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Neutrino mass explanations of solar and atmospheric neutrino deficits and hot dark matter

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If the solar and atmospheric neutrino deficits and the apparent need for a hot dark matter component all result from neutrino mass, one of three neutrino mass patterns is required. Of these, two appear quite unlikely, leading to a unique solution which points to further experimental tests. We briefly outline possible theoretical models which could generate these three neutrino mass matrices without adjustment of parameters.

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Current indications for neutrino mass are [1] (1) the deficit of solar electron neutrinos, (2) the depletion of atmospheric muon neutrinos relative to electron neutrinos, and (3) the apparent need for some hot dark matter. Conventional wisdom is that (1) results from $\nu_e \to \nu_\mu$ oscillations, so on the basis of a seesaw model ν_τ could have the ~ 7 eV mass desired for (3). If (2) is evidence for neutrino mass, however, the small $\nu_\mu - \nu_\tau$ mass difference needed makes this scenario wrong, and there are three different possibilities for the pattern of neutrino masses and mixings.

These possible scenarios of neutrino mass, if the above three observations are indeed manifestations of neutrino mass, are (a) ν_e, ν_μ, and ν_τ all are ~ 2–3 eV, (b) the ν_e, ν_μ, and ν_τ are all very light, and a sterile neutrino ν_s supplies the hot dark matter, or (c) the ν_e and ν_s are light and the ν_μ and ν_τ are ~ 3–4 eV. The consequences of each of these alternatives will be examined below after a short review of the experimental input.

1. Solar neutrinos. Evidence for a solar neutrino deficit comes from four experiments [2]. If the relative values of the Kamiokande and Homestake results are correct, the problem is not astrophysical but must relate to neutrino properties [3], most likely neutrino mass. This would result in ν_e's produced in the Sun oscillating on the way to Earth to another neutrino species, which could be ν_μ, ν_τ, or ν_s (a sterile neutrino). The required mass difference and mixing are [3,4] (a) $\Delta m_{ei}^2 = (0.3-1.2)\times 10^{-5}$ eV^2, $\sin^2 2\theta = (0.4-1.5)\times 10^{-2}$ [nonadiabatic Mikheyev-Smirnov-Wolfenstein (MSW) mechanism], (b) $\Delta m_{ei}^2 = (0.3-3)\times 10^{-5}$ eV^2, $\sin^2 2\theta = 0.6-0.9$ (large-mixing MSW mechanism), or (c) $\Delta m_{ei}^2 = (0.5-1.1)\times 10^{-10}$ eV^2, $\sin^2 2\theta = 0.8-1.0$ (vacuum oscillations).

2. Atmospheric neutrinos. Depletions of ν_μ relative to ν_e produced in the upper atmosphere mainly by pion and subsequent muon decay have been observed by three experiments. The ratio of observed μ events to e events normalized to the flux ratio expected from calculations are $R(\mu/e) = 0.60 \pm 0.07 \pm 0.05$ (Kamiokande) [5], $0.54 \pm 0.05 \pm 0.12$ (IMB) [6], $0.69 \pm 0.19 \pm 0.09$ (Soudan II, preliminary) [7]. Combining these results with the observations of upward-going muons by the Kamiokande [5], IMB [6], and Baksan [8] groups, and the negative Fréjus [9] results, Frati *et al.* [10] have concluded that ν_μ could be oscillating into either ν_e, ν_τ, or ν_s with $\Delta m^2 \approx 0.5$ to 0.005 eV^2 and $\sin^2 2\theta \approx 0.5$.

3. Dark matter neutrinos. Data on the extent of structure in the Universe are now available on a wide range of distance scales. Evidence from the Cosmic Background Explorer (COBE) results on the anisotropy of the cosmic microwave background radiation, galaxy-galaxy angular correlations, large-scale velocity fields, and correlations of galactic clusters can all be fit [11] by a model [12] of the Universe containing 70% cold dark matter and 30% hot dark matter contributed by ~ 7 eV in neutrino mass. Such a model provides a consistent explanation of not only the shape of the density fluctuation spectrum, but also the observed estimates of the absolute density on small and large scales. While the fits have been made with a single neutrino, dividing the mass among more than one neutrino would work even better [13].

4. Other constraints on neutrinos. Two constraints are important here. The first is the nonobservation of neutrinoless double β decay ($\beta\beta_{0\nu}$), which provides an important limit on an effective Majorana mass, $\langle m_\nu \rangle \lesssim 1-2$ eV [14]. Second, since the possibility of four light neutrinos is considered, their effect on nucleosynthesis is of concern, as the 95% C.L. limit on the number of light neutrinos is 3.3 from the primordial ^{4}He abundance [15]. To keep ν_s out of equilibrium at the time of nucleosynthesis requires a limit on its mixing angle with one of the three active species which depends on Δm_{si}^2. In the atmospheric neutrino case, even the most favorable $\Delta m_{\mu s}^2$ (≈ 0.005 eV^2) would have to have $\sin^2 2\theta_{\mu s}$, almost an order of magnitude smaller than the required 0.5 value in order for the $\nu_\mu \to \nu_s$ oscillation not to contribute 0.3 of an effective neutrino at that time, ruling out this possibility [16].

(a) *Case of three similar neutrino masses.* If the solar ν_e deficit is solved by $\nu_e \to \nu_\mu$, the atmospheric ν_μ deficit by $\nu_\mu \to \nu_\tau$, and only these three light neutrinos exist, then

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having hot dark matter requires the three to be nearly degenerate, with a mass ~ 2.5 eV each. The necessary Δm^2 values make it impossible for fewer neutrinos to supply the needed dark matter mass. In this case a mass matrix of the following type would fit all the constraints:

$$M=\begin{bmatrix} m & \delta s_2^2 s_1 & -\delta s_1 s_2 \\ \delta s_2^2 s_1 & m+\delta s_2^2 & -\delta s_2 \\ -\delta s_1 s_2 & -\delta s_2 & m+\delta \end{bmatrix}, \quad (1)$$

where the columns refer, in order, to ν_e, ν_μ, and ν_τ and the rows to the mass eigenstates m_1, m_2, and m_3. For the nonadiabatic MSW solution of the solar neutrino problem, typical parameters of the matrix are $m \approx 2.5$ eV, $\delta \approx 0.08$ eV, $s_1 \approx 0.05$, and $s_2 \approx 0.35$. This highly fine-tuned mass matrix is difficult to obtain naturally in gauge theories.

In this case, ν_e-ν_τ mixing was made negligible by choice, but the matrix can be generalized to allow for nonzero ν_e-ν_τ mixing angle. Unitarity constraints, however, imply that the maximum value of this mixing angle would be 0.05. No experimental limit comes close to even this maximum value of $\sin^2 2\theta_{e\tau}=10^{-2}$. The smallness of this angle and also of $\theta_{e\mu}$, or more exactly the smallness of the matrix elements M_{e2} and M_{e3} above, make it impossible to arrange a cancellation of the effective neutrino mass in $\beta\beta_{0\nu}$, $\langle m_\nu \rangle \approx \sum_j \xi_j M_{ej}^2 m_j$, by choosing suitable CP eigenvalues, $\xi_j = \pm 1$. Instead, $\langle m_\nu \rangle \approx 2.5$ eV. Thus the likelihood that this case represents reality is marginal at best, since Majorana masses are involved. To avoid experimental limits [14], one must rely on improbable nuclear matrix element suppression in $\beta\beta_{0\nu}$, the neutrino mass needed for hot dark matter to be much less than 7 eV, or the theoretically unlikely introduction of Dirac neutrino masses. Viewed more positively, if this were the correct scenario, $\beta\beta_{0\nu}$ experiments could soon demonstrate its existence.

(b) *Case of three very light active neutrinos and one heavier sterile neutrino.* For this logical possibility [17], the matrix in Eq. (1) could apply here as well, with m now very small or even zero. The sterile neutrino, which could be completely decoupled from the active ones, must have a reduced contribution to the energy density at the era of nucleosynthesis to respect the constraint [15] that $\delta N_\nu < 0.3$. The number density of ν_s now must satisfy the inequality $n_{\nu_s} < n_{\nu_e}(\delta N_\nu^{\max})^{3/4}$. If ν_s is to contribute 30% of dark matter, this implies $m_{\nu_s} = 27.6(\delta N_\nu)^{-3/4} h^2$. For $\delta N_\nu = 0.3$, $h=\frac{1}{2}$ (the Hubble constant in units of 100 $\mathrm{km\,s^{-1}\,Mpc^{-1}}$), $m_{\nu_s} \approx 17$ eV. It is more likely that ν_s decouples before the quark-hadron transition temperature ($T \sim 200-400$ MeV), making its contribution $\delta N_\nu = 0.1$ and requiring $m_{\nu_s} \approx 39$ eV. Such large values of ν_s are unlikely to be acceptable as hot dark matter [18]. That this is a very improbable scenario is fortunate, since direct detection of such ν_s dark matter appears impossible, and its coupling to active neutrinos is forced by the nucleosynthesis constraint to be so small ($\sin^2 2\theta_{si} \lesssim 10^{-4}$) as to make neutrino disappearance detection extremely difficult.

(c) *Case of very light ν_e, ν_s and heavier ν_μ, ν_τ.* A much more attractive scheme is to have $\nu_e \to \nu_s$ solve the solar neutrino problem [19], the parameters being almost the same as for $\nu_e \to \nu_\mu$, and have ν_μ and ν_τ both account for the atmospheric ν_μ deficit and be the hot dark matter. For the Δm_{es}^2 and $\sin^2 2\theta_{es}$ involved, the nucleosynthesis constraint is obeyed for the nonadiabatic MSW or vacuum oscillation cases, but most likely violated by the large-mixing MSW case. The simplest mass matrix attaining these goals is

$$M=\begin{bmatrix} \mu_1 & \mu_3 & 0 & 0 \\ \mu_3 & \mu_2 & 0 & 0 \\ 0 & 0 & m & \delta/2 \\ 0 & 0 & \delta/2 & m+\delta \end{bmatrix}, \quad (2)$$

where the columns refer in order to ν_s, ν_e, ν_μ, and ν_τ, $\mu_{1,2}$ are of order 10^{-2} eV, μ_3 is of order 5×10^{-4} eV, and $m=3.5$ eV and $\delta = 0.07$ to 0.0007 eV for the numbers given above. A similar mass matrix was advocated [20] to account for the solar neutrino and atmospheric ν_μ deficits and for the 17-keV neutrino, but with $\delta \gg m$ in Ref. [20].

For the sake of simplicity we have "disconnected" the ν_e-ν_s sector from the ν_μ-ν_τ sector. One could, for example, add a nonzero $\nu_e \nu_\mu$ element leading to ν_e-ν_μ oscillations, important to determining the reality of this scenario. While $\nu_e \to \nu_s$ can be tested vs $\nu_e \to \nu_\mu$ by observing neutral current interactions at SNO or BOREXINO, and $\nu_\mu \to \nu_\tau$ can be checked in proposed long-baseline oscillation experiments, the hope for seeing a mass difference between the ν_e-ν_s and ν_μ-ν_τ sectors lies mainly in $\nu_\mu \to \nu_e$ oscillation experiments. The LSND experiment at Los Alamos by 1994 could have sufficient data to reduce the present limit in this $\Delta m_{e\mu}^2$ region of $\sin^2 2\theta_{e\mu} = 2\times 10^{-3}$ by an order of magnitude. It would be very difficult to attain such sensitivity in a $\nu_e \to \nu_\tau$ experiment.

Another model-dependent issue is the nature of the ν_e-ν_s and ν_μ-ν_τ mixings. While splitting of a light Dirac particle into Majorana ν_e and ν_s can be accomplished to give other than maximal mixing (although that would be appropriate in the vacuum oscillation case), that is very difficult to do in the heavy nearly degenerate ν_μ-ν_τ case. However, we present later an acceptable model based on an $S_3 \times Z_4$ symmetry having this property.

Theoretical model for case (a). The scenario in which all three active neutrinos have the same mass of about 2.5 eV cannot be realized in the conventional seesaw mechanisms generated by the mass matrix

$$\begin{bmatrix} 0 & m_D \\ m_D & M \end{bmatrix}, \quad (3)$$

where m_D is a Dirac mass and M is a large Majorana mass, since neutrino masses scale quadratically with the charged fermion masses (e.g., $m_u^2 : m_c^2 : m_t^2$). On the other hand, it has been noted [21] that both in left-right-symmetric models, as well as SO(10) models, the ν_L-N_R

mass matrix that emerges naturally has the same form as Eq. (3) with 0 replaced by fv_L and M by fv_R, where $v_L=(4m_{W_L}^2\gamma)/(g^2v_R)$. Here, v_R represents the scale at which the local $B-L$ symmetry [subgroup of SO(10)] breaks, and v_L is the vacuum expectation value of the $SU(2)_L$ triplet field [contained in the {126}-dimensional Higgs multiplet in the case of SO(10) theory] that couples to the lepton doublets. For $v_R\approx 10^{12}-10^{13}$ GeV and $f\approx(0.1-1)$, $\gamma\sim 1$, we get contributions to all neutrino masses of order 4 eV. The off-diagonal contributions would be dominated by the usual seesaw terms and are of order 10^{-9} eV, 10^{-3} eV, and a few eV, respectively. If we demanded that, by some symmetry, f be a unit matrix in the generation space, then indeed we would get $m_{\nu_e}\approx m_{\nu_\mu}\approx m_{\nu_\tau}\approx 2.5$ eV without unnatural fine-tuning of parameters. We would also get $\Delta m_{e\mu}^2\approx 5\times 10^{-3}$ eV^2 and $\Delta m_{\mu\tau}^2\approx$few eV^2, values which are too large, but at least close to those desired.

Possible model for case (b). With the solar and atmospheric neutrino puzzles accounted for by the ν_e, ν_μ, and ν_τ, the dark matter would be provided by one or more sterile neutrinos. Three sterile neutrinos, each with mass ≈ 13 eV, is very suggestive of an E_6 grand unified theory, where the {27}-dimensional representation (to which the fermions are usually assigned) contains one sterile neutrino, leading to three for three generations. Under the $SO(10)\times U(1)_X$ subgroup of E_6 we have, $\{27\}=\{16\}_{+1}+\{10\}_{-2}+\{1\}_{+4}$, where the subscript denotes the $U(1)_X$ quantum number. The seesaw mechanism in this case is implemented by the Higgs boson in the {351'} representation of E_6, which could be used to generate the light masses for (ν_e,ν_μ,ν_τ), as usual. (The Dirac masses arise from a {27}-dimensional Higgs boson.) However, the sterile SO(10)-singlet fermion remains massless at the tree level in this minimal scenario. It picks up mass at the one-loop level from a combination of $m_0[\phi\{27\}]^3$ and $b\Psi\{27\}\Psi\{27\}\phi\{27\}$ couplings, where Ψ are fermions and ϕ are Higgs bosons. A typical magnitude of this contribution is $m_{\nu_s}\approx(b^2m_0\langle S\rangle^2)/(16\pi^2M_H^2)$, where S is the SO(10) singlet in $\phi\{27\}$. For $b\approx 10^{-2}$, $m_0\approx$TeV, we need $\langle S\rangle/M_H\approx 10^{-7}$ to get $m_{\nu_s}\approx 13$ eV. If M_H is of the order of the grand unified theory (GUT) scale ($\approx 10^{16}$ GeV), this implies $\langle S\rangle\approx\frac{1}{2}\times 10^{10}$ GeV, which is a plausible intermediate scale.

Theoretical model for case (c). The mass matrix in Eq. (2) for this case can be generated by a combination of seesaw and loop mechanisms without adjustment of parameters. The model is presented as an argument to show that, while the mass matrix may appear highly fine tuned, its existence may have some theoretical basis in terms of hidden symmetries of nature. Consider an extension of the standard model with three sterile neutrinos ν_s, N_μ, and N_τ. Demand that this theory be invariant under an $S_3\times Z_4$ permutation symmetry. The S_3 group has one {2}-dimensional and two {1}-dimensional representations, denoted here by 1^- and 1^+, respectively. The fields then have the representations shown in Table I.

The most general gauge-, S_3-, and Z_4-invariant Yukawa coupling is

$$\mathcal{L}_Y=M(N_\mu^TC^{-1}N_\mu+N_\tau^TC^{-1}N_\tau)+h_0(\overline{\Psi}_\mu\phi_uN_\mu+\overline{\Psi}_\tau\phi_uN_\tau)+h_e\overline{\Psi}_e\phi_{d_3}e_R+h_e'\overline{\Psi}_e(\phi_{d_1}\mu_R+\phi_{d_2}\tau_R)$$
$$+h_\mu[(\overline{\Psi}_\mu\phi_{d_2}+\overline{\Psi}_\tau\phi_{d_1})\mu_R+(\overline{\Psi}_\mu\phi_{d_1}-\overline{\Psi}_\tau\phi_{d_2})\tau_R]+f_1\Psi_\mu^TC^{-1}\tau_2\Psi_\tau\eta_2^++f_2\nu_s^TC^{-1}e_R\eta_2^+$$
$$+f_4k_2^{++}e_R^TC^{-1}e_R+f_3k_1^{++}(\mu_R^TC^{-1}\mu_R+\tau_R^TC^{-1}\tau_R)+\text{H.c.} \tag{4}$$

Similarly, the potential, in addition to the usual terms, consists of

$$V'=\mu_1k_1^{++}\eta_1^-\eta_1^-+\lambda_2k_2^{++}\eta_1^-\eta_2^-\sigma+\mu_3k_1^{++}\eta_2^-\eta_2^-$$
$$+\mu_4\phi_{d_3}^T\tau_2\phi_{d_4}\eta_1^-+\mu_s\eta_1\eta_2^*\sigma' , \tag{5}$$

where σ is a real S_3-odd singlet with a vacuum expectation value.

The first two terms in Eq. (4) lead to degenerate ν_μ,ν_τ Majorana states with masses $m_\nu\approx h_0^2\kappa_u^2/M$, where $\kappa_u=\langle\phi_u^0\rangle$. For $\kappa_u\approx 100$ GeV, $h_0\approx\frac{1}{2}\times 10^{-4}$, and $M\approx 7$ TeV, we get $m_\nu\approx 3.5$ eV, as required. The purpose of explicit numbers is to show that the choice of parameters is not too different in order of magnitude from those of the standard model. At the tree level, $m_{\nu_e}=m_{\nu_s}=0$. At the two-loop level, these masses will arise. Note that even with S_3 symmetry, m_e, m_μ, and m_τ can be very different from each other.

The two-loop graphs are similar to those in Ref. [22] and are shown in Figs. 1(a)–1(c). The $\nu_\mu\nu_\tau$ and the $\nu_\mu\nu_\mu$ elements are roughly [in terms of the parameters in Eqs. (4) and (5)]

$$m_{\nu_\mu\nu_\tau}\approx\frac{2f_1^2}{(16\pi^2)^2}\frac{m_\tau m_{\mu\tau}\mu_3}{M_H^2} , \tag{6}$$

$$m_{\nu_\mu\nu_\mu}\approx\frac{f_1^2}{(16\pi^2)^2}\frac{\mu_3}{M_H^2}(m_\tau^2+m_{\mu\tau}^2) . \tag{7}$$

M_H is a typical Higgs boson mass in the loop, chosen ≈ 100 GeV; $m_{\mu_2}\approx h_\mu\langle\phi_{d_1}^0\rangle$. For $f_1\approx 10^{-2}$, $\mu_3\approx M_H$, $m_{\nu_\mu\nu_\tau}\approx 3\times 10^{-2}$ eV $\approx m_{\nu_\mu\nu_\mu}$, which is roughly of the right order of magnitude. Equations (6) and (7) together predict the ν_μ-ν_τ mixing angle to be approximately $\tan 2\theta_{\mu\tau}\approx 2m_{\mu\tau}/m_\tau$, avoiding maximal mixing.

In the $\nu_e\nu_s$ sector we can select parameters with reasonable values and obtain $m_{\nu_e\nu_s}\approx m_{\nu_s\nu_s}\approx 10^{-2}-10^{-3}$ eV, if we choose $f_2\approx 10^{-3}$, $f_4\approx 10^{-1}$ and $\langle\sigma\rangle\approx\langle\phi_{d_4}\rangle\approx M_H$. This explains the solar neutrino puzzle via the nonadiabatic MSW mechanism. There are

TABLE I. Gauge and discrete symmetry transformation properties of the fermions and bosons in model (c).

Fields		$SU(2)\times U(1)_Y$	S_3	Z_4
Fermions:	$\begin{bmatrix}\Psi_\mu\\ \Psi_\tau\end{bmatrix}$	(2,−1)	2	+1
	$\begin{bmatrix}N_\mu\\ N_\tau\end{bmatrix}$	(1,0)	2	−1
	$\begin{bmatrix}\mu_R\\ \tau_R\end{bmatrix}$	(1,−2)	2	+1
	Ψ_e	(2,−1)	1^+	+1
	e_R	(1,−2)	1^+	$-i$
	ν_s	(1,0)	1^-	$+i$
Higgs bosons:	ϕ_u	(2,−1)	1^+	−1
	$\begin{bmatrix}\phi_{d_1}\\ \phi_{d_2}\end{bmatrix}$	(2,+1)	2	+1
	ϕ_{d_3}	(2,+1)	1^+	i
	ϕ_{d_4}	(2,+1)	1^+	$-i$
	η_1^+	(1,+2)	1^+	+1
	η_2^+	(1,2)	1^-	+1
	k_1^{++}	(1,4)	1^+	+1
	k_2^{++}	(1,4)	1^+	−1
	σ	(1,0)	1^-	−1
	σ'	(1,0)	1^-	+1

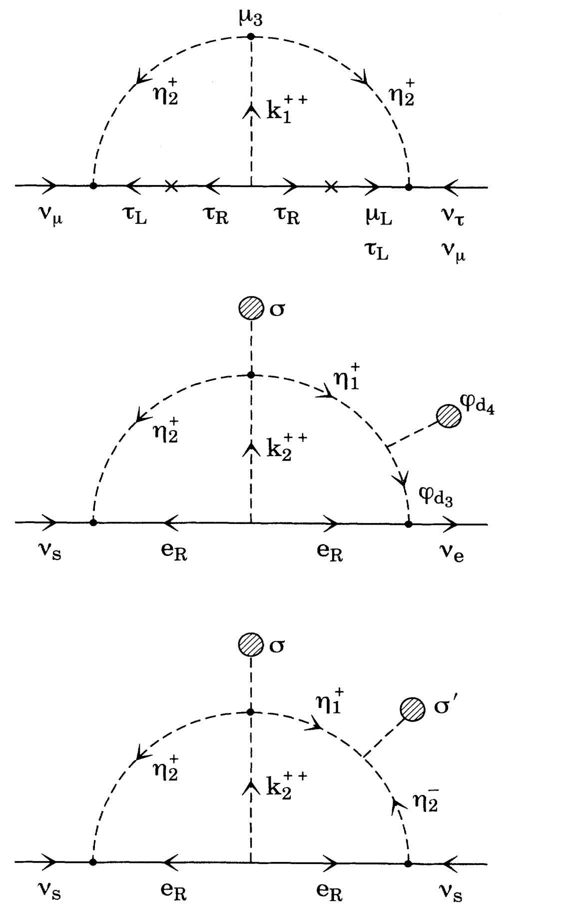

FIG. 1. Two-loop graphs contributing to $m_{\nu_\mu\nu_\mu}$, $m_{\nu_\mu\nu_\tau}$, $m_{\nu_e\nu_s}$, $m_{\nu_s\nu_s}$ in model (c).

also small contributions connecting the ν_e-ν_s and ν_μ-ν_τ sectors proportional to h_e'. The mixing angle cannot be predicted because it depends on the unknown parameter h_e'.

The strength of the ν_s-e interaction in this model is given by $f_2^2/4M_{\eta_2}^2$. For $M_{\eta_2}\simeq 100$ GeV and the above quoted value of $f_2(\approx 10^{-3})$, the strength of ν_s-e scattering is $\simeq 2.5\times 10^{-6}G_F$. This implies that in the early Universe the sterile neutrinos go out of equilibrium around $T\simeq 10$ GeV, resulting in their contribution to energy density at the epoch of nucleosynthesis being an effective $\delta N_\nu < 0.05$.

In summary, if present indications of neutrino mass from solar and atmospheric neutrino experiments, as well as indirect evidence for a component of hot dark matter, are correct, then one of three scenarios for the masses of neutrinos must be realized. The first, in which ν_e, ν_μ, and ν_τ are all ~ 2–3 eV, is either ruled out now, or soon could be, by neutrinoless double β decay experiments. The second, having very light ν_e, ν_μ, and ν_τ, with dark matter being supplied by one or more sterile neutrinos ν_s probably does not provide fast enough dark matter. The third, and by far most likely, in which light $\nu_e\rightarrow\nu_s$ solves the solar neutrino deficit and the heavier ν_μ-ν_τ sector accounts for hot dark matter and the deficiency of atmospheric ν_μ's, is testable by SNO and BOREXINO (for $\nu_e\rightarrow\nu_s$), by long-base-line oscillation experiments (for $\nu_\mu\rightarrow\nu_\tau$), but particularly by detecting $\nu_\mu\rightarrow\nu_e$ or $\nu_e\rightarrow\nu_\tau$ oscillations to see the mass difference between the two sectors. Theoretical possibilities for each of the three scenarios are given to demonstrate that such arrangements of neutrino mass are not impossible to achieve in gauge theories and to give indications for future theoretical work.

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