

Hyperon polarization in a hydrodynamical model

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A possible coherent origin of polarization phenomena in baryon production is suggested. By describing in terms of an optical potential the interaction of the hadronic matter in expansion with a baryon produced in its interior, the main features of the hyperon polarization observed in high-energy hadronic collisions are shown to be well reproduced. In contrast with the existent models, this one can describe the antihyperon polarization just as naturally as the hyperon polarization and may complement the usual quark-rearrangement mechanism in the latter case.

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I. INTRODUCTION

In high-energy proton-proton or proton-nucleus collisions with hyperon (or antihyperon) production,

$$p + A \rightarrow Y(\bar{Y}) + X, \quad (1)$$

where $Y = (\Lambda, \Sigma, \Xi)$, these particles appear to be significantly polarized [1–4]. This is a remarkable fact because from a simple extrapolation of low-energy data as well as two-final-particle-collision data, we would expect that the hadronic interaction becomes less and less spin dependent as the incident energy increases. Thus, before the appearance of the hyperon polarization data, it was believed that hadron polarization is a low-energy phenomenon.

Some models have been proposed such as the semiclassical string model [5] and the parton recombination model [6], both of which are essentially based on a mechanism of flavor recombination of the incident proton. They are found to reproduce the main characteristics of the hyperon polarization data, but they seem to meet trouble with the recently reported antihyperon data [3,4], since antihyperons have nothing in common with the proton and cannot be produced by a quark recombination. Another model has been proposed [7], which is based on Reggeized one-pion exchange and relates, upon factorization, the polarization observed in high-energy $pA \rightarrow YX$ collisions with that observed in low-energy $\pi p \rightarrow YK$ reactions. Also this model will meet trouble with the antihyperon data because we cannot produce antihyperons in low-energy πp collisions.

In this work, we propose an alternative or complementary approach by which the polarization appears as a consequence of a coherent interaction of the produced particle (we shall call it as such although it may just be a

protoparticle or a group of constituents with correct quantum numbers, which eventually evolves into the observed particle) with the surrounding hadronic system. In such a picture, what is relevant is not the incident energy, which is only necessary to produce complex hadronic matter, but the relative energy of the produced hyperon with respect to this matter where it is formed, so that the polarization still appears as a relatively low-energy phenomenon. Also, as far as the polarization is concerned, the specific character of the incident particle, such as its flavor content, does not play any role. Thus, there is no *a priori* restriction for antihyperons, which may be polarized just like hyperons.

The main purpose of our paper is to show how this alternative mechanism works, giving a special emphasis in more qualitative aspects of the problem. It is also our purpose to show that this mechanism may reproduce the correct order of magnitude and the same general x_F as well as k_{\perp} behavior exhibited by the data. Although of great interest, no effort will be made in this paper to give possible explanations for the observed differences among the different kinds of particles.

In the next section, we describe the main idea in terms of a particle emerging from a sharp surface of background hadronic matter at rest. The coherent interaction is simply described by a complex potential there. In Sec. III, we consider a somewhat more realistic version of longitudinally expanding cylindrical matter taking, however, the results of the previous section as valid in the rest frame of each fluid element. Comparisons with the data are carried out having in mind that our mechanism appears more clearly when other ones are clearly absent. Conclusions are drawn in Sec. IV.

II. STATIC BACKGROUND MODEL

To illustrate the idea outlined in the Introduction, let us first consider a simplified picture of an outgoing particle of momentum \mathbf{k} emerging from a sharp surface of background hadronic matter. By making an analogy with optics, we shall represent the interaction by a com-

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plex potential $V = V_1 + iV_2$ inside the matter and $V = 0$ outside. We remark that, although it is always possible to refine the model by introducing a more smooth potential, it does not change the qualitative feature of the results so we prefer the present simple form. Also a spin-dependent interaction is perfectly possible but it will be seen below that polarization appears even without an explicit spin dependence in V .

Writing the Dirac equation for our hyperon

$$E\Psi = (-i\alpha \cdot \nabla + \beta m + V)\Psi, \quad (2)$$

we have (see Fig. 1 for definitions of variables)

$$\begin{aligned} \Psi_I(\mathbf{r}) &= u_{\text{in}}(\mathbf{k}_{\text{in}})e^{ik_{\text{in}} \cdot \mathbf{r}} + u_{\text{ref}}(\mathbf{k}_{\text{ref}})e^{ik_{\text{ref}} \cdot \mathbf{r}}, \\ \Psi_{II}(\mathbf{r}) &= u(\mathbf{k})e^{ik \cdot \mathbf{r}}, \end{aligned} \quad (3)$$

where the continuity of the wave function leads to

$$\mathbf{k}_{\text{in}} = \begin{pmatrix} k_1 \\ 0 \\ k'_3 \end{pmatrix}, \quad \mathbf{k}_{\text{ref}} = \begin{pmatrix} k_1 \\ 0 \\ -k'_3 \end{pmatrix}, \quad \mathbf{k} = \begin{pmatrix} k_1 \\ 0 \\ k_3 \end{pmatrix} \quad (4)$$

and these components are related to the (outgoing) energy E and the potential V by

$$\begin{aligned} (E - V)^2 &= k_1^2 + k_3'^2 + m^2, \\ E^2 &= k_1^2 + k_3^2 + m^2. \end{aligned} \quad (5)$$

Solving the Dirac equation, we express the spinor of the transmitted wave in the form

$$\begin{pmatrix} u^+(\mathbf{k}) \\ u^-(\mathbf{k}) \end{pmatrix} = \begin{pmatrix} T^{++} & T^{+-} \\ T^{-+} & T^{--} \end{pmatrix} \begin{pmatrix} u_{\text{in}}^+(\mathbf{k}_{\text{in}}) \\ u_{\text{in}}^-(\mathbf{k}_{\text{in}}) \end{pmatrix}, \quad (6)$$

where plus and minus represent, respectively, positive and negative polarizations normal to the collision plane (x_2 axis). We verify that $T^{+-} = T^{-+} = 0$, due to the continuity of the wave function.

Then, the transverse polarization P (parallel to x_2 axis) is written as

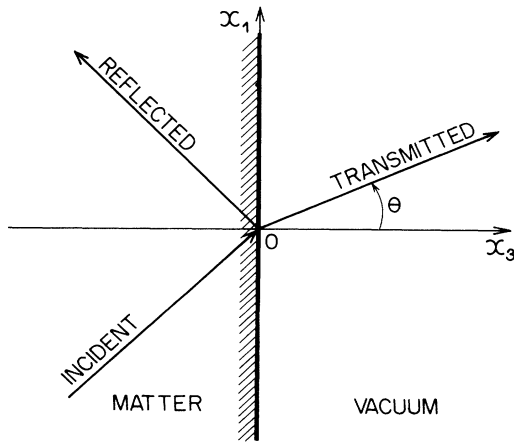


FIG. 1. An incident wave in the matter (I) with \mathbf{k}_{in} is partially reflected at $x_3 = 0$ and partially transmitted to the vacuum (II).

$$P = \frac{|T^{--}|^2 - |T^{++}|^2}{|T^{--}|^2 + |T^{++}|^2}, \quad (7)$$

where T^{++} and T^{--} may easily be computed as

$$\begin{aligned} T^{++} &= \frac{2\sqrt{2m(E+m)}N_+k'_3}{(E - V + m)k_3 + (E + m)k'_3 - iV k_1}, \\ T^{--} &= \frac{2\sqrt{2m(E+m)}N_-k'_3}{(E - V + m)k_3 + (E + m)k'_3 + iV k_1}, \end{aligned} \quad (8)$$

with the normalization constants N_{\pm} given by

$$|N_{\pm}|^2 = \frac{|E - V + m|^2}{2\{m(E - V_1 + m) + V_2^2 - (\text{Im}k'_3 \pm k_1)\text{Im}k'_3\}}. \quad (9)$$

Physically, the polarization appears in our picture due to the well-known spin-orbit interaction which, in the low-energy limit, writes $\hat{\sigma} \cdot \nabla V \times \hat{\mathbf{k}}/4m^2$. In our case, this term is effective only at the surface of the matter.

For the unpolarized incident beam, one sees from Eqs. (7)–(9) that the net polarization of the outgoing flux is nonzero only if the potential V contains an imaginary part ($V_2 \neq 0$). It is found that the sign of the polarization is the same as that of V_2 . We also find that, if the potential contains a positive real part, the polarization effect is enhanced and, for the negative real part, it is reduced. In Fig. 2, we show how the transverse polarization varies as a function of the refraction angle θ for a pure imaginary potential. As seen there, a larger polarization is produced for a larger refraction angle, apparently in accordance with the observed larger polarization for larger values of the Feynman x_F variable. Since the polarization effect we discuss is due to the discontinuity in the potential V , we also expect that it increases with p_T because such particles can leave the hadronic matter more easily and thus polarized, whereas low- k_T particles may not have enough time to escape from the medium, so frozen out unpolarized in this case.

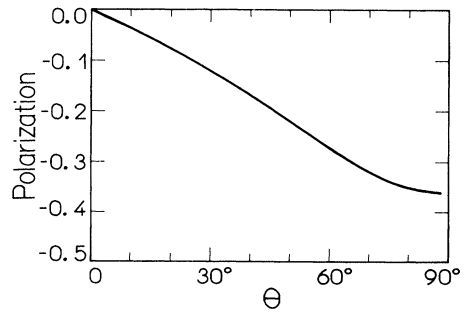


FIG. 2. Polarization computed by using (7) as a function of the refraction angle θ . The potential has been chosen $V = -0.3i$ GeV and the outgoing momentum $k = 1$ GeV.

III. EXPANDING BACKGROUND MODEL AND COMPARISONS WITH DATA

In order to incorporate the mechanism mentioned at the end of the preceding section and give a more realistic description of other aspects of high-energy collisions, let us consider a longitudinally expanding cylindrical hadronic matter with transverse radius R . We shall parametrize the rapidity distribution of the collective flow by a Gaussian

$$\frac{d\rho}{dy_{\text{fluid}}} \propto e^{-\alpha y_{\text{fluid}}^2} \quad (10)$$

and assume that the matter density is constant in the transverse directions. Then, by making a naive assumption that the previous results embodied by Eqs. (2)–(9) remain valid in each rest frame of the matter and that particles are produced in equilibrium at certain temperature T , the overall polarization is written as a convolution of the flow distribution (10) and the partial polarization given by (7), with an appropriate Boltzmann factor $\exp(-E/T)$ and normalization.

The interaction of our baryon with the background matter depends indeed on the matter density, giving rise to a time-varying potential. For the present estimate, we shall replace such a potential just by a time-averaged value. However, there is one effect that we have to explicitly take into account, namely, the freeze-out of the fluid. Since our particle is polarized only when it leaves the matter surface, it will remain unpolarized if the medium becomes rarefied before such an escape occurs. Thus, the number of polarized baryons is proportional to the surface area, the transverse particle velocity and the time interval before the freeze-out occurs, during which our polarization mechanism is effective. Writing this time interval as Δt , the number of polarized baryons is

$$F_{\text{pol}} \propto 2\pi R \Delta t \frac{k_T}{E}. \quad (11)$$

As for the number of unpolarized baryons, it is proportional to the volume of the cylinder at the moment of the freeze-out, so

$$F_{\text{unpol}} \propto \pi R^2. \quad (12)$$

The net polarization shall be computed by taking these weights into account and, as a function of the particle momentum \mathbf{k} , reads

$$P(\mathbf{k}) = \frac{\int d\bar{y} P(k) F_{\text{pol}} e^{-\alpha(y-\bar{y})^2} E e^{-E/T}}{\int d\bar{y} (F_{\text{unpol}} + F_{\text{pol}}) e^{-\alpha(y-\bar{y})^2} E e^{-E/T}}, \quad (13)$$

where $y = y(\mathbf{k})$ is the center-of-mass rapidity of the particle (\bar{y} is the value relative to the fluid element) and $E = E(\mathbf{k}, \bar{y})$ the energy of the particle in the rest frame of the fluid element.

In the present picture, the polarization emerges as a result of the coherent interaction of the produced particle with the surrounding hadronic matter. Thus, such a mechanism should be more effective in the central region, whereas in the fragmentation region, some quark-

recombination mechanism [5–7] could be dominant. Our model is also applicable for antibaryons, which cannot be produced by a recombination from the incident proton. In this sense, it would be nice if we could compare the model with antihyperon data. However, because of the scarceness of such data, we choose the Ξ particles to illustrate how our model works. For Ξ particles the recombination mechanism is likely to be rather ineffective because of the increase of the strangeness quantum number by 2.

In Figs. 3–6, the polarization of Ξ^- in the reaction $p + \text{Be} \rightarrow \Xi^- + X$ calculated with the present model is compared to the corresponding data. In this calculation, we used as input $\alpha = 0.5$, $V_1 = 0.7$ GeV, $V_2 = -0.5$ GeV, $T = 0.15$ GeV, $R/\Delta t = 2.5$. We see the results are quite satisfactory reproducing the main qualitative and quantitative features of the experimental data. We can also see that the antiparticle data (Ξ^+) are consistent with these calculations. The above values of α , T , and $R/\Delta t$ are also consistent with the usual hydrodynamical picture of the multiparticle production process. In fact, the value of α determined by the inclusive rapidity distribution is around 0.5. As for the potential, it may appear that the values obtained here for V_1 and V_2 are somewhat larger than those expected from the usual optical-model potential in the traditional nuclear physics, which are of the order of some tens MeV. However, the relativistic mean-field theories [8] suggest that the apparent optical potential is the difference of the two contributions from the vector meson field (repulsive) and the scalar meson field (attractive). In these models, each of these potentials is as large as several hundred MeV at the central region of the normal nuclei. In our case, we are dealing with very high-density hadronic matter, so it may well be expected that the potential strength comes much stronger than those found in ground state of the normal nuclei.

In our picture, the polarization is produced at the surface of the matter and it decreases slowly for large- k_T values. On the other hand, a low- k_T hyperon hardly

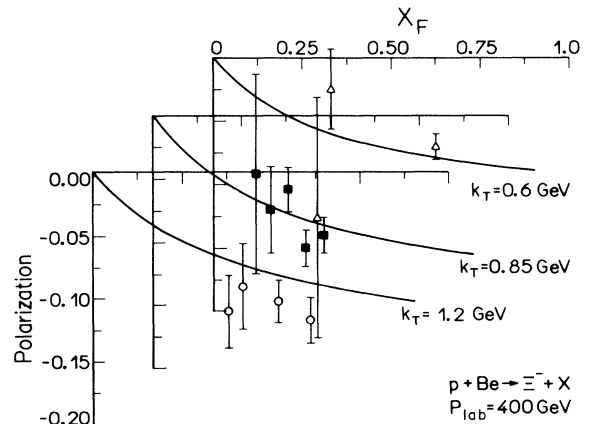


FIG. 3. Polarization of Ξ^- hyperon P_{Ξ^-} , calculated as a function of x_F at $p_{\text{lab}} = 400$ GeV for three values of k_T , is compared with data [2,3]. The latter are classified in three k_T intervals: $0.4 < k_T < 0.55$ GeV (Δ); $0.7 \leq k_T < 1.0$ GeV (\blacksquare), and $1.0 \leq k_T < 1.3$ GeV (\circ).

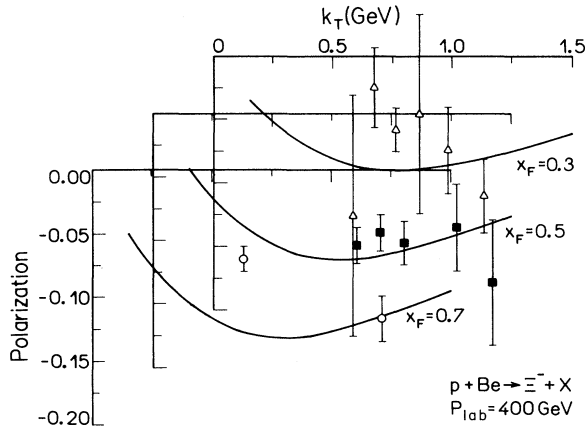


FIG. 4. Polarization of Ξ^- hyperon P_{Ξ^-} , calculated as a function of k_T at $p_{\text{lab}}=400$ GeV for three values of x_F , is compared with data [2,3]. The latter are classified in three x_F intervals: $0.2 < x_F \leq 0.4$ (Δ), $0.4 < x_F \leq 0.6$ (\blacksquare), and $0.6 < x_F \leq 0.8$ (\circ).

leaves the matter before the freeze-out, so its net polarization becomes reduced too. As a result, there appears a maximum of $|P|$ around 0.8 GeV. Looking at Fig. 6, it seems that this value is too small, when compared to data. However, if we include the transverse expansion of the fluid, such a minimum is expected to shift toward a higher- k_T value ($k_T \simeq 1.3$ GeV) [9], improving the agreement.

In the present simplified version, the potential $V = V_1 + iV_2$ represents the effective interaction of the particle with the hadronic matter (including its absorption or creation) and it may vary from one particle to another. If we take $V_1 \simeq 1$ GeV and $V_2 \simeq -1$ GeV, the Λ -particle data are well reproduced, provided that the transverse momentum k_T and x_F are not so large ($k_T < 1.5$ GeV and $x_F < 0.5$). High- k_T particles are more likely to be produced by a hard collision process and so

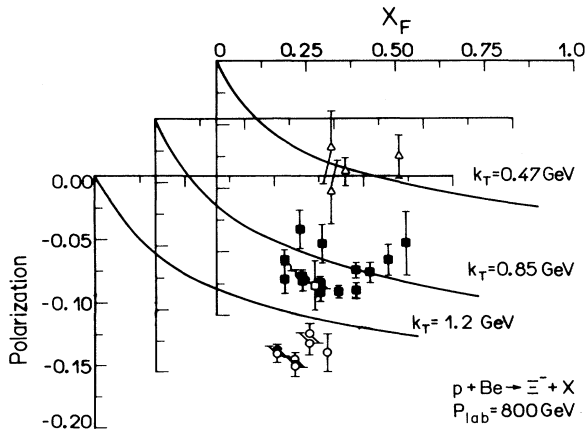


FIG. 5. Polarization of Ξ^- hyperon P_{Ξ^-} , calculated as a function of x_F at $p_{\text{lab}}=800$ GeV for three values of k_T , is compared with data [2,3]. The symbols are the same as in Fig. 3. Ξ^+ data have also been included here (\square).

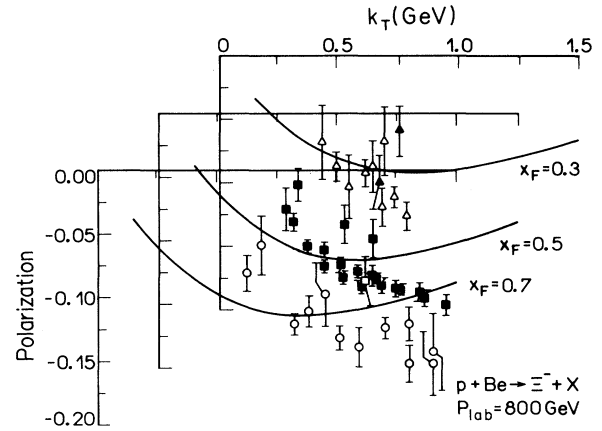


FIG. 6. Polarization of Ξ^- hyperon P_{Ξ^-} , calculated as a function of k_T at $p_{\text{lab}}=800$ GeV for three values of x_F , is compared with data [2,3]. The symbols are the same as in Fig. 4. Ξ^+ data have also been included here: $0.2 < x_F \leq 0.4$ (\blacktriangle) and $0.4 < x_F \leq 0.6$ (\square).

are probably out of the scope of the hydrodynamical description on which our model is based.

For Σ particles, the observed polarization data show an opposite sign compare to those of Λ and Ξ . In our model, this property is achieved with a positive V_2 . In fact, taking $V_2 \simeq 1.0$ GeV, the correct sign and the magnitude of Σ polarization can be obtained. The positive imaginary part of the potential seems to imply that these particles be produced coherently and more in the surface region of the matter.

IV. CONCLUSIONS

In summary, the present simple calculations show that the polarization of hyperons, except in situations such that the quark recombination of the incident particle becomes dominant, can naturally be described as due to a secondary, low-energy, and coherent interaction of the particle produced within a dense hadronic matter with the surroundings. In this case, with an appropriate choice of the parameters, both x_F and p_T dependences of P may be correctly reproduced, not only qualitatively but also quantitatively. In the present picture, the interactions of the produced hyperons with the surrounding hadronic matter are represented by optical potentials V . It will be extremely interesting to understand the microscopic origin of these potentials, in particular, which is the cause of the differences among different kinds of hyperons.

We are making effort in that direction but we find that just to know the existence of an alternative mechanism for polarization is already amusing enough. Our picture is particularly interesting for the antihyperon case, so more data are desirable for antihyperon polarization.

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