Polarized and unpolarized prompt photon production beyond the leading order

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We calculate the complete $O(\alpha\alpha_s^2)$ corrections to the inclusive cross section for hadronic prompt photon production, both for the unpolarized case and for the case of longitudinal polarization for the incoming hadrons. We present analytical expressions for all our results.

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I. INTRODUCTION

The production of large-transverse-momentum prompt photons in pp or $p\bar{p}$ collisions has turned out to be an important tool to uncover the unpolarized gluon distribution of the proton [1–7]. The reason for this is the presence of the $O(\alpha\alpha_s)$ $qg\rightarrow\gamma q$ Compton subprocess through which the gluon distribution enters the cross section already in the leading order (unlike the case of deep-inelastic scattering) and which dominates the cross section over a wide kinematical region. Since the European Muon Collaboration (EMC) measurement of the spin-dependent proton structure function g_1^p [8] (as well as the new Spin Muon Collaboration (SMC) results on the neutron's g_1^n [9]) has left us with the attractive possibility, among others, that gluons inside the proton could be strongly polarized [10], it is compelling to study prompt photon production also for the polarized case, i.e., for longitudinal polarization of both incoming hadrons, in order to examine the sensitivity of this process to the polarized gluon distribution ΔG in the proton. Indeed, leading-order studies of this kind have been presented in the literature [11], indicating that the process, if studied experimentally, should provide a good opportunity to settle the important question whether gluons inside the proton are or are not strongly polarized.

It is true quite, in general, that leading-order QCD calculations can usually only give semiquantitative results for high-energy hadron-hadron reactions. Firm predictions should at least be based on next-to-leading-order calculations. In the unpolarized case such corrections have been calculated in numerous cases. Apart from their importance for studying the perturbative stability, one of the main properties of these corrections is to appreciably decrease the scale dependence of the predictions. In this way the comparison between theoretical predictions and experimental results is put on a much stronger foundation and is generally improved. It is therefore also to be expected in the case of longitudinal polarization that next-to-leading-order corrections to the leading-order process are crucial for more precise predictions. Furthermore, the question is interesting whether the sizable corrections to the individual unpolarized and polarized cross sections tend to cancel out when the ratio of the cross sections, i.e., the asymmetry, is calculated. This feature was recently observed for the asymmetry in polarized deep-inelastic Compton scattering [12] and also for the asymmetry in Drell-Yan dimuon production with transversely polarized protons [13].

In this paper we calculate the complete $O(\alpha\alpha_s^2)$ corrections to inclusive hadronic prompt photon production, both for the unpolarized and the polarized case. Of course, the complete results for the unpolarized case have been obtained twice before: First they were calculated analytically by the authors of Refs. [2,3], later on they were determined numerically using Monte Carlo techniques in Ref. [5]. Nevertheless, the unpolarized results have never been published analytically in a closed form. Since they can be obtained with not much extra effort as a by-product in the calculation of the polarized corrections, where they also serve as a good check on the calculations, we present them in this paper. As far as the next-to-leading-order corrections for the polarized case are concerned, the results are entirely new, and we believe our calculation to be the first full-fledged higher-order calculation in polarized hadron-hadron scattering.

The paper is organized as follows. In Sec. II we go through the details of the calculation step by step. To be more specific, we point out the general framework for our calculations in Secs. II A and II B where we also specify our regularization method which will be dimensional regularization, treating γ_5 and the totally antisymmetric tensor $\epsilon_{\mu\nu\rho\sigma}$ according to the original proposal by 't Hooft and Veltman [14] and Breitenlohner and Maison [15]. In Secs. II C–II E we present our results for the virtual corrections, discuss the contributions from the $2\rightarrow3$ processes and deal with the factorization of mass singularities. In Sec. III we arrive at our final results and discuss some of their properties. Appendixes A–C contain some calculational details. Finally we list our complete results in Appendixes D and E.

II. THE CALCULATION

A. Leading- and next-to-leading-order contributions

At lowest order $[O(\alpha\alpha_s)]$ two $2\rightarrow2$ subprocesses are the dominant source for the hadroproduction of prompt photons. These are the annihilation process $q\bar{q}\rightarrow\gamma g$ and the QCD Compton process $qg\rightarrow\gamma q$. The corresponding Feynman diagrams are shown in Fig. 1.

At next-to-leading-order $[O(\alpha\alpha_s^2)]$ we encounter a

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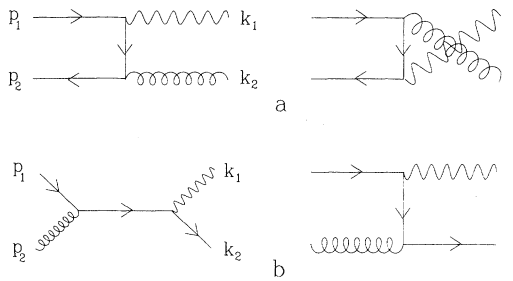

FIG. 1. Born [$O(\alpha\alpha_s)$] graphs for prompt photon production. (a) Annihilation $q\bar{q}\to\gamma g$; (b) Compton $qg\to\gamma q$.

large variety of new graphs, which can be classified as follows: virtual corrections to the Born graphs as shown in Figs. 2(a) and 2(b); real-gluon emission $2\to 3$ corrections to the Born graphs [Figs. 2(c) and 2(d)]; photon bremsstrahlung corrections to any pure QCD $2\to 2$ process (other than $qg\to qg$) involving a quark in the final state. To be more precise, these are

$$q\bar{q}\to\gamma q'\bar{q}' \quad \text{[Fig. 3(a)]} ,$$

$$gg\to\gamma q\bar{q} \quad \text{[Fig. 3(b)]} ,$$

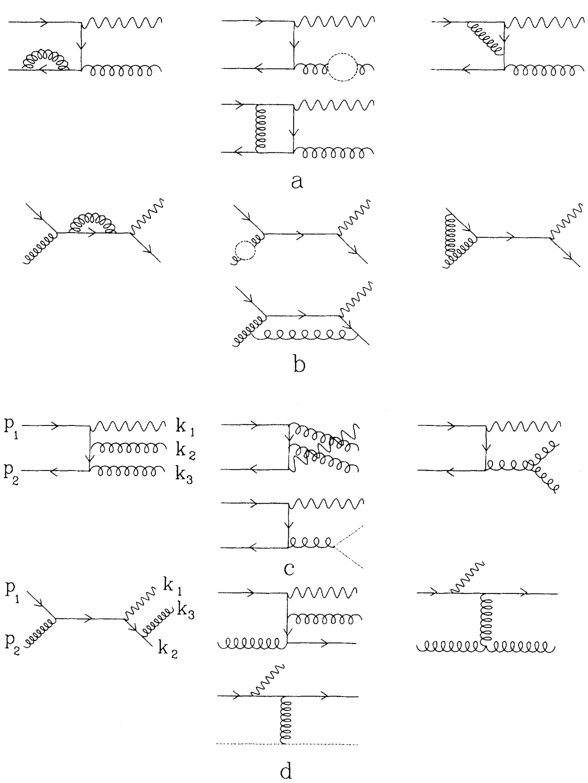

FIG. 2. (a),(b) Some representative Feynman diagrams for the virtual corrections to the Born graphs. The dashed lines can stand either for a quark, a gluon or a ghost loop. (c),(d) Some representative diagrams for the $2\to 3$ real gluon emission corrections to the Born graphs. Dotted lines refer to ghosts, which for $qg\to\gamma qg$ (d) are only present in the unpolarized case. All in all there are $8+2$ ghost graphs for either process.

$$qq\to\gamma qq \quad \text{[Fig. 3(c)]} ,$$

$$q\bar{q}\to\gamma q\bar{q} \quad \text{[Fig. 3(d)]} ,$$

$$qq'\to\gamma qq' \quad \text{[Fig. 3(e)]} ,$$

where the last process also includes $q\bar{q}'\to\gamma q\bar{q}'$.

Figures 2 and 3 show representative Feynman diagrams for each of the subprocesses as well as ghost graph contributions which (in the $2\to 3$ case) have to be taken into account if simply $-g_{\mu\nu}$ is taken for the polarization sum for an unpolarized gluon and two unpolarized external gluons appear in the process under consideration. Of course, ghosts also have to be taken into account in the gluonic self-energy contribution to the virtual corrections [Figs. 2(a) and 2(b)]. Needless to say, the virtual contributions in $O(\alpha\alpha_s^2)$ only arise via the interference of the graphs in Figs. 2(a) and 2(b) with the Born diagrams. When calculating the helicity-dependent matrix elements corresponding to Figs. 1–3, which are needed to derive the contributions to the polarized hadroproduction of prompt photons, we have to project onto definite helicity states of the incoming particles the momenta of which we label by p_1 and p_2. This is achieved by using the relations [16]

$$u(p_1,h)\bar{u}(p_1,h)=\tfrac{1}{2}\not{p}_1(1-h\gamma_5) \qquad (1)$$

for incoming quarks with helicity h (analogously for antiquarks) and

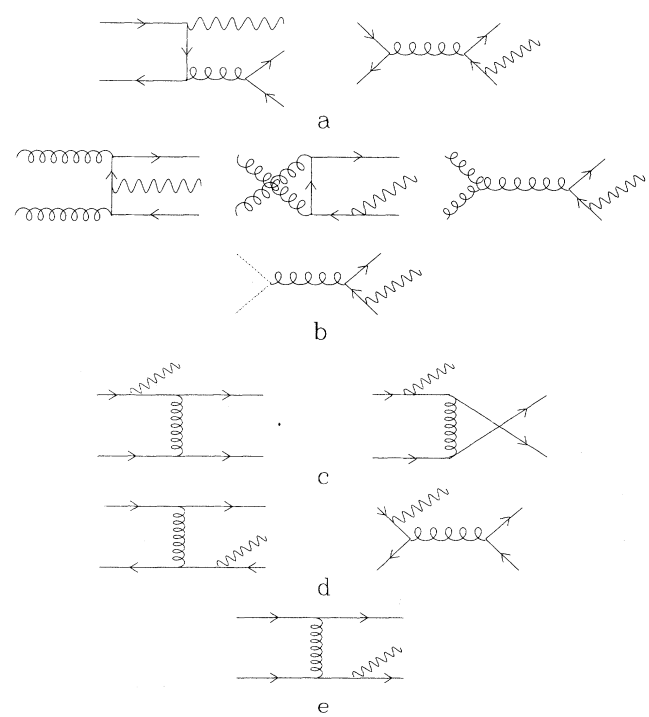

FIG. 3. Some representative diagrams for (a) $q\bar{q}\to\gamma q'\bar{q}'$ (four diagrams altogether), (b) $gg\to\gamma q\bar{q}$. The dotted lines refer to ghosts which are present only for the unpolarized case. All in all there are $8+2$ ghost graphs. (c) $qq\to\gamma qq$ (eight diagrams altogether), (d) $q\bar{q}\to\gamma q\bar{q}$ (eight diagrams), and (e) $qq'\to\gamma qq'$ (or $q\bar{q}'\to\gamma q\bar{q}'$) (four diagrams).

$$\epsilon_\mu(p_2,\lambda)\epsilon_\nu^*(p_2,\lambda)=\frac{1}{2}\left[-g_{\mu\nu}+i\lambda\epsilon_{\mu\nu\rho\sigma}\frac{p_2^\rho p_1^\sigma}{p_1\cdot p_2}\right] \quad (2)$$

for incoming gluons with helicity λ. As stated above, the inclusion of ghost graphs allows us to drop all terms other than $-g_{\mu\nu}$ in the symmetric part of $\epsilon_\mu\epsilon_\nu^*$. With the help of Eqs. (1) and (2) we are in the position to calculate the contributions of Figs. 1–3 to unpolarized and polarized prompt photon production at the same time by taking the sum or the difference of helicity-dependent squared matrix elements:

unpolarized

$$\overline{|M|^2}=\tfrac{1}{2}[|M|^2(++)+|M|^2(+-)] , \quad (3)$$

polarized

$$\Delta|M|^2=\tfrac{1}{2}[|M|^2(++)-|M|^2(+-)] , \quad (4)$$

where $|M|^2(h_1,h_2)$ denotes the squared matrix element for any of the subprocesses in Figs. 1–3 for two incoming particles (quarks or gluons) with helicities h_1 and h_2.

As is well known, when calculating the contributions of loop diagrams [Figs. 2(a) and 2(b)] or when performing the phase-space integrations for the $2\to3$ processes [Figs. 2(c), 2(d), and 3], one encounters singularities. First of all, the loop diagrams contain ultraviolet divergencies which are removed by renormalization. Adding the renormalized loop and the corresponding $2\to3$ contributions, the infrared singularities which are individually present in both ingredients also cancel out, and one is left with collinear singularities which are finally removed by the factorization procedure (of course, for the graphs in Fig. 3 there are only singularities of the latter kind). All these steps are standard by now. In order for them to work one of course has to choose a consistent method of regularizing the singularities so that they become manifest. For this purpose we choose the concept of dimensional regularization, i.e., we calculate Dirac traces, phase-space integrations, etc., in $n=4-2\epsilon$ dimensions [14]. This way of regularizing is certainly the best and most uncomplicated one in the unpolarized case, or, more generally, if no chiral couplings of any kind are present. Problems arise if quantities such as γ_5 and the totally antisymmetric tensor $\epsilon_{\mu\nu\rho\sigma}$ enter the calculations, which have to be regarded as of purely four-dimensional origin with no simple analytic continuation to $n\neq4$ dimensions. Several ways of using γ_5 and $\epsilon_{\mu\nu\rho\sigma}$ in $n\neq4$ dimensions have been proposed and discussed in the literature [12,14,15,17–19]. Those preserving the total anticommutativity of γ_5 in $n\neq4$ dimensions [12,17,18] can easily be shown to lead to algebraic inconsistencies [20,21], unless extra conditions such as giving up the cyclicity of the trace [18] are met. On the contrary, the original scheme of t'Hooft and Veltman [14], afterwards systematized by Breitenlohner and Maison [15] (HVBM scheme), was shown to be internally completely consistent [22,23]. In this scheme explicit definitions for γ_5 and $\epsilon_{\mu\nu\rho\sigma}$ are given, which essentially correspond to the usual four-dimensional ones, e.g., $\gamma_5\equiv i\gamma^0\gamma^1\gamma^2\gamma^3$. In this way the n-dimensional Minkowski space is explicitly divided into two subspaces, a four-dimensional one and an $(n-4)$-dimensional one, each of them equipped with its metric tensor. This means that any vector p is the sum of $\hat{\hat{p}}$ and $\hat{p}$, where $\hat{\hat{p}}$ contains the first four components of p and zeros everywhere else and $\hat{p}$ vice versa. Of course, this property of the HVBM scheme renders it algebraically much more involved, since, e.g., apart from n-dimensional scalar products $p\cdot q$ (Mandelstam variables), their respective subspace counterparts $\widehat{\widehat{p\cdot q}}$ and $\widehat{p\cdot q}$ can also show up in calculations. As far as the actual calculation of the Dirac traces is concerned, helpful routines such as TRACER [24], which we have used, allow for taking into account the algebraic peculiarities of the HVBM scheme. Apart from this, terms such as $\widehat{p\cdot q}$, etc. also deserve special attention when performing phase space integrations. This will be discussed in Sec. II D.

As has been shown in the literature [25], the presence of chiral couplings in loop integrals may lead to anomalous terms in the HVBM scheme, which are usually referred to as "spurious anomalies." These originate from ultraviolet poles encountered in loop integrations multiplied by ϵ terms from trace calculations and are, therefore, of ultraviolet origin. They have to be subtracted by hand by a finite renormalization using appropriate counterterms [26,27]. Of course, the spurious anomalies again make the HVBM scheme more complicated and less handy. Fortunately, however, in the case of polarized prompt photon production no spurious anomalies appear, which is obvious since all self-energies and vertex corrections can be calculated and renormalized independent of any polarization *before* taking the interference with the Born diagrams which only then gives rise to γ_5 traces.

As a consequence of the points discussed above, we have chosen the HVBM scheme for our calculations since, first of all, it is internally consistent unlike those schemes which make use of a naive anticommuting γ_5 which are, of course, simpler algebraically. Nevertheless, the HVBM scheme is not very much harder to deal with; in particular, we find it simpler in our case than the scheme suggested in Ref. [18] since in this scheme cyclicity of the trace is given up, and all traces contributing to a subprocess have to be read from the same vertex, the so-called "reading point." In the case of the virtual graphs this inevitably leads to a reading point which is located *inside* a loop (at least for some graphs) which means that standard loop results can no longer be applied and the calculation becomes more complicated.

Before concluding this subsection let us note that it is well known that gluons can take $n-2=2(1-\epsilon)$ different spin orientations in $n=4-2\epsilon$ dimensions. Therefore, in the calculation for the unpolarized case one should average the spin of each incoming gluon with the factor $1/2(1-\epsilon)$ rather than with $\frac{1}{2}$. This can be achieved by the replacement

$$-\frac{1}{2}g_{\mu\nu}\to-\frac{1}{2(1-\epsilon)}g_{\mu\nu} \quad (5)$$

in Eq. (2) but leaving Eqs. (3) and (4) unchanged. In this point we differ from the calculational method in Refs. [2,3]. For the polarized case we are, of course, only interested in the difference of the matrix elements for the *two* possible helicity states; therefore, no extra factor as in Eq. (5) is needed there.

B. The inclusive cross section $E_\gamma d^3\sigma/d^3p_\gamma$

Before going into the details of the calculation let us write down the invariant cross section $E_\gamma d^3\sigma/d^3p_\gamma$ for inclusive prompt photon production, where E_γ and $\mathbf{p}_\gamma$ stand for the energy and the three-momentum of the produced prompt photon. We shall write down everything in terms of the polarized quantities, the unpolarized ones are then immediately obtained by removing the Δ's, which of course means taking the sum instead of the difference in Eqs. (6) and (7) below. In analogy to Eq. (4) the polarized invariant cross section $E_\gamma d^3\Delta\sigma/d^3p_\gamma$ is given by the difference

$$E_\gamma \frac{d^3\Delta\sigma^{AB}}{d^3p_\gamma} \equiv \frac{1}{2}\left[E_\gamma \frac{d^3\sigma^{AB}(++)}{d^3p_\gamma} - E_\gamma \frac{d^3\sigma^{AB}(+-)}{d^3p_\gamma}\right], \tag{6}$$

where

$$E_\gamma d^3\sigma^{AB}(h_A h_B)/d^3p_\gamma$$

denotes the invariant cross section for two incoming hadrons A and B with helicities h_A and h_B. Introducing the polarized parton distributions $\Delta f_a^A(x)$ by

$$\Delta f_a^A(x) \equiv (f_a^A)_+(x) - (f_a^A)_-(x) , \tag{7}$$

where $(f_a^A)_{+(-)}(x)$ denotes the distribution of parton type a with positive (negative) helicity in hadron A with positive helicity, we can relate the hadronic and the subprocess cross section [28]:

$$E_\gamma \frac{d^3\Delta\sigma^{AB}}{d^3p_\gamma} = \frac{1}{\pi}\sum_{a,b}\int dx_1 \int dx_2 \Delta f_a^A(x_1)\Delta f_b^B(x_2) \times \frac{1}{\hat{s}v}\frac{d\Delta\hat{\sigma}^{ab}}{dv\,dw} , \tag{8}$$

where $\sum_{a,b}$ denotes the sum over the appropriate parton combinations. Here we have introduced the partonic invariant variables v and w which are defined by

$$v \equi 1+\hat{t}_1/\hat{s} ,$$
$$w \equiv -\hat{u}_1/(\hat{s}+\hat{t}_1) \tag{9}$$

with $\hat{t}_1=(x_1P_A-p_\gamma)^2$, $\hat{u}_1=(x_2P_B-p_\gamma)^2$, and

$$\hat{s}=(x_1P_A+x_2P_B)^2=x_1x_2S ,$$

P_A and P_B being the momenta of the incoming hadrons. Furthermore, $d\Delta\hat{\sigma}^{ab}/dv\,dw$ in Eq. (8) is defined in complete analogy with Eq. (6), the Δ now referring to the difference of cross sections for definite *parton* helicities. Since we shall present our final results for the subprocess cross sections for the various processes depicted in Figs. 1–3 in terms of the variables v and w, it is convenient to write the integrations in Eq. (8) as integrations over these:

$$E_\gamma \frac{d^3\Delta\sigma^{AB}}{d^3p_\gamma} = \frac{1}{\pi p_T^4}\sum_{a,b}\int_{x_Te^{\eta}/2}^{1-(x_Te^{-\eta})/2} dv \int_{(x_T/2)e^{\eta}/v}^{1} dw\, x_1\Delta f_a^A(x_1,M^2)x_2\Delta f_b^B(x_2,M^2)vw(1-v)\hat{s}\frac{d\Delta\hat{\sigma}^{ab}}{dv\,dw} , \tag{10}$$

where we have introduced the prompt photon's transverse momentum p_T and its rapidity η in the c.m.s. of the colliding hadrons. In terms of these we have $x_T \equiv 2p_T/\sqrt{S}$, $x_1=x_Te^{\eta}/2vw$, and $x_2=x_Te^{-\eta}/2(1-v)$. Finally, we have also introduced an appropriate mass scale in the parton distributions. It should be noted again that in all cases the corresponding unpolarized results can be obtained by replacing all polarized quantities by their unpolarized counterparts, i.e., $\Delta\hat{\sigma}^{ab}\to\hat{\sigma}^{ab}$, $\Delta f_a^A \to f_a^A$, where $\hat{\sigma}^{ab}$ and f_a^A are the usual unpolarized cross sections and parton distributions.

For the two-body (i.e., Born and virtual) contributions we have of course $\hat{s}+\hat{t}_1+\hat{u}_1=0$, which is equivalent to $w=1$. In fact,

$$\frac{d\hat{\sigma}^{ab}_{2\to2}}{dv\,dw} = \frac{d\hat{\sigma}^{ab}_{2\to2}}{dv}\delta(1-w) . \tag{11}$$

In the case of the three-body processes we have $\hat{s}+\hat{t}_1+\hat{u}_1=s_{23}=\hat{s}v(1-w)$, which is the invariant mass squared of the two unobserved outgoing partons. The $2\to3$ cross sections also display singularities at $w\to1$ which have to be made manifest, as will be explained in Sec. II D. The choice (9) for the variables v and w guarantees that only the variable w will lead to singularities, which arise at its upper integration limit.

C. Born graphs and virtual corrections

In this section we deal with the two-body graphs. To begin with, let us for completeness write down the results for the Born cross sections. The unpolarized cross sections are given by (in four dimensions; see, e.g., Ref. [29])

$$\frac{d\hat{\sigma}}{dv}(q\bar{q}\to\gamma g) = \frac{2C_F}{N_C}\frac{\pi\alpha\alpha_s e_q^2}{\hat{s}}\frac{v^2+(1-v)^2}{v(1-v)} , \tag{12}$$

$$\frac{d\hat{\sigma}}{dv}(qg\to\gamma q) = \frac{1}{N_C}\frac{\pi\alpha\alpha_s e_q^2}{\hat{s}}\frac{1+(1-v)^2}{1-v} , \tag{13}$$

where N_C is the number of colors, $C_F=(N_C^2-1)/2N_C$, and e_q denotes the quark's charge. For the polarized case one has

$$\frac{d\Delta\hat{\sigma}}{dv}(q\bar{q}\to\gamma g) = -\frac{d\hat{\sigma}}{dv}(q\bar{q}\to\gamma g) , \tag{14}$$

due to helicity conservation, and

$$\frac{d\Delta\hat{\sigma}}{dv}(qg\to\gamma q)=\frac{1}{N_C}\frac{\pi\alpha\alpha_s e_q^2}{\hat{s}}\frac{1-(1-v)^2}{1-v} . \quad (15)$$

Let us now turn to the virtual corrections to the Born graphs which were already depicted in Figs. 2(a) and 2(b). The unpolarized results have already been published in Ref. [30] for the $q\bar{q}$ case and in Ref. [31] for the qg case. Of course, the two results are related by crossing, and we do obtain the same results. Since in the polarized case the helicities of the incoming particles are involved, there is no use in applying crossing symmetry, and we have to present separate results for the $q\bar{q}$ and the qg virtual corrections. Nevertheless, of course, the calculations for the two processes proceed along quite the same lines. It should be noted that if we used a totally anticommuting γ_5 in $n\neq 4$ dimensions, the results for the virtual corrections to $q\bar{q}$ annihilation would trivially be the same for the unpolarized and the polarized case apart from a sign, since it would be possible to remove all γ_5 from the traces by using their anticommutativity and their property $\gamma_5^2=1$. Of course, this does not work in the HVBM scheme used by us, and we shall return to this point in Sec. III.

The renormalization scheme we adopt is the modified minimal subtraction ($\overline{\rm MS}$) scheme [32], which requires the subtraction of all the ultraviolet poles together with the attendant Euler constant γ_E and $\ln 4\pi$. In calculating the loop integrals we have partially made use of the results in Ref. [33]. The renormalization procedure and the results for the self-energy and vertex correction diagrams are standard and can be found, e.g., in Ref. [34]. For convenience we list the massless four-point functions needed for the (ultraviolet-finite) box diagrams in Appendix A. For the renormalized virtual corrections to polarized $q\bar{q}\to\gamma g$ [Fig. 2(a)] we find, in the HVBM scheme,

$$\begin{aligned}
&v(1-v)\hat{s}\frac{d\Delta\hat{\sigma}^V}{dv\,dw}(q\bar{q}\to\gamma g)\\
&\quad=\frac{\alpha\alpha_s^2(\mu^2)e_q^2\mu^{2\epsilon}}{\Gamma(1-2\epsilon)}\left[\frac{(4\pi\mu^2)^2}{\hat{t}_1\hat{u}_1}\right]^\epsilon\frac{C_F}{N_C}\delta(1-w)\\
&\quad\times\Bigg[\frac{(2C_F+N_C)T_{q\bar{q}}}{2\epsilon^2}+\frac{1}{\epsilon}\left[-\frac{N_F}{6}T_{q\bar{q}}-N_C T_{q\bar{q}}\ln v+\frac{1}{12}N_C(23-10vv_1)+\frac{1}{2}C_F(7-2vv_1)\right]\\
&\qquad-\frac{1}{12}(4C_F-N_C)\pi^2T_{q\bar{q}}-\frac{1}{2}T_{q\bar{q}}b_0\ln\frac{\mu^2}{\hat{s}}+\frac{1}{2}N_C T_{q\bar{q}}\ln v\ln v_1-N_C(2-v)(1+v)\ln v\\
&\qquad-\frac{1}{2}(2C_F-N_C)(1+v^2)\ln^2 v-C_F(3-v)v_1\ln v+\frac{1}{2}C_F(15-8vv_1)-\frac{N_F}{3}(1+vv_1)\\
&\qquad+\frac{1}{6}N_C(14+11vv_1)\Bigg]+(v\leftrightarrow 1-v) , \qquad (16)
\end{aligned}$$

where we have defined $T_{q\bar{q}}\equiv(1-v)^2+v^2$, $v_1\equiv 1-v$, and $b_0\equiv 11N_C/6-N_F/3$. Furthermore, for polarized $qg\to\gamma q$ [Fig. 2(b)] our result is

$$\begin{aligned}
&v(1-v)\hat{s}\frac{d\Delta\hat{\sigma}^V}{dv\,dw}(qg\to\gamma q)\\
&\quad=\frac{\alpha\alpha_s^2(\mu^2)e_q^2\mu^{2\epsilon}}{\Gamma(1-2\epsilon)}\left[\frac{(4\pi\mu^2)^2}{\hat{t}_1\hat{u}_1}\right]^\epsilon\frac{1}{N_C}\delta(1-w)\\
&\quad\times\Bigg[-\frac{(2C_F+N_C)v\Delta T_{qg}}{2\epsilon^2}+\frac{1}{\epsilon}\Bigg[\frac{N_F}{6}v\Delta T_{qg}+\frac{1}{2}N_C v\Delta T_{qg}\ln v_1+\frac{1}{2}(2C_F-N_C)v\Delta T_{qg}\ln v\\
&\qquad\qquad-\frac{1}{12}N_C(22-5v)v^2-\frac{1}{2}C_F(6-v)v^2\Bigg]\\
&\qquad+\frac{1}{2}b_0\ln\frac{\mu^2}{\hat{s}}v\Delta T_{qg}-\frac{1}{4}(2C_F-N_C)v(1-2v)\ln v_1\ln\frac{v_1}{v^2}-\frac{1}{2}v^3b_0+\frac{1}{12}N_C\pi^2v(3-2v-2v^2)\\
&\qquad-\frac{1}{6}C_F\pi^2v(3-4v-v^2)+\frac{1}{2}N_C v(v^2+v_1)\ln v_1-C_F(7-2v)v^2+\frac{1}{2}(2C_F-N_C)v^3\ln v\\
&\qquad+\frac{1}{2}C_F v(1+2v)\ln v_1\Bigg] \qquad (17)
\end{aligned}$$

with $\Delta T_{qg} \equiv 1-(1-v)^2$. In Eqs. (16) and (17) $\alpha_s(\mu^2)$ is the running strong-coupling constant renormalized at scale μ^2 in the $\overline{\rm MS}$ scheme which satisfies the renormalization-group equation

$$\mu^2 \frac{d}{d\mu^2}\alpha_s = -\alpha_s \left[\beta_0 \frac{\alpha_s}{4\pi} + \beta_1 \left[\frac{\alpha_s}{4\pi}\right]^2 + O(\alpha_s^3)\right] \tag{18}$$

with [35]

$$\beta_0 = 2b_0 = 11N_C/3 - 2N_F/3 ,$$

$$\beta_1 = 34N_C^2/3 - 10N_C N_F/3 - 2C_F N_F ,$$

and the number of active flavors N_F. Of course, Eqs. (16) and (17) still contain infrared and collinear singularities which will only cancel when the contributions from the inelastic (i.e., the $2 \to 3$) graphs are added and the factorization of mass singularities is performed. These points are the topics of the next two sections.

D. Three-body contributions

The $2 \to 3$ processes constitute the main task in this calculation. The corresponding graphs were already shown in Figs. 2(c), 2(d), and 3. We have evaluated the traces arising from these using the program package TRACER [24]. Note that our n-dimensional matrix elements for the unpolarized case for each process are in complete agreement [36] with those published in Ref. [31] after suitable crossing. For the polarized case we have always checked that our matrix elements reproduce those in Ref. [29] in the limit $n \to 4$. The harder part to do then is the phase-space integration. This is conveniently performed in the rest frame of the two outgoing unobserved partons. Denoting the momenta of the process by $p_1 + p_2 \to k_1 + k_2 + k_3$, where k_1 is the momentum of the prompt photon, we have, in this frame,

$$\begin{aligned} k_2 &= (k_0, k_0\sin\theta_1\cos\theta_2, k_y, k_0\cos\theta_1, \hat{k}) , \\ k_3 &= (k_0, -k_0\sin\theta_1\cos\theta_2, -k_y, -k_0\cos\theta_1, -\hat{k}) , \end{aligned} \tag{19}$$

where $k_0 = \sqrt{s_{23}}/2$, $\hat{k}$ stands for the $(n-4)$-dimensional part of k_2, and k_y denotes the unspecified y component which can be trivially integrated over since the matrix element will not depend on it. This is not true for the $\hat{k}$ components since, as explained in Sec. II A, in the HVBM scheme such components will explicitly appear in the matrix element. The other three momenta p_1, p_2, and k_1 can be oriented in such a way that they lie in the x-z plane. Depending on the propagators appearing in the process under consideration one of the three vectors can always be chosen to have only a nonvanishing z component and zeros everywhere else in its spatial part. The resulting three sets of parametrizations for the momenta are listed for convenience in Appendix B. In any case the "sub"vector $\hat{k}$ is the only $(n-4)$-dimensional quantity in the calculation [37]. We can express all $(n-4)$-dimensional scalar products by $\hat{k}^2$:

$$\hat{k}_2^2 = -\hat{k}^2 = \hat{k}_3^2 ,$$

$$\widehat{k_2 \cdot k_3} = \hat{k}^2 .$$

$\hat{k}^2$ appears as an additional integration variable in the $2 \to 3$ phase space, which reads

$$R_3 = \frac{\hat{s}}{(4\pi)^4\Gamma(1-2\epsilon)} \left[\frac{4\pi}{\hat{s}}\right]^{2\epsilon} \times \int_0^1 dv\, v^{1-2\epsilon}(1-v)^{-\epsilon} \int_0^1 dw [w(1-w)]^{-\epsilon} \int_0^\pi d\theta_1 \sin^{1-2\epsilon}\theta_1 \int_0^\pi d\theta_2 \sin^{-2\epsilon}\theta_2 \frac{1}{B(\frac{1}{2},-\epsilon)} \int_0^1 \frac{dx}{\sqrt{1-x}} x^{-(1+\epsilon)} , \tag{20}$$

where $B(a,b)$ is the β function and x is nothing but $\hat{k}^2$ divided by its upper limit:

$$x \equiv \frac{4\hat{k}^2}{s_{23}\sin^2\theta_1\sin^2\theta_2} .$$

Equation (20) is in agreement with the result of Ref. [27]. The last integral in Eq. (20) has been written in such a way that it is unity if no $\hat{k}^2$ dependence occurs in the matrix element, which is the case for the unpolarized matrix elements and for the vast majority of terms in the polarized ones. The remaining terms are proportional to $\hat{k}^2$ for which the last integral gives

$$\frac{1}{B(\frac{1}{2},-\epsilon)} \int_0^1 \frac{dx}{\sqrt{1-x}} x^{-(1+\epsilon)} \hat{k}^2 = -\frac{2\epsilon}{1-2\epsilon}\frac{s_{23}}{4}\sin^2\theta_1\sin^2\theta_2 . \tag{21}$$

After performing the x integration, the matrix element has to be integrated over θ_1 and θ_2. For terms which do not originate from $\hat{k}^2$ terms this procedure is quite standard and has been presented several times (see, e.g., Refs. [38,39]). One has to make extensive use of relations between Mandelstam variables to reduce complex combinations of the variables to simple ones by partial fractioning. In the end, one is left only with expressions containing at most two Mandelstam variables which in turn demand the general integral of the type

$$\int_0^\pi d\theta_1 \sin^{1-2\epsilon}\theta_1 \int_0^\pi d\theta_2 \sin^{-2\epsilon}\theta_2 \frac{1}{(1-\cos\theta_1)^j(1-\cos\theta_1\cos\chi - \sin\theta_1\cos\theta_2\sin\chi)^l} = 2\pi\frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)} 2^{-j-l} \frac{B(1-\epsilon-j, 1-\epsilon-l)}{\Gamma(1-\epsilon)} {}_2F_1\left[j,l,1-\epsilon;\cos^2\frac{\chi}{2}\right] , \tag{22}$$

which has been evaluated in Ref. [40]. The angle χ can stand for ψ, ψ', and ψ'' which we define in Appendix B and, of course, also for $\chi=\psi+\pi$, etc., depending on which Mandelstam variables occur. For $j=1$ or $l=1$, $1/\epsilon$ poles appear in Eq. (22). Note that only the case $j\leq 1$, $l\leq 1$ occurs, and that for j or $l\leq 0$ the hypergeometric series ${}_2F_1(a,b,c;z)$ terminates and the integral becomes very simple. The case $j=l=1$ is the only nontrivial one. Here a double pole arises if $\chi=\psi$, ψ', or ψ'', since the right-hand side of Eq. (22) becomes

$$-\frac{\pi}{\epsilon}\,{}_2F_1\left[1,1,1-\epsilon;\cos^2\frac{\chi}{2}\right] \approx -\frac{\pi}{\epsilon}\left[\sin^2\frac{\chi}{2}\right]^{-1-\epsilon}\left[1+\epsilon^2\mathrm{Li}_2\left[\cos^2\frac{\chi}{2}\right]\right],$$

where $\mathrm{Li}_2(z)$ is the dilogarithm function. From Appendix B it can be seen that the term $[\sin^2(\chi/2)]^{-1-\epsilon}$ produces a pole at $w\rightarrow 1$. This can be made manifest using the identity [41]

$$(1-w)^{-1-\epsilon}=-\frac{1}{\epsilon}\delta(1-w)+\frac{1}{(1-w)_+} -\epsilon\left[\frac{\ln(1-w)}{1-w}\right]_+ +O(\epsilon^2)\ , \qquad (23)$$

where the "plus" distributions are defined as usual, namely, by

$$\int_0^1\frac{f(w)}{(1-w)_+}dw=\int_0^1\frac{f(w)-f(1)}{1-w}dw\ ,$$

and analogously for $(\ln(1-w)/1-w)_+$. Note that Eq. (23) is also needed for $1/s_{23}=1/\hat{s}v(1-w)$ terms in the matrix element since these also diverge for $w\rightarrow 1$.

Let us now briefly turn to the $\hat{k}^2$ terms in the matrix element. According to Eq. (21) these yield an additional factor $\sin^2\theta_1\sin^2\theta_2$ in the integrand in Eq. (22) which is readily taken into account by changing $\epsilon\rightarrow\epsilon-1$ there. With this simple trick all $\hat{k}^2$ terms can be integrated over. Since $\hat{k}^2$ sets a new mass scale in the matrix element it is not surprising that these terms are often accompanied by the inverse *squared* of a Mandelstam variable. It is easy to see from Eqs. (21) and (22) that in fact only such terms with at least either $j=2$ or $l=2$ give nonvanishing contributions in the limit $\epsilon\rightarrow 0$ [27], which is due to the factor of ϵ in Eq. (21) and of course to the shift $\epsilon\rightarrow\epsilon-1$. For the same reason and because of the factor $s_{23}\sim 1-w$ in Eq. (21) no pole terms arise from the $\hat{k}^2$ terms. Finally, the only case where distributions as in Eq. (23) can arise is when $\hat{k}^2/s_{23}^2$ terms are present in the matrix element which give (finite) terms $\sim\delta(1-w)$ in the final answer.

Equipped with all the above formulas we can integrate all $2\rightarrow 3$ matrix elements for the unpolarized and the polarized case. Adding the results for $q\bar{q}\rightarrow\gamma gg$ and $qg\rightarrow\gamma qg$ to the respective results for the virtual corrections [which are proportional to $\delta(1-w)$], all infrared poles drop out (including those $\sim 1/\epsilon^2$), and we are finally left with collinear singularities for all processes which occur as simple poles at $\epsilon=0$.

E. Factorization of mass singularities

The factorization procedure based on the factorization theorem [42] has been outlined in detail, e.g., in Refs. [38,31]. The mass singularities associated with collinear emission arise when either an incoming particle collinearly emits another particle or when a final-state quark is collinear to the outgoing photon. The singular terms attached to the initial legs are separated off at the factorization scale M^2. For the final-state singularities we factorize at the scale M'^2. In this way the scale-dependent "dressed" (polarized) unpolarized structure functions $(\Delta)f_a^A(x,M^2)$ for a parton a in a hadron A and the unpolarized scale-dependent photon fragmentation function $D_\gamma^q(z,M'^2)$ are introduced which obey their respective next-to-leading-order QCD evolution equations. Of course, there is a well-known freedom in choosing the factorization prescription, i.e., in subtracting finite pieces along with the pole terms. In general, we shall present our results in the $\overline{\mathrm{MS}}$ scheme in which only the pole terms and, as above in Sec. II C, the γ_E and $\ln 4\pi$ terms are subtracted. Only when dealing with the initial-state singularities in the polarized case shall we adopt a slightly modified scheme, hereafter referred to as $\overline{\mathrm{MS}}_P$, which takes into account some subtleties connected with the HVBM scheme which will be discussed soon.

As an example, let us briefly discuss the factorization of the $qg\rightarrow\gamma qg$ subprocesses. This is performed easiest by adding a "counter cross section" [38] which, taking into consideration all possible collinear configurations, takes the form

$$\begin{aligned}\frac{1}{\hat{s}v}\frac{d\Delta\sigma^F}{dv\,dw}=&-\frac{\alpha_s}{2\pi}\left[\int_0^1 dx_1\Delta H_{qq}(x_1,M^2)\frac{d\Delta\hat{\sigma}^{qg\rightarrow\gamma q}}{dv}\left[x_1\hat{s},1+\frac{\hat{t}_1}{\hat{s}},\epsilon\right]\delta(x_1(\hat{s}+\hat{t}_1)+\hat{u}_1)\right.\\&+\int_0^1 dx_2\Delta H_{qg}(x_2,M^2)\frac{d\Delta\hat{\sigma}^{q\bar{q}\rightarrow\gamma g}}{dv}\left[x_2\hat{s},1+\frac{\hat{t}_1}{x_2\hat{s}},\epsilon\right]\delta(x_2(\hat{s}+\hat{u}_1)+\hat{t}_1)\\&\left.+\int_0^1 dx_2\Delta H_{gg}(x_2,M^2)\frac{d\Delta\hat{\sigma}^{qg\rightarrow\gamma q}}{dv}\left[x_2\hat{s},1+\frac{\hat{t}_1}{x_2\hat{s}},\epsilon\right]\delta(x_2(\hat{s}+\hat{u}_1)+\hat{t}_1)\right]\\&-\frac{\alpha}{2\pi}\left[\int_0^1\frac{dx_3}{x_3^2}H_{\gamma q}(x_3,M'^2)\frac{d\Delta\hat{\sigma}^{qg\rightarrow qg}}{dv}\left[\hat{s},1+\frac{\hat{t}_1}{x_3\hat{s}},\epsilon\right]\delta\left[\hat{s}+\frac{1}{x_3}(\hat{t}_1+\hat{u}_1)\right]\right]\end{aligned}$$

$$= -\frac{\alpha_s}{2\pi}\left[\frac{1}{\hat{s}v}\Delta H_{qq}(w,M^2)\frac{d\Delta\hat{\sigma}^{qg\to\gamma q}}{dv}(w\hat{s},v,\epsilon)+\frac{1}{\hat{s}(1-vw)}\Delta H_{qg}\left[\frac{1-v}{1-vw},M^2\right]\frac{d\Delta\hat{\sigma}^{q\bar{q}\to\gamma g}}{dv}\left[\frac{1-v}{1-vw}\hat{s},vw,\epsilon\right]\right.$$

$$\left.+\frac{1}{\hat{s}(1-vw)}\Delta H_{gg}\left[\frac{1-v}{1-vw},M^2\right]\frac{d\Delta\hat{\sigma}^{qg\to\gamma q}}{dv}\left[\frac{1-v}{1-vw}\hat{s},vw,\epsilon\right]\right]$$

$$-\frac{\alpha}{2\pi}\left[\frac{1}{\hat{s}(1-v+vw)}H_{\gamma q}(1-v+vw,M'^2)\frac{d\Delta\hat{\sigma}^{qg\to qg}}{dv}\left[\hat{s},\frac{vw}{1-v+vw},\epsilon\right]\right]\right] . \quad (24)$$

Again Eq. (24) has been written down for the polarized case; it is again valid for the unpolarized case if the Δ's are removed. The

$$d(\Delta)\hat{\sigma}^{ab\to cd}/dv(\hat{s},v,\epsilon)$$

are the *n-dimensional* $2\to 2$ cross sections for the processes $ab\to cd$ and are listed in Appendix C for the polarized case, in which they of course also have to be calculated in the HVBM scheme [37]. For the unpolarized case these cross sections can be obtained from Ref. [31]. Furthermore, in Eq. (24),

$$(\Delta)H_{ij}(z,M^2)\equiv -\frac{1}{\hat{\epsilon}}(\Delta)P_{ij}(z)\left[\frac{\mu^2}{M^2}\right]^{\epsilon}+(\Delta)f_{ij}(z) , \quad (25)$$

where $1/\hat{\epsilon}\equiv 1/\epsilon-\gamma_E+\ln 4\pi$ (as usual in the $\overline{\text{MS}}$ scheme). In Eq. (25) the $(\Delta)P_{ij}(z)$ denote the well-known unpolarized (polarized) one-loop splitting functions for the transitions $j\to i$ [43]. The functions $(\Delta)f_{ij}(z)$ represent the above-mentioned freedom in choosing a factorization prescription. In the $\overline{\text{MS}}$ scheme (which we adopt for the unpolarized case) these functions vanish. In the polarized case we shall slightly deviate from this scheme. To see the reason for this we briefly turn to the results for deep-inelastic scattering (DIS).

The spin-dependent DIS structure function $g_1^p(x,Q^2)$ including $O(\alpha_s)$ corrections in general reads

$$g_1^p(x,Q^2)=\frac{1}{2}\sum_q e_q^2\int_x^1\frac{dz}{z}\left\{\left[\delta(1-z)+\frac{\alpha_s(Q^2)}{2\pi}\Delta f_q(z)\right]\left[\Delta q\left[\frac{x}{z},Q^2\right]+\Delta\bar{q}\left[\frac{x}{z},Q^2\right]\right]\right.$$

$$\left.+2\frac{\alpha_s(Q^2)}{2\pi}\Delta f_g(z)\Delta G\left[\frac{x}{z},Q^2\right]\right\} \quad (26)$$

with coefficient functions $\Delta f_{q,g}(z)$ which can be calculated from simple parton model graphs but which are subject to the factorization prescription chosen. The results for such a calculation depend, of course, on the method of regularization which is adopted. This has led to some debate concerning the gluonic coefficient $\Delta f_g(z)$ in the literature [44–49], mainly about the question whether its first moment, $\Delta f_g^1\equiv\int_0^1\Delta f_g(z)dz$, vanishes or is finite, $\Delta f_g^1=-\frac{1}{2}$. In the latter case, gluons could significantly contribute to the first moment of $g_1^p(x,Q^2)$, leading to an attractive explanation of the surprising EMC result in terms of a large ΔG [10,44]. As has been shown in Refs. [47,48], the HVBM scheme in $\overline{\text{MS}}$ yields $\Delta f_g^1=0$. This value, however, comes about because of an exact cancellation of contributions from a region where the emitted partons in the relevant process $\gamma^* g\to q\bar{q}$ have a large transverse momentum with respect to the incoming particles and a region where they are collinear with them [48]. This latter contribution should rather be absorbed into the definition of the quark densities beyond the leading order than be regarded as part of the hard gluonic contribution to g_1^p. In Bjørken-x space we have in the HVBM scheme in $\overline{\text{MS}}$ factorization [47,48],

$$\Delta f_g(z)=T_R\left[(2z-1)\left[\ln\frac{1-z}{z}-1\right]+2(1-z)\right] , \quad (27)$$

where $T_R=\frac{1}{2}$. The term $2T_R(1-z)$ is the one which has a collinear origin and cancels the contribution of the first term when the z integration is performed [48]. The first term (which integrates to $-\frac{1}{2}$) comes entirely from the noncollinear region and can, e.g., be calculated using a transverse momentum cutoff as the regulator [46]. It therefore seems expedient to factorize the term $\sim(1-z)$ present in the HVBM scheme into the polarized quark distributions. This view is supported when considering the coefficient $\Delta f_q(z)$ in Eq. (26). This coefficient has been calculated by Ratcliffe [50] who used a totally anticommuting γ_5. The result for the first moment, $\Delta f_q^1=-2$, agrees with the one obtained by Kodaira *et al.*

[51] in the operator product expansion (OPE). It should be noted that $\Delta f_q^1 = -2$ according to Eq. (26) corresponds to the well-established $[1-\alpha_s(Q^2)/\pi]$ correction in the quark sector of the first moment of $g_1^p(x,Q^2)$. In the HVBM scheme one finds in $\overline{\rm MS}$ factorization $\Delta f_q^1 = -\frac{14}{3}$ which in Bjørken x space comes about due to an additional term $-4C_F(1-z)$ [52] when comparing with the result of Ratcliffe [50]:

$$\Delta f_q(z) = C_F \left[(1+z^2)\left[\frac{\ln(1-z)}{1-z}\right]_+ - \frac{3}{2}\frac{1}{(1-z)_+} - \left[\frac{1+z^2}{1-z}\right]\ln z + 2 + z - \left[\frac{9}{2}+\frac{1}{3}\pi^2\right]\delta(1-z)\right] - 4C_F(1-z) \ . \quad (28)$$

Again the last term can be traced back to have a collinear origin in the process $\gamma^* q \to gq$. For this reason we decide to also factorize it into the polarized quark distributions. Since the structure of the collinear terms in $\Delta f_g(z)$ and $\Delta f_q(z)$ in Eqs. (27) and (28) which appear in the HVBM scheme is the same in both cases, namely, $\sim(1-z)$, the question arises whether terms of such a kind are also present in the transitions $q \to g$ and $g \to g$ which are not accessible in $O(\alpha_s)$ deep-inelastic scattering since there is no coupling of photons to gluons. A possible way to study this problem is to examine the behavior of our polarized n-dimensional $2 \to 3$ matrix elements in the limit when one parton becomes collinear to another one. For example, again regarding the process $q(p_1)g(p_2) \to \gamma(k_1)q(k_2)g(k_3)$, we can study the case $k_2 = (1-x)p_2$ which means that the outgoing quark is emitted collinearly by the incoming gluon with the momentum fraction $1-x$ [Fig. 4(a)]. In this limit the polarized matrix element reduces to

$$\Delta|M|^2_{qg\to\gamma qg} \sim (-\epsilon)\left[-\frac{1}{\epsilon}\Delta P_{qg}(x) + 2T_R(1-x)\right] \times \Delta|M|^2_{q\bar{q}\to\gamma g} \ , \quad (29)$$

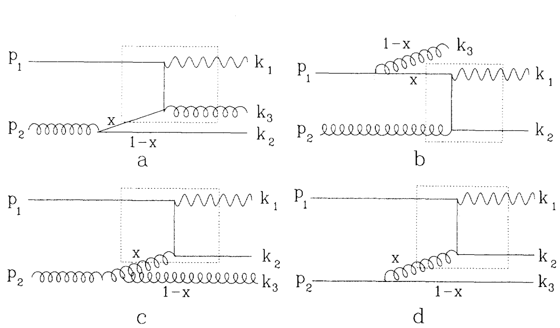

FIG. 4. Collinear configurations in $qg \to \gamma qg$ and $qq' \to \gamma qq'$ (as examples) which reveal the amount of $(1-x)$ contributions accompanying the splitting functions. (a) For the $g \to q$ transition, (b) for the $q \to q$ transition, (c) for the $g \to g$ transition, and (d) for the $q \to g$ transition. The dotted boxes frame the n-dimensional $2 \to 2$ processes which, when multiplied with the appropriate splitting terms, give the collinear limit of the $2 \to 3$ matrix element under consideration. Crossed diagrams are not shown.

which, according to the previous discussion is not unexpected. In Eq. (29) $\Delta|M|^2_{q\bar{q}\to\gamma g}$ denotes the *n-dimensional* matrix element for the process $q\bar{q} \to \gamma g$ which is proportional to the corresponding n-dimensional cross section $d\Delta\hat{\sigma}^{q\bar{q}\to\gamma g}/dv(\hat{s},v,\epsilon)$ in Appendix C [Eq. (C1)]. Analogously, for $k_3 = (1-x)p_1$ [outgoing gluon collinear to incoming quark, Fig. 4(b)] we can recover the splitting function $-\Delta P_{qq}(x)/\epsilon$, together with the desired term $-4C_F(1-x)$. The interesting case is now $k_3 = (1-x)p_2$, when the outgoing gluon is collinearly emitted by the incoming one via the triple gluon vertex [Fig. 4(c)]. Here we find

$$\Delta|M|^2_{qg\to\gamma qg} \sim (-\epsilon)\left[-\frac{1}{\epsilon}\Delta P_{gg}(x) - 4N_C(1-x)\right] \times \Delta|M|^2_{qg\to\gamma q} \ . \quad (30)$$

To study the transition $q \to g$, we have to turn, for example, to the process

$$q(p_1)q'(p_2) \to \gamma(k_1)q(k_2)q'(k_3)$$

in the limit $k_3 = (1-x)p_2$ [Fig. 4(d)]. The result is

$$\Delta|M|^2_{qq'\to\gamma qq'} \sim (-\epsilon)\left[-\frac{1}{\epsilon}\Delta P_{gq}(x) - 2C_F(1-x)\right] \times \Delta|M|^2_{qg\to\gamma q} \ . \quad (31)$$

Equations (29)–(31) strongly suggest that the presence of additional terms $\sim(1-x)$ is peculiar for the transitions $j \to i$ in the HVBM scheme. In fact, no matter which collinear limit we study in one of our subprocesses, the respective splitting function is always accompanied by the amount of $(1-x)$ terms shown in Eqs. (29)–(31), which seems to render these terms in some sense universal in the HVBM scheme.

Summarizing the above discussion, we find it most plausible to choose the finite pieces $\Delta f_{ij}(x)$ in the initial-state factorization for the polarized case as ($\overline{\rm MS}_P$ scheme)

$$\Delta f_{ij}(z) = a_{ij}(1-z) \ , \quad (32)$$

where

$$a_{qq} = -4C_F, \quad a_{qg} = 2T_R \ , \qquad a_{gq} = -2C_F, \quad a_{gg} = -4N_C \ . \quad (33)$$

The terms $a_{ij}(1-z)$ are thus absorbed into the definition of the polarized parton distributions. For most of the subprocesses the effect of this choice on the final result is nothing but an exact cancellation of all terms which originate from $\hat{k}^2$ terms in the $2 \to 3$ matrix elements. This does not happen by chance: It is well known [48] that in the HVBM scheme calculation of the DIS coefficient $\Delta f_g(z)$ the additional $2T_R(1-z)$ term [see Eq. (27)] also originates from the $(n-4)$-dimensional scalar products in the matrix element for $\gamma^* g \to q\bar{q}$, and quite a similar statement is true for the term $-4C_F(1-z)$ in $\Delta f_q(z)$ [Eq. (28)]. Furthermore, the additional $(1-x)$ terms in Eqs. (30) and (31) can also be shown to completely originate from "hat" momenta integrations. Only in the case of the $q\bar{q}$ annihilation processes, some $\delta(1-w)$ terms that originate from $\hat{k}^2/s_{23}^2$ terms in the matrix elements as described in Sec. II D, survive after factorization and appear in the final result. It should be noted that in spite of the attractiveness of the $\overline{\text{MS}}_P$ scheme any other choice is possible. Of course, physical results must be independent of the choice of the factorization scheme. Any change in the functions $(\Delta)f_{ij}(z)$ leads to a corresponding modification of the two-loop Altarelli-Parisi splitting functions in such a way that physical results remain unchanged. Unfortunately, the latter are not known up to now in the polarized case.

A final remark is necessary concerning the function $H_{\gamma q}(z,M'^2)$ appearing in Eq. (24) which we take to be

$$H_{\gamma q}(z,M'^2)=-\frac{1}{\hat{\epsilon}}P_{\gamma q}(z)\left[\frac{\mu^2}{M'^2}\right]^{\epsilon} \tag{34}$$

with

$$P_{\gamma q}(z)=[1+(1-z)^2]/z \ .$$

Note that even for the polarized case only the unpolarized splitting function appears since the $H_{\gamma q}$ term in Eq. (24) belongs to a *final*-state collinearity. Equation (34) shows that we do not subtract any finite pieces, i.e., we stick to the $\overline{\text{MS}}$ scheme as for all unpolarized calculations. A transformation to any other scheme (e.g., DIS_γ scheme [53]) is straightforward.

We have now collected all technical details. Adding the factorization counter cross sections to the previous results we can eliminate all remaining singularities and arrive at the final result which is presented in the next section.

III. FINAL RESULTS

Before we point out the general structure of our final results we have to make a short comment concerning the cancellation of mass singularities. Since we are considering an inclusive photon cross section, the outgoing photon may be accompanied by different partons in the final state, even for the same combination of incoming partons. For $q\bar{q}$ scattering there are contributions from $q\bar{q}\rightarrow\gamma gg$, $q\bar{q}\rightarrow\gamma q'\bar{q}'$, and $q\bar{q}\rightarrow\gamma q\bar{q}$. Only when the cross sections for all these processes are added can a finite result be expected. Indeed, after adding all (i.e., virtual, $2\rightarrow3$ and factorization counter term) contributions to $q\bar{q}\rightarrow\gamma gg$ the pole term

$$\frac{C_F N_F}{3N_C\epsilon}T_{q\bar{q}}\delta(1-w) \tag{35}$$

remains in the unpolarized and (apart from the sign) the polarized cross sections. This term is canceled by corresponding terms from $q\bar{q}\rightarrow\gamma q'\bar{q}'$ and $q\bar{q}\rightarrow\gamma q\bar{q}$. This works since the cross section for the process $q\bar{q}\rightarrow\gamma q'\bar{q}'$ can be split up into contributions which are either $\sim e_q^2$ or $\sim e_q'^2$ where e_q and e_q' are the charges of q and q', respectively. The terms proportional to e_q^2 simply lead to contributions $\sim(N_F-1)$ when the sum over the different possible quark flavors in the final state is taken. Separating off the same terms from the process $q\bar{q}\rightarrow\gamma q\bar{q}$ (for identical flavors) therefore yields N_F contributions. There is a pole term among these which cancels the term in Eq. (35). This procedure is not only suited for the above pole term but also for *all* terms $\sim e_q^2$ in the cross section for $q\bar{q}\rightarrow\gamma q'\bar{q}'$. We have absorbed all of these (always together with corresponding terms from $q\bar{q}\rightarrow\gamma q\bar{q}$) as contributions $\sim N_F$ into the $q\bar{q}\rightarrow\gamma gg$ cross section. This has several advantages. First of all, there are $\delta(1-w)$ and $1/(1-w)_+$ terms among these terms, collecting all these in one place makes them easier to handle numerically. The remaining "truncated" cross sections for $q\bar{q}\rightarrow\gamma q^{(\prime)}\bar{q}^{(\prime)}$ then no longer contain any δ functions or plus distributions. Note that this is also automatically the case for all other processes shown in Fig. 3. Second, we find that the final result for $q\bar{q}\rightarrow\gamma gg$ for the polarized case is (apart from the sign) exactly equal to the one for the unpolarized case. This feature only develops after absorbing the e_q^2 terms from $q\bar{q}\rightarrow\gamma q^{(\prime)}\bar{q}^{(\prime)}$. It should be noted that the equality of the polarized and the unpolarized cross sections for $q\bar{q}\rightarrow\gamma gg$ is trivial in any scheme which uses a totally anticommuting γ_5 since in such a scheme all γ_5 can be removed from any of the traces (i.e., in the virtual as well as in the $2\rightarrow3$ or the factorization piece), leaving a factor $(1-hh')$ in front of the final helicity-dependent result where h and h' are the helicities of the incoming particles. In the HVBM scheme the result comes about in a rather nontrivial way, namely, due to a complicated interlude of the virtual corrections, the $2\rightarrow3$ contributions [including the surviving $\delta(1-w)$ terms originating from the $\hat{k}^2$ integrations, cf. Secs. II D and II E], the factorization piece [*including* the absorption of the terms $\sim(1-z)$ mentioned in Sec. II E], and the e_q^2 terms from $q\bar{q}\rightarrow\gamma q^{(\prime)}\bar{q}^{(\prime)}$, all of which are individually different from the corresponding unpolarized result. Only in their sum do these differences drop out. The same is true for the remaining terms $(\sim e_q'^2)$ in the final result for $q\bar{q}\rightarrow\gamma q'\bar{q}'$ which are also the same (apart from the sign) for the polarized and the unpolarized cases, which is again trivial in a scheme with an anticommuting γ_5. Of course, for our choice of factorization scheme, in particular, regarding Eq. (32) and its preceding discussion, the final results for the polarized cross sections when calculated in the HVBM scheme or in a scheme with an anticommuting γ_5 must be the same. At this point one might argue whether it was necessary to use the more complicated HVBM scheme instead of using a scheme with an anticommuting γ_5 which allows one to obtain certain results in a trivial way. Nevertheless, we think that it is much safer to use the HVBM scheme when dealing with the other processes such as $qg\rightarrow\gamma qg$, where γ_5 really develops its algebraic peculiarities, since this scheme, as mentioned above, has the property of being internally consistent.

As far as the unpolarized case is concerned, we have been able to compare our results with those listed in the

FORTRAN code of Ref. [3]. We find complete agreement in every detail, apart from the fact that we use a different spin average for incoming gluons *whenever* they are present in a subprocess, as discussed in Sec. II A. This leads to slight differences between the two results which are, however, calculable and under control.

For all processes the final cross section can be cast in the form

$$vw(1-v)\hat{s}\frac{d\Delta\hat{\sigma}}{dv\,dw}$$
$$=\alpha\alpha_s^2(\mu^2)\Bigg[\left[\Delta c_a\delta(1-w)+\Delta c_b\frac{1}{(1-w)_+}+\Delta c_c\right]\ln\frac{M^2}{\hat{s}}+\Delta c_d\ln\frac{M'^2}{\hat{s}}+\Delta c_1\delta(1-w)+\Delta c_2\frac{1}{(1-w)_+}$$
$$+\Delta c_3\left[\frac{\ln(1-w)}{1-w}\right]_+ +\Delta c_4\ln v+\Delta c_5\ln(1-v)+\Delta c_6\ln w+\Delta c_7\frac{\ln w}{1-w}+\Delta c_8\ln(1-w)$$
$$+\Delta c_9\ln(1-vw)+\Delta c_{10}\frac{\ln[(1-v)/(1-vw)]}{1-w}+\Delta c_{11}\ln(1-v+vw)+\Delta c_{12}\frac{\ln(1-v+vw)}{1-w}+\Delta c_{13}\Bigg]\,. \quad (36)$$

Again the unpolarized results are obtained by removing the Δ's from Eq. (36). The coefficients $(\Delta)c_i(v,w)$ which are rather lengthy even when abbreviations are introduced are listed in Appendix D for the unpolarized case and in Appendix E for the polarized [54]. Because of our choice to absorb all e_q^2 terms from $q\bar{q}\to\gamma q'\bar{q}'$ into $q\bar{q}\to\gamma gg$ the coefficients $(\Delta)c_a$ and $(\Delta)c_b$ and $(\Delta)c_1$, $(\Delta)c_2$, and $(\Delta)c_3$ are nonvanishing only for the processes $q\bar{q}\to\gamma gg$ and $qg\to\gamma qg$. These coefficients constitute the so-called dominant part [55] of the next-to-leading-order corrections. It should be noted that the "plus" distributions associated with them have to be modified if the lower limit of the w-integration is different from zero [as for our case, cf. Eq. (10)] [38]:

$$\frac{1}{(1-w)_+}=\frac{1}{(1-w)_A}+\ln(1-A)\delta(1-w)\,,$$
$$\left[\frac{\ln(1-w)}{1-w}\right]_+=\left[\frac{\ln(1-w)}{1-w}\right]_A+\frac{1}{2}\ln^2(1-A)\delta(1-w)\,, \quad (37)$$

where $1/(1-w)_A$ is defined for the lower limit A of the w integration by

$$\int_A^1\frac{f(w)}{(1-w)_A}dw\equiv\int_A^1\frac{f(w)-f(1)}{1-w}dw\,, \quad (38)$$

and analogously for

$$\left[\frac{\ln(1-w)}{1-w}\right]_A.$$

These latter equations are important for a numerical evaluation of our next-to-leading-order cross sections.

Let us finally note that first numerical calculations indicate that the asymmetry

$$A_\gamma\equiv\frac{E_\gamma d^3\Delta\sigma^{AB}/d^3p_\gamma}{E_\gamma d^3\sigma^{AB}/d^3p_\gamma} \quad (39)$$

is stable under the $O(\alpha\alpha_s^2)$ corrections, although the individual cross sections receive sizable corrections in some kinematical regions. Furthermore, A_γ, in fact, turns out to be strongly dependent on the size of the polarized gluon distribution ΔG. It should be noted, however, that the validity of the numerical results for the polarized case is still limited since, as mentioned above, the polarized two-loop splitting functions are not known up to now. These are needed for a consistent Q^2 evolution of the polarized parton distributions, unless one sticks to a fixed scale M^2. Furthermore the numerical results depend on further assumptions concerning the treatment of the poorly understood fragmentation of a final-state parton into a photon. As the content of this present paper is completely untouched by these uncertainties (apart from the choice of the factorization scheme which is trivial to change), we want to keep it self-contained and reserve a more detailed quantitative analysis to a forthcoming publication.

Note added in proof. After completing this work we became aware of the recent paper [57] in which the $O(\alpha\alpha_s^2)$ corrections for polarized prompt photon production were calculated independently and quantitative results were presented for them. In contrast to the HVBM scheme which we used the authors of Ref. [57] obtained the corrections using partly the γ_5 scheme of Ref. [12] and partly dimensional reduction [19].

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APPENDIX A: FOUR-POINT FUNCTIONS

In this appendix we present the results for the scalar, vectorial, and tensorial four-point functions for massless particles [33] which are needed to calculate the box-graph contributions [see Figs. 2(a) and 2(b)]:

$$\int \frac{d^n r}{(2\pi)^n} \frac{[1; r^\mu; r^\mu r^\nu; r^\mu r^\nu r^\rho]}{r^2(p-r)^2(q-r)^2(q+k-r)^2}$$

$$= \frac{i(-4\pi)^{(4-n)/2}}{(4\pi)^2} \Gamma\left[\frac{4-n}{2}\right] B\left[\frac{n}{2}-1, \frac{n}{2}-1\right]$$

$$\times [2(n-3)A_1 ; \quad \text{(A1)}$$

$$(n-3)(p^\mu A_2 + k^\mu \overline{A}_2 + q^\mu A_1) ; \quad \text{(A2)}$$

$$\tfrac{1}{2}(n-2)(p^\mu p^\nu A_3 + k^\mu k^\nu \overline{A}_3 + q^\mu q^\nu A_1 + [kq]^{\mu\nu} \overline{A}_2) + \tfrac{1}{2}(n-4)[pq]^{\mu\nu} A_2 + \tfrac{1}{2}[pk]^{\mu\nu} A_4 + \tfrac{1}{2} g^{\mu\nu} A_5 ; \quad \text{(A3)}$$

$$(n/4)(p^\mu p^\nu p^\rho A_7 + k^\mu k^\nu k^\rho \overline{A}_7 + q^\mu q^\nu q^\rho A_1 + [kkq]^{\mu\nu\rho} \overline{A}_3 + [qqk]^{\mu\nu\rho} \overline{A}_2) + \tfrac{1}{4}(n-4)[pqq]^{\mu\nu\rho} A_2$$

$$+ \tfrac{1}{4}(n-4)[ppq]^{\mu\nu\rho} A_3 + \tfrac{1}{2}[ppk]^{\mu\nu\rho} A_8 + \tfrac{1}{2}[pkk]^{\mu\nu\rho} \overline{A}_8 + \tfrac{1}{4}([pkq]^{\mu\nu\rho} + [pqk]^{\mu\nu\rho}) A_4 + \tfrac{1}{4}[gq]^{\mu\nu\rho} A_5$$

$$+ \tfrac{1}{4}[gp]^{\mu\nu\rho} A_6 + \tfrac{1}{4}[gk]^{\mu\nu\rho} \overline{A}_6 + O((n-4)^2)] , \quad \text{(A4)}$$

where

$$[kq]^{\mu\nu} \equiv k^\mu q^\nu + k^\nu q^\mu ,$$

$$[kpq]^{\mu\nu\rho} \equiv k^\mu p^\nu q^\rho + q^\mu k^\nu p^\rho + p^\mu q^\nu k^\rho ,$$

and

$$[gp]^{\mu\nu\rho} \equiv g^{\mu\nu} p^\rho + g^{\mu\rho} p^\nu + g^{\nu\rho} p^\mu .$$

Defining $s \equiv 2qk$, $t \equiv -2pq$, and $u \equiv -2kp$, the vectors p, q, and k must satisfy the relations $p^2 = q^2 = k^2 = 0$ and $s+t+u=0$. With

$$\omega \equiv 1 + s/t, \quad \Omega \equiv 1 + t/s = \omega/(\omega - 1) ,$$

and $n = 4 - 2\epsilon$ the coefficients A_i are given by [33]

$$A_1 = \frac{s^{-1-\epsilon}}{t}\left[\frac{1}{\epsilon} - \omega \mathcal{I}(\omega)\right] + \frac{t^{-1-\epsilon}}{s}\left[\frac{1}{\epsilon} - \Omega \mathcal{I}(\Omega)\right] ,$$

$$A_2 = \frac{s^{-1-\epsilon}}{t}\left[\frac{1}{\epsilon} + (1-\omega)\mathcal{I}(\omega)\right] - \frac{t^{-1-\epsilon}}{s}\mathcal{I}(\Omega) ,$$

$$A_3 = \frac{s^{-1-\epsilon}}{t}\left[\frac{1-\omega}{\Omega}\mathcal{I}(\omega) + \frac{1}{\omega(1-\epsilon)} + \frac{1}{\epsilon}\right]$$

$$- \frac{t^{-1-\epsilon}}{s}\frac{1}{\Omega}\left[\mathcal{I}(\Omega) - \frac{1}{1-\epsilon}\right] ,$$

$$A_4 = -2\frac{s^{-1-\epsilon}}{t}\left[\frac{1}{\omega} + (1-\epsilon)\frac{1}{\Omega}\mathcal{I}(\omega)\right]$$

$$-2\frac{t^{-1-\epsilon}}{s}\left[\frac{1}{\Omega} + (1-\epsilon)\frac{1}{\omega}\mathcal{I}(\Omega)\right] ,$$

$$A_5 = \frac{s^{-1-\epsilon}}{t} s \mathcal{I}(\omega) + \frac{t^{-1-\epsilon}}{s} t \mathcal{I}(\Omega) ,$$

$$A_6 = \frac{s^{-1-\epsilon}}{t} s\left[\frac{1}{\Omega}\mathcal{I}(\omega) + \frac{1}{\omega(1-\epsilon)}\right]$$

$$+ \frac{t^{-1-\epsilon}}{s} t \frac{1}{\Omega}\left[\mathcal{I}(\Omega) - \frac{1}{1-\epsilon}\right] ,$$

$$A_7 = \frac{s^{-1-\epsilon}}{t}\left[\frac{1-\omega}{\Omega^2}\mathcal{I}(\omega) - \frac{1}{\omega}\left[\frac{1}{2-\epsilon} - \frac{3}{1-\epsilon}\right] + \frac{1}{\epsilon} - \frac{1}{\omega^2(1-\epsilon)}\right]$$

$$- \frac{t^{-1-\epsilon}}{s}\frac{1}{\Omega}\left[\frac{1}{\Omega}\left[\mathcal{I}(\Omega) - \frac{1}{1-\epsilon}\right] - \frac{1}{2-\epsilon}\right] ,$$

$$A_8 = -\frac{s^{-1-\epsilon}}{t}\left[\frac{2-\epsilon}{\omega\Omega(1-\epsilon)} + \frac{1}{\omega(1-\epsilon)} + (2-\epsilon)\frac{1}{\Omega^2}\mathcal{I}(\omega)\right]$$

$$- \frac{t^{-1-\epsilon}}{s}\left[\frac{1}{\Omega} - \frac{2-\epsilon}{\omega\Omega(1-\epsilon)} + (2-\epsilon)\frac{1}{\omega\Omega}\mathcal{I}(\Omega)\right] ,$$

and $\overline{A}_i(s,t) = A_i(t,s)$. Here the function $\mathcal{I}$ is defined by

$$\mathcal{I}(y) \equiv \int_0^1 dx \frac{x^{-\epsilon}}{1-xy} = -\frac{\ln(1-y)}{y} + \epsilon\frac{1}{y}\mathrm{Li}_2(y) + O(\epsilon^2) .$$

Note that the last equality is also valid for $y > 1$, using in this case [56] $\ln(1-y) = \ln(y-1) + i\pi$ and

$$\mathrm{Li}_2(y) = -\mathrm{Li}_2[y/(y-1)] - \tfrac{1}{2}\ln^2(y-1) + \tfrac{1}{2}\pi^2 - 2i\pi \ln y + i\pi \ln(y-1) .$$

As can be seen in Eqs. (16) and (17), all dilogarithms Li_2 drop out in the final results for the virtual corrections.

APPENDIX B: PARAMETRIZATION OF THE MOMENTA

In this appendix we give the parametrizations for the vectors p_1, p_2, and k_1 introduced in Sec. II D in the rest frame of $k_2 + k_3$. There are three possible sets, depending on which vector is chosen to point in the z direction. With $p_1^0 = \hat{s}v/2\sqrt{s_{23}}$,

$$p_2^0 = \hat{s}(1 - vw)/2\sqrt{s_{23}} ,$$

and

$$k_1^0 = \hat{s}(1 - v + vw)/2\sqrt{s_{23}}$$

we have the following.

Set 1:

$$p_1=p_1^0(1,0,0,1,0,\ldots)\,,$$
$$p_2=p_2^0(1,-\sin\psi'',0,\cos\psi'',0,\ldots)\,,$$
$$k_1=k_1^0(1,-\sin\psi,0,\cos\psi,0,\ldots)\,;$$

Set 2:

$$p_1=p_1^0(1,\sin\psi'',0,\cos\psi'',0,\ldots)\,,$$
$$p_2=p_2^0(1,0,0,1,0,\ldots)\,,$$
$$k_1=k_1^0(1,\sin\psi',0,\cos\psi',0,\ldots)\,;$$

Set 3:

$$p_1=p_1^0(1,\sin,\psi,0,\cos\psi,0,\ldots)\,,$$
$$p_2=p_2^0(1,-\sin\psi',0,\cos\psi',0,\ldots)\,,$$
$$k_1=k_1^0(1,0,0,1,0,\ldots)\,,$$

where the ellipses indicate $n-5$ zero components. The angles ψ, ψ', and ψ'' are given by

$$\sin\psi=\frac{2\sqrt{w(1-v)(1-w)}}{1-v+vw}\,,$$
$$\sin\psi'=\frac{2v\sqrt{w(1-v)(1-w)}}{(1-vw)(1-v+vw)}\,,$$
$$\sin\psi''=\frac{2\sqrt{w(1-v)(1-w)}}{1-vw}\,.$$

Note that these expressions vanish for $w\to1$. It is now easy to obtain the Mandelstam variables from these sets of parametrizations. For example, defining $s_{12}\equiv(k_1+k_2)^2$ and $\hat{t}_2\equiv(p_1-k_2)^2$ we conveniently choose set 1 where we have, using Eq. (19),

$$\frac{1}{s_{12}\hat{t}_2}=-\frac{4}{\hat{s}^2v(1-v+vw)}\,\frac{1}{(1-\cos\theta_1)(1+\sin\psi\sin\theta_1\cos\theta_2-\cos\psi\cos\theta_1)}$$

which leads to the integral of the type (22).

APPENDIX C: *n*-DIMENSIONAL CROSS SECTIONS FOR THE POLARIZED 2→2 PROCESSES

Here we present the polarized cross sections $d\Delta\hat{\sigma}^{ab\to cd}/dv(\hat{s},v,\epsilon)$ as calculated in the HVBM scheme [37] needed for the factorization of mass singularities. Defining the common factor

$$\mathcal{N}\equiv\frac{\pi}{\hat{s}}\,\frac{\mu^{2\epsilon}}{\Gamma(1-\epsilon)}\left[\frac{4\pi\mu^2}{\hat{s}v(1-v)}\right]^{\epsilon}\,,$$

we have, using $\hat{t}\equiv(p_a-p_c)^2=-\hat{s}(1-v)$ and $\hat{u}\equiv(p_b-p_c)^2=-\hat{s}v$,

$$\frac{d\Delta\hat{\sigma}^{q\bar{q}\to\gamma g}}{dv}(\hat{s},v,\epsilon)=-\frac{2C_F}{N_C}\alpha\alpha_s e_q^2\mathcal{N}\left[(1+\epsilon)^2\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}+6\epsilon+2\epsilon^2\right]\,, \tag{C1}$$

$$\frac{d\Delta\hat{\sigma}^{qg\to\gamma q}}{dv}(\hat{s},v,\epsilon)=\frac{1}{N_C}\alpha\alpha_s e_q^2\mathcal{N}\left[-\frac{\hat{s}^2-\hat{t}^2}{\hat{s}\hat{t}}-\epsilon\frac{\hat{u}^2}{\hat{s}\hat{t}}\right]\,, \tag{C2}$$

$$\frac{d\Delta\hat{\sigma}^{q\bar{q}\to q'\bar{q}'}}{dv}(\hat{s},v,\epsilon)=-\frac{C_F}{N_C}\alpha_s^2\mathcal{N}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}+\epsilon\right]\,, \tag{C3}$$

$$\frac{d\Delta\hat{\sigma}^{gg\to q\bar{q}}}{dv}(\hat{s},v,\epsilon)=-\frac{1}{2C_FN_C}\alpha_s^2\mathcal{N}(\hat{t}^2+\hat{u}^2)\left[\frac{C_F}{\hat{t}\hat{u}}-\frac{N_C}{\hat{s}^2}\right]\,, \tag{C4}$$

$$\frac{d\Delta\hat{\sigma}^{qg\to qg}}{dv}(\hat{s},v,\epsilon)=\frac{1}{N_C}\alpha_s^2\mathcal{N}(\hat{s}^2-\hat{u}^2+\epsilon\hat{t}^2)\left[\frac{N_C}{\hat{t}^2}-\frac{C_F}{\hat{s}\hat{u}}\right]\,, \tag{C5}$$

$$\frac{d\Delta\hat{\sigma}^{qq'\to qq'}}{dv}(\hat{s},v,\epsilon)=\frac{C_F}{N_C}\alpha_s^2\mathcal{N}\left[\frac{\hat{s}^2-\hat{u}^2}{\hat{t}^2}\right]\,, \tag{C6}$$

$$\frac{d\Delta\hat{\sigma}^{qq\to qq}}{dv}(\hat{s},v,\epsilon)=\frac{C_F}{N_C}\alpha_s^2\mathcal{N}\left[\frac{\hat{s}^2-\hat{u}^2}{\hat{t}^2}+\frac{\hat{s}^2-\hat{t}^2}{\hat{u}^2}-\frac{2}{N_C}\left[(1+\epsilon)\frac{\hat{s}^2}{\hat{t}\hat{u}}-\epsilon(3+\epsilon)\right]\right]\,, \tag{C7}$$

$$\frac{d\Delta\hat{\sigma}^{q\bar{q}\to q\bar{q}}}{dv}(\hat{s},v,\epsilon)=\frac{C_F}{N_C}\alpha_s^2\mathcal{N}\left[\frac{\hat{s}^2-\hat{u}^2}{\hat{t}^2}-\epsilon-\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}+\frac{2}{N_C}\left[(1+\epsilon)\frac{\hat{u}^2}{\hat{s}\hat{t}}-\frac{\epsilon}{\hat{s}}[\hat{t}(1+\epsilon)-\hat{u}(1-\epsilon)]\right]\right]\,. \tag{C8}$$

APPENDIX D: COEFFICIENTS FOR THE UNPOLARIZED CASE

Here we list the coefficients c_i $(i=a,\ldots,d;\ 1,\ldots,13)$ [to be used in the unpolarized version of Eq. (36)] for all contributing subprocesses. We have introduced the abbreviations

$$\begin{aligned}
&T_{q\bar{q}}=(1-v)^2+v^2\ ,\\
&T_{qg}=1+(1-v)^2\ ,\\
&X=1-vw\ ,\\
&Y=1-v+vw\ ,\\
&v_i=i-v\ \ (i=1,\ldots,5)\ ,\\
&b_0=\tfrac{11}{6}N_C-\tfrac{1}{3}N_F\ ,
\end{aligned}\tag{D1}$$

where N_F is the number of active flavors. As usual we have $C_F=\frac{4}{3}$ and $N_C=3$. Furthermore, we have already set $T_R=\frac{1}{2}$ wherever it appeared. As discussed in the main text, we have absorbed the $\sim e_q^2$ contributions from $q\bar{q}\to\gamma q'\bar{q}'$ (together with the corresponding contributions from $q\bar{q}\to\gamma q\bar{q}$) into the cross section for $q\bar{q}\to\gamma gg$ where they appear as N_F contributions in the coefficients c_1, c_2, and c_{13}. Therefore, the coefficients for $q\bar{q}\to\gamma q'\bar{q}'$ are all proportional to $e_q'^2$ which we have left explicitly in the formulas for the sake of clarity. In all processes involving only one quark flavor we omit the trivial charge factor e_q^2. The cross section for $q\bar{q}'\to\gamma q\bar{q}'$ can be obtained from the one for $qq'\to\gamma qq'$ by simply changing $e_q'\to -e_q'$, where e_q' is the charge of the q'. Finally it should be noted that the cross section for the process $g(p_1)q(p_2)\to\gamma qg$ can be obtained from the one for $q(p_1)g(p_2)\to\gamma qg$ presented below by making the replacements

$$v\to 1-vw\ ,$$
$$w\to\frac{1-v}{1-vw}$$

and introducing the Jacobian $v/(1-vw)$ in the integrand in Eq. (10).

1. $q\bar{q}\to\gamma gg$

$$c_a=-\frac{C_F^2}{N_C}T_{q\bar{q}}\left[3-2\ln\frac{v_1}{v}\right],\quad c_b=-4\frac{C_F^2}{N_C}T_{q\bar{q}},\quad c_c=\frac{C_F^2}{N_C}\left[(1-2v)^2-\frac{(1-2v)v}{X}+\frac{vv_1}{X^2}+(3v^2+v_1^2)w\right],\quad c_d=0\ ,$$

$$\begin{aligned}
c_1=&\frac{C_F}{N_C}T_{q\bar{q}}b_0\ln\frac{\mu^2}{\hat{s}}-\frac{C_FN_F}{9N_C}T_{q\bar{q}}(5-3\ln v)\\
&+\frac{C_F^2}{N_C}[-(7-\pi^2)T_{q\bar{q}}-2T_{q\bar{q}}\ln v\ln v_1+v(2+v)\ln v_1+v_1v_3\ln v+(3v^2+2v_1)\ln^2 v+(2v+3v_1^2)\ln^2 v_1]\\
&+C_F\left[\frac{(67-6\pi^2)T_{q\bar{q}}}{18}-vv_1\ln v_1-\frac{11-16vv_1}{6}\ln v-\frac{2v+3v_1^2}{2}\ln^2 v_1-\frac{vv_2}{2}\ln^2 v\right],
\end{aligned}$$

$$c_2=T_{q\bar{q}}\left[\frac{C_FN_F}{3N_C}-\frac{C_F}{6}(11-12\ln v_1)-4\frac{C_F^2}{N_C}\ln\frac{v_1}{v}\right],\quad c_3=2\frac{C_F}{N_C}(4C_F-N_C)T_{q\bar{q}},\quad c_4=-c_c\ ,$$

$$c_5=C_F\left[1-v^2-\frac{1+v_1^2}{X}+vv_1w\right]+2\frac{C_F^2}{N_C}\frac{v_1^2+v^2w^2}{X},\quad c_6=c_5,\quad c_7=C_FT_{q\bar{q}}\ ,$$

$$c_8=C_F\left[-1-2vv_1+\frac{1+v_1^2}{X}-(1-2v)vw\right]+\frac{C_F^2}{N_C}\left[1+8vv_1-\frac{vv_1}{X^2}-\frac{3v+4v_1^2}{X}-(1-4v+8v^2)w\right],$$

$$c_9=-4\frac{C_F^2}{N_C}v(v_1-vw)+C_Fv(v_2-vw),\quad c_{10}=\frac{C_F}{N_C}(4C_F-N_C)T_{q\bar{q}},\quad c_{11}=c_{12}=0\ ,$$

$$\begin{aligned}
c_{13}=&\frac{C_FN_F}{3N_C}v(v_2-vw)+\frac{C_F^2}{N_C}\left[-4vv_1+\frac{vv_1}{X^2}+\frac{vv_1}{X}+(1+v-4v^2)w\right]\\
&+C_F\left[-\frac{v(1-4v)}{2X}+\frac{vv_1}{2X^2}-\frac{2-11v_1^2}{6}+\frac{(v_3-11vv_1)w}{6}\right].
\end{aligned}$$

2. $q\bar{q}\rightarrow\gamma q'\bar{q}'$

$$c_a=c_b=c_c=c_1=c_2=c_3=c_5=c_6=c_7=c_9=c_{10}=c_{12}=0\ ,$$

$$c_d=\frac{C_F}{N_C}e_q'^2v^2v_1w\left[-1-\frac{4v_1^2}{Y^4}+\frac{2v_2}{Y}+\frac{4v_1v_2}{Y^3}-\frac{2v_2^2}{Y^2}\right],\quad c_4=c_8=-c_d,\quad c_{11}=-2c_d\ ,$$

$$c_{13}=\frac{C_F}{N_C}e_q'^2\frac{2v^3v_1}{Y^2}(1-w)w\left[-1+\frac{6vv_1w}{Y^2}\right].$$

3. $qg\rightarrow\gamma qg$

$$c_a=T_{qg}v\left[-3\frac{C_F}{4N_C}+\frac{N_F}{6N_C}-\frac{1}{12}\left[11-12\ln\frac{v_1}{v}\right]\right],\quad c_b=-\frac{1}{N_C}(C_F+N_C)T_{qg}v\ ,$$

$$c_c=\frac{C_F}{N_C}\left[-\frac{vv_1^2}{X^3}+\frac{vv_1(1+2v_1)}{X^2}-\frac{v(1+2v_1)^2}{2X}+\frac{v(v^2+6v_1)}{2}+\frac{v(2+v^2)w}{2}\right]$$
$$+\left[-v+\frac{v(v+v_2^2)}{X}+\frac{vv_1^2}{X^3}-\frac{vv_1v_2}{X^2}-\frac{v^2(2v^2+3v_1)w}{v_1}+\frac{v^3(1+v)w^2}{v_1}-\frac{v^4w^3}{v_1}\right],$$

$$c_d=-\frac{v}{2Y^3v_1}\left[1+v^2(1-w)^2\right]\left[\frac{C_F}{N_C}v_1^2+Yvw\right](v_1^2+2Yvw)\ ,$$

$$c_1=T_{qg}v\frac{b_0}{2N_C}\ln\frac{\mu^2}{\hat{s}}-\frac{v}{4}(v^2-2v_1)\ln^2v+\frac{vv_1}{2}\ln v_1-vv_1\ln v+\frac{1}{4}\pi^2vv_1(1+v)+\frac{vv_1}{4}(1+v)\ln^2v_1$$
$$-\frac{vv_1}{2}(1+v)\ln v\ \ln v_1+\frac{C_F}{N_C}\left[-\frac{7v}{4}T_{qg}+\frac{v}{2}(1+2v)\ln v_1+\frac{\pi^2}{6}v(1-4v+5v^2)-\frac{v}{4}(3v^2-2v_1)\ln v\right.$$
$$\left.+\frac{v}{2}(3v^2+2v_1)\ln^2v+\frac{v}{2}(v^2+v_1^2)\ln^2v_1-v(v^2+v_1^2)\ln v\ \ln v_1\right],$$

$$c_2=T_{qg}v\left[-\ln\frac{v_1}{v}-\frac{C_F}{4N_C}(3-4\ln v)\right],\quad c_3=\frac{1}{N_C}(C_F+2N_C)T_{qg}v\ ,$$

$$c_4=\frac{C_F}{N_C}v\left[\frac{3v^2-10v_1}{2}+\frac{v_1v_3^2}{2Y}+\frac{v_1^2}{X^3}-\frac{v_1^2v_3}{Y^2}+\frac{v_1^3}{Y^3}-\frac{v_1(1+2v_1)}{X^2}+\frac{(1+2v_1)^2}{2X}-\frac{(11v^2+2v_1)w}{2}+4v^2w^2\right]$$
$$+\frac{v}{v_1}\left[-v_1(v+v_2^2)+\frac{v_1^2v_4}{Y}-\frac{v_1^3}{X^3}-\frac{v_1^3}{Y^2}-\frac{v_1v_2}{X}+\frac{v_1^2v_2}{X^2}+\frac{v(7+v^2)w}{2}-2v^2w^2(1+X)\right],$$

$$c_5=-\frac{C_F}{N_C}v^3(1+w)-\frac{v}{2v_1}[(1+v)v_1^2-4vw(2X-vv_1+Yvw)]\ ,$$

$$c_6=-\frac{C_F}{N_C}v^3(1+4w-2w^2)+\frac{v}{2}\left[\frac{2T_{qg}}{X}+v^2-2v_1-vw(4v_1-X)\right],$$

$$c_7=\frac{1}{2N_C}(2C_F-N_C)v(3v^2+2v_1)\ ,$$

$$c_8=c_4-\frac{C_F}{N_C}v^3(1-2w+2w^2)-v\left[\frac{T_{qg}}{X}-\frac{vw}{2}(4-3v)-\frac{v^2w^2}{2}\right],$$

$$c_9=\frac{C_F}{N_C}v^3(3-2w)w+\frac{1}{2v_1}v[v_1(1+v_2^2)-v(8-3vv_1)w+v^2w^2(7+v-4vw)]\ ,$$

$$c_{10}=\frac{v}{2}(1+v_2^2)+\frac{C_F}{N_C}v(v^2+v_1^2)\ ,$$

$$c_{11}=\frac{C_F}{N_C}v\left[-4v_1+\frac{v_1v_3^2}{Y}-\frac{2v_1^2v_3}{Y^2}+\frac{2v_1^3}{Y^3}+v(2X+v)w\right]+v\left[-\frac{11}{2}+v+\frac{2v_1v_4}{Y}-\frac{2v_1^2}{Y^2}+\frac{3vw}{2}+\frac{v^2w^2}{2}\right],$$

$$c_{12}=-\frac{1}{2}(1-2v)v-\frac{C_F}{N_C}v(1+v^2)\ ,$$

$$c_{13}=\frac{vw}{v_1}\left[-\frac{1}{2}T_{qg}v_1-\frac{2vv_1^2}{Y^2}-\frac{(9-4v)vv_1^2}{2X^2}+\frac{3vv_1^3}{X^3}+\frac{vv_1(1+3v_1)}{2X}+\frac{vv_1v_2}{Y}+Yv^2w\right]$$
$$+\frac{C_F}{N_C}\left[-\frac{v}{2X}(22-35v+11v^2)+\frac{vv_1}{2X^2}(23-16v)-\frac{vv_1}{2Y}(13-3v-3v^2)-\frac{4vv_1^2}{X^3}\right.$$
$$\left.+\frac{(17-5v)vv_1^2}{2Y^2}-\frac{3vv_1^3}{Y^3}+\frac{3v(v^2+10v_1)}{4}+\frac{v(4-7v^2)w}{4}\right].$$

4. $gg\to\gamma q\bar{q}$

$$c_a=c_b=c_1=c_2=c_3=c_7=c_{10}=c_{12}=0\ ,$$

$$c_c=\frac{1}{N_C}v\left[-\frac{(3v_1^2+2v_2)}{2}-\frac{2v_1^2}{X^2}+\frac{2v_1v_2}{X}+w(v+2v_1^2)-\frac{w^2}{2}(3v^2+4v_1)\right],$$

$$c_d=-\frac{v}{2}\left[\frac{1}{N_C}-\frac{1}{C_F}\frac{vv_1w}{Y^2}\right]\left[X^2+v_2^2-\frac{4v_1v_2}{Y}+\frac{4v_1^2}{Y^2}\right],$$

$$c_4=\frac{1}{C_F}\frac{v^2v_1w}{2}\left[3-\frac{4v_1^2}{Y^4}+\frac{4v_1}{Yv_2}-\frac{2(1+v_1^2)}{Xv_2}+\frac{4v_1v_2}{Y^3}-\frac{2v_2^2}{Y^2}\right]$$
$$+2\frac{1}{N_C}v\left[\frac{v_1^2}{X^2}+\frac{v_1^2}{Y^2}-\frac{2v_1^2}{Xv_2}-\frac{2v_1^2}{Yv_2}+(v^2+v_1)(1-w+w^2)\right],$$

$$c_5=\frac{1}{C_F}\frac{v^2v_1w}{2}\left[1-2w+\frac{2(1+v_1^2)w}{XY}\right]+\frac{1}{N_C}v\left[\frac{4-3v+v^3}{v}-\frac{2v_1(1+v_1^2)}{XYv}-2vw(1-w)\right],$$

$$c_6=\frac{1}{C_F}\frac{vv_1w}{2}\left[1-3v_1+\frac{2v_1(1+v_1^2)}{XY}\right]+\frac{1}{N_C}v\left[-2v_1(1-w)+\frac{2v_1(1+v_1^2)(1-w)}{XY}+v^2w^2\right],$$

$$c_8=\frac{1}{C_F}vv_1w\left[1-\frac{2vv_1^2}{Y^4}-\frac{1+v_1^2}{Xv_2}+\frac{2vv_1v_2}{Y^3}-\frac{vv_2^2}{Y^2}-\frac{v_1(2v_1-vv_2)}{Yv_2}\right]$$
$$+\frac{1}{N_C}v\left[\frac{2v_1^2}{X^2}+\frac{2v_1^2}{Y^2}+3(1+v_1^2)-\frac{2(2-v^2)v_1}{Yvv_2}+\frac{2v_1^2(2v_1-vv_2)}{Xvv_2}-4(v^2+v_1)w+(3v^2+2v_1)w^2\right],$$

$$c_9=-\frac{1}{N_C}v[v^2+v_1^2+vw(2v_1+vw)]\ ,$$

$$c_{11}=\frac{1}{C_F}v^2v_1w\left[-3-\frac{4v_1^2}{Y^4}+\frac{2v_2}{Y}+\frac{4v_1v_2}{Y^3}-\frac{2v_2^2}{Y^2}\right]-4\frac{1}{N_C}\frac{v^3v_1(1-w)w}{Y^2}\ ,$$

$$c_{13}=\frac{1}{C_F}\frac{vv_1w}{2}\left[\frac{-2v}{Y}+\frac{2(7-6v)v}{Y^2}-\frac{4vv_1}{X^2}+\frac{12vv_1^2}{Y^4}+v_2+\frac{2vv_2}{X}-\frac{12vv_1v_2}{Y^3}-2v_2w\right]$$
$$+\frac{1}{N_C}\frac{v}{2}\left[-v_1v_3-\frac{4v_1^2}{X^2}+\frac{4v_1v_2}{X}+2(v+2v_1^2)w-v_2^2w^2\right].$$

5. $qq \to \gamma qq$

$$c_a = c_b = c_1 = c_2 = c_3 = c_7 = c_{10} = c_{12} = 0 ,$$

$$c_c = \frac{C_F}{N_C}\frac{1}{2v_1 w}\left[-\frac{v_1^2}{X^2} - v_1(5-3v+2v^2v_1) + \frac{2v_1v_2}{X} - 2v_1(v^3-v_1^2)w - (4v^2-2v^3v_1+v_1^2)w^2 + 2(2-Y)v^3w^3 \right] ,$$

$$c_d = \frac{1+v^2(1-w)^2}{Y^2 w(1-v)}\left[\frac{C_F}{N_C^2} Y^2 v v_1 w - \frac{C_F}{N_C}(v_1^4 + v v_1^3 w + v^2 v_1^2 w^2 + Y v^3 w^3) \right] ,$$

$$c_4 = \frac{C_F}{N_C^2} v \left[-(1+v)^2 + \frac{4v_1}{Yv_2} - \frac{2(1+v_1^2)}{Xv_2} + 2v(1+v)w - v^2w^2 \right]$$
$$+ \frac{C_F}{N_C}\frac{1}{2v_1 w}\left[(43-33v+4v^2v_3)v_1 + \frac{v_1^2}{X^2} - \frac{8(1+v_2^2)v_1^2}{Y} + \frac{4v_1^4}{Y^2} - \frac{2v_1v_2}{X} \right.$$
$$\left. - 2v_1[1+11v+v^2(1+2v_1)]w + (22v^2-18v^3+4v^4+v_1^2)w^2 - 4(2-Y)v^3w^3 \right] ,$$

$$c_5 = \frac{C_F}{N_C^2}\left[v_1 - \frac{4v_1}{Yv_2} + \frac{v(1+v_1^2)}{Xv_2} - v^3(1-w)^2 - vw \right] + \frac{C_F}{N_C}\frac{v}{v_1}\left[-2\frac{2-Y}{Y}(v_1^2 + Yv^2w^2) + vw(3+v^2) \right] ,$$

$$c_6 = \frac{C_F}{N_C}\frac{1}{w}\left[2(3+v^2)v_1 - \frac{4v_1^2}{Y} - 2v(1+v)v_2 w + v_3 v^2 w^2 \right]$$
$$+ \frac{C_F}{N_C^2}\left[-(1+v+v^3) + \frac{4v_1}{Yv_2} - \frac{v(1+v_1^2)}{Xv_2} + vw[1+v^2(2-w)] \right] ,$$

$$c_8 = c_4 - \frac{C_F}{N_C}\frac{1}{Y}(X-v)v(X+v) - \frac{C_F}{N_C^2}\frac{1}{XY}[1+v^2(1-w)^2]\{v_1^2 - vw[1+v^2(1-w)]\} ,$$

$$c_9 = \frac{C_F}{N_C}\frac{v}{v_1}[(1+v)v_1 - vw(4-vv_1) + 2(2-Y)v^2w^2], \quad c_{11} = 4\frac{C_F}{N_C}\frac{v^3v_1w(1-w)}{Y^2} ,$$

$$c_{13} = -\frac{C_F}{N_C^2}Y^2 v + \frac{C_F}{N_C}\frac{1}{2v_1 w}\left[-(27-30v+2v^2+2v^3)v_1 - \frac{v_1^2}{X^2} + \frac{8v_1^3v_5}{Y} - \frac{16v_1^4}{Y^2} + \frac{v_1(1+5v_1)}{X} + v_1(1+2vv_1v_3)w \right.$$
$$\left. - v_1[1-4v(1+v)v_1]w^2 + 2Yv^3w^3 \right] .$$

6. $q\bar{q} \to \gamma q\bar{q}$

$$c_a = c_b = c_1 = c_2 = c_3 = 0 ,$$

$$c_c = \frac{C_F}{N_C}\frac{1}{2v_1 w}\left[-\frac{v_1^2}{X^2} - v_1(5-3v+2v^2v_1) + \frac{2v_1v_2}{X} - 2v_1(v^3-v_1^2)w - (4v^2-2v^3v_1+v_1^2)w^2 + 2(2-Y)v^3w^3 \right] ,$$

$$c_d = \frac{1+v^2(1-w)^2}{Y^4 v_1 w}\left[-\frac{C_F}{N_C^2} Y^2 v v_1 w (v_1^2 - vv_1 w + v^2 w^2) \right.$$
$$\left. - \frac{C_F}{N_C}(v_1^6 + 3vwv_1^5 + 5v^2w^2v_1^4 + 4v^3w^3v_1^3 + 5v^4w^4v_1^2 + 3v^5w^5v_1 + v^6w^6) \right] ,$$

$$c_4 = \frac{C_F}{N_C^2} v \left[-2 - \frac{2(1+vv_1)v_1}{Y} - v_1^2 + \frac{6v_1^2}{Y^2} + v(5+v)w - 3v^2w^2 \right]$$
$$+ \frac{C_F}{N_C}\frac{1}{2v_1 w}\left[(11-53v+40v^2-12v^3)v_1 + \frac{v_1^2}{X^2} - \frac{4(10-27v+16v^2-3v^3)v_1^2}{Y} + \frac{4v_3v_5v_1^4}{Y^2} - \frac{8v_1^5v_4}{Y^3} + \frac{8v_1^6}{Y^4} \right.$$
$$\left. - \frac{2v_1v_2}{X} - 2(1-5v-7v^2)v_1 w + (2v^3+6v^4+v_1^2)w^2 - 4(2-Y)v^3w^3 \right] ,$$

$$c_5=\frac{C_F}{N_C^2}v\left[4-3v-\frac{2v_1(1+v^2+2v_1)}{Y}-v(X+2v_1)w\right]+\frac{C_F}{N_C}\frac{v}{v_1}\left[-2\frac{2-Y}{Y}(-v_1^2+Yv^2w^2)+vw(3+v^2)\right],$$

$$c_6=\frac{C_F}{N_C^2}v\left[-(1+v_2^2)+\frac{2v_1(1+v^2+2v_1)}{Y}+2vw(1+X)\right]$$
$$+\frac{C_F}{N_C}\frac{1}{w}\left[\frac{4v_1^2}{Y}-2(1+v)v_1^2+2vw(v^2+v_2)-v^2w^2(1+v)\right],$$

$$c_7=-\frac{C_F}{N_C^2}(1+v^2),\quad c_8=c_4+\frac{C_F}{N_C}\frac{v}{Y}(2-Y)(Y-2vw)-\frac{C_F}{N_C^2}\frac{v^2}{Y}(Y-2vw)(2v-w-3vw+2vw^2),$$

$$c_9=\frac{C_F}{N_C^2}v[v_2-(3-Y)vw]-\frac{C_F}{N_C}\frac{v}{v_1}[(1+v)v_1+2Xvw+v^2w(1+X-Y+2vw^2)],\quad c_{10}=-\frac{C_F}{N_C^2}(1+v_1^2),$$

$$c_{11}=2\frac{C_F}{N_C}v\left[-2(v+v_2^2)-\frac{2v_1^2v_3^2}{Y^2}+\frac{4v_1^3v_3}{Y^3}-\frac{4v_1^4}{Y^4}+\frac{4v_1v_2^2}{Y}+vwv_3\right]$$
$$+\frac{C_F}{N_C^2}v\left[15-13v+3v^2+\frac{12v_1^2}{Y^2}-\frac{12v_1v_2}{Y}-7vw+3v^2w^2\right],$$

$$c_{12}=\frac{C_F}{N_C^2}(3-2vv_1),$$

$$c_{13}=\frac{C_F}{2N_C^2}\left[-3-\frac{(5-2v)v}{X}+\frac{vv_1}{X^2}+\frac{4vv_1v_3}{Y}+2vv_1^2-\frac{8vv_1^2}{Y^2}+(v^2+2v^3+v_1^2)w+2v^3w^2\right]$$
$$+\frac{C_F}{N_C}\frac{1}{2v_1w}\left[-(31-54v+22v^2+2v^3)v_1-\frac{v_1^2}{X^2}+\frac{4(27-19v)v_1^3}{Y}-\frac{4(1+18v_2)v_1^4}{Y^2}+\frac{24v_1^5v_4}{Y^3}-\frac{24v_1^6}{Y^4}+\frac{v_1v_2}{X}\right.$$
$$\left.+v_1(1+2vv_1v_3)w-v_1(1-4v^2v_1)w^2+2Yv^3w^3\right].$$

7. $qq'\rightarrow\gamma qq'$

$$c_a=c_b=c_1=c_2=c_3=c_7=c_{10}=c_{12}=0,$$

$$c_c=e_q^2\frac{C_F}{N_C}\frac{v^2w}{v_1}\left[\frac{-(3+v^2)}{2}+\frac{v_1}{X}-\frac{v_1^2}{2X^2}+(2-Y)vw\right]-e_q'^2\frac{C_F}{2N_C}\frac{1}{w}(1+v^2)v_1(2-2w+w^2),$$

$$c_d=\frac{C_F}{N_C}\frac{(1+v^2(1-w)^2)}{2Y^2wv_1}\{-e_q^2v^2w^2(v_1^2+2Yvw)-e_q'^2v_1^2[2v_1^2+vw(2Y-vw)]\},$$

$$c_4=2e_qe_q'\frac{C_F}{N_C}v\left[-(5+v)+\frac{3+v_2^2}{Y}+3vw\right]$$
$$+e_q^2\frac{C_F}{2N_C}\frac{v^2w}{v_1}\left[\frac{-2v_1}{X}-\frac{2v_1v_3}{Y}+\frac{v_1^2}{X^2}+\frac{2v_1^2}{Y^2}+2(4-vv_1)-4(2-Y)vw\right]$$
$$+e_q'^2\frac{C_F}{N_C}\frac{v_1}{2w}\left[2(2v^2+v_2)+\frac{2vv_1^2(1-w)}{Y^2}-(1+2v^2)(2-w)w\right],$$

$$c_5=2e_qe_q'\frac{C_F}{N_C}\left[1-\frac{2}{Y}\right]vv_1+e_q^2\frac{C_F}{N_C}\frac{v^2w}{v_1}[3+v^2-2(2-Y)vw],$$

$$c_6=-2e_qe_q'\frac{C_F}{N_C}v^2w\frac{2-Y}{Y}+e_q'^2\frac{C_F}{N_C}\frac{v_1}{w}[2+v^2(2-2w+w^2)],$$

$$c_8=c_4-e_qe_q'\frac{C_F}{N_C}\frac{v}{Y}(2-Y)(v_1-vw)\ ,$$

$$c_9=e_qe_q'\frac{C_F}{N_C}(2-Y)v-e_q^2\frac{C_F}{N_C}\frac{v^2w}{v_1}[3+v^2-2(2-Y)vw]\ ,$$

$$c_{11}=2e_qe_q'\frac{C_F}{N_C}v(2-Y)+2e_q'^2\frac{C_F}{N_C}vv_1\left[-\frac{v}{Y}-\frac{v_1}{Y^2}\right]+2e_q^2\frac{C_F}{N_C}v^2w\left[1-\frac{v_3}{Y}+\frac{v_1}{Y^2}\right]\ ,$$

$$c_{13}=e_qe_q'\frac{C_F}{N_C}vv_1w(2-Y)\left[\frac{1}{X}-\frac{2v}{Y^2}\right]+e_q^2\frac{C_F}{2N_C}\frac{v^2w}{v_1}\left[(1-2v)v_1-\frac{v_1^2}{X^2}-\frac{4v_1^2}{Y^2}+\frac{v_1v_2}{X}+\frac{2v_1v_2}{Y}+2Yvw\right]$$
$$+e_q'^2\frac{C_F}{2N_C}\frac{v_1}{w}\left[\frac{2v_1v_4}{Y}-\frac{4v_1^2}{Y^2}-2(1+vv_1)+(v_3-2v_1^2)w-(1-2v)(1+v)w^2\right]\ .$$

APPENDIX E: COEFFICIENTS FOR THE POLARIZED CASE

Here we present the Δc_i $(i=a,\ldots,d;1,\ldots,13)$. We use the abbreviations (D1) of Appendix D, but with T_{qg} replaced by

$$\Delta T_{qg}=1-(1-v)^2 \tag{E1}$$

as in Sec. II C. The coefficients for the processes $q\bar{q}\to\gamma gg$ and $q\bar{q}\to\gamma q'\bar{q}'$ need not be repeated, they are exactly the negatives of the corresponding unpolarized ones. All remarks made at the beginning of Appendix D also apply to this appendix.

1. $qg\to\gamma qg$

$$\Delta c_a=\Delta T_{qg}v\left[-3\frac{C_F}{4N_C}+\frac{N_F}{6N_C}-\frac{1}{12}\left[11-12\ln\frac{v_1}{v}\right]\right],\quad \Delta c_b=-\frac{1}{N_C}(C_F+N_C)\Delta T_{qg}v\ ,$$

$$\Delta c_c=v^2\left[2-\frac{3-2v}{Xv}+\frac{2v_1}{X^2v}+\frac{(1+v)v_1}{v}+(1+X-Y)w\right]+\frac{C_F}{N_C}\frac{v}{2}\left[-(v^2-2v_2)+\frac{2v_1}{X^2}-\frac{1+4v_1}{X}+vv_4w\right]\ ,$$

$$\Delta c_d=-v[1+v^2(1-w)^2]\frac{Y+vw}{2Y^3}\left[\frac{C_F}{N_C}v_1^2+Yvw\right]\ ,$$

$$\Delta c_1=\Delta T_{qg}v\frac{b_0}{2N_C}\ln\frac{\mu^2}{\hat{s}}+\frac{1}{4}\Delta T_{qg}v\ln^2v+\frac{1}{2}vv_1\ln v_1+\frac{1}{4}\pi^2vv_1(1+v)+\frac{1}{4}vv_1(1+v)\ln^2v_1-\frac{1}{2}vv_1(1+v)\ln v\ln v_1$$
$$+\frac{C_F}{N_C}\left[-\frac{7v}{4}\Delta T_{qg}-\frac{3}{4}\Delta T_{qg}v\ln v+\frac{1}{2}\Delta T_{qg}v\ln^2v+\frac{1}{2}v(1+2v)\ln v_1-\frac{\pi^2}{6}v(3-4v-v^2)\right.$$
$$\left.+\frac{1}{2}v(v^2-v_1^2)\ln^2v_1-v(v^2-v_1^2)\ln v\ln v_1\right]\ ,$$

$$\Delta c_2=\Delta T_{qg}v\left[-\ln\frac{v_1}{v}-\frac{C_F}{4N_C}(3-4\ln v)\right],\quad \Delta c_3=\frac{1}{N_C}(C_F+2N_C)\Delta T_{qg}v\ ,$$

$$\Delta c_4=\frac{C_F}{N_C}\frac{v^2}{2}\left[\frac{5-4v}{Xv}-\frac{2+2v-7v^2}{v}-\frac{2v_1}{X^2v}+\frac{2v_3v_1^2}{Y^2v}-\frac{v_5v_1^2}{Yv}-\frac{2v_1^3}{Y^3v}-w(4+7v)\right]$$
$$+\frac{v^2}{2}\left[\frac{2v_2}{v}-\frac{4v_1}{X^2v}+\frac{2v_3v_1}{Xv}-\frac{2v_4v_1}{Yv}+\frac{2v_1^2}{Y^2v}-v_5w+6vw^2\right]\ ,$$

$$\Delta c_5=-\frac{C_F}{N_C}v^3(1+w)+\frac{v}{2}\left[-1-4v+3v^2+\frac{2vv_2}{X}-2(1+2X-Y)vw\right]\ ,$$

$$\Delta c_6=\frac{C_F}{N_C}v^3(1-2w)+\frac{v^2}{2}[2-v(1-w)^2-w],\quad \Delta c_7=\frac{1}{2N_C}(2C_F-N_C)v\Delta T_{qg}\ ,$$

$$\Delta c_8=\Delta c_4-\frac{C_F}{N_C}v^3(1-2w)+\frac{v^2}{2}[-2v_2-(3-Y)w],\quad \Delta c_9=3\frac{C_F}{N_C}v^3w+\frac{v}{2}[1+4v-3v^2-v(1-7X+v)w]\ ,$$

$$\Delta c_{10}=\frac{v}{2}(1+4v-3v^2)+\frac{C_F}{N_C}v(v^2-v_1^2)\ ,$$

$$\Delta c_{11}=\frac{v}{2}\left[11-2v-\frac{4v_1v_4}{Y}+\frac{4v_1^2}{Y^2}-(4-X)vw\right]+\frac{C_F}{N_C}v\left[2(1-2v)+\frac{2v_1^2v_3}{Y^2}-\frac{v_1^2v_5}{Y}-\frac{2v_1^3}{Y^3}+v^2w\right]\ ,$$

$$\Delta c_{12}=-\frac{C_F}{N_C}vv_1(1+v)-\frac{v}{2}(v^2-v_1^2)\ ,$$

$$\Delta c_{13}=\frac{C_F}{4N_C}\left[\frac{-2(8-9v)v}{X}+\frac{6vv_1}{X^2}+\frac{2v(13-3v-3v^2)v_1}{Y}-\frac{2(17-5v)vv_1^2}{Y^2}+\frac{12vv_1^3}{Y^3}+vv_2(X+3v_1)\right]$$
$$+\frac{v^2w}{2}\left[2Y+v-\frac{v}{X}-\frac{2v_2}{Y}+\frac{3v_1}{X^2}+\frac{4v_1}{Y^2}\right]\ .$$

2. $gg\to\gamma q\bar{q}$

$$\Delta c_a=\Delta c_b=\Delta c_1=\Delta c_2=\Delta c_3=\Delta c_7=\Delta c_{10}=\Delta c_{12}=0,\quad \Delta c_c=-\frac{1}{2N_C}v[-6X+(7+3v)v_1+v^2(2-w)^2]\ ,$$

$$\Delta c_d=\frac{v}{2}\left[X^2+v_2^2-\frac{4v_2v_1}{Y}+\frac{4v_1^2}{Y^2}\right]\left[\frac{1}{N_C}-\frac{1}{C_F}\frac{vv_1w}{Y^2}\right]\ ,$$

$$\Delta c_4=\frac{1}{2C_F}v^2v_1w\left[-3+\frac{2v_2^2}{Y^2}-\frac{4v_1}{Yv_2}-\frac{4v_2v_1}{Y^3}+\frac{4v_1^2}{Y^4}+\frac{2(1+v_1^2)}{Xv_2}\right]$$
$$+\frac{2}{N_C}v^2\left[\frac{1-2v}{v}-\frac{v_1^2}{Y^2v}+\frac{2v_1^2}{Yvv_2}-\frac{v_1(1+v_1^2)}{Xvv_2}+v_2w\right]\ ,$$

$$\Delta c_5=\frac{1}{2C_F}vv_1w\left[-(2+v)+\frac{2v_1(1+v_1^2)}{XY}\right]+\frac{1}{N_C}v\left[-(2+v_1^2)+\frac{2v_1(1+v_1^2)(1-w)}{XY}+2w\right]\ ,$$

$$\Delta c_6=\frac{1}{2C_F}v^2v_1w\left[-3-2w+\frac{2(1+v_1^2)w}{XY}\right]+\frac{1}{N_C}v^2\left[\frac{4v_1}{v^2}-\frac{2v_1(1+v_1^2)}{XYv^2}+v_2w^2\right]\ ,$$

$$\Delta c_8=\frac{1}{N_C}v^2\left[\frac{2(2-v^2)v_1}{Yv^2v_2}-\frac{2v_1^2}{Y^2v}-\frac{4-v^3-6vv_1}{v^2}+\frac{2v_1^2(1+v_1^2)}{Xv^2v_2}+v_2w(4-w)\right]$$
$$+\frac{1}{C_F}v^2v_1w\left[-2+\frac{v_2^2}{Y^2}-\frac{2v_2v_1}{Y^3}+\frac{2v_1^2}{Y^4}+w-\frac{v_1(v+2v_1w)}{XY}\right]\ ,$$

$$\Delta c_9=\frac{1}{N_C}(2-X)v(X-2v_1)+\frac{2}{C_F}v^2v_1w\ ,$$

$$\Delta c_{11}=-\frac{1}{C_F}v^2v_1w\left[-3+\frac{2v_2}{Y}-\frac{2v_2^2}{Y^2}+\frac{4v_1v_2}{Y^3}-\frac{4v_1^2}{Y^4}\right]+\frac{4}{N_C}\frac{v^3v_1w(1-w)}{Y^2}\ ,$$

$$\Delta c_{13}=-\frac{1}{2N_C}v(v_1^2+v^2w^2)+\frac{1}{2C_F}v^2v_1w\left[-3+\frac{2}{Y}+\frac{-14+12v}{Y^2}+\frac{12v_1(v_1^2+vv_2w)}{Y^4}\right]\ .$$

3. $qq\to\gamma qq$

$$\Delta c_a=\Delta c_b=\Delta c_1=\Delta c_2=\Delta c_3=\Delta c_7=\Delta c_{10}=\Delta c_{12}=0\ ,$$

$$\Delta c_c=\frac{C_F}{2N_C}\left[-\frac{vv_3}{X}+\frac{vv_1}{X^2}-2(1-2v-v^2v_1)+(1+v)(v^2+v_1^2)w-2v^3w^2\right],$$

$$\Delta c_d=\frac{v}{Y^2}(1+v^2(1-w)^2)\left[\frac{C_F}{N_C^2}Y^2-\frac{C_F}{N_C}(v_1^2+Yvw)\right],$$

$$\Delta c_4=-\frac{C_F}{N_C^2}\frac{v}{XY}(1+v^2(1-w)^2)[v_1+(3-Y)vw]$$
$$+\frac{C_F}{2N_C}\left[2(1+7v-5v^2+2v^3)+\frac{vv_3}{X}-\frac{vv_1}{X^2}+\frac{4v(-5+2v)v_1}{Y}\right.$$
$$\left.+\frac{4vv_1^2}{Y^2}-(6v^2+4v^3+v_1)w+4v^3w^2\right],$$

$$\Delta c_5=\frac{C_F}{N_C}v\left[-2v_1+\frac{4v_1}{Y}-(1+X-Y)vw\right]+\frac{C_F}{N_C^2}\frac{1}{XY}[1+v^2(1-w)^2][-Xv_1(1+v)+v^3w^2],$$

$$\Delta c_6=\frac{C_F}{N_C}v^2\left[\frac{2(v^2+v_2)}{v}-\frac{4v_1}{Yv}-w(1+v)\right]$$
$$+\frac{C_F}{N_C^2}\frac{1}{XY}[1+v^2(1-w)^2]\{v_1^2-vw[1+v^2(1-w)]\},$$

$$\Delta c_8=\Delta c_4+\frac{C_F}{N_C}\frac{(X-v)v(X+v)}{Y}-\frac{C_F}{N_C^2}\frac{1}{XY}[1+v^2(1-w)^2]\{v_1^2-vw[1+v^2(1-w)]\},$$

$$\Delta c_9=-\frac{C_F}{N_C}v[X(2-Y)+v^2w^2],\quad \Delta c_{11}=-4\frac{C_F}{N_C}\frac{v^3v_1w(1-w)}{Y^2},$$

$$\Delta c_{13}=-\frac{C_F}{N_C^2}Y^2v+\frac{C_F}{2N_C}\left[-1-2v-\frac{(6-5v)v}{X}+2v^3+\frac{2vv_1}{X^2}+\frac{8vv_3v_1}{Y}-\frac{16vv_1^2}{Y^2}\right.$$
$$\left.+v_2w(v^2+v_1^2)+2v^3w^2\right].$$

4. $q\bar{q}\to\gamma q\bar{q}$

$$\Delta c_a=\Delta c_b=\Delta c_1=\Delta c_2=\Delta c_3=0,$$

$$\Delta c_c=\frac{C_F}{2N_C}\left[-\frac{vv_3}{X}+\frac{vv_1}{X^2}+2(v^2v_2-v_1^2)+(1+v)(v^2+v_1^2)w-2v^3w^2\right],$$

$$\Delta c_d=\frac{v(1+v^2(1-w)^2)}{Y^4}\left[\frac{C_F}{N_C^2}Y^2(v_1^2-vv_1w+v^2w^2)-\frac{C_F}{N_C}[v_1^4+v^4w^4+2vv_1w(v_1+vw)^2]\right],$$

$$\Delta c_4=\frac{C_F}{N_C^2}v\left[2+v_1^2-\frac{6v_1^2}{Y^2}+\frac{2v_1(1+vv_1)}{Y}-v(5+v)w+3v^2w^2\right]$$
$$+\frac{C_F}{2N_C}\left[\frac{vv_3}{X}-2(7v^2-4v^3-v_1)-\frac{vv_1}{X^2}-\frac{4(5-2v)vv_1^2}{Y}+\frac{4vv_3^2v_1^2}{Y^2}-\frac{8vv_3v_1^3}{Y^3}\right.$$
$$\left.+\frac{8vv_1^4}{Y^4}-(1+v)(v^2+v_1^2)w+4v^3w^2\right],$$

$$\Delta c_5=\frac{C_F}{N_C}v\left[2v_1-\frac{4v_1}{Y}-(1+X-Y)vw\right]+\frac{C_F}{N_C^2}v\left[-4+3v+\frac{2v_1(2+v_1^2)}{Y}+(2+X)vw-2v^2w\right],$$

$$\Delta c_6=-\frac{C_F}{N_C}\frac{v^2}{Y}(2v_1^2+(4-3v_1^2)w-vv_3w^2)-\frac{C_F}{N_C^2}\frac{v}{Y}[2(2-Y)v^2w^2+(1+v^2)(Y-2vw)],$$

$$\Delta c_7=\frac{C_F}{N_C^2}(1+v^2)\ ,$$

$$\Delta c_8=\Delta c_4-\frac{C_F}{N_C}\frac{(2-Y)v(Y-2vw)}{Y}+\frac{C_F}{N_C^2}\frac{v^2}{Y}(Y-2vw)(2v-w-3vw+2vw^2)\ ,$$

$$\Delta c_9=-\frac{C_F}{N_C^2}v[v_2-(3-Y)vw]+\frac{C_F}{N_C}v(1+v+v^2w-2v^2w^2),\quad \Delta c_{10}=\frac{C_F}{N_C^2}(1+v_1^2)\ ,$$

$$\Delta c_{11}=2\frac{C_F}{N_C}v\left[2(v+v_2^2)-\frac{4v_2^2v_1}{Y}+\frac{2v_3^2v_1^2}{Y^2}-\frac{4v_3v_1^3}{Y^3}+\frac{4v_1^4}{Y^4}-vv_3w\right]$$
$$+\frac{C_F}{N_C^2}v\left[-(15-13v+3v^2)+\frac{12v_2v_1}{Y}-\frac{12v_1^2}{Y^2}+7vw-3v^2w^2\right]\ ,$$

$$\Delta c_{12}=-\frac{C_F}{N_C^2}(3-2vv_1)\ ,$$

$$\Delta c_{13}=\frac{C_F}{2N_C}\left[-1-\frac{vv_2}{X}+\frac{2vv_1}{X^2}+\frac{4(8-7v)vv_1}{Y}-2v(1+v)v_1-\frac{4(19-12v)vv_1^2}{Y^2}\right.$$
$$\left.+\frac{24vv_3v_1^3}{Y^3}-\frac{24vv_1^4}{Y^4}+2Yv^2w+v_2w\right]$$
$$+\frac{C_F}{2N_C^2}\left[3+\frac{(5-2v)v}{X}-\frac{vv_1}{X^2}-\frac{4vv_3v_1}{Y}-2vv_1^2+\frac{8vv_1^2}{Y^2}-(v^2+2v^3+v_1^2)w-2v^3w^2\right]\ .$$

5. $qq'\to\gamma qq'$

$$\Delta c_a=\Delta c_b=\Delta c_1=\Delta c_2=\Delta c_3=\Delta c_7=\Delta c_{10}=\Delta c_{12}=0\ ,$$

$$\Delta c_c=-e_q^2\frac{C_F}{2N_C}\frac{(1+X)(1+X-Y)v^3w^2}{X^2}-e_q'^2\frac{C_F}{2N_C}(1+v)v_1^2(2-w)\ ,$$

$$\Delta c_d=-\frac{C_F}{N_C}\frac{v}{2Y^2}[1+v^2(1-w)^2][e_q^2vw(Y+vw)+e_q'^2(2v_1^2+vv_1w)]\ ,$$

$$\Delta c_4=e_q^2\frac{C_F}{2N_C}v^2w\left[\frac{2}{X}-2(2X+v)+\frac{2v_3}{Y}-\frac{v_1}{X^2}-\frac{2v_1}{Y^2}\right]+e_q'^2\frac{C_F}{2N_C}v_1\left[\frac{2v^2}{Y}+\frac{2vv_1}{Y^2}+(1-2v^2)(2-w)\right]$$
$$+2e_qe_q'\frac{C_F}{N_C}\frac{(1+X)v^2w}{Y}\ ,$$

$$\Delta c_5=-e_q^2\frac{C_F}{N_C}(1+X-Y)v^2w+2e_qe_q'\frac{C_F}{N_C}\frac{(2-Y)vv_1}{Y},\quad \Delta c_6=-e_q'^2\frac{C_F}{N_C}v^2v_1(2-w)+2e_qe_q'\frac{C_F}{N_C}\frac{(2-Y)v^2w}{Y}\ ,$$

$$\Delta c_8=\Delta c_4+e_qe_q'\frac{C_F}{N_C}\frac{(2-Y)v(v_1-vw)}{Y},\quad \Delta c_9=e_q^2\frac{C_F}{N_C}(1+X-Y)v^2w-e_qe_q'\frac{C_F}{N_C}(2-Y)v\ ,$$

$$\Delta c_{11}=2e_q^2\frac{C_F}{N_C}\frac{v^2w}{Y^2}[Y+v^2w(1-w)]+2e_q'^2\frac{C_F}{N_C}\frac{vv_1}{Y^2}[1-v^2(1-w)]-2e_qe_q'\frac{C_F}{N_C}(2-Y)v\ ,$$

$$\Delta c_{13}=e_q^2\frac{C_F}{2N_C}v^2w\left[v_2-\frac{v_2}{X}-\frac{2v_2}{Y}+\frac{2v_1}{X^2}+\frac{4v_1}{Y^2}+2vw\right]$$
$$+e_q'^2\frac{C_F}{2N_C}v_1\left[-2v^2+\frac{2vv_2}{Y}-v_1-\frac{4vv_1}{Y^2}+(v^2+v_2)w\right]+e_qe_q'\frac{C_F}{N_C}vv_1w\left[-1-\frac{v}{X}+\frac{4v}{Y^2}-\frac{2v}{Y}\right]\ .$$

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[22] This means that no algebraic contradictions arise in this scheme, which, e.g., makes it possible to derive general proofs of Ward identities [15]. For instance, the HVBM scheme automatically reproduces [14] the well-known result [23] for the triangle anomaly, preserving the canonical vector Ward identities. This is quite in contrast to, e.g., the scheme of Ref. [17] where Ward identities have to be enforced by hand order by order in perturbation theory by appropriately fixing the inherent ambiguity present in this scheme.

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