theoretical support. For this reason we shall now consider more modern approaches, where the interactions among pseudoscalars, vector mesons, and photons are dictated by well-defined and QCD-rooted Lagrangians. In these contexts, the octet containing the lightest pseudoscalar mesons plays the role of the set of Goldstone bosons originated through the spontaneous symmetry breaking of the QCD Lagrangian for vanishing u, d, and s quark masses. We shall discuss three types of models: The first refers to improved and updated versions of the nonlinear  $\sigma$  model, such as chiral perturbation theory (ChPT) [14], and the other two refer to more recent attempts to include vector mesons in these chiral Lagrangians, particularly, the "massive Yang-Mills approach" [15] and the "hidden symmetry scheme" [16]. We shall argue that in the context of these three models the VMD form factor in (12) and the corresponding predictions (with the quoted theoretical errors) in Eqs. (13)-(17) are fully justified.

Let us start discussing the "massive Yang-Mills approach" proposed mainly by Meissner and extensively discussed in [15]. Much as in the VMD case, vec-

<sup>&</sup>lt;sup>1</sup>To further appreciate this effect, we rewrite our older result [9] (correcting a misprint)  $\operatorname{Re} R_{\pi e e}(m_{\pi}^2) - \operatorname{Re} R_{\eta \mu \mu}(m_{\eta}^2) \simeq 3 \ln(\Lambda_{\eta}/\Lambda_{\pi}) - 3 \ln(m_{\mu}/m_e) + \ln(m_e m_{\mu}/m_{\pi} m_{\eta}) \ln(m_e m_{\eta}/m_{\mu} m_{\pi}) - r \ln(m_{\mu}/\Lambda_{\eta}) \ln(1 - m_{\eta}^2/\Lambda_{\eta}^2) + \cdots$ , where  $\Lambda_{\eta}, \Lambda_{\pi}$  are cutoffs needed to regularize the integrals and the form factor model dependence is effectively parametrized by the term proportional to r. (The dots refer to negligible contributions.) Invoking SU(3) symmetry  $\Lambda_{\eta} = \Lambda_{\pi}$  and allowing  $0 \leq r \leq 1$  we obtained [9]  $\operatorname{Re} R_{\pi e e}(m_{\pi}^2) - \operatorname{Re} R_{\eta \mu \mu}(m_{\eta}^2) \simeq +12 \pm 2$ , compatible with (18) but with larger uncertainties.

tor mesons are introduced as a nonet of gauge bosons through conventional covariant derivatives in ungauged, chiral Lagrangians with a Wess-Zumino (WZ) term. Axial vectors can be introduced with the same procedure, but (appropriate) mass terms for both types of spin-1 mesons have to be incorporated unsatisfactorily by hand. When gauging the WZ term, several possibilities are open concerning the relative weights of axial-vector vs vector mesons. The most attractive one is due to Bardeen and concentrates all the effects of the anomaly in the axialvector sector. This is also the choice favored by Meissner in his extensive review [15], where it is also shown the total equivalence of this Bardeen version of the WZ term with conventional VMD for the case in hand. Accordingly, in the most favored version of the "massive Yang-Mills" approach the  $\pi^0$  and  $\eta$  couplings to  $\gamma^* \gamma^*$ are considered to proceed through the  $F_{VV}$  form factor (12) and therefore to reproduce precisely our previous, VMD predictions (13)–(17).

The alternative but related "hidden symmetry scheme" by Bando et al. [16] looks more interesting for our present discussion. Vector mesons are introduced as "dynamical" gauge bosons of the hidden local  $U(3)_V$  symmetry in the  $U(3)_L \times U(3)_R/U(3)_V$  nonlinear  $\sigma$  model. The corresponding vector-meson masses  $M_V$ are now automatically generated inside the model, while quantum or QCD effects are expected to generate "dynamically" their kinetic terms. Photons and weak bosons can be finally incorporated as external gauge fields. One is then lead to a well defined theory describing the strong interactions of pseudoscalar and vector mesons at low energy, as well as their electroweak ones. In the WZ sector the scheme contains three free parameters, two of which  $(a_2 \text{ and } a_3 \text{ in the notation of Refs. [17] and [18]}$ where more details can be found) are relevant for our purposes. Their role is to fix the relative weight of the direct  $P \rightarrow \gamma \gamma$  amplitude to the (singly or doubly) V mediated ones,  $P \to V\gamma \to \gamma\gamma$  and  $P \to VV \to \gamma\gamma$ . According to the general analysis by Bando et al. [16], the preferred values for these parameters are [17]

$$a_2 = 2a_3 = -3/16\pi^2 , \qquad (20)$$

which reproduce complete VMD, i.e., they lead to a cancellation of the  $P\gamma\gamma$  and  $PV\gamma$  vertices containing direct couplings of photon(s) to hadrons.

In our specific context we are forced to fix  $a_3$  to the above value (20) by simply requiring that the present approach has to lead to a finite result, i.e., by assuming that vector mesons alone are enough to render convergent the otherwise divergent two-photon loop. The most appropriate way to fix the remaining parameter  $a_2$  consists in adjusting the recent data coming from  $\gamma\gamma^* \rightarrow P$  production and involving one (essentially) real photon and a virtual one. The  $k^{*2}$  dependence of the latter requires a VMD-like form factor with averaged (see Ref. [19]) mass parameters  $\Lambda = 0.75 \pm 0.03, 0.77 \pm 0.04$ , and  $0.81 \pm 0.04$ GeV for  $\pi^0$ ,  $\eta$ , and  $\eta'$  production, respectively. These values are immediately interpreted in Bando's context just fixing  $a_2 + 2a_3 = -3/8\pi^2$ , thus reproducing the complete VMD result (20), and identifying the mass parameter  $\Lambda$  with the vector masses  $M_V$ . The numerical coincidence between these masses shows that we can safely use the physical, Particle Data Group (PDG) values [3] for the  $\rho$ ,  $\omega$ , and  $\phi$  masses (the latter being responsible for the slight increase in  $\Lambda$  when going from  $\pi^0$  to  $\eta$  and  $\eta'$ ), affected by errors smaller than some 10%. Alternatively, we can interpret the above  $\gamma \gamma^* \to P$  results as requiring the use of the physical, PDG vector masses but allowing for slight variations of the  $a_2$  parameter (again, of some 10%) around its VMD central value (20). In this case, our  $P \rightarrow l^+ l^-$  amplitude proceeds mainly through the  $F_{VV}$  form factor (12) but it is then allowed to have small contaminations of a similar (single) VMD form factor,  $F_V = M_V^2/(M_V^2 - k^2)$ . The latter has been discussed by several authors in Refs. [6] and [11] showing that it leads to just slightly smaller values of  $\operatorname{Re} R$ . In any one of these two alternative interpretations we obtain for  $\operatorname{Re} R$  our central values [Eqs. (13), (15), and (17)], affected with errors which are roughly one-half of the quoted ones. The present analysis can be confirmed invoking the complete set of data on radiative vector meson decays,  $V \rightarrow P\gamma$  [the most clean and accurate being  $\Gamma(\omega \rightarrow \pi^0 \gamma) = 720 \pm 50$  keV [3]], as well as the (less conclusive) data coming from  $\pi^0, \eta \to \gamma l^+ l^$ decays. Somewhat conservatively, however, we have enlarged our input error bars on  $M_V$  for two main reasons. One is due to a single (unconfirmed) measurement of the form factor in  $\omega \to \pi^0 \mu^+ \mu^-$  leading to a mass parameter  $\Lambda = 0.65 \pm 0.03$  GeV, well below the expected  $M_{\rho}$ . The second reason refers to recent theoretical analyses suggesting values for  $a_2$  and  $a_3$  somewhat different from the VMD ones (20) as required by the last mentioned value of  $\Lambda$  or as preferred by the attractive and simplifying "minimal coupling" principle of Pallante and Petronzio [20]. Accordingly, we have adopted  $M_{\rho} = 0.77 \pm 0.10$ GeV thus generating the error estimates quoted in our main results (13)–(17).

Let us finally turn to consider our previous results from the point of view of chiral perturbation theory (ChPT). As is well known, ChPT is a successful effective theory accounting for strong and electroweak interactions of pseudoscalar mesons at low energy. It is a nonrenormalizable theory containing an infinite set of counterterms needed to cancel the divergencies appearing when computing loop corrections. Very recently, Savage, Luke, and Wise [10] have discussed the  $P \rightarrow l^+ l^-$  decays using preliminary data from Saturne to fix the required local counterterms and then predicting the  $\pi^0, \eta \to e^+ e^$ branching ratios. An alternative way to proceed consists in assuming that the relevant, finite part of the ChPT counterterms are saturated (dominated) by the contributions of meson resonances. This resonance saturation hypothesis was already suggested in the original papers by Gasser and Leutwyler [14], was further discussed in [21] and has been fully confirmed by several authors. Vector mesons usually play the central role, thus realizing VMD in a modern context which turns out to be particularly successful in the anomalous sector of the ChPT Lagrangian. As shown in Refs. [17] and [18], vector-meson contributions are fully dominant in this sector, well above other ChPT corrections such as the finite part of the chiral loops. In this sense, our previous VMD

results on  $P \rightarrow l^+ l^-$  decays can also be considered as rather safe calculations in the context of ChPT with resonance saturation. To further illustrate this point we have computed the "reduced" amplitude R in terms of the local counterterms proposed in Ref. [10] within the same renormalization scheme, obtaining

$$\operatorname{Re} R(q^{2} = m_{P}^{2}) = -\frac{\chi_{1}(\Lambda) + \chi_{2}(\Lambda)}{4} - \frac{5}{2} + 3\ln\frac{m_{l}}{\Lambda} + \frac{1}{4\beta}\ln^{2}\frac{1-\beta}{1+\beta} + \frac{\pi^{2}}{12\beta} - \frac{1}{\beta}\operatorname{Li}_{2}\left(\frac{\beta-1}{\beta+1}\right), \qquad (21)$$

where  $\Lambda$  is the subtraction point. We have checked that this result agrees with the amplitude A in [10] (which is related to our R by  $A = -\alpha R/\pi^2$ ) with a minor modification: The term +11 in Eq. (2.8) of Ref. [10] is now found to be +7. This preserves all the relevant results in [10] except that the new values of the Saturne experiment should require a counterterm (for our  $\Lambda = M_{\rho} = 0.77$  GeV) given by

$$\chi_1(M_{\rho}) + \chi_2(M_{\rho}) = \begin{cases} -7^{+4}_{-5}, \\ -31^{+5}_{-4}. \end{cases}$$
(22)

In turn, the less precise  $\pi^0 \to e^+ e^-$  available experimental data translate into

$$\chi_1(M_{\rho}) + \chi_2(M_{\rho}) = \begin{cases} -22^{+25}_{-16}, \\ +81^{+16}_{-25}. \end{cases}$$
(23)

The first (second) value in (22) and (23) corresponds to fixing the sign ambiguities for Re*R* according (contrary) to our amplitudes. Notice that the first values are consistent with the existence of a unique counterterm, while the second ones are not and, consequently, have to be discarded. This further confirms that our am-

- [1] R. S. Kessler et al., Phys. Rev. Lett. 70, 892 (1993).
- [2] R.I. Dzhelyadin et al., Phys. Lett. 87B, 471 (1980).
- [3] Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D 45, S1 (1992).
- [4] A. Deshpande (private communication); A. Deshpande et al., Phys. Rev. Lett. 71, 27 (1993).
- [5] Y. Wah (private communication); and (unpublished).
- [6] S.D. Drell, Nuovo Cimento 11, 471 (1959); S.M. Berman and D.A. Geffen, *ibid.* 18, 1192 (1960); B.L. Young, Phys. Rev. 161, 1620 (1967); C. Quigg and J.D. Jackson, UCRL Report No. 18487, 1968 (unpublished); A. Ivanov and V. M. Shekhter, Yad. Fiz. 32, 796 (1980) [Sov. J. Nucl. Phys. 32, 410 (1980)]; L. Bergström, Z. Phys. C 14, 129 (1982); L. Bergström *et al.*, Phys. Lett. 126B 117 (1983); L. Bergström and E. Ma, Phys. Rev. D 29, 1029 (1984); G.B. Tupper and M.A. Samuel, *ibid.* 26, 3302 (1982); 29, 1031 (1984).
- M. Pratap and J. Smith, Phys. Rev. D 5, 2020 (1972);
   Ll. Ametller et al., Nucl. Phys. B228, 301 (1983); A. Pich and J. Bernabéu, Z. Phys. C 22, 197 (1984).
- [8] B. Margolis et al., Phys. Rev. D 47, 1942 (1993).
- [9] Ll. Ametller, A. Bramon, and E. Massó, Phys. Rev. D 30, 251 (1984).
- [10] M.J. Savage, M. Luke, and M.B. Wise, Phys. Lett. B 291, 48 (1992).
- [11] Bergström et al. [6].

plitudes represent a good description for the  $P \rightarrow l^+ l^$ processes. Thanks to the resonance saturation hypothesis, we can go one step further and predict the value of the finite part of these counterterms. This amounts to choose  $M_{
ho,\omega} = 0.77$  GeV as both the subtraction point  $\Lambda$  and the mass  $M_V$  appearing in our  $F_{VV}$  form factor (12). Our previous results for  $\eta \rightarrow \mu^+\mu^-$  (15) and  $\pi^0 \to e^+e^-$  (13) can now be presented as leading to  $\chi_1(M_{\rho}) + \chi_2(M_{\rho}) \simeq -14$  and -12, respectively, close to the experimental values displayed in the first row of (22) and (23). Notice that we obtain slightly different cutoff values for the two processes. This is related to the fact that our VMD saturation hypothesis does not strictly lead to a constant counterterm, as explicitly required in (10), but to a function smoothly depending on  $M_V$ ,  $m_P$ , and  $m_l$ .

In conclusion, we have performed a careful calculation of the  $\pi^0 \rightarrow e^+e^-$  and  $\eta \rightarrow \mu^+\mu^-$  decay rates in conventional vector-meson dominance. We have shown that the calculation is equivalent to those coming from favored versions of more modern approaches such as the "massive Yang-Mills approach" and "hidden symmetry schemes." Similarly, we have predicted the appropriate value for the finite part of the corresponding ChPT counterterms under the resonance saturation hypothesis. Special care has been taken when estimating the theoretical errors, particularly in the rather precise prediction (18) for the difference between the real part of the two decay amplitudes. The other two relevant results of our calculation, Eqs. (14) and (17), are in reasonable agreement with recent data.

Discussions with and comments from M. Garçon, R.S. Kessler, B. Mayer, A. Deshpande, and Y. Wah (from the Saturne Collaboration, BNL, and Fermilab) are warmly acknowledged.

- [12] R.E. Mischke et al., Phys. Rev. Lett. 48, 1153 (1982);
   J. Fishcher et al., Phys. Lett. 73B, 364 (1978); 76B, 663(E) (1978).
- [13] F. J. Gilman and R. Kaufman, Phys. Rev. D 36, 2761 (1987); A. Bramon, Phys. Lett. 51B, 87 (1974).
- [14] J. Gasser and H. Leutwyler, Nucl. Phys. B250, 465 (1985); B250, 517 (1985).
- [15] U-G. Meissner, Phys. Rep. C 161, 213 (1988); see also Ö.
   Kaymakcalan and J. Schechter, Phys. Rev. D 31, 1109 (1985).
- [16] M. Bando *et al.*, Phys. Rep. C **164**, 217 (1988); Phys. Lett. B **297**, 151 (1992).
- [17] J. Bijnens, A. Bramon, and F. Cornet, Z. Phys. C 46, 599 (1990); Phys. Lett. B 237, 488 (1990).
- [18] A. Bramon, A. Grau, and G. Pancheri, Phys. Lett.
   B 277, 353 (1992); A. Bramon, E. Pallante, and R. Petronzio, Phys. Lett. B 271, 237 (1991).
- [19] CELLO Collaboration, H. J. Behrend et al., Z. Phys. C 49, 401 (1991).
- [20] E. Pallante and R. Petronzio, Phys. Lett. B 292, 143 (1992); Rome Preprint No. ROM2F 92/37 (unpublished); and Nucl. Phys. (to be published).
- [21] G. Ecker et al., Nucl. Phys. B321, 311 (1989); G. Ecker et al., Phys. Lett B 223, 425 (1989); J.F. Donoghue, C. Ramirez, and G. Valencia, Phys. Rev. D 39, 1947 (1989).