Seesaw enhancement of lepton mixing

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The seesaw mechanism of neutrino mass generation may enhance lepton mixing up to maximal even if the Dirac mass matrices of leptons have a structure similar to that in the quark sector. Two sets of conditions for such an enhancement are found. The first one includes the seesaw generation of heavy Majorana masses for right-handed neutrinos and a universality of Yukawa couplings which can follow from the unification of neutrinos with new superheavy neutral leptons. The second set is related to the lepton number symmetry of the Yukawa interactions in the Dirac basis of neutrinos. Models which realize these conditions have a strong hierarchy or strong degeneration of Majorana masses of the right-handed neutrinos.

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I. INTRODUCTION

The seesaw mechanism of neutrino mass generation naturally relates the smallness of the neutrino masses with the neutrality of neutrinos [1]. According to the seesaw mechanism the following mass terms are introduced in the fermion family basis:

$$\mathcal{L}_m = -\bar{\nu}_L m_D \nu_R - \nu_R^T C^{-1} \tfrac{1}{2} M \nu_R + \text{H.c.} \quad (1)$$

Here m_D is the Dirac mass matrix (Dirac sector) and M is the majorana mass matrix for right-handed neutrino components (Majorana sector). At $M \gg m_D$ the terms (1) generate the Majorana mass matrix for left (active) components ($\approx \nu_L$) [1]:

$$m^{\text{Maj}} = m_D M^{-1} m_D^T . \quad (2)$$

An attractive feature of this mechanism is that the Dirac matrix m_D can be similar (in scale and structure) to that in the quark sector which is naturally implied by quark-lepton symmetry and grand unification. The difference in scales of neutrino masses, and probably in lepton mixing, follows from the structure of expression (2) and from the Majorana mass matrix which has no analogy in the quark sector.

Mixing of leptons in the seesaw mechanism has been widely discussed previously [2–14] (see [2] for review). For m_D(leptons) $\sim m_D$(quarks) the lepton mixing turns out to be typically of the same order of magnitude as quark mixing [3–13]; i.e., lepton mixing is relatively small at least between the first two generations. This was illustrated, in particular, by Monte Carlo studies of different configurations of the Dirac and Majorana mass matrices [8], although some textures of matrices result in large mixing [4,6,8].

At the same time it is difficult to expect that lepton mixing coincides precisely with quark mixing. Mixing is related to masses but the masses of the charged leptons and down quarks from the same fermion generations are different. Neutrino masses, if they exist, are much smaller than the masses of up quarks. The results of the gallium experiments [15] with solar neutrinos show that Cabibbo mixing cannot explain the solar neutrino problem.

Moreover, there are several hints that lepton mixing might be much larger than that in the quark sector. The solar neutrino problem [15,16] can be solved by long length vacuum oscillations ("just so") [17,18] or by resonant flavor conversion, the Mikheyev-Smirnov-Wolfenstein effect [19,20]. The former requires the values of neutrino mixing angles θ and masses squared difference Δm^2: $\sin^2 2\theta = 0.85-1.0$, $\Delta m^2 = (0.8-1.1) \times 10^{-10}$ eV^2 [18]. The latter picks up two regions of neutrino parameters, one of which involves large mixing angles: $\sin^2 2\theta = 0.6-0.9$ at $\Delta m^2 = (10^{-7}-10^{-5})$ eV^2 [20]. The deficit of the muon neutrinos in the atmospheric neutrino flux can be explained by ν_μ-ν_e oscillations with parameters: $\sin^2 2\theta = 0.5-0.9$, $\Delta m^2 = (10^{-3}-10^{-2})$ eV^2 [21].

As follows from (2) the masses of light neutrinos in these regions correspond to Majorana masses of the right-handed neutrinos of the order of $(10^{10}-10^{12})$ GeV. This is much smaller than a possible grand unification scale which may testify for a strong hierarchical structure in the Majorana mass sector. Such a hierarchy (as we will show) may result in an enhancement of lepton mixing.

In this paper we assume that the Dirac mass matrices in the lepton sector are similar to those in the quark sector and find conditions at which lepton mixing is enhanced by the seesaw mechanism itself. The paper is organized as follows. In Sec. II, considering the two-neutrino case, we formulate general conditions for lepton-mixing enhancement. Two sets of conditions are found. In Secs. III and IV we consider two scenarios which realize these conditions. Section V contains a discussion and summary of results. Some technical details are explained in the Appendix.

II. SEESAW ANGLE. MECHANISMS OF ENHANCEMENT

We will first explain the mechanism of enhancement for the two-neutrino case and then generalize the results for three-neutrino mixing.

The mixing angle for two generations of leptons, $\theta^{\rm lept}$, can be written in the form

$$\theta^{\rm lept} = \theta_L^D - \theta_L^l + \theta_{\rm ss} \ . \tag{3}$$

The first two terms in the right-hand side (RHS) of this equation are the direct analogies of mixing angles in the quark sector: θ_L^l is the angle of rotation of the left charged lepton components which diagonalizes the mass matrix of charged leptons; θ_L^D is the angle of rotation of the left neutrino components ν_L, which diagonalizes the Dirac mass matrix of neutrinos, m_D (see Appendix). The last term in (3), $\theta_{\rm ss}$, is the additional angle that specifies the effect of the seesaw mechanism itself. If there are no Majorana mass terms ($M=0$), then $\theta_{\rm ss}=0$. We will call $\theta_{\rm ss}$ the seesaw angle.

Let us suggest that the Dirac matrix contribution to $\theta^{\rm lept}$ is of the order of quark mixing angle: $\theta_L^D - \theta_L^l \lesssim \theta_C \approx 13^\circ$ (θ_C is the Cabibbo angle), and that θ_L^D and θ_L^l are separately also small $\approx \sqrt{m_e/m_\mu}$. We will find the conditions at which $\theta_{\rm ss}$ is appreciably larger than θ_L^D and θ_L^l, so that $\theta^{\rm lept} \sim \theta_{\rm ss}$ and the latter may be close to 45°.

The angle $\theta_{\rm ss}$ is determined by the properties of both the Majorana M as well as of the Dirac m_D mass matrices, and it is convenient to write down $\theta_{\rm ss}$ in terms of mixing angles and mass hierarchies that characterize M and m_D. Namely, let θ^M and θ_R^D be the angles of rotation which diagonalize M and m_D correspondingly (see Appendix). Let us define the mass hierarchies of M and m_D as

$$\epsilon^D \equiv \frac{m_1}{m_2} \ , \quad \epsilon^M \equiv \frac{M_1}{M_2} \ , \tag{4}$$

where m_i and M_i ($i=1, 2$) are the eigenvalues of m_D and M, respectively. Then $\theta_{\rm ss}$ is determined from (see Appendix)

$$\tan 2\theta_{\rm ss} = -2\tan(\theta^M - \theta_R^D)\frac{\epsilon^D(1-\epsilon^M)}{\tan^2(\theta^M-\theta_R^D)+\epsilon^M-\delta} \ , \tag{5}$$

where δ is a small term of second order in the Dirac mass hierarchy:

$$\delta \equiv (\epsilon^D)^2[1+\epsilon^M \tan^2(\theta^M - \theta_R^D)] \ . \tag{6}$$

Two remarks are in order. As follows from (5,6), the angle $\theta_{\rm ss}$ is precisely zero at $\theta^M = \theta_R^D$ (i.e., when the rotations of right-handed components that diagonalize M and m_D are the same), or at $\epsilon^M = 1$ (when the masses of Majorana sector are degenerate). The angle $\theta_{\rm ss}$ is proportional to the mass hierarchy in the Dirac sector (rather than $\sqrt{\epsilon^D}$), and since ϵ^D is small one obtains typically small values of $\theta_{\rm ss}$, unless the denominator in the RHS of Eq. (5) is strongly suppressed.

Let us consider the conditions for $\theta_{\rm ss}$ enhancement. According to (5) there are two possibilities: (i) all terms in the denominator of the RHS of the Eq. (5) are very small, (ii) there is a strong cancellation among two first terms.

(i) The first case corresponds to *a strong mass hierarchy in the Majorana sector.* Indeed, according to Eq. (5), one obtains $\tan 2\theta_{\rm ss} \gtrsim 1$, when

$$\tan(\theta_R^D - \theta^M) \lesssim \epsilon^D \ , \tag{7}$$

and

$$\epsilon^M \lesssim (\epsilon^D)^2 \ . \tag{8}$$

Note, if $\epsilon^M > 0$, then the maximal value of $\theta_{\rm ss}$,

$$\tan 2\theta_{\rm ss}^{\rm max} \approx -\frac{\epsilon^D}{\sqrt{\epsilon^M - (\epsilon^D)^2}} \ , \tag{9}$$

is achieved at $\tan(\theta_R^D - \theta^M) = \sqrt{\epsilon^M - (\epsilon^D)^2}$. Consequently, the equality $\tan 2\theta_{\rm ss}^{\rm max} \approx 1$ implies $\epsilon^M \sim (\epsilon^D)^2$; the mass hierarchy in the Majorana sector should be of the order of the Dirac mass hierarchy squared.

The condition (7) means that the angles θ^M and θ_R^D are close to each other: since $\theta_R^D \sim \sqrt{\epsilon^D}$, we get from (7) $\theta_R^D - \theta^M \ll \theta_R^D$.

(ii) The condition of *strong cancellation in the denominator* of the RHS of Eq. (5) reads

$$\tan^2(\theta^M - \theta_R^D) \approx -\epsilon^M \ . \tag{10}$$

Now a strong mass hierarchy is not needed and, moreover (as we will show), an approximate mass degeneration in the Majorana sector naturally results in cancellation. If the equality in Eq. (10) is exact, then, according to (5),

$$\tan 2\theta_{\rm ss} \approx \frac{2\sqrt{|\epsilon^M|}}{\epsilon^D} \ . \tag{11}$$

Consequently, at $\epsilon^M \approx \epsilon^D$, one has $\tan 2\theta_{\rm ss} \sim 1/\sqrt{\epsilon^D} \gg 1$; i.e., the angle $\theta_{\rm ss}$ can be close to the maximal mixing value. The weaker the hierarchy ϵ^M, the larger $\theta_{\rm ss}$ and, consequently, the larger the lepton mixing, in contrast with the previous case.

Since θ^M is determined by the Majorana matrix M whereas θ_R^D is determined by the Dirac matrix m_D, the equality (10) implies, in general, the relation between the structures of these matrices. It is instructive to express this relation in the following form. Let us define the *Dirac basis* of neutrinos, ν_R^D, as the basis of the neutrino states in which $\theta_R^D = 0$, i.e., in which the Dirac mass matrix m_D is diagonal. Then the condition (10) means that in the Dirac basis the Majorana mass matrix should satisfy the condition

$$\tan^2\theta^M \approx -\epsilon^M \quad (\theta_R^D = 0) \ . \tag{12}$$

In turn, Eq. (12) implies the following inequality of the matrix elements of M:

$$M_{11}M_{22} \ll M_{12}^2 \ . \tag{13}$$

If $M_{11}M_{22} = 0$, the equality (12) is exact.

Let us comment on some special cases.

(1) $M_{11} = 0$, $M_{22} \neq 0$. (Inequality $M_{22} \gg M_{12}$ corresponds to the Fritzsch ansatz [22] for M.) One gets near

to maximal mixing according to Eq. (11), and an inverse mass hierarchy in the light neutrino sector.

(2) $M_{11}\neq 0$, $M_{22}=0$. These conditions result in an inverse mass hierarchy of the heavy Majorana neutrinos: $\epsilon^M > 1$, and in an inequality $\tan^2\theta^M = -1/\epsilon^M$. Therefore, a strong cancellation will take place only at strong mass degeneration, $\epsilon^M \approx 1$ ($M_{11} \ll M_{12}$). In this case, the expression for the angle θ_{ss} can be written as

$$\tan 2\theta_{ss} \approx \frac{2\epsilon^D}{1-|\epsilon^M|} = -\frac{2\epsilon^D M}{\Delta M} , \tag{14}$$

where ΔM is the difference in absolute values of Majorana masses. [At $M_{11}=M_{22}=0$ there are no mass splitting and no mixing; the light components form a Zeldovich-Konopinsky-Mahmoud (ZKM) neutrino.]

(3) In the more natural case, $M_{11}\sim M_{22} \ll M_{12}$, one can get both mixing enhancement and a normal mass hierarchy of light neutrinos (see Sec. IV).

Mixing is enhanced also if the condition (12) is satisfied at nonzero θ_R^D, but this angle should be small. Indeed substituting (12) in (5) we get

$$\tan 2\theta_{ss} \approx \frac{\epsilon^D}{\tan\theta_R^D} ,$$

i.e., for $\tan 2\theta_{ss}$ to be of the order of 1, one needs $\tan\theta_R^D \leq \epsilon^D \approx 10^{-2}$, which is much smaller than the typical rotation angle in the Dirac sector. Consequently, a strong nondiagonality (13) of the Majorana mass matrix of right-handed neutrinos alone is not enough to get strong enhancement of mixing.

The above conditions can be immediately generalized for three-neutrino mixing. In the first case one should suggest a strong mass hierarchy in the Majorana sector: $M_i \propto (m_i^D)^2$ and that the Majorana and the Dirac mass matrices are diagonalized by approximately the same transformations of right-handed neutrino components S_R [$S_R \equiv S(\theta_R^D)$ for two neutrinos]. In the second case, at least some of the elements of the Majorana mass matrix in the *Dirac basis of neutrinos* should satisfy the condition $M_{ij}^2 \gg |M_{ii}M_{jj}|$. Note that such a nondiagonality of the Majorana mass matrix is a generic feature of models where neutrino components have nonzero lepton numbers and a symmetry related to this lepton number is preserved (or approximately preserved) by the Yukawa couplings and possible bare mass terms.

III. ENHANCEMENT OF MIXING IN THE CASE OF STRONG MASS HIERARCHY IN THE MAJORANA SECTOR

Both conditions of the enhancement—strong hierarchy in the Majorana sector (8), and approximate equality of the rotation angles (7), are satisfied simultaneously if

$$M \sim \frac{1}{\mu} m_D^T m_D \sim \frac{1}{\mu} S_R (m_D^{\text{diag}})^2 S_R^T . \tag{15}$$

Here μ is some mass parameter and S_R is the transformation which diagonalizes m_D. [Strictly speaking, the last equality in (15) is true for the *CP*-conserving m_D.] In case of exact equality (15), the matrix M is also diagonalized by S_R, i.e., $\theta^M = \theta_R^D$, and $\epsilon^M = (\epsilon^D)^2$ for the two-neutrino mixing. In fact, the equality (15) should be slightly broken because at $\theta^M = \theta_R^D$ one gets $\theta_{ss}=0$.

The relation (15) can be obtained if the right-handed neutrino components acquire Majorana masses via the seesaw mechanism too. This implies the existence of new heavy neutral leptons and the "cascade," or extended seesaw (see e.g., [6], where the extended seesaw was used for another purpose). The first (high-mass scale) seesaw induced by the interactions of ν_R with new leptons gives the Majorana masses to ν_R's, and the second one generates the masses to the light neutrinos.

Let us introduce three (one for each fermion family) neutral leptons $N_L = (N_{1L}, N_{2L}, N_{3L})$, singlets of the electroweak symmetry, with bare Majorana masses m_N at some large scale (e.g., m_N can coincide with grand unification scale M_{GU}). Let us introduce also the scalar field σ singlet of SU(2)×U(1), which acquires a vacuum expectation value $\sigma_0 \gtrsim 10^{12}$ GeV. Then the Yukawa interactions and the bare mass terms of the model are

$$\bar{\nu}_L h_\nu \nu_R \phi + \bar{N}_L h_N \nu_R \sigma + N_L^T C^{-1} \tfrac{1}{2} m_N N_L + \text{H.c.} \tag{16}$$

Here ϕ is the usual Higgs doublet, and h_ν and h_N are the matrices of the Yukawa couplings. We suggest that the bare Majorana masses of ν_R's are small or forbidden by some symmetry G. For example, one can introduce the global $G=\text{U}(1)$ and prescribe the following G charges: $G(\nu_R)=G(\nu_L)=G(\sigma)=1$, $G(N_L)=G(\phi)=0$. Another possibility is that the ν_R enters the doublet of $\text{SU}(2)_R$ symmetry which is broken at the scale $V_R \ll M_{GU}$. In this case ν_R may acquire a mass $\ll V_R$ [1].

To reproduce the relation (15) one should suggest that the matrices of the Yukawa interactions, h_ν and h_N, are correlated. The simplest possibility, $h_\nu = h_N$, means a universality of Yukawa interactions: the couplings of ν_R with ν_L and N_L are the same. Such a universality implies unification; it can be realized if ν_L and N_L as well as ϕ and σ enter the same fermion and scalar multiplets correspondingly. In fact, the equality of h_ν and h_N is not necessary; the enhancement takes place if the matrices of the Yukawa couplings have the form

$$h_\nu \equiv S_L \frac{m_D^{\text{diag}}}{v} S_R^T , \quad h_N = S_N \frac{m_D^{\text{diag}}}{v} S_R^T . \tag{17}$$

Here v is the vacuum expectation value of ϕ and S_N is some almost arbitrary matrix which may be related to features of N_L interactions. In particular, it can be equal to I or S_L.

When the fields ϕ and σ acquire nonzero vacuum expectation values, neutrino masses are generated and the mass matrix in the block basis (ν_L, ν_L^c, N_L) can be written as

$$\begin{bmatrix} 0 & m_D & 0 \\ m_D^T & 0 & \frac{\sigma_0}{v} m_D^T S_N^T \\ 0 & \frac{\sigma_0}{v} S_N m_D & m_N \end{bmatrix} . \tag{18}$$

It gives the Majorana mass matrix for right-handed neutrino components,

$$M=-\left[\frac{\sigma_0}{v}\right]^2 m_D^T S_N^T m_N^{-1} S_N m_D \ , \tag{19}$$

and then the Majorana mass matrix for light components:

$$m^{\mathrm{Maj}}=\left[\frac{v}{\sigma_0}\right]^2 S_N^T m_N S_N \tag{20}$$

(for real S_N). As follows from Eq. (20), at $S_N=I$, the matrix m^{Maj} is proportional to the mass matrix of superheavy leptons, N_L. We may suggest that the Yukawa couplings or/and bare mass terms at the highest mass scale have no hierarchy; all elements of the matrix m_N are of the same order. The matrix S_N with small nondiagonal elements gives only small corrections to the above picture.

Note that at $\sigma_0 \sim m_N \sim 10^{16}$ GeV, the typical mass scale for the lightest components is about $m \sim v^2 m_N/\sigma_0^2 \approx 10^{-2}$ eV. Therefore a small spread of parameters in m_N allows us to explain the scales of both the solar ($m \sim 0.3\times 10^{-2}$ eV) and atmospheric [$m \sim (3-10)\times 10^{-2}$ eV] neutrino problems. The Majorana masses of right-handed neutrinos are naturally in the intermediate mass scale region ($10^{10}-10^{14}$) GeV.

Two remarks are in order. A Goldstone boson which could appear due to spontaneous violation of G symmetry by the vacuum expectation of σ has negligibly small interactions with active neutrinos. Moreover, it can acquire a nonzero mass due to explicit violation of this symmetry at low scales. Additional scalars can be introduced to generate m_N spontaneously.

The above mixing enhancement based on the extended seesaw mechanism and the universality of the Yukawa couplings allows us to systematically compensate for the smallness in the lepton mixing related to the hierarchical structure of m_D. In the extreme case the lepton mixing is defined by the structure of the mass matrix of superheavy leptons and does not depend on the Dirac matrix at all.

IV. ENHANCEMENT OF MIXING DUE TO LEPTON SYMMETRY IN THE DIRAC BASIS. DEGENERATION OF HEAVY MAJORANA MASSES

The enhancement of mixing takes place due to the cancellation in the denominator on the RHS of Eq. (5). The condition of cancellation [the relation (12) for the Majorana mass matrix in Dirac basis of neutrons] implies a correlation between structures of the Dirac and the Majorana matrices. The possible deviation from this correlation quantified by the angle θ_R^D should be small. These features again imply the universality of the Yukawa couplings. The neutrino family states, at least ν_R, should enter both the Yukawa interactions and the bare mass terms in the same combinations, which coincide with eigenstates of the Dirac mass matrix: $S_R^T \nu_R \equiv \nu_R^D$. However, in contrast with the previous case, the couplings of ν_R^D need not be proportional to m_D^{diag}/v. To naturally satisfy the condition (12), the Yukawa interaction of ν_R^D should obey definite symmetry.

It is possible to formulate these conditions in another form. Usually lepton symmetry and lepton numbers are introduced in the Dirac basis of the *charged leptons* (i.e., in the basis where the mass matrix of charged leptons is diagonal; see e.g., [7] in context of the seesaw mechanism). Lepton number symmetry introduced in such a way is violated by neutrino mass terms. In contrast, one can impose the lepton symmetry in the *neutrino* Dirac basis and suggest that this symmetry is violated by mass terms of charged leptons. These two cases are physically equivalent if there are only Dirac masses, but they give different results in presence of the Majorana mass terms. In the neutrino Dirac basis, the lepton mixing matrix can be written as

$$S^{\mathrm{lept}}=(S_L^l)^+ S_{\mathrm{ss}} \ , \tag{21}$$

where S_L^l is the transformation of the left components which diagonalizes the charged lepton mass matrix and S_{ss} is the seesaw transformation which diagonalizes the mass matrix for light components in the Dirac basis (see Appendix). In the two-neutrino cases, Eq. (21) corresponds to Eq. (3) at $\theta_L^D=0$; by definition, S_L^l relates the Dirac and the flavor bases: $\nu_f=S_L^l \nu_D$; S_L^l is similar to the Cabibbo-Kobayashi-Maskawa matrix of the quark mixing.

Consider neutrino mixing in the Dirac basis, ν_D. Let us suppose that the Yukawa interactions as well as the bare mass terms obey a global $G=\mathrm{U}(1)$ symmetry. We prescribe zero G-charge for the usual Higgs doublet, $G(\phi)=0$, and different G-charges for different pairs of left- and right-handed neutrino components: $G(\nu_{iL})=G(\nu_{iR})=G_i$, $G_i \neq G_j$ $(i,j=1,\,2,\,3)$. Then the Yukawa couplings of neutrinos with ϕ and consequently the Dirac mass matrix are diagonal (as is demanded by definition of the Dirac basis). We will also introduce a scalar field σ with nonzero charge G_σ, which acquires nonzero vacuum expectation value σ_0 and thus breaks G symmetry. Now Yukawa interactions and bare mass terms of the model can be written as

$$\bar{\nu}_L \frac{m_D^{\mathrm{diag}}}{v} \nu_R \phi + \nu_R^T C^{-1} \frac{1}{2} h \nu_R \sigma + \nu_R^T C^{-1} \frac{1}{2} M_b \nu_R + \mathrm{H.c.} \tag{22}$$

Matrices of the Yukawa couplings h and bare masses M_b are determined by the G-charge prescription and in general are nondiagonal. The neutrino mass matrix which results from the spontaneous breaking of the electroweak and G symmetries can be written in the basis (ν_L^c, ν_R) as

$$\begin{bmatrix} 0 & m_D^{\mathrm{diag}} \\ m_D^{\mathrm{diag}} & M \end{bmatrix} . \tag{23}$$

The matrix of right-handed neutrino components, M, should satisfy the cancellation condition [see Eqs. (12) and (13) in two neutrino case]. According to the seesaw mechanism the mass matrix for light neutrinos is

$$m_{\mathrm{ss}}=-m_D^{\mathrm{diag}} M^{-1} m_D^{\mathrm{diag}} \ . \tag{24}$$

Depending on G_σ and on the charge prescription for ν_R^D: $G_\nu \equiv (G_1, G_2, G_3)$, one gets different realizations of M needed for mixing enhancement. Let us comment on the simplest possibilities.

(1) For the charge prescription $G_\nu = (1, 0, -1)$, $G_\sigma = 2$ the Majorana mass matrix is equal to

$$M = \begin{bmatrix} h_{11}\sigma_0 & 0 & M_{13} \\ 0 & M_{22} & 0 \\ M_{13} & 0 & h_{33}\sigma_0 \end{bmatrix} . \tag{25}$$

The state ν_{2R} decouples and the task is reduced to the two-neutrino case. At $h_{11}\sigma_0 \sim h_{33}\sigma_0 \ll M_{13}$, one naturally obtains a direct mass hierarchy in the light sector and a strong cancellation in the denominator. The seesaw transformation diagonalizing matrix (24) is

$$S_{ss} = \begin{bmatrix} \cos\theta_{ss} & 0 & \sin\theta_{ss} \\ 0 & 1 & 0 \\ -\sin\theta_{ss} & 0 & \cos\theta_{ss} \end{bmatrix} ,$$

where, according to (14),

$$\tan 2\theta_{ss} = \frac{2\epsilon^D M_{13}}{[(\epsilon^D)^2 h_{33} - h_{11}]\sigma_0} \approx \frac{2\epsilon^D M_{13}}{h_{11}\sigma_0} . \tag{26}$$

Here $\epsilon^D \equiv m_1/m_3$. The ν_e-ν_τ mixing turns out to be enhanced, whereas ν_e-ν_μ and ν_μ-ν_τ mixings induced only by Dirac matrices are small.

For the charge prescription $G_\nu = (1, -1, 0)$, $G_\sigma = 2$ one gets similarly an enhancement of $1-2$ and, consequently, ν_e-ν_μ mixing. For $G_\nu = (0, 1, -1)$, ν_μ-ν_τ mixing is enhanced.

(2) For $G_\nu = (1, 0, -1)$, $G_\sigma = 1$ the Majorana mass matrix has a form

$$M = \begin{bmatrix} 0 & h_{12}\sigma_0 & M_{13} \\ h_{12}\sigma_0 & M_{22} & h_{23}\sigma_0 \\ M_{13} & h_{23}\sigma_0 & 0 \end{bmatrix} . \tag{27}$$

A straightforward calculation of the m_{ss} according to (24) gives that at $h_{23}\sigma_0 \gg M_{13}$ the enhancement of $1-2$ (ν_e-ν_μ) mixing takes place:

$$\tan 2\theta_{ss} \approx \frac{2\epsilon^D (M_{13}/h_{23}\sigma_0)}{(\epsilon^D)^2 - (M_{13}/h_{23}\sigma_0)^2} , \tag{28}$$

where $\epsilon^D \equiv m_1/m_2$.

(3) For $G_\nu = (0, 1, 2)$, $G_\sigma = 2$, the Majorana mass matrix is

$$M = \begin{bmatrix} M_{11} & 0 & h_{13}\sigma_0 \\ 0 & h_{22}\sigma_0 & 0 \\ h_{13}\sigma_0 & 0 & 0 \end{bmatrix} . \tag{29}$$

The state with charge $G = 1$ decouples and the task is reduced again to the two-neutrino case. In contrast with version (1), here mixing is induced by the Yukawa couplings with σ. The enhancement of mixing implies $h_{13}\sigma_0 \gg M_{11}$ and according to (14) one has

$$\tan 2\theta_{ss} = 2\epsilon_{13}^D \frac{h_{13}\sigma_0}{M_{11}} . \tag{30}$$

The opposite order of charges $G_\nu = (2, 1, 0)$ gives strong enhancement of $1-3$ mixing, but an inverse mass hierarchy of light neutrinos. In this case, large mixing angles, i.e., $\sin^2 2\theta > 0.6$, are disfavored by SN 1987A data [23].

Changing the neutrino component which has charge $G = 1$ and, consequently, decouples, one can enhance mixings of other pairs of neutrinos states.

(4) For $G_\nu = (2, 1, 0)$, $G_\sigma = 1$, the Majorana mass matrix

$$M = \begin{bmatrix} 0 & h_{12}\sigma_0 & 0 \\ h_{12}\sigma_0 & 0 & h_{23}\sigma_0 \\ 0 & h_{23}\sigma_0 & M_{33} \end{bmatrix} \tag{31}$$

results in a strong enhancement of $1-2$ (ν_e-ν_μ) mixing at $M_{33} \gtrsim h_{12}\sigma_0$,

$$\tan 2\theta_{ss} = -\frac{2}{\epsilon_{12}^D} \frac{M_{33} h_{12}}{\sigma_0 h_{23}} , \tag{32}$$

and in an inverse mass hierarchy of light neutrinos. At $h_{12}/h_{23} \lesssim \epsilon^D$ one can get simultaneously the $(1-3)$ mixing enhancement:

$$\tan 2\theta_{ss} \approx \frac{2\epsilon^D (h_{23}/h_{12})}{1 - (\epsilon^D)^2 (h_{23}/h_{12})^2} . \tag{33}$$

For $G_\nu = (0, 1, 2)$, and $G_\sigma = 1$, ν_μ-ν_τ mixing is enhanced at $M \gtrsim h\sigma_0$.

All elements of the Majorana mass matrix M can be generated spontaneously by introducing additional scalar bosons, so that the hierarchy of masses in the Majorana sector is related to the hierarchy of the vacuum expectation values of different scalars. Moreover, the extended global symmetry and/or discrete symmetry can be imposed to obtain the needed texture of mass matrices.

To explain the universality of the Yukawa interactions one can also develop the scenario of Sec. III. Namely, it is possible to generate the Majorana mass matrix of ν_R via interactions with additional superheavy leptons.

V. DISCUSSION AND CONCLUSIONS

(1) The most natural scenario of lepton mass and mixing generation implies that the Dirac mass matrices in the lepton sector are similar to those in the quark sector. The difference in the neutrino masses and probably in mixing follows from the Majorana mass matrix of the right-handed neutrinos.

(2) The effect of the seesaw mechanism itself in lepton mixing can be described by the seesaw matrix or the seesaw angle (when the task is reduced to the two-neutrino case). The seesaw angle is proportional to the mass hierarchy in the Dirac sector ϵ^D, rather than $\sqrt{\epsilon^D}$. Therefore, usually the seesaw corrections to lepton mixing are small, or are of the same order as mixing induced by the Dirac matrices. However, the smallness of the

seesaw angle related to such a proportionality can be systematically compensated. At definite conditions, the seesaw angle can be larger than the angles induced by Dirac matrices and the lepton mixing is determined mainly by the seesaw angle.

(3) There are two sets of conditions at which the seesaw angle dominates and the lepton mixing is large. The first set demands a strong mass hierarchy in the Majorana sector as well as approximately the same transformations of the right-handed neutrino components which diagonalize the Dirac and Majorana mass matrices. The latter implies the universality of the Yukawa couplings of the right-handed components.

These conditions can be realized in models with additional superheavy leptons N_L, where right-handed neutrinos acquire Majorana masses via the seesaw mechanism induced by interactions with N_L. The universality of Yukawa couplings of ν_R could be explained by the unification of ν_L and N_L in one multiplet. As a result of the extended (cascade) seesaw, the mixing of the light neutrinos is determined essentially by mixing of N_L. The typical mass scale of light neutrinos, $(10^{-3}-10^{-1})$ eV, allows us to explain both the solar and atmospheric neutrino deficits.

(4) The second set of conditions demands definite form of the Majorana mass matrix of right-handed components in the Dirac basis of neutrinos (where Dirac mass matrix is diagonal). Namely, dominance of the nondiagonal elements is needed and in most natural cases the Majorana masses of right-handed neutrinos turn out to be degenerate. Such a nondiagonality of M (or suppression of the diagonal elements) is a generic feature of models in which neutrinos have nonzero charges of some symmetry, G. For this scenario an enhancement of mixing for two neutrinos is typical. Different G-charge prescriptions result in enhancement of mixing of different components.

(5) Future solar neutrino experiments will be able to confirm (or reject) the solutions based on large lepton mixing and therefore check (at least partly) the possibilities considered in this paper.

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APPENDIX: SEESAW ANGLE AND SEESAW MATRIX

In the two-neutrino case, the mass matrix is diagonalized by the unitary transformation $S(\theta_i)$ which is determined by a rotation angle θ_i:

$$S(\theta_i) \equiv \begin{pmatrix} \cos\theta_i & \sin\theta_i \\ -\sin\theta_i & \cos\theta_i \end{pmatrix} . \tag{A1}$$

Let θ_L^D and θ_R^D be the angles of rotations of the left- and right-handed neutrino components, $\nu_L = S(\theta_L^D)\nu_L^D$ and $\nu_R = S(\theta_R^D)\nu_R^D$, which diagonalize the Dirac mass matrix:

$$S^T(\theta_L^D) m_D S(\theta_R^D) = m_D^{\text{diag}} \equiv \text{diag}(m_1, m_2) . \tag{A2}$$

Here ν_L^D, ν_R^D and m_1, m_2 are the eigenstates and the eigenvalues of the Dirac mass matrix correspondingly (subscript T means transponent). Let θ^M be the angle of rotation of the right-handed neutrino components which diagonalizes the Majorana mass matrix of right-handed neutrino components, $\nu_R = S(\theta^M)\nu_R^M$:

$$S^T(\theta^M) M S(\theta^M) = M^{\text{diag}} \equiv \text{diag}(M_1, M_2) , \tag{A3}$$

where M_i are the eigenvalues of the Majorana mass matrix. Then taking into account that the inverse matrix M^{-1} is diagonalized by the same transformation (A3) we can write the Majorana mass matrix for light neutrino components in terms of diagonalized matrices and rotations in the form

$$m^{\text{Maj}} = S(\theta_L^D) m_D^{\text{diag}} S(\theta^M - \theta_R^D)(M^{\text{diag}})^{-1} \times S^T(\theta^M - \theta_R^D) m_D^{\text{diag}} S(\theta_L^D)^T . \tag{A4}$$

This matrix can be diagonalized by transformation (A1) with the angle

$$\theta_\nu \equiv \theta_L^D + \theta_{\text{ss}} , \tag{A5}$$

where θ_{ss} is the angle of rotation which diagonalizes according to (A4) the matrix

$$m_{\text{ss}} \equiv m_D^{\text{diag}} S(\theta^M - \theta_R^D)(M^{\text{diag}})^{-1} S^T(\theta^M - \theta_R^D) m_D^{\text{diag}} . \tag{A6}$$

Now the lepton mixing angle can be written as $\theta^{\text{lept}} = \theta_L^D + \theta_{\text{ss}} - \theta_L^l$; here θ_L^l is the angle of rotation which diagonalizes the Dirac mass matrix of charged leptons. At $\theta_{\text{ss}} = 0$ the form of lepton mixing coincides with that in quark sector. So, θ_{ss} specifies the features of the seesaw mechanism and we will call it the seesaw angle.

In the general case (three-neutrino mixing), one can introduce the seesaw matrix, S_{ss}, so that the lepton mixing matrix can be written in the form

$$S^{\text{lept}} = (S_L^l)^+ S_L^D S_{\text{ss}} , \tag{A7}$$

where the transformation S_{ss} diagonalizes the matrix

$$m_{\text{ss}} = m_D^{\text{diag}} S_R^T M^{-1} S_R m_D^{\text{diag}} .$$

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