Semileptonic decays $D \to \pi(\rho)e\nu$ and $B \to \pi(\rho)e\nu$ from QCD sum rules

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We investigate the semileptonic decays of B and D mesons into π and ρ mesons, respectively, by means of QCD sum rules. We find that for the vector form factors involved the pole dominance hypothesis is valid to good accuracy with pole masses in the expected range. Pole dominance, however, does not apply to the axial form factors which results in specific predictions for the predominant polarization of the ρ meson and the shape of the lepton spectrum. For the total decay rates we find $\Gamma(\bar{B}^0 \to \pi^+ e^- \bar{\nu}) = (5.1 \pm 1.1)\,|V_{ub}|^2 \times 10^{12}\ \mathrm{s}^{-1}$, $\Gamma(D^0 \to \pi^- e^+ \nu) = (8.0 \pm 1.7)\,|V_{cd}|^2 \times 10^{10}\ \mathrm{s}^{-1}$, $\Gamma(\bar{B}^0 \to \rho^+ e^- \bar{\nu}) = (1.2 \pm 0.4\,)\,|V_{ub}|^2 \times 10^{13}\ \mathrm{s}^{-1}$, and $\Gamma(D^0 \to \rho^- e^+ \nu) = (2.4 \pm 0.7)\,|V_{cd}|^2 \times 10^{9}\ \mathrm{s}^{-1}$.

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I. INTRODUCTION

Semileptonic weak decays of heavy mesons have proved to be a very important tool in exploring the Higgs sector of the standard model and, in particular, the strength of weak decays of quarks, parametrized by the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Once the relevant hadronic matrix elements are known, the CKM matrix can be extracted from experimental measurements of the decay rates. Whereas the situation (both experimental and theoretical) now seems rather settled for the dominant decays $B \to D^{(*)}e\nu$ and $D \to Ke\nu$, at least at a level of accuracy of $\sim 10\%$ for the CKM matrix elements [1, 2], the experimental results for the Cabibbo-suppressed decays $B, D \to \pi, \rho e\nu$ at present still suffer from large statistical uncertainties. This situation, however, will improve in the near future, since the exploration of these decays is motivated by the quest for $|V_{ub}|$ which of all the CKM matrix elements still is the one most poorly known [3–5].

There exist several theoretical calculations employing relativistic [6,7] or nonrelativistic [8] quark models as well as (for D decays) lattice calculations [9, 10], and, quite recently, attempts to relate form factors of D decays to those of the corresponding B decays by means of the heavy-quark effective theory [11]. All these models are, however, for conceptional or, as for lattice calculations, for economical reasons not capable of calculating the full dynamics of the decay process, even apart from model limitations. A quite standard procedure is to determine a form factor f at some fixed point of t, the momentum transfer squared to the leptons, and then to assume either some polelike t dependence,

$$f(t) \sim \frac{1}{m_{\mathrm{pol}}^2 - t}, \tag{1.1}$$

where m_{pol} is the mass of the lowest-lying resonance coupling to the corresponding current (e.g., [6]), or an exponentially increasing form factor as in the nonrelativistic model of [8]. Indeed, at the level of a desired accuracy of, say, $\sim 20\%$ for the rates, the details of the functional dependence do not matter as long as $t_{\max}$, the maximum value of t allowed by kinematics, is much smaller than m_{pol}^2 and the form factors vary only slowly (as for $B \to D^{(*)}e\nu$). A certain deviation from that insensitivity is noticeable in the decay $D \to K^*e\nu$ where all "conventional" (quark model and lattice) calculations are not capable of reproducing either the absolute value of the rate or the small value of the ratio of rates of longitudinal to transversal polarized K^* (cf. [1]). The actual functional form of the t dependence becomes crucial if $t^{\max}/m_{\mathrm{pol}}^2 \approx 1$ as in the decays $B \to \pi, \rho e\nu$ which in the future will provide us with the most accurate information on V_{ub}. Thus a point seems to be reached where increased attention should be paid to the investigation of the t dependence of form factors.

In fact, there is another method for calculating hadronic matrix elements, including nonperturbative effects, which relies on the field-theoretical aspects and features of QCD and was designed to make maximum use of known manifestations of nonperturbative QCD: the QCD sum rules method [12]. Originally invented for the calculation of vacuum-to-meson transition amplitudes, it soon found application to the calculation of the electromagnetic form factor of the pion [13] and other meson-to-meson transition amplitudes (cf. [14] for a review). Although this method in general yields less detailed results than fine-tuned models, it has the advantage that only a small number of parameters is needed that have an evident physical meaning (e.g., quark masses) and/or characterize the nonperturbative regime of QCD (e.g., the so-called quark condensate, the order parameter of chiral symmetry breaking). Once these parameters are fixed from well known processes, they can be used to calculate, for example, heavy-meson decays.

In previous publications [15–18], we have shown that the t dependence of the form factors of $D \to K^{(*)}$ and $B \to D^{(*)}$ can reliably be calculated by means of QCD sum rules. As a general pattern to be followed, form factors determined by vector currents observe a pole-type behavior with pole masses in the expected range, those determined by axial-vector currents in general do not. In view of the above stated advantages of the QCD sum rules method, we feel it worthwile to apply it likewise to the Cabibbo-suppressed decays $B, D \to \pi, \rho e\nu$ and to go beyond the existing QCD sum rule calculations [19–24]

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which for reasons to be explained in the next section were restricted to a determination of the form factors at $t=0$. We expect to gain a reliable picture of the dynamics of these decays and well founded predictions of their decay rates within the scope of accuracy to be obtained by QCD sum rules; i.e., at the level of 20% at best.

Our paper is organized as follows. In Sec. II we present the QCD sum rules method, improve the existing calculations, and collect all necessary kinematics. In Sec. III we evaluate the sum rules and give results for the decays $D\to\pi,\rho e\nu$ and $B\to\pi,\rho e\nu$. Finally, in Sec. IV we discuss the results and compare them to experiment and to other calculations. Formulas and technical details are collected in the appendixes.

II. THE METHOD

We consider the three-point functions

$$\Pi^{\mu\nu}=i^2\int d^4x\,d^4y\,e^{-ip_Hx+i(p_H-p_L)y}\,\langle 0|Tj_L^{A\nu}(0)V_{hl}^{\mu}(y)j_H^{\dagger}(x)|0\rangle$$
$$=i(p_H+p_L)^{\mu}p_L^{\nu}\Pi_+ +\cdots \tag{2.1}$$

and

$$\Gamma^{\mu\nu}=i^2\int d^4x\,d^4y\,e^{-ip_Hx+i(p_H-p_L)y}\,\langle 0|Tj_L^{\nu}(0)(V_{hl}-A_{hl})^{\mu}(y)j_H^{\dagger}(x)|0\rangle$$
$$=ig^{\mu\nu}\Gamma_0-i(p_H+p_L)^{\mu}p_H^{\nu}\Gamma_+ -\epsilon^{\mu\nu}{}_{\rho\sigma}p_H^{\rho}p_L^{\sigma}\Gamma_V+\cdots, \tag{2.2}$$

respectively. Here $(V_{hl}-A_{hl})_\mu=\bar{l}\gamma_\mu(1-\gamma_5)h$ is the weak current mediating the weak decay of the heavy quark h with mass m_h into the light quark l with mass m_l, $j_H=\bar{q}i\gamma_5h$ is an interpolating field describing the pseudoscalar meson H (B or D) built up from h and the light antiquark $\bar{q}$, and $j_L^{(A)\nu}=\bar{q}\gamma^{\nu}(\gamma_5)l$ interpolates the light vector (pseudoscalar) meson L (π or ρ). p_H and p_L are the momenta of the heavy and the light meson, respectively. In the above equations, we have made explicit only those Lorentz structures that actually contribute to the decays under consideration.

The correlation functions (2.1) and (2.2) are functions of the scalars p_H^2, p_L^2, and $t=(p_H-p_L)^2$ and can be calculated in perturbation theory for Euclidean values $p_H^2-m_h^2,\,p_L^2-m_l^2\ll 0$. On the other hand, the singularity structure of the correlation functions is known, and thus we can represent them by double dispersion relations in p_H^2 and p_L^2, e.g.,

$$\Pi_+=\int ds_H\,ds_L\,\frac{\rho_+^{\rm phys}(s_H,s_L,t)}{(s_H-p_H^2)(s_L-p_L^2)}+\text{subtractions}. \tag{2.3}$$

The spectral function $\rho_+^{\rm phys}$ can be expressed in terms of physical observables as

$$\rho_+^{\rm phys}\sim(2\pi)^6\sum_{m,n}\int\prod_{i,j}^{m,n}\left[\frac{d^3p_{Li}}{(2\pi)^3 2E_{Li}}\frac{d^3p_{Hj}}{(2\pi)^3 2E_{Hj}}\right]\delta^4\left(q_L-\sum p_{Li}\right)\delta^4\left(q_H-\sum p_{Hj}\right)\langle 0|j_L^{A\nu}|m\rangle\langle m|V_{hl}^{\mu}|n\rangle\langle n|j_H^{\dagger}|0\rangle \tag{2.4}$$

where one has to take the appropriate Lorentz-structure on the right-hand side and $q_H^2=s_H$ and $q_L^2=s_L$. The sum runs over all m- and n-particle states coupling to the currents $j_L^{A\nu}$ and j_H, respectively. In particular, we shall single out the ground states and write

$$\rho_+^{\rm phys}\sim\langle 0|j_L^{A\nu}|L\rangle\langle L|V_{hl}^{\mu}|H\rangle\langle H|j_H^{\dagger}|0\rangle+\rho^{\rm cont}, \tag{2.5}$$

where $\rho^{\rm cont}$ contains both the contributions of higher resonances with appropriate quantum numbers and of many-particle states. The first term on the right-hand side contains exactly the quantities in which we are interested:

$$\langle\pi|V_{hl}^{\mu}|H\rangle=f_+(t)(p_H+p_\pi)_\mu+f_-(t)(p_H-p_\pi)_\mu, \tag{2.6}$$

$$\langle\rho,\lambda|V_{hl}^{\mu}-A_{hl}^{\mu}|H\rangle=-i(m_H+m_\rho)A_1(t)\epsilon_\mu^{*(\lambda)}+\frac{iA_2(t)}{m_H+m_\rho}(\epsilon^{*(\lambda)}p_H)(p_H+p_\rho)$$
$$+\frac{iA_3(t)}{m_H+m_\rho}(\epsilon^{*(\lambda)}p_H)(p_H-p_\rho)+\frac{2V(t)}{m_H+m_\rho}\epsilon_\mu{}^{\nu\rho\sigma}\epsilon_\nu^{*(\lambda)}p_{H\rho}p_{\rho\sigma}. \tag{2.7}$$

These are the relevant matrix elements governing the hadronic part of the decays in question, decomposed in terms of the form factors $f_\pm$, A_i, and V, where m_H and m_ρ are the masses of the H and the ρ meson, respectively; λ denotes the polarization state of the ρ. In the limit of vanishing lepton mass, the form factors f_- and A_3 do not contribute to the decay rates and henceforth will not be considered. Expressed in terms of the above form factors, the spectra with respect to the electron energy E measured in the rest system of H read

$$\frac{d\Gamma(H\to\pi^+e^-\bar\nu)}{dE} = \frac{G_F^2|V_{hl}|^2}{16\pi^3 m_H}\int_0^{t_{\max}} dt\{2E(m_H^2-m_\pi^2+t)-m_H(t+4E^2)\}f_+^2(t), \tag{2.8}$$

$$\frac{d\Gamma(H\to\rho^+e^-\bar\nu)}{dE} = \frac{G_F^2|V_{hl}|^2}{128\pi^3 m_H^2}\int_0^{t_{\max}} dt\, t\{(1-\cos\theta)^2H_-^2+(1+\cos\theta)^2H_+^2+2(1-\cos^2\theta)H_0^2\} \tag{2.9}$$

with the helicity amplitudes

$$H_\pm = (m_H+m_\rho)A_1(t)\mp\frac{\lambda^{1/2}}{m_H+m_\rho}V(t), \tag{2.10}$$

$$H_0 = \frac{1}{2m_\rho\sqrt{t}}\left\{(m_H^2-m_\rho^2-t)(m_H+m_\rho)A_1(t)-\frac{\lambda}{m_H+m_\rho}A_2(t)\right\}. \tag{2.11}$$

$t_{\max}$, the maximum value of t, the invariant mass squared of the lepton pair, is given by

$$t_{\max} = 2E\left(m_H-\frac{m_{\pi,\rho}^2}{m_H-2E}\right). \tag{2.12}$$

θ is the angle between the ρ and the charged lepton in the $(e^-\bar\nu)$ c.m. system and given by

$$\cos\theta = \frac{1}{\sqrt{\lambda}}(m_H^2-m_\rho^2+t-4m_HE) \tag{2.13}$$

where $\lambda=(m_H^2+m_\rho^2-t)^2-4m_H^2m_\rho^2$.

Returning to (2.1) and (2.2), it was the idea of Shifman, Vainshtein, and Zakharov [12] to account for nonperturbative corrections to correlation functions by expressing them via an operator product expansion (OPE) including terms that vanish in the perturbative vacuum, but acquire finite values in the QCD vacuum. These so-called condensates characterize the long-distance behavior of the correlation function and we are led to write, e.g.,

$$\Pi_+(p_H^2,p_L^2,t) = \sum_n \Pi_+^{(n)}(p_H^2,p_L^2,t)\langle 0|\mathcal{O}_n|0\rangle. \tag{2.14}$$

The Wilson coefficients $\Pi_+^{(n)}$ can be calculated with the aid of perturbation theory for negative values of $p_H^2-m_h^2$ and $p_L^2-m_l^2$. The $\langle 0|\mathcal{O}_n|0\rangle$ are vacuum expectation values of gauge invariant operators, the above-mentioned condensates (cf. Appendix A). The first term in the series just covers usual perturbation theory, the others are nonperturbative corrections. In our analysis we will take into account the lowest-dimensional condensates up to dimension 6, where we improve existing calculations [25] by the inclusion of the contribution of the gluon condensate (Appendix A). Equating (2.3) and the OPE (2.14) yields expressions for the form factors determining (2.1) and (2.2) in terms of QCD parameters (such as quark masses) and condensates. Before, however, we can start to evaluate these sum rules, we have to specify how to treat ρ^{cont} in (2.5). As for that, we employ the argument of quark-hadron-duality (e.g., [12]) and model ρ^{cont} by the contribution of usual perturbation theory above some thresholds s_H^0 and s_L^0:

$$\rho^{\text{cont}} = \rho^{\text{pert}}[1-\Theta(s_L^0-s_L)\,\Theta(s_H^0-s_H)]. \tag{2.15}$$

The calculation of ρ^{pert} for $t>0$ involves some delicate points connected with the possibility of the spectral function to become singular. For the discussion of the additional "non-Landau" contributions caused by these singularities we refer to [15]. In the numerical analysis we will tacitly include those contributions whenever necessary.

The dependence of the sum rule on the continuum model as well as the error induced by truncating the OPE series can be diminished by the application of a Borel transformation. For an arbitrary function of Euclidean momentum, $f(P^2)$ with $P^2=-p^2$, that transformation is defined by

$$\hat f \equiv \widehat{B}_{P^2}(M^2)\,f = \lim_{\substack{P^2\to\infty,N\to\infty\\ P^2/N=M^2\ \text{fixed}}} \frac{1}{N!}(-P^2)^{N+1}\frac{d^{N+1}}{(dP^2)^{N+1}}f, \tag{2.16}$$

where M^2 is a new variable, called Borel parameter. For a typical term appearing in the OPE, the transformation yields

$$\widehat{B}_{P^2}(M^2)\frac{1}{(p^2-m^2)^n} = \frac{1}{(n-1)!}(-1)^n\frac{1}{(M^2)^n}e^{-m^2/M^2}. \tag{2.17}$$

Since condensates with high dimension get multiplied by high powers of (p^2-m^2) in the denominator, their contributions get suppressed by factorials. In addition, the contribution of higher resonances and the continuum, ρ^{cont}, gets exponentially suppressed relative to the contribution of the ground state, which is just the desired effect.

We are now in a position to write down sum rules for the relevant form factors. From (2.1) and (2.2) we find

$$f_+^{H\to L}(t)=\frac{m_h}{f_H f_\pi m_H^2}\exp\left\{\frac{m_H^2}{M_h^2}+\frac{m_\pi^2}{M_u^2}\right\}M_h^2M_u^2\hat{\Pi}_+(M_h^2,M_u^2,t), \tag{2.18}$$

$$A_1^{H\to L}(t)=\frac{m_h}{f_H f_\rho (m_H+m_\rho) m_H^2 m_L}\exp\left\{\frac{m_H^2}{M_h^2}+\frac{m_\rho^2}{M_u^2}\right\}M_h^2M_u^2\hat{\Gamma}_0(M_h^2,M_u^2,t), \tag{2.19}$$

$$A_2^{H\to L}(t)=\frac{m_h(m_H+m_\rho)}{f_H f_\rho m_H^2 m_L}\exp\left\{\frac{m_H^2}{M_h^2}+\frac{m_\rho^2}{M_u^2}\right\}M_h^2M_u^2\hat{\Gamma}_+(M_h^2,M_u^2,t), \tag{2.20}$$

$$V(t)^{H\to L}=\frac{m_h(m_H+m_\rho)}{2f_H f_\rho m_H^2 m_L}\exp\left\{\frac{m_H^2}{M_h^2}+\frac{m_\rho^2}{M_u^2}\right\}M_h^2M_u^2\hat{\Gamma}_V(M_h^2,M_u^2,t). \tag{2.21}$$

Here M_h and M_u are the Borel parameters that come from the Borelization of the correlation functions in the virtualities p_H^2 and p_L^2 of the heavy and the light quark, respectively. Parts of the explicit formulas for the correlation functions can be found in [15]. For the present analysis, we in addition have calculated the contributions of the gluon condensate and the contribution of the four-quark condensate to f_+; the formulas can be found in the Appendixes. Note that we have expressed the vacuum-to-meson transition amplitudes in terms of the corresponding leptonic decay constants as

$$\langle 0|\,\bar{d}i\gamma_5 b\,|\,\bar{B}^0\,\rangle=f_B\frac{m_B^2}{m_b}, \tag{2.22}$$

$$\langle 0\,|\,\bar{d}\gamma_\nu u\,|\,\rho^+,\lambda\,\rangle=f_\rho m_\rho \epsilon_\nu^{(\lambda)}, \tag{2.23}$$

$$\langle 0|\,\bar{d}\gamma_\nu\gamma_5 u\,|\,\pi^+\,\rangle=if_\pi p_{\pi\nu}. \tag{2.24}$$

The question of how to treat these quantities will be discussed in the next section.

III. EVALUATION OF THE SUM RULES

In the numerical evaluation of the sum rules (2.18) to (2.21) we use the following values of the condensates at a renormalization scale of 1 GeV (taken from [14] except for the value of the quark condensate, where we take the average value of the results obtained in [12] and [26]):

$$\begin{aligned}\langle\bar{q}q\rangle(1\,\text{GeV})&=(-0.24\,\text{GeV})^3,\\ \left\langle\frac{\alpha_s}{\pi}G^2\right\rangle&=0.012\,\text{GeV}^4\,,\\ \langle\bar{q}\sigma gGq\rangle(1\,\text{GeV})&=0.8\,\text{GeV}^2\langle\bar{q}q\rangle(1\,\text{GeV}),\\ \pi\alpha_s\left\langle\bar{q}\gamma^\tau\lambda^A q\sum_{u,d,s}\bar{q}\gamma_\tau\lambda^A q\right\rangle&\approx-\frac{16}{9}\,\pi\alpha_s\langle\bar{q}q\rangle^2,\\ 4\pi\alpha_s\langle\bar{d}\bar{u}ud\rangle&\approx4\pi\alpha_s\langle\bar{q}q\rangle^2.\end{aligned} \tag{3.1}$$

We use leading-order anomalous dimensions of the quark and the mixed condensate to evaluate them at a scale μ which is given by the harmonic mean of the Borel parameters, $\mu^2=\sqrt{M_h^2M_u^2}$. For the four-quark condensates we assume vacuum saturation. Since their contributions are tiny, we neglect the scale dependence. In general the sensitivity of the form factors to the actual values of the condensates will be smaller than 10% when changing $(-\langle\bar{q}q\rangle)^{1/3}$ by 10 MeV and most pronounced for the axial form factors A_2. The smallness of the contributions of the contributions of the four-quark condensates indicates that higher order power corrections are well under control. Concerning quark masses, we put the masses of the u and the d quark to zero, for the heavy quarks we use the renormalization-group and -scheme invariant pole mass. Its connection to the running mass in the modified minimal subtraction ($\overline{\text{MS}}$) scheme is given by (for scales $\mu\ll m_{\overline{\text{MS}}}$)

$$m_{\text{pole}}=m_{\overline{\text{MS}}}(\mu)\left\{1+\frac{\alpha_s(\mu)}{\pi}\left(\frac{4}{3}+\ln\frac{\mu^2}{m_{\overline{\text{MS}}}^2}\right)\right\}. \tag{3.2}$$

The numerical values are (cf. [14]; a recent determination of m_b is given in [27])

$$m_b=(4.6\text{–}4.8)\,\text{GeV},\qquad m_c=(1.3\text{–}1.4)\,\text{GeV}. \tag{3.3}$$

For the leptonic decay constants we use the experimental values $f_\pi=0.133$ GeV and $f_\rho=0.216$ GeV. For f_B and f_D we employ two-point sum rules (e.g., [28]), discarding radiative corrections. We expect the accuracy of the sum rules for the form factors to be increased by that, since both in the limit of infinitely heavy quarks and for the matrix-element $\langle B|V_\mu|B\rangle$, where charge conservation fixes the form factor at zero recoil, QCD sum rules yield the correct normalization independent of the values of quark masses, continuum thresholds, and Borel parameter [17, 29, 30], provided the continuum thresholds in the two-point and the three-point sum rules are chosen equal and the Borel parameter in the three-point sum rule takes twice the value of that of the two-point sum rule. We will take these prescriptions over to the case where the outgoing meson is light, and actually the sensitivity of the resulting sum rules to m_b or m_c is greatly reduced as compared to the sum rule for f_B or f_D. In addition, the effect of the unknown radiative corrections to the three-point function should tend to cancel against the radiative corrections to the two-point function. Concluding, we take both the range of Borel parameters, the "sum rule window," and the values of the continuum thresholds from the two-point sum rules, i.e., we evaluate (2.18) to (2.21) in the range $7\,\text{GeV}^2\le M_b^2\le10\,\text{GeV}^2$ and $2\,\text{GeV}^2\le M_c^2\le4\,\text{GeV}^2$ and for continuum thresholds $s_D^0=(6\text{–}7)\,\text{GeV}^2$, $s_B^0=(34\text{–}36)\,\text{GeV}^2$, $s_\pi^0=(0.75\text{–}1.0)\,\text{GeV}^2$, $s_\rho^0=(1.25\text{–}1.5)\,\text{GeV}^2$. In addition, we choose a fixed ratio of the Borel parameters,

$$\frac{M_b^2}{M_u^2}=4,\quad\frac{M_c^2}{M_u^2}=2. \tag{3.4}$$

This procedure ensures that perturbative and nonperturbative corrections in both the heavy and the light channel are equally weighted. The sum rules are rather insensitive to the actual value of that ratio, and changing for example M_b^2/M_u^2 from 3 to 5 results in changes of the results of at most 10%.

Our aim is to extract the t dependence of the form factors from the sum rules, and thus we are restricted to a range of values where the correlation function can be expected to be reliable in that variable. That is, we have to stay approximately 1 GeV^2 below the perturbative cut starting at $t = m_{b,c}^2$. Thus we can trust the sum rules up to $t \approx 20\,\mathrm{GeV}^2$ for the B decays and $t \approx 0.9\,\mathrm{GeV}^2$ for the D decays. The maximum values allowed by kinematics are $t_{\max}^{B\to\pi} = 26.4\,\mathrm{GeV}^2$, $t_{\max}^{B\to\rho} = 20.3\,\mathrm{GeV}^2$, $t_{\max}^{D\to\pi} = 3.0\,\mathrm{GeV}^2$, and $t_{\max}^{D\to\rho} = 1.2\,\mathrm{GeV}^2$, so apart from $B \to \rho$ we cannot cover the full range of t. Although the accessible range is sufficient to determine the shape of the form factor and the total rate, it is not for the calculation of the electron spectrum. We thus extrapolate the form factors to $t_{\max}$ to get the electron spectrum for large values of the electron energy E. This procedure does not introduce too large an uncertainty for $D \to \rho$ since according to the above remarks at least 80% of the integration range in t is covered, and for $D \to \pi$ and $B \to \pi$ we will find a pole-type behavior of the form factors which facilitates the applicability of the extrapolation.

In Fig. 1(a) the form factor $f_+^{D\to\pi}(t = 0)$ is shown as function of the Borel parameter M_c^2. The different curves correspond to different choices of the set of input parameters. To be specific, we use $m_c = 1.3\,\mathrm{GeV}$, $s_D^0 = 6\,\mathrm{GeV}^2$, $s_\pi^0 = 0.75\,\mathrm{GeV}^2$ (set C1), $m_c = 1.3\,\mathrm{GeV}$, $s_D^0 = 6\,\mathrm{GeV}^2$, $s_\pi^0 = 1\,\mathrm{GeV}^2$ (set C2), $m_c = 1.4\,\mathrm{GeV}$, $s_D^0 = 7\,\mathrm{GeV}^2$, $s_\pi^0 = 0.75\,\mathrm{GeV}^2$ (set C3), $m_c = 1.4\,\mathrm{GeV}$, $s_D^0 = 7\,\mathrm{GeV}^2$, $s_\pi^0 = 1\,\mathrm{GeV}^2$ (set C4). The value of s_D^0 is taken as the best-fit continuum threshold for the sum rules for f_D, s_π^0 is taken from [12]. In the "sum rule window" $2\,\mathrm{GeV}^2 \leq M_c^2 \leq 4\,\mathrm{GeV}^2$ the $f_+^{D\to\pi}(0)$ is quite stable and the dependence on the values of both the mass of the c quark and the continuum thresholds as represented by the spread of curves is well under control, yielding $f_+(0) = 0.5 \pm 0.1$ where the error is an educated guess based on both the dependence of the sum rule on the input parameters and the intrinsic uncertainty of the whole method. Perturbation theory and the quark condensate give the dominant contribution, the other condensates contributing at the level of ~10%. Thus the series of power corrections is well under control.

$f_+^{B\to\pi}(0)$ as function of the Borel parameter M_b^2 is shown in Fig. 1(b). Here we use the parameter sets B1 ($m_b = 4.6\,\mathrm{GeV}$, $s_B^0 = 36\,\mathrm{GeV}^2$, $s_\pi^0 = 0.75\,\mathrm{GeV}^2$), B2 ($m_b = 4.8\,\mathrm{GeV}$, $s_B^0 = 36\,\mathrm{GeV}^2$, $s_\pi^0 = 1\,\mathrm{GeV}^2$), B3 ($m_b = 4.8\,\mathrm{GeV}$, $s_B^0 = 34\,\mathrm{GeV}^2$, $s_\pi^0 = 0.75\,\mathrm{GeV}^2$), B4 ($m_b = 4.8\,\mathrm{GeV}$, $s_B^0 = 34\,\mathrm{GeV}^2$, $s_\pi^0 = 1\,\mathrm{GeV}^2$). Again the value is remarkably stable against variation in the quark mass, in the continuum thresholds, and the Borel parameter. From Fig. 1(b) we find $f_+^{B\to\pi}(0) = 0.26 \pm 0.02$. This value is higher than obtained in [16] which is due to the contribution of the gluon condensate not included there.

In Fig. 2(a) we show $f_+^{D\to\pi}(t)$ as function of t, normalized to its value at $t = 0$, which representation emphasizes the differences in shape. We have chosen $M_c^2 = 3\,\mathrm{GeV}^2$ and give curves for all parameter sets. We find a rise in t which is very well compatible with a pole-type behavior as suggested by the pole dominance hypothesis (1.1). From a pole fit we get $m_{\mathrm{pol}} = (1.95 \pm 0.10)\,\mathrm{GeV}$ (including all sets and Borel parameters within the window). If pole dominance were exactly valid, the pole mass would be $m_{D^*} = 2.01\,\mathrm{GeV}$, so QCD sum rules confirm pole dominance for $D \to \pi$.

Pole dominance is likewise valid for the $B \to \pi$ transition whose normalized form factor is depicted in Fig. 2(b) as function of t and for all parameter sets at $M_b^2 = 8\,\mathrm{GeV}^2$. Pole fits yield pole masses of ~5.1 GeV for the sets B1 and B2 and ~5.2 GeV for B3 and B4. From that we are forced to exclude the lower value of the b-quark mass, $m_b = 4.6\,\mathrm{GeV}$, from our analysis (since the form factor would become singular at $t_{\max}$) and stick to $m_b = 4.8\,\mathrm{GeV}$. The "physical" pole is at $m_{B^*} = 5.33\,\mathrm{GeV}$ which nicely agrees with the fit value $(5.25 \pm 0.10)\,\mathrm{GeV}$ from sets B3 and B4.

In Fig. 3 we show the electron spectra $d\Gamma/dE$ as func-

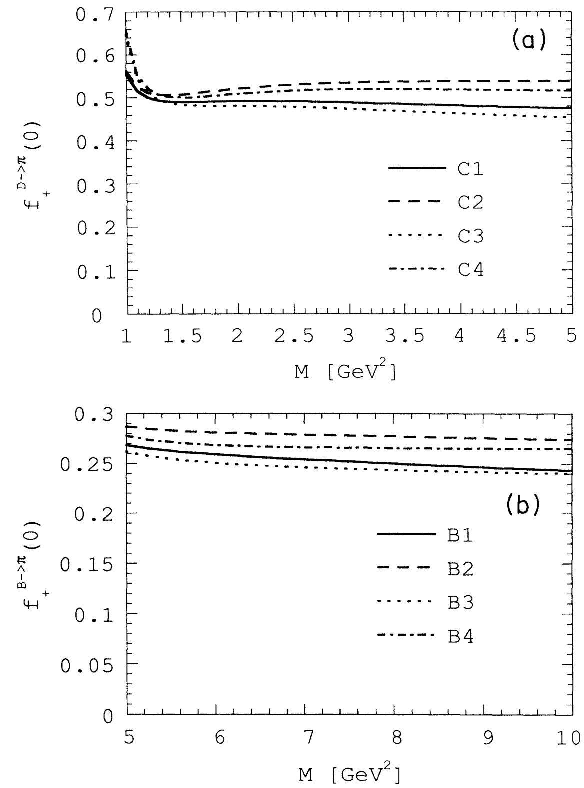

FIG. 1. (a) The form factor $f_+^{D\to\pi}(0)$ as function of the Borel parameter M_c^2. The parameter sets in the legend are $m_c = 1.3\,\mathrm{GeV}$, $s_D^0 = 6\,\mathrm{GeV}^2$, $s_\pi^0 = 0.75\,\mathrm{GeV}^2$ (set C1); $m_c = 1.3\,\mathrm{GeV}$, $s_D^0 = 6\,\mathrm{GeV}^2$, $s_\pi^0 = 1\,\mathrm{GeV}^2$ (set C2); $m_c = 1.4\,\mathrm{GeV}$, $s_D^0 = 7\,\mathrm{GeV}^2$, $s_\pi^0 = 0.75\,\mathrm{GeV}^2$ (set C3); $m_c = 1.4\,\mathrm{GeV}$, $s_D^0 = 7\,\mathrm{GeV}^2$, $s_\pi^0 = 1\,\mathrm{GeV}^2$ (set C4). (b) $f_+^{B\to\pi}(0)$ as function of the Borel parameter M_b^2 with the parameter sets B1 ($m_b = 4.6\,\mathrm{GeV}$, $s_B^0 = 36\,\mathrm{GeV}^2$, $s_\pi^0 = 0.75\,\mathrm{GeV}^2$), B2 ($m_b = 4.8\,\mathrm{GeV}$, $s_B^0 = 36\,\mathrm{GeV}^2$, $s_\pi^0 = 1\,\mathrm{GeV}^2$), B3 ($m_b = 4.8\,\mathrm{GeV}$, $s_B^0 = 34\,\mathrm{GeV}^2$, $s_\pi^0 = 0.75\,\mathrm{GeV}^2$), B4 ($m_b = 4.8\,\mathrm{GeV}$, $s_B^0 = 34\,\mathrm{GeV}^2$, $s_\pi^0 = 1\,\mathrm{GeV}^2$).

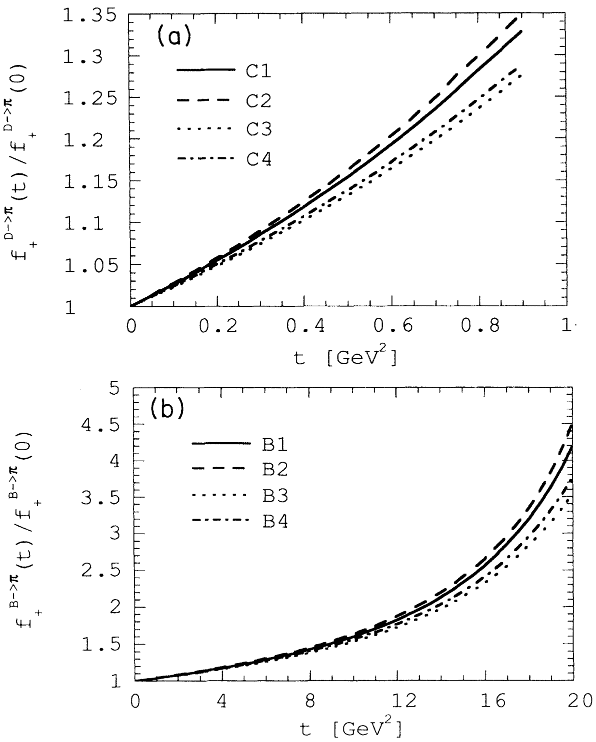

FIG. 2. (a) The form factor $f_+^{D\to\pi}(t)/f_+^{D\to\pi}(0)$ as function of t for all parameter sets at $M_c^2 = 3\,\text{GeV}^2$. A pole fit yields $m_{\text{pol}} = (1.95 \pm 0.10)\,\text{GeV}$, the maximum physical value of t is $t^{\max} = 2.96\,\text{GeV}^2$. (b) Like (a) for $f_+^{B\to\pi}(t)/f_+^{B\to\pi}(0)$. $m_{\text{pol}} = (5.25 \pm 0.10)\,\text{GeV}$ (for B3 and B4), $t^{\max} = 26.4\,\text{GeV}^2$.

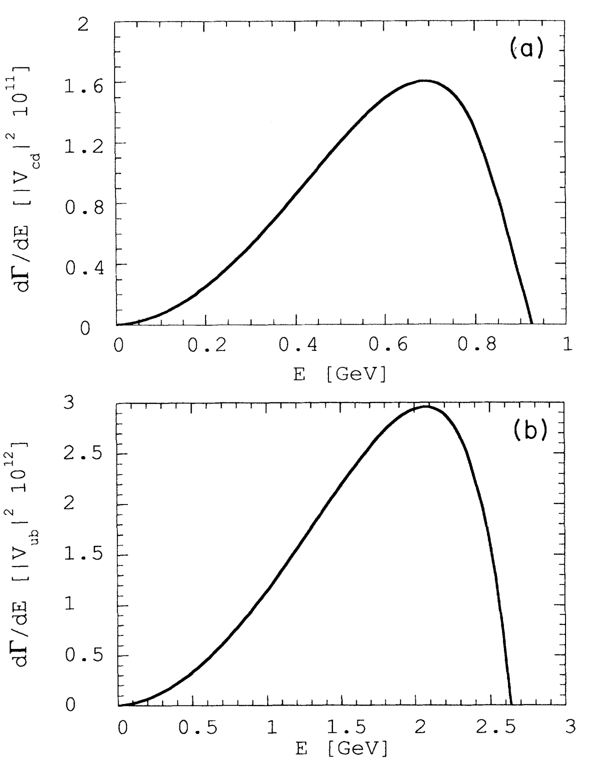

FIG. 3. The electron spectra $d\Gamma/dE$ as function of the electron energy E for (a) the decay $D \to \pi e\nu$ (set C1, $M_c^2 = 3\,\text{GeV}^2$), (b) $B \to \pi e\nu$ (set B3, $M_b^2 = 8\,\text{GeV}^2$).

tions of the electron energy E which can be obtained from the form factors $f_+^{D\to\pi}$ and $f_+^{B\to\pi}$, extrapolated up to $t_{\max}$ according to pole dominance.

Let us now turn turn to the decays $H \to \rho e\nu$. In Fig. 4 we show the form factors of $D \to \rho$ at $t = 0$ as functions of the Borel parameter for the parameter sets C5 ($m_c = 1.3\,\text{GeV}$, $s_D^0 = 6\,\text{GeV}^2$, $s_\rho^0 = 1.25\,\text{GeV}^2$), C6 ($m_c = 1.3\,\text{GeV}$, $s_D^0 = 6\,\text{GeV}^2$, $s_\rho^0 = 1.5\,\text{GeV}^2$), C7 ($m_c = 1.4\,\text{GeV}$, $s_D^0 = 7\,\text{GeV}^2$, $s_\rho^0 = 1.25\,\text{GeV}^2$), C8 ($m_c = 1.4\,\text{GeV}$, $s_D^0 = 7\,\text{GeV}^2$, $s_\rho^0 = 1.5\,\text{GeV}^2$). All form

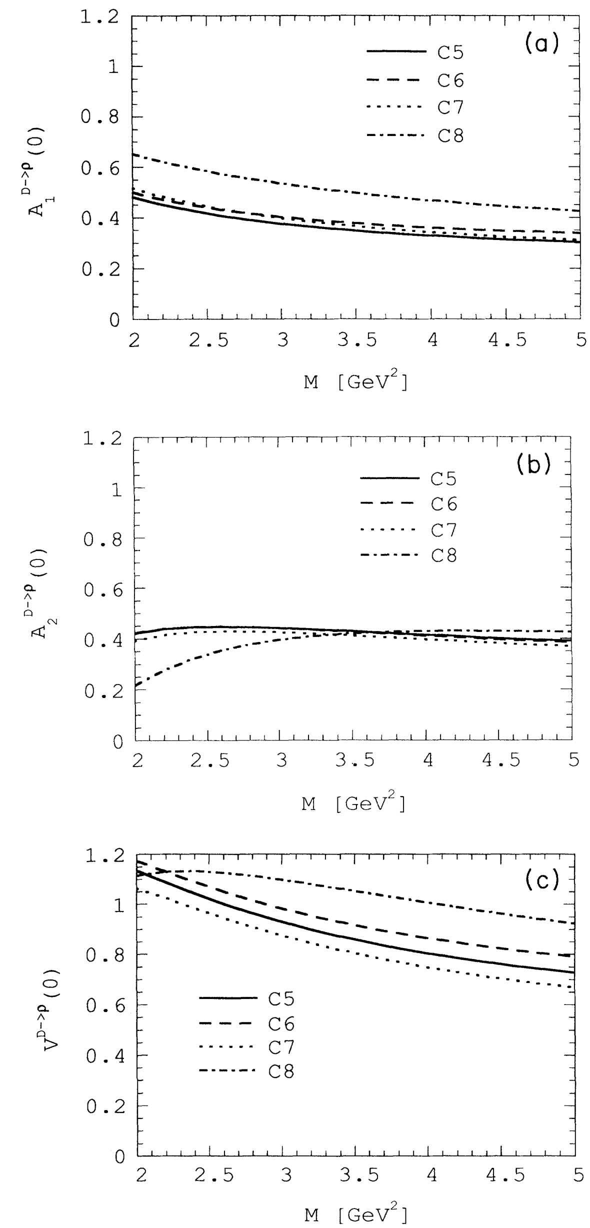

FIG. 4. The form factors of $D \to \rho$ at $t = 0$ as functions of the Borel parameter M_c^2. The parameter sets are C5 ($m_c = 1.3\,\text{GeV}$, $s_D^0 = 6\,\text{GeV}^2$, $s_\rho^0 = 1.25\,\text{GeV}^2$), C6 ($m_c = 1.3\,\text{GeV}$, $s_D^0 = 6\,\text{GeV}^2$, $s_\rho^0 = 1.5\,\text{GeV}^2$), C7 ($m_c = 1.4\,\text{GeV}$, $s_D^0 = 7\,\text{GeV}^2$, $s_\rho^0 = 1.25\,\text{GeV}^2$), C8 ($m_c = 1.4\,\text{GeV}$, $s_D^0 = 7\,\text{GeV}^2$, $s_\rho^0 = 1.5\,\text{GeV}^2$). (a) $A_1^{D\to\rho}(0)$, (b) $A_2^{D\to\rho}(0)$, (c) $V^{D\to\rho}(0)$.

factors are quite stable and we find $A_1^{D\to\rho}(0) = 0.5 \pm 0.2$, $A_2^{D\to\rho}(0) = 0.4 \pm 0.1$ and $V^{D\to\rho}(0) = 1.0 \pm 0.2$ where as in the previous cases the error is intended to include systematic uncertainties.

For $B \to \rho$ we find from Fig. 5 $A_1^{B\to\rho}(0) = 0.5 \pm 0.1$, $A_2^{B\to\rho}(0) = 0.4 \pm 0.2$, $V^{B\to\rho}(0) = 0.6 \pm 0.2$ with the parameter sets B5 ($m_b = 4.6\,\text{GeV}$, $s_B^0 = 36\,\text{GeV}^2$, $s_\rho^0 = 1.25\,\text{GeV}^2$), B6 ($m_b = 4.6\,\text{GeV}$, $s_B^0 = 36\,\text{GeV}^2$, $s_\rho^0 = 1.5\,\text{GeV}^2$), B7 ($m_b = 4.8\,\text{GeV}$, $s_B^0 = 34\,\text{GeV}^2$, $s_\rho^0 = 1.25\,\text{GeV}^2$), B8 ($m_b = 4.8\,\text{GeV}$, $s_B^0 = 34\,\text{GeV}^2$, $s_\rho^0 = 1.5\,\text{GeV}^2$). All these form factors depend only slightly on quark masses and continuum thresholds and are stable in the Borel parameter, except for $A_2^{B\to\rho}(0)$. Here we observe for B7 and B8 a rather strong dependence on M_b^2 the reason being the extremely small contribution of perturbation theory which is only of order 10%.

In Fig. 6 the normalized form factors of the $D \to \rho$ transition are plotted as functions of t for $M_c^2 = 3\,\text{GeV}^2$ and all sets of parameters. We find a *decrease* of

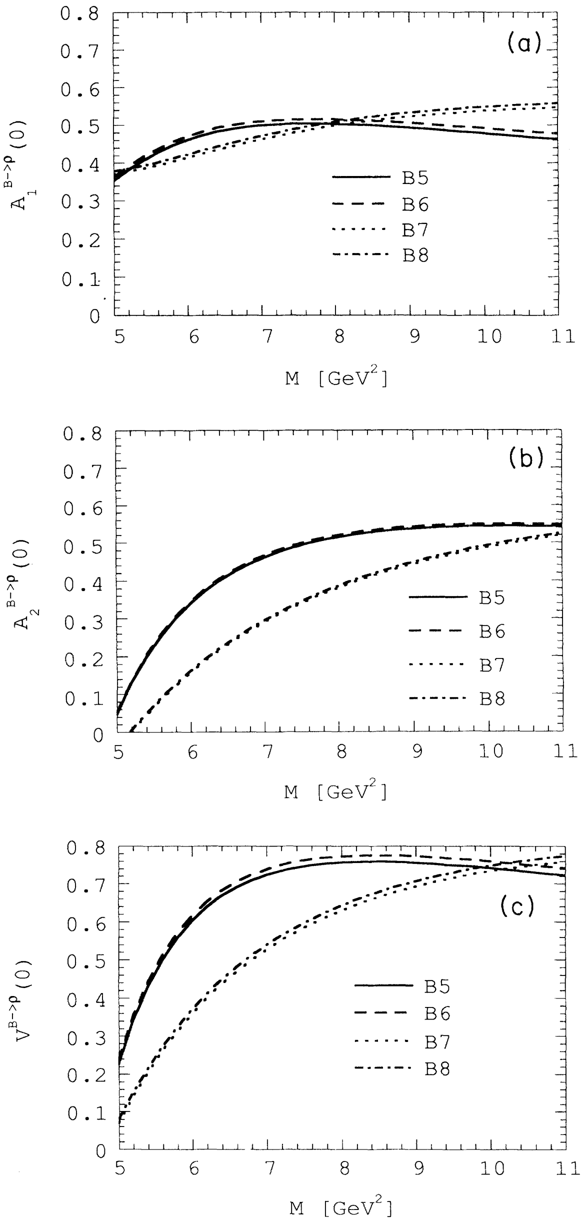

FIG. 5. The form factors of $B \to \rho$ at $t = 0$ as functions of the Borel parameter M_b^2. The parameter sets are B5 ($m_b = 4.6\,\text{GeV}$, $s_B^0 = 36\,\text{GeV}^2$, $s_\rho^0 = 1.25\,\text{GeV}^2$), B6 ($m_b = 4.6\,\text{GeV}$, $s_B^0 = 36\,\text{GeV}^2$, $s_\rho^0 = 1.5\,\text{GeV}^2$), B7 ($m_b = 4.8\,\text{GeV}$, $s_B^0 = 34\,\text{GeV}^2$, $s_\rho^0 = 1.25\,\text{GeV}^2$), B8 ($m_b = 4.8\,\text{GeV}$, $s_B^0 = 34\,\text{GeV}^2$, $s_\rho^0 = 1.5\,\text{GeV}^2$). (a) $A_1^{B\to\rho}(0)$, (b) $A_2^{B\to\rho}(0)$, (c) $V^{B\to\rho}(0)$.

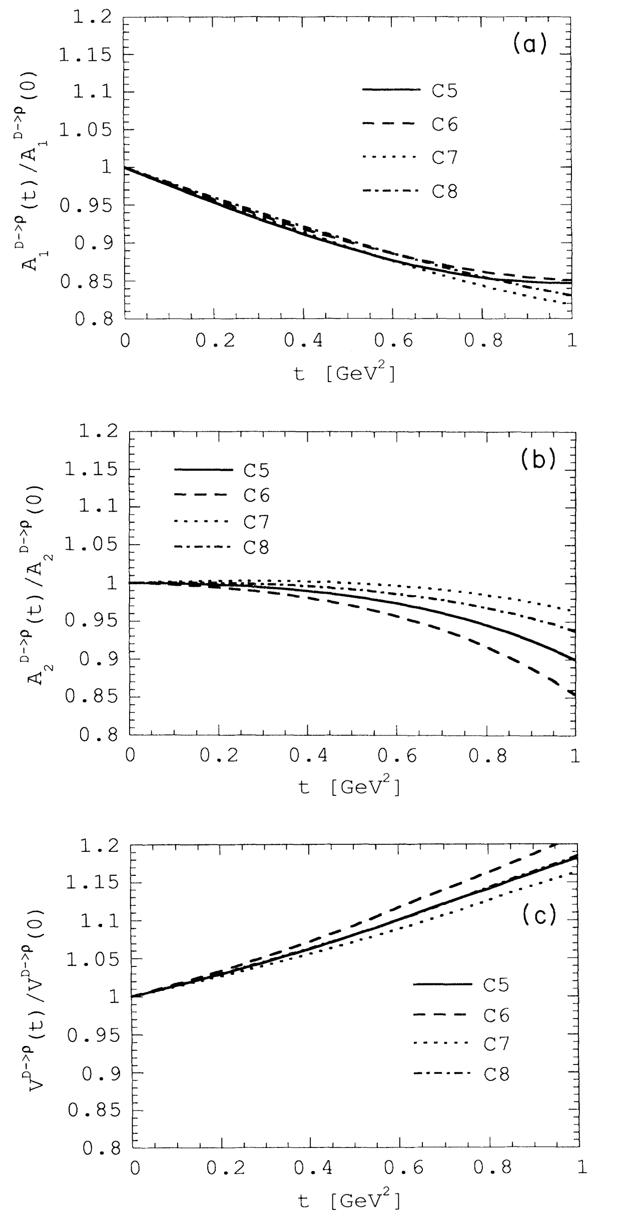

FIG. 6. The form factors of $D \to \rho$, normalized to their values at $t = 0$, as functions of t for all parameter sets and $M_c^2 = 3\,\text{GeV}^2$. A pole fit is sensible only for the vector form factor and yields $m_{\text{pol}} = (2.5 \pm 0.2)\,\text{GeV}$. $t_{\max} = 1.21\,\text{GeV}^2$. (a) $A_1^{D\to\rho}(t)/A_1^{D\to\rho}(0)$, (b) $A_2^{D\to\rho}(t)/A_2^{D\to\rho}(0)$, (c) $V^{D\to\rho}(t)/V^{D\to\rho}(0)$.

$A_1^{D\to\rho}(t)$ in t which is nearly independent of the parameter set used. This behavior is in clear contradiction with pole dominance, which predicts an increase determined by the pole mass $m_{D^{1+}} = 2.42\,\text{GeV}$ corresponding to $A_1^{D\to\rho}(t)/A_1^{D\to\rho}(0) = 1.21$. A similar behavior is encountered for $A_2^{D\to\rho}$ where we find a decrease of about 10% at $t = 1\,\text{GeV}^2$ depending on the parameter set used. For the vector form factor we have an increase in t with a best-fit pole mass of $m_{\text{pol}} = (2.5 \pm 0.2)\,\text{GeV}$, which is a little bit larger than predicted by pole dominance.

For the normalized form factor $A_1^{B\to\rho}(t)/A_1^{B\to\rho}(0)$, shown in Fig. 7(a) at $M_b^2 = 8\,\text{GeV}^2$ for all parameter sets, we find a rather unexpected shape with a minimum at $t \approx 15\,\text{GeV}^2$. Formally, this minimum is due to the interplay between decreasing contributions of perturbation theory and quark condensate and an increasing one of the gluon condensate which becomes effective at large t. For $A_2^{B\to\rho}(t)/A_2^{B\to\rho}(0)$ [Fig. 7(b)] we find a moderate increase in t which at large t is again compensated by a negative contribution of the gluon condensate. For

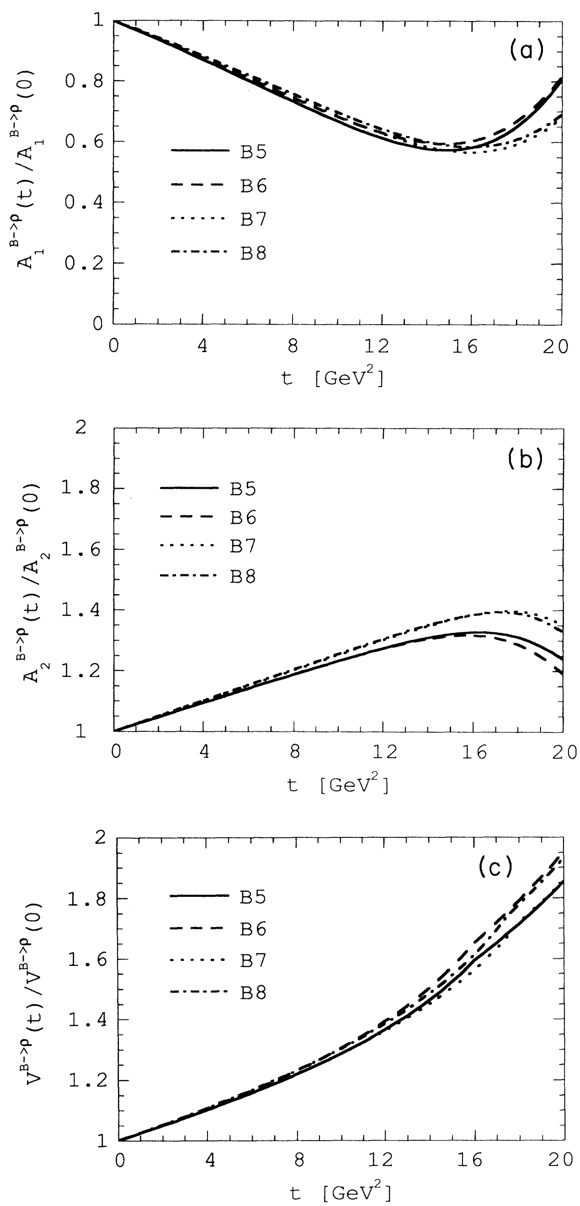

FIG. 7. The form factors of $B \to \rho$, normalized to their values at $t = 0$, as functions of t for all parameter sets and $M_b^2 = 8\,\text{GeV}^2$. A pole fit is sensible only for the vector form factor and yields $m_{\text{pol}} = (6.6 \pm 0.6)\,\text{GeV}$. $t_{\max} = 20.3\,\text{GeV}^2$. (a) $A_1^{B\to\rho}(t)/A_1^{B\to\rho}(0)$, (b) $A_2^{B\to\rho}(t)/A_2^{B\to\rho}(0)$, (c) $V^{B\to\rho}(t)/V^{B\to\rho}(0)$.

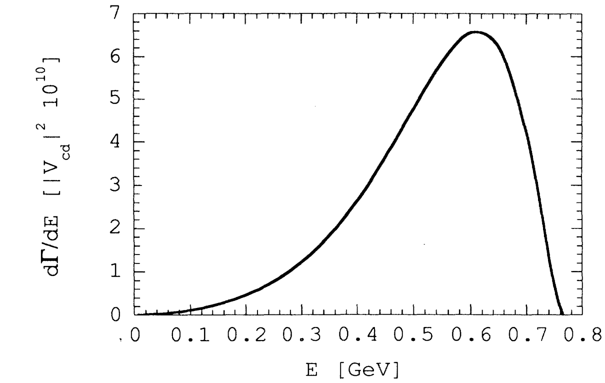

FIG. 8. The electron spectrum $d\Gamma/dE$ for $D \to \rho e\nu$ as function of the electron energy E (set C5, $M_c^2 = 3\,\text{GeV}^2$).

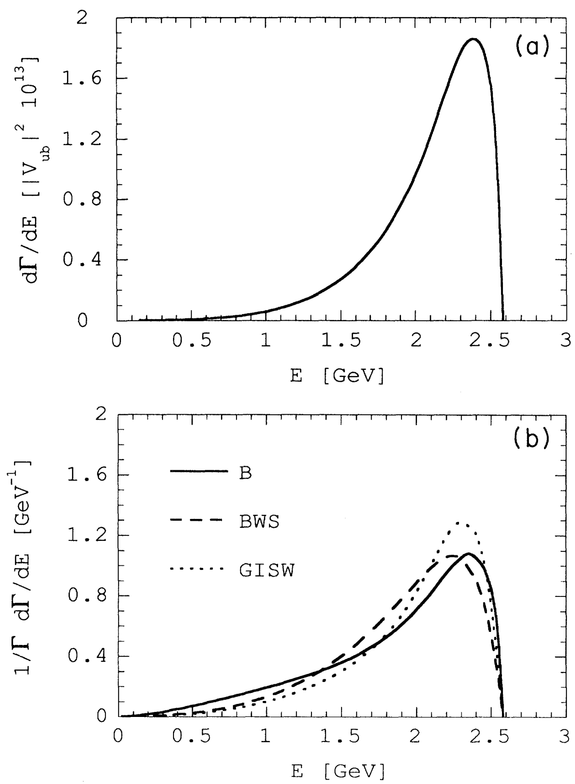

FIG. 9. (a) The electron spectrum $d\Gamma/dE$ for $B \to \rho e\nu$ as function of the electron energy E (set B8, $M_b^2 = 8\,\text{GeV}^2$). (b) Comparison of the spectra $\frac{1}{\Gamma}\frac{d\Gamma}{dE}$ as functions of E as obtained in this paper [parameters as in (a)] with those obtained in the WSB [6] and the ISGW models [8]. The chosen normalization emphasizes the shape of the spectra.

TABLE I. The form factors of the $c \to d$ transitions at $t = 0$ in different models.

Reference	$f_+^{D\to\pi}$	$A_1^{D\to\rho}$	$A_2^{D\to\rho}$	$V^{D\to\rho}$
This paper	0.5±0.1	0.5±0.2	0.4±0.1	1.0±0.2
[19][a]	0.7±0.2			
[20][a]	0.75±0.05			
[6][b]	0.69	0.78	0.92	1.23
[8][b]	0.51	0.59	0.23	1.34
[9][c]	0.58±0.09	0.45±0.04	0.02±0.26	0.78±0.12
[10][c,d]	0.84±0.12±0.35	$0.65\pm0.15\pm^{+0.24}_{-0.23}$	$0.59\pm0.31\pm^{+0.28}_{-0.25}$	1.07±0.49±0.35
[11][e]	0.79	0.55	0.28	1.01
[31][f]	$0.80^{+0.21}_{-0.14}$			

[a] QCD sum rules.
[b] Quark model.
[c] Lattice calculation.
[d] First error statistical, second systematical.
[e] HQET + chiral perturbation theory; value of f_+ taken from experiment.
[f] Experiment (using pole dominance).

$V^{B\to\rho}(t)/V^{B\to\rho}(0)$ Fig. 7(c) shows the usual increase in t corresponding to a pole mass of (6.6 ± 0.6) GeV, about 1 GeV larger than predicted by pole dominance.

Finally, in Fig. 8 we show the electron-spectrum $d\Gamma/dE$ of the decay $D \to \rho e\nu$ as function of the electron energy E for set C5 and $M_c^2 = 3\,\text{GeV}^2$, where the form factors are extrapolated in the range $t \geq 1\,\text{GeV}^2$. Figure 9(a) shows the electron spectrum $d\Gamma/dE$ of $B \to \rho e\nu$ as function of E for set B7 and $M_b^2 = 8\,\text{GeV}^2$ as quite sharp and concentrated around large electron energies.

IV. RESULTS AND DISCUSSION

In the previous section we have given a careful analysis of the semileptonic heavy-light decays $D \to \pi e\nu$, $B \to \pi e\nu$, $D \to \rho e\nu$, and $B \to \rho e\nu$. We have put some stress on the calculation of the t dependence of the form factors which for the vector form factors in general can well be described by a pole-dominance formula, whereas the axial form factors tend to decrease in t and even develop extrema. The numerical results of our calculation as well as other models are collected in the tables. The form factors at $t = 0$ can be found in Tables I and II, the rates in Tables III and IV. The rates were calculated either using pole dominance (as indicated in the table notes) or some other model for the t dependence. In addition to the total rates we give for $H \to \rho e\nu$ the ratios Γ_L/Γ_T and Γ_+/Γ_- where the index denotes the polarization state of the ρ (longitudinal, transversal, positive, and negative helicity, respectively). The corresponding electron spectra are shown in Figs. 3 ($D, B \to \pi e\nu$), 8 ($D \to \rho e\nu$), and 9(a) ($B \to \rho e\nu$).

The only decay where a comparison with experiment is possible so far is $D \to \pi e\nu$. In addition there exist several model calculations in literature, using QCD sum rules [19, 20], quark models [6–8], and some lattice calculations [9, 10]. One calculation relying on the heavy-quark effective theory [11] takes the experimental result [31] as input to their values of the form factors of $B \to \pi, \rho e\nu$. The theoretical predicitions of $\Gamma(D \to \pi e\nu)$ differ by a factor of two, and assuming $|V_{cd}| = 0.22$, which can be inferred from the unitarity of the CKM matrix with high accuracy, we find that the central value of our rate is two standard deviations smaller than the experimental value. This discrepancy is not strong enough to be conclusive and might be due to the neglect of radiative corrections to our sum sules. Still further experimental effort in improving statistics is to be desired to clarify this point.

TABLE II. The form factors of the $b \to u$ transitions at $t = 0$ in different models.

Reference	$f_+^{B\to\pi}$	$A_1^{B\to\rho}$	$A_2^{B\to\rho}$	$V^{B\to\rho}$
This paper	0.26±0.02	0.5±0.1	0.4±0.2	0.6±0.2
[21][a,b]	0.26±0.01			
[22][a,b]		0.96±0.15	1.21±0.18	1.27±0.12
[23][a]	0.23±0.02	0.35±0.16	0.42±0.12	0.47±0.14
[24][a]	0.4±0.1			
[6][c]	0.33	0.28	0.28	0.33
[8][c]	0.09	0.05	0.02	0.27
[11][d]	0.89	0.21	0.20	1.04

[a] QCD sum rules.
[b] Analysis suffering from a missing factor 12 in the perturbative contribution.
[c] Quark model.
[d] HQET + chiral perturbation theory.

TABLE III. Decay rates of the $c \to d$ transitions in units $|V_{cd}|^2\, 10^{11}\ \mathrm{s}^{-1}$. Γ_L denotes the portion of the rate with a longitudinal polarized ρ, Γ_T with a transversely polarized ρ, Γ_+ with a ρ with positive, Γ_- with a ρ with negative helicity.

Reference	$\Gamma(D^0 \to \pi^- e^+ \nu)$	$\Gamma(D^0 \to \rho^- e^+ \nu)$	Γ_L/Γ_T	Γ_+/Γ_-
This paper	0.80±0.17	0.24±0.07	1.31±0.11	0.24±0.03
[19]	$1.45^{+0.95}_{-0.71}$			
[20][a]	$1.66^{+0.23}_{-0.21}$			
[22][b]		1.4±1.0	0.9	
[6][a]	1.41	1.38	0.91	0.19
[7][c]	1.41	1.40	0.80	0.13
[8]	0.77	1.35	1.33	0.11
[9][a,d]	$0.99^{+0.34}_{-0.28}$	0.83±0.19	1.86±0.56	0.16
[10][a,d]	$2.09^{+2.24}_{-1.44}$	1.09	1.10	0.18
[11][a,e]	1.9	0.93	1.40	0.14
[31][f]	$\lvert 0.22/V_{cd}\rvert^2 \times (1.9^{+1.1}_{-0.6})$	$1/\lvert V_{cd}\rvert^2\, \Gamma(D^+ \to \rho^0 e^+ \nu) < 0.71 \times 10^{11}\ \mathrm{s}^{-1}$		

[a] Rates calculated using pole dominance with $m_{1^-} = 2.01$ GeV, $m_{1^+} = 2.42$ GeV.
[b] No values of form factors given.
[c] Values of form factors at $t = 0$ identical to [6].
[d] Values without errors from central values of Table I.
[e] Value of $\Gamma(D^0 \to \pi^- e^+ \nu)$ taken from experiment.
[f] Experiment.

For $D \to \rho e\nu$ experiment only has set an upper bound for the total rate so far [22]. Our value is a factor of 5 smaller than these. For the ratio $\Gamma(D \to \rho)/\Gamma(D \to \pi)$ we get 0.3 which again is smaller than the predictions in other models which yield a maximum value of 1.8 [8]. We recall that the corresponding ratio for the Cabibbo-favored decays, $\Gamma(D \to K^*)/\Gamma(D \to K)$, is approximately 0.5 [1] and that we do not expect flavor SU(3) to be broken by a factor of 2 or more. The form factors at $t = 0$ roughly agree in all models except for the lattice calculation [10], which predicts vanishing $A_2^{D\to\rho}(0)$ and a small value of $V^{D\to\rho}(0)$ yielding a large value of Γ_L/Γ_T.

For the $b \to u$ decays we do not dare to quote any experimental upper bound for the total rates due to the uncertainty in $|V_{ub}|$ (but cf. [5]). We remark that the total rates for $B \to \pi e\nu$ summarized in Table IV and obtained by QCD sum rules [22–24], the quark models [6–8], and the HQET calculation [11] differ by a factor of 26. For $B \to \rho e\nu$ this value shrinks to 4. That spread in predictions clearly shows the neccesity for an accurate investigation of the t dependence of the form factors that we have concentrated on in this paper. With the t dependence obtained by QCD sum rules we obtain $\Gamma(\bar{B}^0 \to \rho^+ e\bar{\nu}) = (1.2 \pm 0.4)\, |V_{ub}|^2 \times 10^{13}\ \mathrm{s}^{-1}$ where the ρ has mainly negative helicity. Furthermore, we find $\Gamma(B \to \rho)/\Gamma(B \to \pi) = 2.4$ which is smaller than all other model predictions ranging from 3.1 to 11 except for [11] which predicts 0.6. In Fig. 9(b) we give the electron spectrum $1/\Gamma\, d\Gamma/dE$ for $B \to \rho e\nu$ as obtained in this paper [same parameters as in Fig. 9(a)], in the Wirbel-Stech-Bauer (WSB) model [6] (using pole dominance) and the nonrelativistic Isgur-Scora-Grinstein-Wise (ISGW) model [8]. The chosen normalization emphasizes the difference in shape rather than in the absolute normalization. Although the WSB spectrum is softer in the end-point region above the threshold for

TABLE IV. Decay rates of the $b \to u$ transitions in units $|V_{ub}|^2\, 10^{13}\ \mathrm{s}^{-1}$. Γ_L denotes the portion of the rate with a longitudinal polarized ρ, Γ_T with a transversely polarized ρ, Γ_+ with a ρ with positive, Γ_- with a ρ with negative helicity.

Reference	$\Gamma(\bar{B}^0 \to \pi^+ e^- \bar{\nu})$	$\Gamma(\bar{B}^0 \to \rho^+ e^- \bar{\nu})$	Γ_L/Γ_T	Γ_+/Γ_-
This paper	0.51±0.11	1.2±0.4	0.06±0.02	0.007±0.004
[21]	0.68±0.23			
[22]		0.77±0.42		
[23][a]	0.302± 0.005	3.3±0.3	$0.88^{+0.39}_{-0.20}$	$0.12^{+0.04}_{-0.02}$
[24][b]	1.45±0.59			
[6][a]	0.74	2.6	1.34	0.16
[7][c]	0.74	2.30	0.54	0.02
[8]	0.21	1.63	0.75	0.08
[11][a]	5.4	3.4	0.36	0.14

[a] Rates calculated using pole dominance with $m_{1^-} = 5.33$ GeV, $m_{1^+} = 5.71$ GeV.
[b] Rate calculated using a modified pole dominance with $m_{1^-} = 5.33$ GeV.
[c] Values of form factors at $t = 0$ identical to [6].

charm production, the only region where $b \to u$ transitions can be observed, our spectrum and that of ISGW are nearly indistinguishable for $E \leq 2.4\,\text{GeV}$. If, however, a detection of the polarization of the ρ was feasible, one could test the considerable deviations of the corresponding spectra in the different models. We predict the ρ to have predominantly negative helicity (as indicated by the very small values of Γ_L/Γ_T and Γ_+/Γ_-) whereas in other models the ratio Γ_L/Γ_T is closer to one.

In Fig. 10 we give a comparison of the inclusive $b \to u$ semileptonic spectrum calculated by means of QCD sum rules in [32] with the exclusive decay spectra $B \to \pi e\nu$ and $B \to \rho e\nu$ calculated with the same parameters ($m_b = 4.8\,\text{GeV}$, $s_B^0 = 34\,\text{GeV}^2$, $M_b^2 = 8\,\text{GeV}^2$). Figure 10(a) shows the spectrum in the rest frame of the decaying B meson, Fig. 10(b) in the laboratory system of an e^+e^- collider operating on the $\Upsilon(4S)$ resonance. From both we find that at high electron energies $B \to \rho e\nu$ constitutes nearly the whole differential inclusive rate, so it is worthwhile to concentrate on measurements of the exclusive channels, where theoretical predictions are still not at their best precision, but are much more accurate than calculations of the inclusive spectrum (cf. [32]).

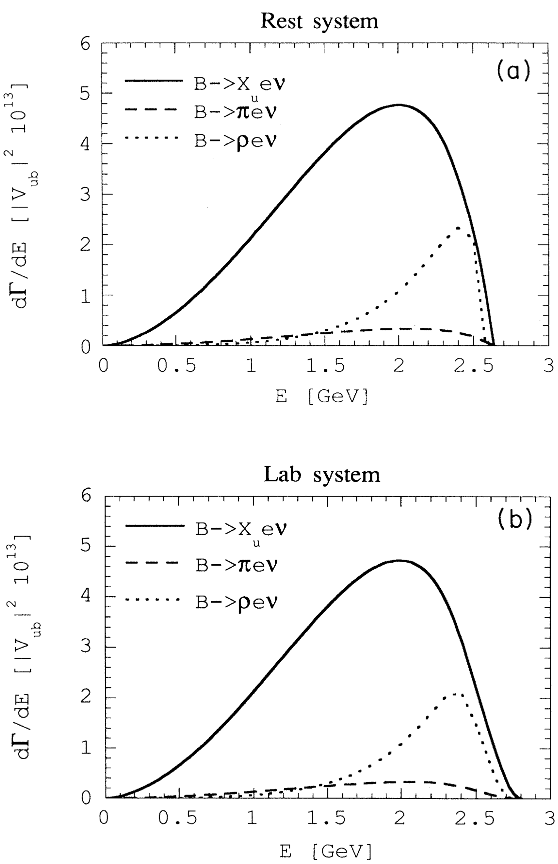

FIG. 10. Comparison of the spectra $d\Gamma/dE$ as functions of E of the exclusive decays $B \to \pi, \rho e\nu$ [parameters as in Figs. 3 and 9(a)] with the spectrum of the inclusive decay $B \to X_u e\nu$, taken from [32] and calculated with the same parameters. (a) rest system of the decaying B; (b) lab system of a collider working at the $\Upsilon(4S)$ resonance.

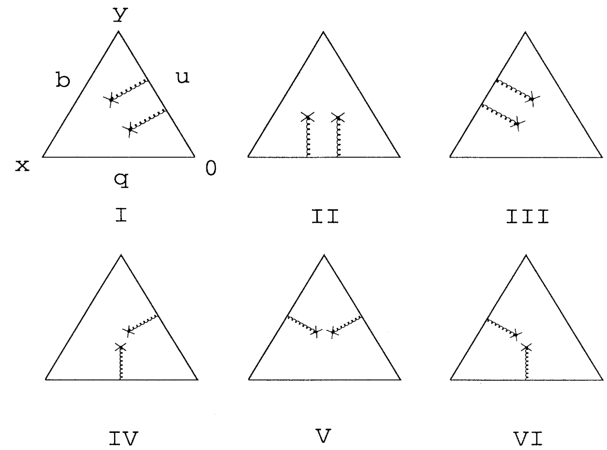

FIG. 11. Diagrams contributing to the Wilson coefficient of the gluon condensate. Lines with a cross denote vacuum expectation values. 0, x, y are space-time coordinates; b, u, q denote quark flavors, q being a light quark (u or d). The weak vertex is at y.

ACKNOWLEDGMENTS

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APPENDIX A: THE WILSON COEFFICIENT OF THE GLUON CONDENSATE

In the following we present a technique for calculating Borel transformed Wilson coefficients directly from the loop integrals. This method does not allow for the subtraction of continuum contributions, which, however, does no harm in our case as the total contribution of the gluon condensate to the three-point sum rule is small by itself ($\lesssim 10\%$), and so is its continuum portion. Besides, one would expect typical continuum contributions to show up as incomplete Γ functions in the Wilson coefficients, which, however, are absent in our formulas (i.e., in the sum of all diagrams, but are encountered in each diagram separately). Thus one is led to conclude that those contributions are actually absent in the processes under consideration.

We calculate the diagrams shown in Fig. 11 that contribute to the Wilson coefficient of the gluon condensate $\langle \alpha_s G^2/\pi \rangle$ in the fixed point gauge:

$$x^\mu A_\mu^a(x) = 0 \tag{A1}$$

with the gluon field A_μ^a, $a \in \{1, 2, \ldots, 8\}$. For massless light quarks and with the coordinates chosen as indicated in the first diagram, diagrams I and II evaluate to zero. Note that for massless light quarks there is no mixing of the gluon with the quark condensate.

In the evaluation of the remaining diagrams we encounter integrals of the type (since the Borel transform removes UV divergences, there is no need for dimensional regularization of these divergences and we thus stay with four-dimensional integrals)

$$I_{\mu_1\mu_2\cdots\mu_n}(a,b,c)=\int\frac{d^4k}{(2\pi)^4}\,\frac{k_{\mu_1}k_{\mu_2}\cdots k_{\mu_n}}{[k^2]^a[(k+p_\rho)^2]^b[(k+p_B)^2-m_b^2]^c}. \tag{A2}$$

Although the sum of all diagrams is IR convergent, IR divergent terms occur at each step of the calculation and need proper regularization. We let the mass of the u quark be finite in the denominator of its propagator, $m_u^2>0$ (but let $m_u=0$ in the traces) and regularize the singularities in the q-quark line (for $a=2$) by shifting the power of the q-quark propagator to $2-\epsilon$ (which from a technical point of view is simpler than introducing dimensional regularization). This procedure has the advantage that all integrals with b, $c>1$ can be obtained from the case $b=c=1$ by taking derivatives with respect to the quark masses:

$$I_{\mu_1\mu_2\cdots\mu_n}(a,b,c)=\frac{1}{\Gamma(b)\Gamma(c)}\,\frac{d^{b-1}}{d(m_u^2)^{b-1}}\,\frac{d^{c-1}}{d(m_b^2)^{c-1}}\,I_{\mu_1\mu_2\cdots\mu_n}(a,1,1). \tag{A3}$$

Continuing to Euclidean space-time and employing the Schwinger representation for propagators,

$$\frac{1}{[P^2+m^2]^a}=\frac{1}{\Gamma(a)}\int_0^\infty d\alpha\,\alpha^{a-1}\,e^{-\alpha(P^2+m^2)}, \tag{A4}$$

we find for the scalar integral $n=0$ with $a=b=c=1$ (with capital letters denoting Euclidean momenta),

$$I(1,1,1)=-i\int_0^\infty d\alpha\,d\beta\,d\gamma\int\frac{d^4\widetilde{K}}{(2\pi)^4}\,\exp\left(-\Sigma\widetilde{K}^2-\frac{\alpha\beta}{\Sigma}P_\rho^2-\frac{\alpha\gamma}{\Sigma}P_B^2-\frac{\beta\gamma}{\Sigma}T-\gamma m_b^2\right) \tag{A5}$$

where

$$\widetilde{K}=K+\frac{1}{\Sigma}(\beta P_\rho+\gamma P_B), \tag{A6a}$$

$$\Sigma=\alpha+\beta+\gamma, \tag{A6b}$$

$$T=-t. \tag{A6c}$$

The above representation proves very convenient for applying the Borel transformation with

$$\widehat{B}_{P^2}(M^2)\,e^{-\alpha P^2}=\delta(1-\alpha M^2). \tag{A7}$$

From that, we get

$$\hat{I}(1,1,1)\equiv\widehat{B}_{P_\rho^2}(M_u^2)\,\widehat{B}_{P_B^2}(M_b^2)I(1,1,1)=\frac{i}{16\pi^2}\,\frac{1}{M_b^2+M_u^2}\,e^{-t/(M_b^2+M_u^2)}\,\mathrm{Ei}(-z) \tag{A8}$$

where the exponential integral function is given by

$$\mathrm{Ei}(x)=-\int_{-x}^\infty dt\,\frac{e^{-t}}{t} \tag{A9}$$

and

$$z=\frac{m_u^2}{M_u^2}+\frac{m_b^2}{M_b^2}-\frac{t}{M_b^2+M_u^2}. \tag{A10}$$

Actually IR divergent diagrams only occur for the scalar case where we find

$$\hat{I}(2-\epsilon,1,1)=\frac{i}{16\pi^2}\,\frac{e^{-m_u^2/M_u^2-m_b^2/M_b^2}}{M_b^2M_u^2}\left\{\frac{1}{\epsilon}+1-2\gamma_E-\ln\left(-\frac{\mu^2}{M_u^2}-\frac{\mu^2}{M_b^2}\right)-\ln z\right\}. \tag{A11}$$

Here μ is some arbitrary scale introduced to render the canonical dimension of the integral, which, however, cancels in the complete expressions for Wilson coefficients, as it should.

For larger values of n, we get

$$\hat{I}_{\mu_1}(1,1,1)=\frac{-i}{16\pi^2}\left(\frac{M_b^2M_u^2}{M_b^2+M_u^2}\right)^2\frac{1}{M_u^2M_b^2}\left(\frac{p_{\rho\mu_1}}{M_u^2}+\frac{p_{B\mu_1}}{M_b^2}\right)$$
$$\times\left(e^{-m_b^2/M_b^2-m_u^2/M_u^2}+e^{-t/(M_b^2+M_u^2)}[1+z\,\mathrm{Ei}(-z)]\right), \tag{A12}$$

$$\hat{I}_{\mu_1}(2,1,1)=\frac{i}{16\pi^2}\frac{1}{M_b^2+M_u^2}\left(\frac{p_{\rho\mu_1}}{M_u^2}+\frac{p_{B\mu_1}}{M_b^2}\right)e^{-t/(M_b^2+M_u^2)}\,\mathrm{Ei}(-z), \tag{A13}$$

$$\hat{I}_{\mu_1\mu_2}(a\le 2,1,1)=\frac{i}{16\pi^2}\frac{(-1)^a}{\Gamma(a)}\left(\frac{M_b^2M_u^2}{M_b^2+M_u^2}\right)^{4-a}\frac{e^{-m_b^2/M_b^2-m_u^2/M_u^2}}{M_u^2M_b^2}\,\Gamma(4-a)$$
$$\times\left\{-\frac{1}{2(3-a)}\left(\frac{1}{M_b^2}+\frac{1}{M_u^2}\right)g_{\mu_1\mu_2}\,U(3-a,0;z)\right.$$
$$\left.+\left(\frac{p_{\rho\mu_1}p_{\rho\mu_2}}{M_u^4}+\frac{p_{\rho\mu_1}p_{B\mu_2}+p_{B\mu_1}p_{\rho\mu_2}}{M_b^2M_u^2}+\frac{p_{B\mu_1}p_{B\mu_2}}{M_b^4}\right)U(4-a,1;z)\right\}, \tag{A14}$$

where we have continued back to Minkowski space. U is the confluent hypergeometric function defined as

$$U(i,j;x) = \frac{1}{\Gamma(i)} \int_0^\infty dt\,(1+t)^{j-i-1}\,t^{i-1}\,e^{-xt}. \tag{A15}$$

In addition, we use

$$\widehat{B}_{P_\rho^2}(M_u^2)\,\widehat{B}_{P_B^2}(M_b^2)\,[p_\rho^2]^{m_1}[p_B^2]^{m_2} I_{\mu_1\mu_2\cdots\mu_n}(a,b,c) = [M_u^2]^{m_1}[M_b^2]^{m_2}\frac{d^{m_1}}{d(M_u^2)^{m_1}}\frac{d^{m_2}}{d(M_b^2)^{m_2}}[M_u^2]^{m_1}[M_b^2]^{m_2}\hat{I}_{\mu_1\mu_2\cdots\mu_n}(a,b,c). \tag{A16}$$

We now decompose the Lorentz invariants $\hat\Lambda$ occurring in the Borel transformed correlation functions (2.1) and (2.2) as

$$\hat\Lambda = \sum_n \hat\Lambda^{(n)}\langle\mathcal{O}_n\rangle \tag{A17}$$

where $\langle\mathcal{O}_1\rangle = \langle\mathbb{1}\rangle = 1$, $\langle\mathcal{O}_3\rangle = \langle\bar qq\rangle$, $\langle\mathcal{O}_4\rangle = \langle\alpha_s G^2/\pi\rangle$, $\langle\mathcal{O}_5\rangle = \langle\bar q\sigma gGq\rangle$, $\langle\mathcal{O}_{6_1}\rangle = \pi\alpha_s\langle\bar q\gamma^\tau\lambda^A q\sum_{u,d,s}\bar q\gamma_\tau\lambda^A q\rangle$, and $\langle\mathcal{O}_{6_2}\rangle = 4\pi\alpha_s\langle\bar d\bar u ud\rangle$ are the condensates taken into account. The formulas for $n\in\{1,3,5,6\}$ can be found in [15] (with the same notations), where vacuum saturation for the condensates with dimension 6 is assumed; the formulas for $n=4$ are new and read for the relevant invariants:

$$\hat\Pi_+^{(4)} = \frac{m_b e^{-m_b^2/M_b^2}}{M_b^2M_u^2}\left[\frac{1}{96M_b^2}+\frac{1}{48M_u^2}-\frac{m_b^2M_u^2\{M_b^2(M_b^2+3M_u^2)-m_b^2(M_b^2+M_u^2)\}}{24M_b^6(M_b^2+M_u^2)^2z^3}\right.$$
$$-\frac{8M_b^4M_u^4+4m_b^2M_b^2(M_b^4+4M_b^2M_u^2+2M_u^4)-m_b^4M_u^2(2M_b^2+M_u^2)}{96M_b^6(M_b^2+M_u^2)^2z^2}$$
$$\left.+\frac{4M_b^2(M_b^2+M_u^2)-m_b^2(2M_b^2+M_u^2)}{48M_b^4(M_b^2+M_u^2)z}+\frac{m_b^4M_u^4}{16M_b^6(M_b^2+M_u^2)^2z^4}\right], \tag{A18}$$

$$\hat\Gamma_0^{(4)} = \frac{m_b e^{-m_b^2/M_b^2}}{M_b^2M_u^2}\left[-\frac{m_b^6M_u^6}{4M_b^8(M_u^2+M_b^2)^2z^5}+\frac{m_b^4M_u^4\{M_b^2(3M_b^2+10M_u^2)-3m_b^2M_u^2\}}{16M_b^8(M_u^2+M_b^2)^2z^4}\right.$$
$$+\frac{m_b^2M_u^4\{m_b^2M_b^2(23M_u^2+7M_b^2)-2M_b^4(13M_u^2+5M_b^2)-3m_b^4M_u^2\}}{48M_b^8(M_u^2+M_b^2)^2z^3}$$
$$+\frac{M_u^2\{8M_b^6M_u^2(M_b^2+2M_u^2)+4m_b^2M_b^4(M_b^4-4M_b^2M_u^2-11M_u^4)-m_b^6M_u^4\}}{96M_b^8(M_u^2+M_b^2)^2z^2}$$
$$+\frac{m_b^4M_u^4(7M_b^2+16M_u^2)}{96M_b^6(M_u^2+M_b^2)^2z^2}+\frac{M_u^2\{4M_b^2(2M_u^2-M_b^2)+m_b^2(2M_b^2-9M_u^2)\}}{48M_b^4(M_u^2+M_b^2)z}$$
$$\left.+\frac{m_b^4M_u^4}{32M_b^6(M_u^2+M_b^2)z}+\frac{M_b^2(8M_u^2-5M_b^2)-3m_b^2M_u^2}{96M_b^4}+\frac{(M_b^2+M_u^2)z}{96M_b^2}\right], \tag{A19}$$

$$\hat\Gamma_+^{(4)} = \frac{m_b e^{-m_b^2/M_b^2}}{M_b^2M_u^2}\left[\frac{1}{96M_b^2}+\frac{m_b^2M_u^2(M_b^2+3M_u^2)}{24M_b^4(M_b^2+M_u^2)^2z^3}+\frac{M_u^2(4M_b^2-m_b^2)}{48M_b^4(M_b^2+M_u^2)z}\right.$$
$$\left.+\frac{M_u^4(m_b^4-4m_b^2M_b^2-8M_b^4)}{96M_b^6(M_b^2+M_u^2)^2z^2}-\frac{m_b^4M_u^4}{16M_b^6(M_b^2+M_u^2)^2z^4}\right], \tag{A20}$$

$$\hat\Gamma_V^{(4)} = \frac{m_b e^{-m_b^2/M_b^2}}{M_b^2M_u^2}\left[\frac{m_b^4M_u^4}{8M_b^6(M_b^2+M_u^2)^2z^4}+\frac{m_b^2M_u^2\{m_b^2M_u^2-M_b^2(M_b^2+3M_u^2)\}}{12M_b^6(M_b^2+M_u^2)^2z^3}\right.$$
$$\left.+\frac{M_u^2\{8M_b^4M_u^2-4m_b^2M_b^2(M_b^2+2M_u^2)+m_b^4M_u^2\}}{48M_b^6(M_b^2+M_u^2)^2z^2}+\frac{1}{48M_b^2}+\frac{2M_b^2(2M_u^2-M_b^2)-m_b^2M_u^2}{24M_b^4(M_b^2+M_u^2)z}\right]. \tag{A21}$$

Note that we have checked our method for calculating Borel-transformed Wilson coefficients for the matrix element $\langle B\,|\,V_\mu\,|\,B\rangle$ at $t=0$ where the result is uniquely determined by charge conservation.

APPENDIX B: THE WILSON COEFFICIENT OF THE FOUR-QUARK CONDENSATE

In addition to the formulas given in [15], we also have calculated the contributions of the four-quark condensate to as the decays $B, D\to\pi e\nu$ which read [in the notation of (A17)]

$$\hat{\Pi}_+^{(6_1)} = \frac{m_b e^{-m_b^2/M_b^2}}{M_b^2 M_u^2}\left(\frac{1}{9M_b^2M_u^2} - \frac{m_b^2}{72M_b^6} - \frac{1}{18M_b^4} + \frac{t}{36M_b^4M_u^2} - \frac{m_b^2-t}{18M_b^2M_u^4}\right),$$
$$\hat{\Pi}_+^{(6_2)} = \frac{m_b e^{-m_b^2/M_b^2}}{M_b^2 M_u^2}\left(\frac{1}{9M_u^4} + \frac{2}{9M_u^2(m_b^2-t)}\right). \tag{B1}$$

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