

CP -odd correlation in the decay of a neutral Higgs boson into ZZ , W^+W^- , or $t\bar{t}$

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We investigate the possibility of detecting CP -odd angular correlations in the various decay modes of the neutral Higgs boson including the modes of a ZZ pair, a W^+W^- pair, or a heavy quark pair. It is a natural way to probe the CP character of the Higgs boson once it is identified. Final state interactions (i.e., the absorptive decay amplitude) are not required in such correlations. As an illustrative example we take the fundamental source of CP nonconservation to be in the Yukawa couplings of the Higgs boson to the heavy fermions. A similar correlation in the process $e^+e^- \rightarrow l^+l^-H$ is also proposed. Our analysis of these correlations will be useful for experiments in future colliders such as the CERN LEP II, SSC, CERN LHC or NLC.

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I. INTRODUCTION

The Higgs boson sector remains the most mysterious part of theories of electroweak unification. To probe this illusive sector one is well advised to keep an open mind. Most analyses of the Higgs sector, in the standard model or beyond, ignore potential CP violation. However, in many of these models, even the lightest neutral Higgs boson can have interesting CP -violating phenomena. In many models of CP violation, including the standard model, CP violation is a consequence of simultaneous existence of many coupling constants. Such requirement results in the presence of many coupling constants and/or loop suppression factors in the observables. The neutral Higgs sector is unique in the sense that a single Higgs boson coupling to a massive fermion is enough to manifest CP violation as long as the Yukawa coupling contains both scalar and pseudoscalar components. Therefore, in many ways the CP -violating aspect may be the most interesting part of the Higgs boson physics beyond the standard model once a neutral Higgs boson is identified.

In a previous paper, we investigated the signatures of various CP -odd asymmetries in different polarized decay modes of the neutral Higgs boson [1]. Among the interesting modes are the CP asymmetries in the event rate differences $N(Q_L\bar{Q}_L) - N(Q_R\bar{Q}_R)$, $N(W_L^+W_L^-) - N(W_R^+W_R^-)$, or $N(Z_LZ_L) - N(Z_RZ_R)$. As expected, the CP asymmetries will manifest themselves only after the final state interactions are taken into account. For the heavy quark pair $Q\bar{Q}$ mode or the W^+W^- pair mode, the relative energy of the final state charged leptons can be used as the polarization analyzers of the heavy quarks

or the W 's [2, 3]. However, for the case of the ZZ mode, the Z coupling to charged leptons is mainly axial vectorial. Therefore the efficiency of using its leptonic energy spectrum as the polarization analyzer is suppressed by a factor of $c_V/c_A = -1 + 4\sin^2\theta_W \sim -0.08$.

In this paper we consider a different kind of signal of CP violation: the CP -odd correlations of final state momenta. Note that this is different from the more conventional T -odd signals which can be imitated by CP -conserving final state interactions. No final state interaction is required to obtain the signals. Therefore, unlike the case of the CP asymmetry considered in Ref. [1], it is not necessary to cross a heavy fermion threshold to obtain a large signal in the W^+W^- or the ZZ channel. For earlier discussions of CP -odd effects in different processes, see Refs. [4-8].

II. $H \rightarrow ZZ$

Let us start with the ZZ mode whose CP -odd asymmetry does not translate into large lepton energy asymmetry. After the Z 's decay there are three modes of final state: (1) $l^-l^+l'^-l'^+$, (2) $l^-l^+q\bar{q}$, (3) $q\bar{q}q'\bar{q}'$. The purely leptonic modes are of course the easy ones in which to get a CP -odd correlation provided that we collect enough events. In the bigger samples of the semileptonic modes and the hadronic ones, it is difficult to tell the charge of the leading quark when it hadronizes into a jet. However, the most intriguing part of our result is that it is not necessary to identify charge to detect the CP -odd signal, an argument we will elaborate on later. This ob-

ervation will be even more crucial in trying to decode CP -odd signals in the W^+W^- or $t\bar{t}$ modes of the Higgs boson decay. Another important point is that in modes (1) and (3), the lepton pairs or the quark pairs do not have to be distinct. As we will show later, the identical particle effect does not wash out the main CP -odd signal. We shall discuss these modes in order.

Consider $H \rightarrow ZZ \rightarrow l^-l^+l'^-l'^+$. If H is scalar, the two Z polarizations will be parallel; if H is pseudoscalar, the two Z polarizations will be perpendicular. The correlation between the polarizations of the two Z bosons translates into the correlation between the two planes of the l^+l^- pairs. In fact this observation was used a long time ago to measure the parity of the π^0 through its decay into two photons [9]. When the Higgs coupling is neither scalar nor pseudoscalar, one has a source of CP violation. We shall explore this case.

The effective interaction for the ZZH vertex can be written as

$$\mathcal{L}_{HZZ} = \frac{1}{2}g_H H [BZ^\nu Z_\nu + \frac{1}{4}DM_H^{-2}\epsilon_{\mu\nu\rho\sigma}Z^{\mu\nu}Z^{\rho\sigma}], \quad (1)$$

where $g_H = (g_2 M_Z / \cos \theta_W)$, $Z^{\mu\nu}$ is the field strength of Z boson and B and D are dimensionless form factors. For the Higgs boson in the standard model, $D = 0, B = 1$. The D term is CP odd while the B term is CP even. Simultaneous presence of D and B is CP violating. We have ignored higher-dimensional CP -even operators in the above Lagrangian and a possible CP -violating $HZ\gamma$ vertex which could also contribute to four fermions final states. Their effect should be negligible at the pole of the final state Z boson. In momentum space, these effective vertices give rise to the covariant amplitude

$$iM_{H \rightarrow Z(P,\eta), Z(P',\eta')} = ig_H [B\eta \cdot \eta' + (D/M_H^2)\epsilon(\eta, \eta', P, P')]. \quad (2)$$

Here η, η' are the polarizations and P, P' are the momenta of the two Z bosons. We define $\epsilon(\eta, \eta', P, P') \equiv \epsilon_{\mu\nu\alpha\beta}\eta^\mu\eta'^\nu P^\alpha P'^\beta$, with the convention $\epsilon_{0123} = 1$. The helicity amplitudes in $H \rightarrow ZZ$ are represented as

$$M_{+,+} = g_H(B - \frac{i}{2}D\beta), \quad M_{-,-} = g_H(B + \frac{i}{2}D\beta), \\ M_{0,0} = -g_H B \frac{1 + \beta^2}{1 - \beta^2}, \quad (3)$$

in the H rest frame, with $\beta = 1 - 4M_Z^2/M_H^2$. Here the subscripts $(+, -, 0)$ denote, respectively, the helicities $(R, L, ||)$ of the Z bosons. Conservation of angular momentum implies only the above three helicity configurations for this decay process. Under CP transformation, $M_{+,+} \rightarrow M_{-,-}$, and $M_{00} \rightarrow M_{00}$. Thus, D is CP odd; however $\text{Re } D$ is CPT even, while $\text{Im } D$ is CPT odd. The detailed definition of CPT can be found in Ref. [8]. It is roughly the CPT symmetry without reversing the initial and the final states, that is, the symmetry reverses only the kinematic variables of the states according to their CPT property. The effect of $\text{Im } D$ will give an asymmetry in the production of polarized states:

$$N(Z_L Z_L) - N(Z_R Z_R) \\ = (-\beta \text{Im } D/B)[N(Z_L Z_L) + N(Z_R Z_R)]. \quad (4)$$

Its consequence in the energy asymmetry of the final leptons has been discussed in Ref. [1]. In this paper, we focus our attention only on the effect of the real part of D through the angular asymmetry of the final decay products. In the decay process $H \rightarrow ZZ \rightarrow l^-(\mathbf{k}_-)l^+(\mathbf{k}_+)l'^-(\mathbf{p}_-)l'^+(\mathbf{p}_+)$, the angular correlation of the final four fermions is encoded in the 3×3 density matrix

$$\rho_{\lambda\lambda'}^\lambda = M_{\lambda,\lambda} M_{\lambda',\lambda'}^* \quad (\text{no dummy summation}), \quad (5)$$

which is folded with the Z decay amplitudes to produce the event distribution. The CP -odd, CPT -even combinations are $\rho_+^- - \rho_+^+$ and $\rho_+^0 - \rho_+^- + \rho_0^- - \rho_0^+$ while the ones that are CP and CPT odd are $\rho_+^+ - \rho_+^-$ and $\rho_+^0 - \rho_+^- - \rho_0^- + \rho_0^+$. Only the first two are relevant to our discussion below.

Similar notation will be used for the modes of $W^{+-}W^-$.

III. CP -VIOLATING OBSERVABLES

We will show observables which are related to the CP -odd correlations among the momenta, $\mathbf{k}_-, \mathbf{k}_+, \mathbf{p}_-$, and \mathbf{p}_+ . To simplify our discussion, we start with the purely leptonic case and we arbitrarily label the lepton pair from one of the Z bosons with primes. The process $Z \rightarrow \bar{l}l$ can be parametrized by the vertex

$$ie\bar{u}(l)\gamma_\mu(c_L L + c_R R)v(\bar{l}).$$

Naively one may simply construct the CP -odd correlation $O_{\text{odd}} = \mathbf{p}_- \times \mathbf{p}_+ \cdot \mathbf{k}_-$ in the H rest frame. It can also be written in a Lorentz invariant form

$$O_{\text{odd}} = -M_H^{-1}\epsilon(p_-, p_+, k_-, k_+).$$

However, as we will show later, the expectation value of this observable $\langle \mathbf{p}_- \times \mathbf{p}_+ \cdot \mathbf{k}_- \rangle$ is proportional to $c_V c_A$ where $c_V = \frac{1}{2}(c_L + c_R)$ and $c_A = \frac{1}{2}(-c_L + c_R)$ are the vector and axial-vector couplings of the Z boson. Since the vector coupling of the Z boson to the charged leptons in the standard model is relatively small, this observable turns out to be rather unimportant. It can be understood as the consequence of an approximate symmetry when the vector coupling (or the axial-vector coupling) is ignored completely, so that there is no distinction between l^+ and l^- as far as the Z boson is concerned. Therefore the differential decay rate is symmetric under two separate partial charge conjugation symmetries: \hat{C}_1 and \hat{C}_2 . Symmetry \hat{C}_1 interchanges l^+ and l^- while leaving l'^+ , l'^- unchanged, while the symmetry \hat{C}_2 interchanges l'^+ and l'^- and leaves l^+ , l^- unchanged instead. The usual charge conjugation operation C is the product of the two \hat{C} 's. It is easy to check that the correlation O_{odd} is odd under either \hat{C}_1 or \hat{C}_2 . Therefore it has nonvanishing expectation value only when both c_V and c_A couplings are

nonzero. It is certainly more desirable to use an observable which is CP odd but even under each \hat{C}_1 or \hat{C}_2 . It is not too difficult to construct a quantity. A possibility is

$$O'_{\text{odd}} = (\mathbf{p}_- \times \mathbf{p}_+ \cdot \mathbf{k}_-)[(\mathbf{p}_- \times \mathbf{p}_+) \cdot (\mathbf{k}_- \times \mathbf{k}_+)]. \quad (6)$$

It is easier to understand these angular correlations in the geometry of the reaction. A typical reaction is shown in Fig. 1. Under CP transformation, (E_{p_-}, E_{k_-}, ϕ) is transformed into $(E_{p_+}, E_{k_+}, -\phi)$. If one defines a plane using the cross product of one of the lepton pairs, say $\mathbf{p}_- \times \mathbf{p}_+$, as the direction of the normal vector, the configurations with the momentum of the other lepton (k_-) coming out of the plane is the CP conjugate of the configurations in which k_- is going into the plane. Therefore the asymmetry, $\langle \text{sgn}(O_{\text{odd}}) \rangle$, can be interpreted as a kind of up-down asymmetry. More explicitly, define the polar and azimuthal angles for the two leptons in their respective Z boson rest frames to be θ, ϕ for $l(\mathbf{k}_-)$, and θ', ϕ' for $l'(\mathbf{p}_-)$. Here all the polar angles are defined relative to the same \hat{z} axis, say the $\mathbf{k}_- + \mathbf{k}_+$ axis. The distribution depends on the relative azimuthal angle; therefore, we can set azimuthal angle of l' to $\phi' = 0$ as shown in the Fig. 1. Then one can label the event configuration by the angles $(\cos \theta, \cos \theta', \phi)$. The energies E_{k_-} and E_{p_-} in the Higgs boson rest frame for the leptons l and l' , respectively, are determined by their polar angles θ and θ' . Under \hat{C}_1, \hat{C}_2 , parity and CP , the configuration transforms as

$$\begin{aligned} \hat{C}_1 : (\cos \theta, \cos \theta', \phi) &\rightarrow (\cos(\pi - \theta), \cos \theta', \phi + \pi), \\ \hat{C}_2 : (\cos \theta, \cos \theta', \phi) &\rightarrow (\cos \theta, \cos(\pi - \theta'), \phi + \pi), \\ P : (\cos \theta, \cos \theta', \phi) &\rightarrow (\cos \theta, \cos \theta', -\phi), \\ CP : (\cos \theta, \cos \theta', \phi) &\rightarrow (\cos(\pi - \theta), \cos(\pi - \theta'), -\phi). \end{aligned} \quad (7)$$

$$n(E_{p_-}, E_{k_-}, \phi) \equiv \frac{dN}{dE_{p_-} dE_{k_-} d\phi}$$

$$= \mathcal{N} \sum_{h, h'} \sum_{\lambda, \lambda'} \rho_{\lambda}^{\lambda'} f_{\lambda}^h(\theta, \phi) f_{-\lambda}^{h'}(\theta', 0) f_{\lambda}^{h*}(\theta, \phi) f_{-\lambda'}^{h'*}(\theta', 0) c_h^2 c_{h'}^2. \quad (8)$$

We denote the helicity amplitude $f_{\lambda}^h(\theta, \phi)$ describing the process $Z(\lambda) \rightarrow l(h)\bar{l}(-h)$ in the Z rest frame, where the spin projection of the Z boson along the \hat{z} axis is $\lambda\hbar$ and the helicity of l is specified by $h = +(R)$ or $-(L)$. One can relate the helicity amplitudes with the spin-1 rotation matrix elements, $f_{\lambda}^h \sim d_{\lambda, h}^1 e^{i\lambda\phi}$,

$$f_{\pm}^h(\theta, \phi) = (1 \pm h \cos \theta) e^{\pm i\phi} / 2, \quad f_0^h = h \sin \theta / \sqrt{2} \quad (h = \pm 1). \quad (9)$$

\mathcal{N} is the normalization to be specified later. One can see from Eqs. (8) and (9) that in order to get the complex phase in angular distribution to expose the CP -odd, CPT -even effect, only the combinations $\rho_{-}^{\pm} - \rho_{+}^{\pm}$

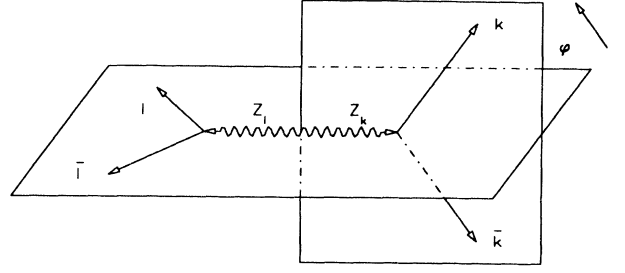


FIG. 1. A typical decay configuration, $H \rightarrow ZZ \rightarrow l(k_-)l(k_+)l'(p_-)l'(p_+)$.

To understand observables such as $\langle O'_{\text{odd}} \rangle$, one can divide the azimuthal angle ϕ into four quadrants, I, II, III, IV. Since \hat{C}_1 transforms (I, II) into (III, IV), it is clear that the up-down asymmetry $\langle \text{sgn}(O_{\text{odd}}) \rangle$ which corresponds to the angular integration of $(I + II) - (III + IV)$ is odd under \hat{C}_1 or \hat{C}_2 . On the other hand, an observable similar to $\langle \text{sgn}(O'_{\text{odd}}) \rangle$ can be constructed by the alternative angular integration $(I+III) - (II+IV)$.

IV. ANGULAR DEPENDENCE

To calculate these asymmetries, we need the differential distribution

and $\rho_{+}^0 - \rho_{-}^0 + \rho_{0}^{-} - \rho_{0}^{+}$ contribute. That is consistent with our general discussion after Eq. (5). Note that in the $H \rightarrow ZZ \rightarrow \bar{l}l\bar{l}l$ mode of identical final lepton pairs, the identical-particle symmetry implies $n(E_{p_-}, E_{k_-}, \phi) = n(E_{k_-}, E_{p_-}, \phi)$. Therefore it does not pose any restriction on the azimuthal, ϕ , angular distribution at all. The overall distribution can be divided into two parts, namely, the CP -even piece n_0 and the CP -odd piece Δn ,

$$n(E_{p_-}, E_{k_-}, \phi) = n_0(E_{p_-}, E_{k_-}, \phi) + \Delta n(E_{p_-}, E_{k_-}, \phi). \quad (10)$$

Using the form factors D, B and the couplings c_L, c_R , we obtain

$$\begin{aligned}\Delta n(E_{p_-}, E_{k_-}, \phi) &= \mathcal{N} D M_H^{-2} \epsilon(p_-, p_+, k_-, k_+) [-(c_L^4 + c_R^4)(p_- \cdot k_- + p_+ \cdot k_+) + 2c_L^2 c_R^2 (p_- \cdot k_+ + p_+ \cdot k_-)], \\ n_0(E_{p_-}, E_{k_-}, \phi) &= \mathcal{N} B [(c_L^4 + c_R^4) p_- \cdot k_- p_+ \cdot k_+ + 2c_L^2 c_R^2 p_- \cdot k_+ p_+ \cdot k_-].\end{aligned}\quad (11)$$

From now on, except for Sec. IX, it is understood that we have dropped the Re prefix for the form factor D . We have only kept the linear piece in D , as a result of perturbation. Kinematically, we can express the ϵ symbol and various scalar products in terms of angles, θ, ϕ, θ' ,

$$\begin{aligned}\epsilon(p_-, p_+, k_-, k_+) &= -\frac{1}{8} M_H^2 M_Z^2 \beta \sin \theta' \sin \theta \sin \phi, \\ p_- \cdot k_- &= \frac{M_H^2}{16} [(1 + \beta^2)(1 - \cos \theta \cos \theta') + 2\beta(\cos \theta - \cos \theta')] - \frac{M_Z^2}{4} \sin \theta \sin \theta' \cos \phi, \\ p_+ \cdot k_+ &= \frac{M_H^2}{16} [(1 + \beta^2)(1 - \cos \theta \cos \theta') - 2\beta(\cos \theta - \cos \theta')] - \frac{M_Z^2}{4} \sin \theta \sin \theta' \cos \phi, \\ p_- \cdot k_+ &= \frac{M_H^2}{16} [(1 + \beta^2)(1 + \cos \theta \cos \theta') - 2\beta(\cos \theta + \cos \theta')] + \frac{M_Z^2}{4} \sin \theta \sin \theta' \cos \phi, \\ p_+ \cdot k_- &= \frac{M_H^2}{16} [(1 + \beta^2)(1 + \cos \theta \cos \theta') + 2\beta(\cos \theta + \cos \theta')] + \frac{M_Z^2}{4} \sin \theta \sin \theta' \cos \phi.\end{aligned}\quad (12)$$

The covariant expression in Eq. (11) derived from the usual Dirac matrix calculation agrees with the result by the helicity amplitude method in Eqs.(3), (5), (8), and (9) with substitutions of Eq. (12). The differential CP -odd asymmetry then can be defined as

$$\begin{aligned}\mathcal{A}_{ud}(E_{p_-}, E_{k_-}, \phi) &\equiv \frac{n(E_{p_-}, E_{k_-}, \phi) - n(E_{p_+}, E_{k_+}, -\phi)}{n(E_{p_-}, E_{k_-}, \phi) + n(E_{p_+}, E_{k_+}, -\phi)} \\ &= \frac{\Delta n(E_{p_-}, E_{k_-}, \phi)}{n_0(E_{p_-}, E_{k_-}, \phi)}.\end{aligned}\quad (13)$$

To simulate realistic detector acceptance, one has to use a Monte Carlo calculation based on the differential distribution in Eq. (11). However, it is instructive to look at the overall asymmetry integrating over the full ranges of $\cos \theta$ and $\cos \theta'$. For the CP -odd numerator in \mathcal{A}_{ud} of Eq. (13), we have

$$\begin{aligned}\Delta n(\phi) &\equiv \int \Delta n d \cos \theta d \cos \theta' \\ &= -\frac{\mathcal{N} B D M_H^4}{2 \cdot 9} z(1 - 4z)^{\frac{1}{2}} \left[z \sin 2\phi (c_L^2 + c_R^2)^2 - \frac{9\pi^2}{64} (1 - 2z) \sin \phi (c_L^2 - c_R^2)^2 \right],\end{aligned}\quad (14)$$

with $z = M_Z^2/M_H^2$. The first term in square brackets is due to the interference between $M_{+,+}$ and $M_{-,-}$ and is proportional to $\rho_+^{\frac{1}{2}} - \rho_+^-$. The second term in square brackets is due to the interference between $M_{0,0}$ and $M_{\pm,\pm}$ and is proportional to $\rho_+^0 - \rho_+^- + \rho_0^- - \rho_0^+$. For the CP -even denominator, we have

$$n_0(\phi) \equiv \int n_0 d \cos \theta d \cos \theta' = \frac{\mathcal{N} B^2 M_H^4}{4 \times 9} \left[(1 - 4z + 12z^2 + 2z^2 \cos 2\phi)(c_L^2 + c_R^2)^2 - \frac{9\pi^2}{16} z(1 - 2z) \cos \phi (c_L^2 - c_R^2)^2 \right].\quad (15)$$

The CP -conserving part $n_0(\phi)$ has been calculated before [10, 11]. Our result agrees with Ref. [11]. A sign difference between ours and Ref.[10] can be due to different definitions of ϕ . The normalization \mathcal{N} can be chosen to be

$$\mathcal{N} = 18/[\pi B^2 M_H^4 (1 - 4z + 12z^2)(c_L^2 + c_R^2)^2], \quad (16)$$

such that the distribution $n(\phi) = n_0(\phi) + \Delta n(\phi)$ is normalized to one after integration over ϕ . Measuring these $\sin \phi$ or $\sin 2\phi$ dependences in the event distribution will establish the CP nonconservation. To enhance statistics, we can look at the up-down asymmetry by integrating again over ϕ from 0 to π :

$$\begin{aligned}\mathcal{A}_{ud} &= \frac{\left(\int_0^\pi - \int_\pi^{2\pi} \right) n(\phi) d\phi}{\int_0^{2\pi} n(\phi) d\phi} \\ &= \frac{D}{B} \frac{9\pi z (1 - 4z)^{\frac{1}{2}} (1 - 2z)}{16(1 - 4z + 12z^2)} \left(\frac{c_L^2 - c_R^2}{c_L^2 + c_R^2} \right)^2.\end{aligned}\quad (17)$$

After the integration, the $\sin 2\phi$ term of the combination $c_L^2 + c_R^2 = 2(c_A^2 + c_V^2)$ gives zero while the surviving $\sin \phi$ term is relatively suppressed by the factor $c_L^2 - c_R^2 = -4c_V c_A$ as we anticipated before. A more realistic integrated asymmetry should be the one that can preserve the $\sin 2\phi$ term. That can be obtained by taking the difference in the integration over $[0, \pi/2]$ and the

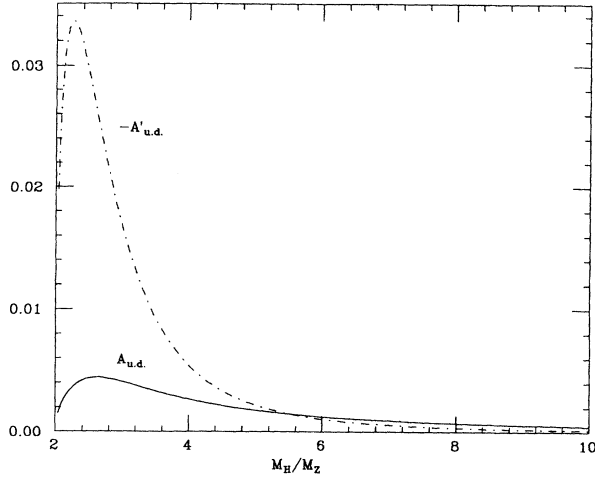


FIG. 2. \mathcal{A}_{ud} , \mathcal{A}'_{ud} versus M_H/M_Z per unit of D/B .

integration over $[\pi/2, \pi]$

$$\begin{aligned} \mathcal{A}'_{ud} &= \frac{\left(\int_0^{\pi/2} - \int_{\pi/2}^{\pi} + \int_{\pi}^{3\pi/2} - \int_{3\pi/2}^{2\pi} \right) n(\phi) d\phi}{\int_0^{2\pi} n(\phi) d\phi} \\ &= -\frac{D}{B} \frac{4z^2(1-4z)^{\frac{1}{2}}}{\pi(1-4z+12z^2)}. \end{aligned} \quad (18)$$

Figure 2 shows \mathcal{A}_{ud} , \mathcal{A}'_{ud} versus M_H per unit of D/B . Following the above formalism, experiments can search for CP nonconservation, or at least, put a constraint on the parameter D . Note that, in the heavy Higgs boson limit $z \ll 1$, the $\sin \phi$ contribution becomes relatively important because of the kinematic suppression factor z in the other contribution of $\sin 2\phi$ in Eq. (14).

V. GENERALIZATION

We can generalize our formalism to the case that l and l' are different fermions of different couplings c_L, c_R and c'_L, c'_R . The replacements in Eqs. (11) and (14)–(17) are given by the rules

$$\begin{aligned} c_L^4 + c_R^4 &\rightarrow c_L^2 c'_L{}^2 + c_R^2 c'_R{}^2, \\ 2c_L^2 c_R^2 &\rightarrow c_L^2 c'_R{}^2 + c_R^2 c'_L{}^2, \\ (c_L^2 + c_R^2)^2 &\rightarrow (c_L^2 + c_R^2)(c_L'^2 + c_R'^2), \\ (c_L^2 - c_R^2)^2 &\rightarrow (c_L^2 - c_R^2)(c_L'^2 - c_R'^2). \end{aligned} \quad (19)$$

This applies also to the case $H \rightarrow ZZ \rightarrow l^- l^+ q \bar{q}$ and $H \rightarrow ZZ \rightarrow q \bar{q} q' \bar{q}'$.

One of the main problems of using the hadronic or semihadronic decay modes of the Higgs boson in searching for CP -violating signals is that experimentally it is impossible to identify charges of jets originating from hadronization of the partons. Therefore one essentially has to smear over the charge information. Naively one may think that it will make CP information impossible to disentangle. This turns out not to be the case in the

observable O'_{odd} , which is even under both \hat{C}_1 and \hat{C}_2 separately. Thus, in O'_{odd} the contribution of an event and its \hat{C}_1 -conjugate event will add, while in O_{odd} they will cancel. Therefore, charge identification is not necessary for the observable O'_{odd} .

To construct the observable, we revisit the definition of ϕ in the general case. Each pair of fermions defines a plane. Given two unoriented planes one can define the angle ϕ_0 between planes to be a or $\pi - a$ ($0 \leq a \leq \pi$ by definition). These two choices are not resolved yet. However, if the common line of the two planes can be physically assigned a direction associated with one of the planes, then the two angles a and $\pi - a$ can be distinguished. For example in the $H \rightarrow ZZ$ decay, see Fig. 1, we can simply use the vector $\mathbf{k}_- + \mathbf{k}_+ = \mathbf{k}_Z$ to define a direction associated with the k_-, k_+ plane. Then, using the right hand rule, one can rotate along the axis $\mathbf{k}_- + \mathbf{k}_+$ and sweep the plane of p_-, p_+ toward the plane of k_-, k_+ and define the resulting angle $\phi_0 \in [0, \pi]$. It is important to notice that choosing $\mathbf{p}_- + \mathbf{p}_+$ to define the angle, instead of $\mathbf{k}_- + \mathbf{k}_+$, gives the same result. (Therefore the two fermion pairs can be identical without smearing out the effect.) On the other hand, the azimuthal angle ϕ defined in Fig. 1 with full identification varies from 0 to 2π , and the relation between them is simply $\phi_0 = \text{mod}(\phi, \pi)$. So the event rate at $\phi_0 = a$ ($0 \leq a \leq \pi$ by definition) is the sum of the elementary rates $n(\phi = a) + n(\phi = a + \pi)$; thus the dependences on $\sin \phi$ in Eq. (14) and $\cos \phi$ in Eq. (15) will be washed away. However, the $\sin 2\phi$ term survives to signal the CP violation. Of course, if one can identify the charges of the final fermions, as in the case of leptonic modes, then one can also decode the $\sin \phi$ term as well. To detect the $\sin 2\phi$ dependence, one can use the same integrated asymmetry, \mathcal{A}'_{ud} , as in Eq. (18) except in this case the sums $\int_0^{\pi/2} + \int_{\pi}^{3\pi/2}$ and $\int_{\pi/2}^{\pi} + \int_{3\pi/2}^{2\pi}$ are automatically taken care of by the definition of ϕ_0 . Therefore Eq. (18) is still valid even in the case of $l^- l^+ q \bar{q}$ or $q \bar{q} q' \bar{q}'$ decay modes. Note that for these modes the asymmetries corresponding to Eq. (17) are not doubly suppressed by the small c_v as in the purely leptonic modes. Unfortunately, \mathcal{A}_{ud} is not observable due to the lack of hadronic charge identification. However, one should keep in mind that if one is willing to zoom into the dependence of the amplitudes on the jet energies (or, equivalently, on the angles θ , or θ') then it may be possible to use energy identification to replace charge identification to construct an observable similar to \mathcal{A}_{ud} .

Note that for the CP -conserving decay, the angular correlation we just described can also be used to detect the important $\cos 2\phi$ distribution even for the semihadronic modes, $l^- l^+ q \bar{q}$, or the hadronic modes, $q \bar{q} q' \bar{q}'$, of the $H \rightarrow ZZ$ decay.

VI. $H \rightarrow W^+ W^-$

Once one understands how the CP -odd correlation can be decoded in the hadronic modes of $H \rightarrow ZZ$ decay, it is easy to apply it to the process $H \rightarrow W^+ W^-$ also. In this case one can write down a similar vertex with the Z fields replaced by the W fields. Since the coupling of the

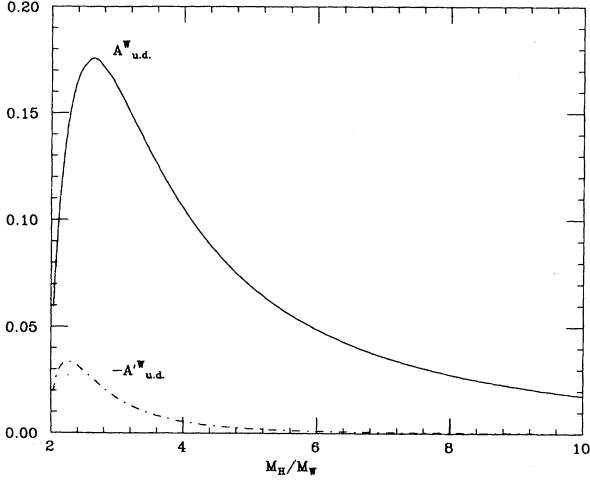


FIG. 3. $\mathcal{A}_{ud}^W, \mathcal{A}'_{ud}^W$ versus M_H/M_Z per unit of D^W/B^W .

W boson to a fermion pair is always left handed, we just set $c_R = c'_R = 0$ and $c_L = c'_L = 1$. All substitutions are straightforward. We can simply add the superscript W to $B, D, n, \Delta n, \mathcal{A}_{ud}$, and \mathcal{A}'_{ud} in Sec. IV to label the WW mode in the above formulas.

In order to determine the decay planes of both W^\pm bosons, we cannot use the purely leptonic mode $l\bar{\nu}l$, where too much kinematic information is carried away by the missing ν and $\bar{\nu}$. However, we can use the mixed modes $l\bar{\nu}q\bar{q}$ or $\bar{l}\nu q\bar{q}$, where kinematics can be fully reconstructed provided that we have good resolution on jets. Without identifying the nature of the leading quarks in jets, we are still able to define uniquely the ϕ_0 as defined in Sec. V. This is good enough to measure the $\sin 2\phi$ dependence, which is related to \mathcal{A}'_{ud}^W , a quantity generalized from the definition in Eq. (18). In Fig. 3, we show $\mathcal{A}_{ud}^W, \mathcal{A}'_{ud}^W$ versus M_H per unit of D^W/B^W . Note that, unlike the purely leptonic modes of $H \rightarrow ZZ$ case, \mathcal{A}_{ud}^W is much larger than \mathcal{A}'_{ud}^W . Unfortunately, \mathcal{A}_{ud}^W is not observable due to lack of charge identification. However, as commented earlier for the ZZ case, it may be possible to replace charge identification by energy identification to construct an observable similar to \mathcal{A}'_{ud}^W .

For the purely hadronic mode, there is tremendous background from nonresonant contributions in the hadronic environment. The e^-e^+ collider may be a cleaner machine in which the Higgs boson may be produced via the process $e^-e^+ \rightarrow ZH$. Then, one can use

the decay $Z \rightarrow l^-l^+$ to tag the recoiling Higgs boson H which turns into a W^+W^- pair and finally becomes four jets, whose CP character can be decoded using the angular correlation defined earlier.

In addition, our study of the angular correlation can be very useful in investigating the general CP -odd or CP -even correlations in other reactions, for example, the hadronic final states of the process $e^+ + e^- \rightarrow W^+ + W^-$ [14].

VII. $H \rightarrow t\bar{t}$

The possibility of detecting CP violation in $H \rightarrow t\bar{t}$ is very interesting in the sense that the corresponding CP -violating couplings can occur at the tree level. Let us look at the phenomenological form of the Yukawa interaction:

$$\mathcal{L}_{CPX}^{\text{Yukawa}} = -(m_t/v)\bar{t}(AP_L + A^*P_R)tH. \quad (20)$$

Here $v = (\sqrt{2}G_F)^{-\frac{1}{2}} \simeq 246$ GeV. The complex coefficient A is a combination of model-dependent mixing angles. The CP -violating effect is proportional [12, 13] to $\text{Im} A^2 = 2\text{Im} A \text{Re} A$. In Higgs boson production at a high-energy collider, one expects the $CP\bar{T}$ -even, CP -violating effect to be observable in the angular asymmetry as before. It can in principle be a tree-level effect and therefore can be very significant. However, in reality, its signal is harder to decode. The $t\bar{t}$ pair decays into $W^+bW^-\bar{b}$. If both W bosons decay leptonically, one faces the problem of identifying the Higgs boson event with the missing neutrinos. If a W boson decays hadronically one has to deal with its multijet final state. Assuming that one can identify the jets associated with the W 's then one can use the momenta of W 's and b, \bar{b} to define the decay plane of $t\bar{t}$. Then the C -even angular correlation discussed in the previous section can be used to decode CP violation. In hadronic colliders this is probably too hard to achieve. In a leptonic machine this may be possible only if the Higgs boson is produced in the futuristic Next Linear Collider (NLC) (such as EE500) or $\gamma\gamma$ colliders [15].

In this section, we will give the full differential form of the decay probability, which is useful for future experimental simulation. Without loss of generality, we look at the process $H \rightarrow t\bar{t}$, with $t \rightarrow b\bar{\nu}$ and $\bar{t} \rightarrow \bar{b}\nu$. Formulas for other processes can be obtained by simple substitutions. Our result is based on the standard $V-A$ coupling for the t decay:

$$\sum_{spin} |M|^2 = 8g^8 m_t^2 b \cdot \nu \bar{\nu} \cdot \bar{\nu} \left\{ (2e \cdot H \bar{e} \cdot H - e \cdot \bar{e} H^2) A_I^2 + [2e \cdot (t - \bar{t}) \bar{e} \cdot (t - \bar{t}) - e \cdot \bar{e} (t - \bar{t})^2] A_R^2 + 4\epsilon(e, \bar{e}, t, \bar{t}) A_I A_R \right\} (m_t/v)^2 |\mathcal{P}_t \mathcal{P}_{\bar{t}} \mathcal{P}_{W^+} \mathcal{P}_{W^-}|^2. \quad (21)$$

For brevity, it is understood that we use the particle symbol to denote its corresponding momentum. The last four propagators \mathcal{P} 's can be treated quite easily in the narrow-width approximation; simply replace the virtual momenta by their on-shell values, and the integrations

over the squares of virtual momenta will give an overall factor

$$\left(\frac{\pi}{m_t \Gamma_t} \frac{\pi}{M_W \Gamma_W} \right)^2.$$

Inspecting Eq. (21), we know that $\epsilon(e, \bar{e}, t, \bar{t})$ is an appropriate CP -odd observable. Since

$$\epsilon(e, \bar{e}, t, \bar{t}) = -\frac{1}{2} M_H \mathbf{p}_e \times \mathbf{p}_{\bar{e}} \cdot (\mathbf{p}_t - \mathbf{p}_{\bar{t}}),$$

in the H rest frame, CP information resides in the relative azimuthal angle between \mathbf{p}_e and $\mathbf{p}_{\bar{e}}$ with respect to the axis $\mathbf{p}_t - \mathbf{p}_{\bar{t}}$. In contrast with Eq. (14) there is only one CP -odd angular distribution in Eq. (21). This is because the top quark is a spin- $\frac{1}{2}$ object and has only two helicity states. The corresponding CP -odd variable is thus proportional only to $\rho_+^+ - \rho_+^-$ where \pm represents the sign of the top helicities.

In the future it may be possible to detect other similar CP -odd correlations in process such as $\gamma + \gamma \rightarrow t + \bar{t} + H$.

VIII. $e^+e^- \rightarrow Z + H$

The analysis given above can be easily applied to the equally important process of the Higgs production as well. Here we shall simply investigate a typical one which is promising. The Higgs boson may be produced [16] through the Z bremsstrahlung in future high energy e^-e^+ colliders and the CP information can be carried over by the lepton pair from the Z decay. To visualize the process of $e^-(p_-)e^+(p_+) \rightarrow Z(P)H$ where $Z \rightarrow l(k_-)\bar{l}(k_+)$, we sketch the reaction configuration in Fig. 4. The final on-shell Z boson is produced at a scattering angle Θ . Its subsequent decay into a lepton pair is described by the polar angle θ and the azimuthal angle ϕ of l in the Z rest frame with its \hat{z} axis opposite to $\mathbf{P} = \mathbf{k}_- + \mathbf{k}_+$. We focus our study in the case that the Z boson, produced simultaneously with the Higgs boson, is on shell and we will use the narrow-width approximation to handle its decay process.

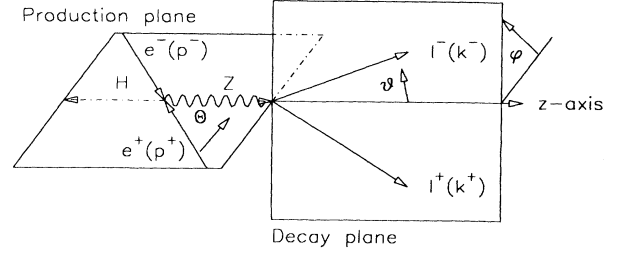


FIG. 4. The configuration of $e^-(p_-)e^+(p_+) \rightarrow Z(P)H$ where $Z \rightarrow l(k_-)\bar{l}(k_+)$. Note that ϑ is defined in the rest frame of Z and is only represented schematically here.

The virtual gauge boson in the s channel can be γ^* or Z^* . It has the momentum $P^* = p_- + p_+$. First, we write down the relevant form factors

$$iM_{Z^*(P^*, \eta_*) \rightarrow Z(P, \eta)H}$$

$$= ig_H [B\eta \cdot \eta_* + D^{Z^*} M_H^{-2} \epsilon(\eta, \eta_*, P, -P^*)], \quad (22)$$

and

$$iM_{\gamma^*(P^*, \eta_*) \rightarrow Z(P, \eta)H} = ig_H [D^{\gamma^*} M_H^{-2} \epsilon(\eta, \eta_*, P, -P^*)]. \quad (23)$$

The form factor D^{Z^*} is related to D in Sec. II when one of the Z 's becomes virtual. We put in a negative sign in front of P^* so that the momentum flow is consistent with that defined in Eq. (2). The CP -odd term of D^{γ^*} provides another source of CP violation. Other higher-dimensional CP -even terms are omitted in our discussion. The overall transition probability is

$$\begin{aligned} \sum_{\text{spin}} |M|^2 &= \left(\frac{4e^2 g_H}{s - M_Z^2} \right)^2 \left| \frac{1}{P^2 - M_Z^2 + i\Gamma_Z M_Z} \right|^2 \\ &\times \left\{ B^2 [(c_L^4 + c_R^4) p_- \cdot k_- p_+ \cdot k_+ + 2c_L^2 c_R^2 p_- \cdot k_+ p_+ \cdot k_-] \right. \\ &\quad + B\epsilon(p_-, p_+, k_-, k_+) \left[(c_L^2 \bar{c}_L^2 + c_R^2 \bar{c}_R^2) (p_- \cdot k_- + p_+ \cdot k_+) \right. \\ &\quad \left. \left. - (c_L^2 \bar{c}_R^2 + c_R^2 \bar{c}_L^2) (p_- \cdot k_+ + p_+ \cdot k_-) \right] / M_H^2 \right\}, \quad (24) \end{aligned}$$

where the D form factors have been lumped into the modified couplings

$$\begin{aligned} \bar{c}_L^2 &= c_L^2 D^{Z^*} - c_L (1 - M_Z^2/s) D^{\gamma^*}, \\ \bar{c}_R^2 &= c_R^2 D^{Z^*} - c_R (1 - M_Z^2/s) D^{\gamma^*}. \end{aligned} \quad (25)$$

The Z couplings for the electron or the muon are $c_L = (-\frac{1}{2} + \sin^2 \theta_W)/(\sin \theta_W \cos \theta_W)$ and $c_R = \tan \theta_W$. The differential cross section is

$$\frac{d\sigma}{d\cos \Theta d\cos \theta d\phi} = \frac{\int \sum |M|^2 \lambda^{\frac{1}{2}} dP^2}{8192\pi^4 s}. \quad (26)$$

The kinematic factor $\lambda^{\frac{1}{2}} = 2|\mathbf{p}_H|/\sqrt{s}$ is defined in the e^-e^+ c.m. frame. The integration over P^2 can be simplified in the narrow-width approximation. One just replaces P^2 by its on-shell value M_Z^2 and substitutes the corresponding propagator squared by $\pi/(M_Z \Gamma_Z)$. Under CP transformation, the configuration transforms as

$$(\cos \Theta, \cos \theta, \phi) \rightarrow (\cos(\pi - \Theta), \cos(\pi - \theta), -\phi). \quad (27)$$

If the differential cross section does not respect the symmetry of this transformation, CP is violated. The presence of the ϵ term in Eq. (24) will produce such CP

violation. One can integrate Eq. (24) over the polar angle θ of l , and obtain a ϕ distribution:

$$\frac{d\sigma}{d\cos\Theta d\phi} = \frac{\alpha^2}{64s} \left(\frac{\alpha M_Z (c_L^2 + c_R^2)}{6\Gamma_Z} \right) \times \frac{(c_L^2 + c_R^2) B^2}{\sin^2\theta_W \cos^2\theta_W} \frac{\Sigma_0 + \Delta\Sigma}{(1 - M_Z^2/s)^2} \lambda^{\frac{1}{2}}. \quad (28)$$

The second factor involving the Z width is just the branching fraction $B(Z \rightarrow l\bar{l})$. We purposely separate the CP -even part Σ_0 and the CP -odd part $\Delta\Sigma$ in the above formula. The CP -even part is

$$\begin{aligned} \Sigma_0 = & \lambda \sin^2\Theta + \frac{2M_Z^2}{s} (4 + \sin^2\Theta \cos 2\phi) \\ & + \frac{3\pi M_Z}{2\sqrt{s}} \left(1 + \frac{M_Z^2}{s} - \frac{M_H^2}{s} \right) \sin\Theta \cos\phi \frac{(c_L^2 - c_R^2)^2}{(c_L^2 + c_R^2)^2}. \end{aligned} \quad (29)$$

$$\mathcal{A}_{ud}^{ZH} \equiv \frac{1}{\sigma} \int_{-1}^1 d\cos\Theta \left(\int_0^\pi - \int_\pi^{2\pi} \right) d\phi \frac{d\sigma}{d\cos\Theta d\phi} = \frac{3a_1 \lambda^{\frac{1}{2}}}{4(\lambda + 12M_Z^2/s)}, \quad (32)$$

$$\begin{aligned} \mathcal{A}'_{ud}{}^{ZH} & \equiv \frac{1}{\sigma} \int_{-1}^1 d\cos\Theta \left(\int_0^{\pi/2} - \int_{\pi/2}^\pi + \int_\pi^{3\pi/2} - \int_{3\pi/2}^{2\pi} \right) d\phi \frac{d\sigma}{d\cos\Theta d\phi} \\ & = \frac{2a_2 \lambda^{\frac{1}{2}}}{\pi(\lambda + 12M_Z^2/s)}. \end{aligned} \quad (33)$$

In Fig. 5, we plot \mathcal{A}_{ud}^{ZH} and $\mathcal{A}'_{ud}{}^{ZH}$ per unit of D^{Z^*}/B or D^{γ^*}/B versus \sqrt{s} for the case $M_H = 80$ GeV. Both the CP -even and the CP -odd contributions to the cross section decrease with increasing s when s is far above the threshold. However, the CP -even part decreases slightly faster and therefore both \mathcal{A}_{ud}^{ZH} and $\mathcal{A}'_{ud}{}^{ZH}$ are increasing functions of s . In addition, \mathcal{A}_{ud}^{ZH} increases faster than

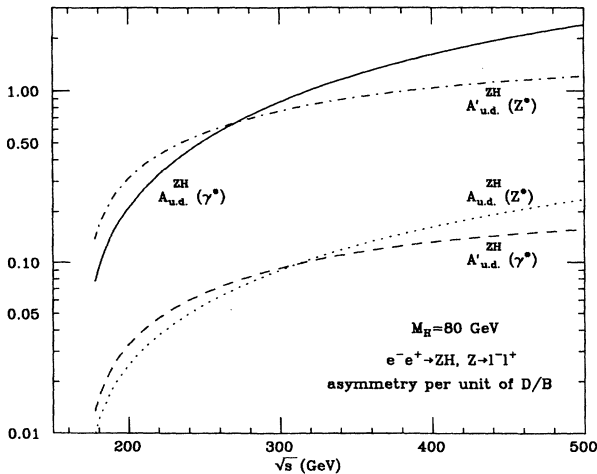


FIG. 5. \mathcal{A}_{ud}^{ZH} and $\mathcal{A}'_{ud}{}^{ZH}$ per unit of D^{Z^*}/B or D^{γ^*}/B versus \sqrt{s} for $M_H = 80$ GeV.

The CP -odd part consists of two terms,

$$\Delta\Sigma = (a_2 \sin^2\Theta \sin 2\phi + a_1 \sin\Theta \sin\phi) \lambda^{\frac{1}{2}}. \quad (30)$$

Both terms of $\sin\phi$ and $\sin 2\phi$ are CP violating. Their coefficients are

$$a_1 = \frac{(c_L^2 - c_R^2)(\bar{c}_L^2 - \bar{c}_R^2)}{B(c_L^2 + c_R^2)^2} \frac{3\pi M_Z \sqrt{s}}{4M_H^2} \left(1 + \frac{M_Z^2}{s} - \frac{M_H^2}{s} \right), \quad (31)$$

$$a_2 = \frac{2 M_Z^2 \bar{c}_L^2 + \bar{c}_R^2}{B M_H^2 c_L^2 + c_R^2}.$$

Note that $\lambda = (1 + M_Z^2/s - M_H^2/s)^2 - 4M_Z^2/s$. We can define the integrated asymmetries as before:

$\mathcal{A}'_{ud}{}^{ZH}$ because of the factor \sqrt{s} in a_1 . The process rate is proportional to s^{-1} for large s , but since every reaction, including the background, is proportional to s^{-1} , we conclude that higher energy is preferable to detect CP violation provided one can get high enough luminosity.

An analysis similar to what we have done here can also be applied to processes such as $e^-e^+ \rightarrow e^-e^+ + (\gamma^*, Z^*) + (\gamma^*, Z^*) \rightarrow e^-e^+H$. Of course even in $e^-e^+ \rightarrow ZH$ or $l\bar{l}b\bar{b}$, one can also analyze CP violation in the Higgs boson decay by zooming into the angular correlations of decay products from the final b quark jets.

IX. HIGGS MODEL

In renormalizable models the CP -odd couplings for both the $H \rightarrow ZZ$ and the $H \rightarrow W^+W^-$ modes can be induced only at the loop level because they are higher-dimensional operators. As the ϵ symbol in Eq. (3) only occurs through a fermion loop in perturbative calculations, it is natural to use the top quark as the internal fermion for its potentially large Yukawa coupling; see Eq. (20).

In this section, we will show the CP -violating form factors D and D^W in processes $H \rightarrow ZZ$ and $H \rightarrow WW$ based on the CP -nonconserving Yukawa interaction in Eq. (20). The one-loop diagram of interest is shown in Fig. 6. We use the method of dispersion relations to find the form factor D for the process $H \rightarrow ZZ$. The virtual mass s of the Higgs boson is analytically extended

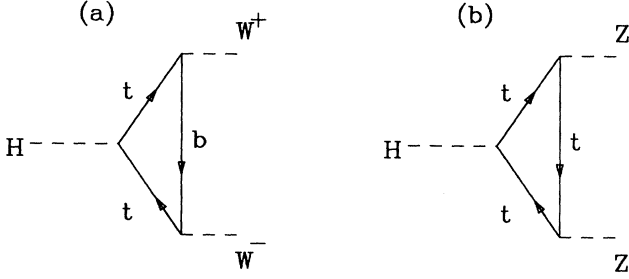


FIG. 6. The one-loop diagrams which give the CP -violating form factors D via the intermediate t quark.

beyond the on-shell value M_H^2 . In Ref. [1], we have the absorptive part

$$\begin{aligned} \text{Im } D(s) = & -\frac{3\sqrt{2}}{4\pi} G_F m_t^2 \text{Im } A \frac{\beta_t}{\beta_Z^2} \\ & \times \left[1 + \frac{K}{4\beta_t \beta_Z} - \left(1 - \frac{8}{3} \sin^2 \theta_W \right)^2 \frac{K \beta_Z}{4\beta_t} \right] \\ & \times \frac{M_H^2}{s}, \end{aligned} \quad (34)$$

with $\beta_t^2 = 1 - 4m_t^2/s$, $\beta_Z^2 = 1 - 4M_Z^2/s$, and the logarithmic factor

$$K = \ln \left| \frac{1 + \beta_Z^2 - 2\beta_t \beta_Z}{1 + \beta_Z^2 + 2\beta_t \beta_Z} \right|. \quad (35)$$

The dispersive part is obtained by the Kramers-Kronig relation

$$\text{Re } D(s) = \frac{1}{\pi} P \int_{4M_Z^2}^{\infty} \frac{\text{Im } D(s')}{s' - s} ds'. \quad (36)$$

The symbol P denotes the principal value of the singular integral. Figure 7 shows typical sizes of $\text{Re } D$ for various cases. It is of order of 10^{-2} to 10^{-3} in general. We must

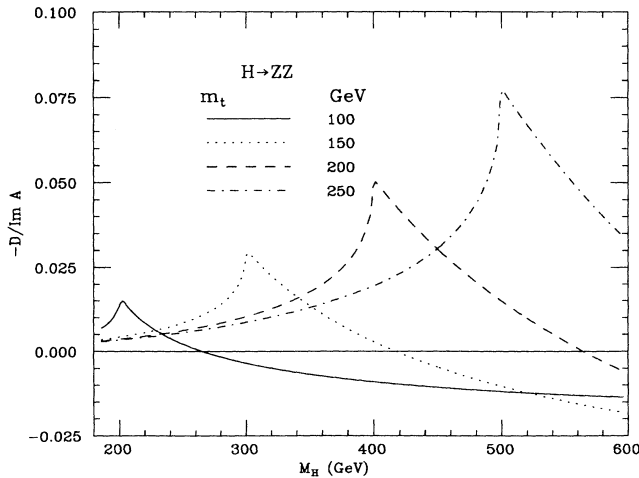


FIG. 7. Typical sizes of $\text{Re } D^W$ for various cases.

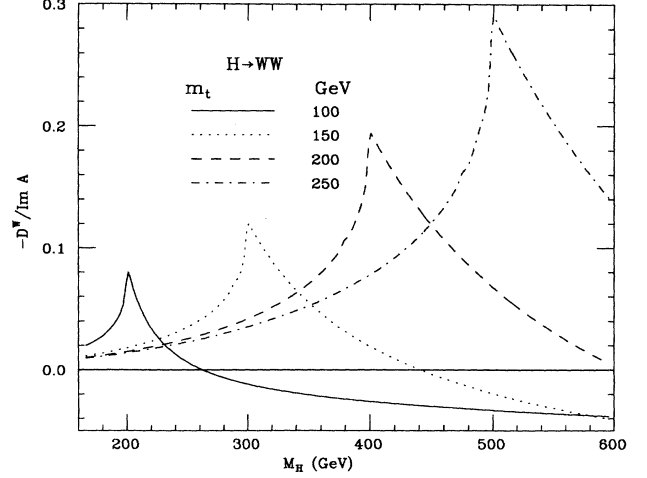


FIG. 8. Typical sizes of $\text{Re } D^W$ for various cases.

keep in mind that we only show the Higgs boson contribution in the perturbative regime. In scenarios where the Higgs bosons interact strongly, the CP -violating effect in the colliders could be much larger.

Similarly, the form factor D^W for the process $H \rightarrow W^+W^-$ can be obtained with the following absorptive amplitude in Ref. [1]:

$$\text{Im } D^W(s) = -\frac{3\sqrt{2}}{4\pi} G_F m_t^2 \text{Im } A \frac{\beta_t}{\beta_W^2} L(\beta_t, \beta_W) \frac{M_H^2}{s}, \quad (37)$$

with $\beta_W^2 = 1 - 4M_W^2/s$ and the corresponding logarithmic expression

$$L(x, y) \equiv 1 + \frac{x^2 - y^2}{2xy} \ln \left| \frac{x - y}{x + y} \right|. \quad (38)$$

Figure 8 shows typical sizes of $\text{Re } D^W$ for various cases.

X. CONCLUSION

We have shown that the angular asymmetry of the prompt lepton events originating in Higgs boson decay are sensitive to CP violation. Useful CP -odd variables are constructed in probing the dispersive parts of the CP -violating form factors. We have demonstrated that the charge identification of jets is not necessary in order to look for CP signals in $H \rightarrow ZZ$ or $H \rightarrow W^+W^-$. The expected event distributions which we give in their full differential forms are useful for future experimental simulation. Most of our approach to the problem is model independent, and so it is suitable for experimental search for the CP -odd form factors.

Note added. While we were finalizing this manuscript, we came across two recent papers on the subject of CP violation in t Higgs boson decay [17, 18]. Both papers have some overlap with our work. We have noticed that

our conclusion regarding the effect of having the identical particles ($H \rightarrow ZZ \rightarrow \bar{l}l\bar{l}$) differs from Ref. [17]. Also, some sign differences between our Eqs. (14) and (15) and the corresponding ones in [17] can be attributed to the difference in definitions of ϕ .

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