

## Supersymmetric QCD corrections to quark pair production in $e^+e^-$ annihilation

A. Djouadi

*Laboratoire de Physique Nucléaire, Université de Montréal, Case 6128A, H3C 3J7 Montréal PQ, Canada*

M. Drees

*Physics Department, University of Wisconsin, Madison, Wisconsin 53706*

H. König

*Department of Physics, Carleton University, Ottawa, Canada K1S 5B6*

(Received 19 May 1993)

We calculate supersymmetric QCD corrections (squark and gluino loops) to quark pair production in  $e^+e^-$  annihilation, allowing for mixing between left- and right-handed squarks and taking into account the effects of nonzero quark masses. Corrections to the  $Z$  boson partial widths are generally small and positive, except in the case of large  $\bar{b}$  squark mixing, where they become negative. At high-energy  $e^+e^-$  colliders, larger corrections to the total cross sections are possible. Corrections to forward-backward asymmetries are negligible except possibly for top quarks, where they are sensitive to  $\bar{t}$  squark mixing. We also comment on the possibility that the gluino mass is only a few GeV.

PACS number(s): 13.65.+i, 12.38.Bx, 14.80.Ly

### I. INTRODUCTION

The introduction of supersymmetry [1] (SUSY) is one of the most attractive extensions of the standard model (SM). It not only stabilizes [2] the huge hierarchy between the weak scale and the grand unification or Planck scale against radiative corrections, if SUSY is broken at a sufficiently large scale, as is the case, e.g., in supergravity (SUGRA) models [3], it might allow us to understand the origin of the hierarchy in terms of radiative gauge symmetry breaking [4]. Moreover, SUSY models offer a natural solution of the cosmological dark matter problem [5], and allow for a consistent grand unification of all known gauge couplings, in contrast with the nonsupersymmetric SM [6]. All these attractive features are already present in the minimal supersymmetric extension of the SM (MSSM) to which we will stick in this article.

Unfortunately, no direct signal for the production of superparticles has yet been observed; experimental searches so far have only resulted in lower bounds on particle masses, the most stringent ones coming from the CERN  $e^+e^-$  collider LEP [7] and the Fermilab Tevatron [8]. It is therefore tempting to look for SUSY through precision measurements, where quantum corrections involving superparticles might alter SM predictions. The potentially largest corrections can be expected from corrections involving strong interactions, i.e., from squark and gluino loops. Given the inherent uncertainties of cross-section calculations as well as measurements at hadron colliders, the most promising (and also the simplest) process where such corrections can be probed is quark pair production in  $e^+e^-$  annihilation.<sup>1</sup>

<sup>1</sup>Squark and gluino loops also contribute to rare  $K$  and  $B$  meson decays and oscillations. However, these corrections always involve flavor-changing couplings, which in the MSSM are induced only through weak interactions. As a result, in the MSSM supersymmetric QCD loops in  $K$  and  $B$  meson physics are actually smaller [9] than loops involving electroweak gauginos or Higgs bosons.

In this paper, we calculate the supersymmetric QCD corrections to quark pair production in  $e^+e^-$  annihilation, allowing for mixing between left- and right-handed squarks and taking into account the effects of nonzero quark masses. At LEP 1 energies, we find that these corrections are small and positive for  $Z$  decays into light quarks; however, for  $b\bar{b}$  final states, mixing in the  $\bar{b}$  squark sector can affect the correction to the cross section, and can even change its sign. In the case of top quark pair production at high-energy  $e^+e^-$  colliders, the effect of mixing in the  $\bar{t}$  squark sector on the total cross section is less significant, since the dominant photon exchange contribution is not sensitive to it. The correction to the top forward-backward symmetry does depend on the details of  $\bar{t}$  squark mixing but, unfortunately, the correction is always very small and will therefore be difficult to measure.

Supersymmetric QCD corrections to quark pair production in  $e^+e^-$  annihilation were first discussed in Ref. [10] for LEP1 energies in the approximation of negligible quark masses and squark mixing and of equal masses of the superpartners of left- and right-handed quarks. In Ref. [11] the effect of squark mixing has been included at LEP1 energies and found to be small. However, in that paper only corrections to the  $Z$ -quark couplings present in the SM at the tree level are considered, while we compute all  $CP$ -conserving form factors for both the  $Z$  boson and the photon (the latter are needed for c.m. energies away from the  $Z$  resonance). In the limit of zero quark mass and squark mixing, our results for the total cross section fully agree with Ref. [10] both numerically and analytically; we also find general numerical agreement with Ref. [11]. Finally, we also compute corrections to the forward-backward asymmetry, while the previous papers [10,11] focused on corrections to total rates.

The rest of this paper is organized as follows. In Sec. II we set up the formalism and present our analytical results for the corrections to the most general set of  $CP$ -conserving  $\gamma q\bar{q}$  and  $Zq\bar{q}$  couplings. In Sec. III we show numerical examples both for LEP1 and for a future

high-energy  $e^+e^-$  linear collider operating at  $\sqrt{s}=500$  GeV. Section IV contains a summary and some conclusions. For the convenience of the reader explicit expressions for the scalar two- and three-point functions appearing in our results are listed in the Appendix.

## II. FORMALISM

The most general  $Zq\bar{q}$  and  $\gamma q\bar{q}$  vertices compatible with  $CP$  invariance can be written as

$$\Gamma_\mu^{Z,\gamma} = -ie_0 g^{Z,\gamma} \left[ \gamma_\mu V_q^{Z,\gamma} - \gamma_\mu \gamma_5 A_q^{Z,\gamma} + \frac{1}{2m_q} P_\mu S_q^{Z,\gamma} \right], \quad (1)$$

where  $e_0$  is the electric charge of the proton,  $P = p_1 - p_2$  with  $p_1$  and  $p_2$  the momenta of the quark and antiquark and  $g^\gamma = 1$  and  $g^Z = 1/(4s_W c_W)$  with  $s_W^2 = 1 - c_W^2 = \sin^2 \theta_W$ . Because  $CP$  is conserved by strong interactions, terms proportional to  $P_\mu \gamma_5$  should be absent and this fact provides a good check of the calculation. In principle, one can also have scalar and pseudoscalar couplings  $q_\mu$  and  $q_\mu \gamma_5$  where  $q = p_1 + p_2$  is the momentum of the gauge boson; but in  $e^+e^-$  collisions these terms give contributions which are proportional to the electron mass and are therefore totally negligible. At the tree level,  $S_q^{Z,\gamma}$  vanish, while the vector and axial-vector couplings take the usual form

$$\begin{aligned} (V_q^Z)^0 &\equiv v_q = 2I_q^{3L} - 4s_W^2 e_q, & (A_q^Z)^0 &\equiv a_q = 2I_q^{3L}, \\ (V_q^\gamma)^0 &= e_q, & (A_q^\gamma)^0 &= 0, \end{aligned} \quad (2)$$

with  $I_q^{3L} = \pm \frac{1}{2}$  the weak isospin and  $e_q$  the electric charge of the quark. When loop corrections are included,  $S_q^{Z,\gamma}$  terms appear and the bare vector and axial-vector couplings are shifted by an amount

$$\begin{aligned} \delta V_q^{Z,\gamma} &= V_q^{Z,\gamma} - (V_q^{Z,\gamma})^0, \\ \delta A_q^{Z,\gamma} &= A_q^{Z,\gamma} - (A_q^{Z,\gamma})^0. \end{aligned} \quad (3)$$

In previous work [10,11] only the corrections to  $V_q^Z$  and  $A_q^Z$  were considered. We find that even for heavy (top) quarks the corrections coming from the scalar form factors  $S_q^{Z,\gamma}$  are indeed somewhat less important than the corrections to the couplings that are already present at the tree level.

Since we are interested in radiative corrections involving strong interactions, we only need to consider dia-

$$\begin{aligned} \mathcal{L}_{\bar{q}qV} &= -ie A^\mu e_q (\bar{q}_1^* \vec{\partial}_\mu \bar{q}_1 + \bar{q}_2^* \vec{\partial}_\mu \bar{q}_2) \\ &\quad - \frac{ie}{s_W c_W} Z^\mu [ -I_q^{3L} \sin \bar{\theta} \cos \bar{\theta} (\bar{q}_1^* \vec{\partial}_\mu \bar{q}_2 + \bar{q}_2^* \vec{\partial}_\mu \bar{q}_1) + (I_q^{3L} \cos^2 \bar{\theta} - s_W^2 e_q) \bar{q}_1^* \vec{\partial}_\mu \bar{q}_1 + (I_q^{3L} \sin^2 \bar{\theta} - s_W^2 e_q) \bar{q}_2^* \vec{\partial}_\mu \bar{q}_2 ]. \end{aligned} \quad (7)$$

Finally, the squark-quark-gluino interaction Lagrangian in the presence of squark mixing is given by

$$\mathcal{L}_{\bar{g}qg} = -i\sqrt{2}g_s T^a \bar{q} \left[ (\cos \bar{\theta} \bar{q}_1 - \sin \bar{\theta} \bar{q}_2) \frac{1+\gamma_5}{2} - (\sin \bar{\theta} \bar{q}_1 + \cos \bar{\theta} \bar{q}_2) \frac{1-\gamma_5}{2} \right] \bar{g}^a + \text{H. c.}, \quad (8)$$

grams involving squark and gluino loops. As stated in the Introduction, we will include effects proportional to the mass of the produced quarks. As is well known [12], the supersymmetric partners of left- and right-handed massive quarks mix, the mass eigenstates  $\bar{q}_1$  and  $\bar{q}_2$  being related to the current eigenstates  $\bar{q}_L$  and  $\bar{q}_R$  by

$$\bar{q}_1 = \bar{q}_L \cos \bar{\theta} + \bar{q}_R \sin \bar{\theta}, \quad \bar{q}_2 = -\bar{q}_L \sin \bar{\theta} + \bar{q}_R \cos \bar{\theta}. \quad (4)$$

The mixing angle  $\bar{\theta}$  as well as the masses  $m_{\bar{q}_1}$  and  $m_{\bar{q}_2}$  of the physical squarks can be calculated from the mass matrices<sup>2</sup>

$$\begin{aligned} \mathcal{M}_{\bar{t}}^2 &= \begin{bmatrix} m_{\bar{t}_L}^2 + m_t^2 + 0.35 D_Z & -m_t (A_t + \mu \cot \beta) \\ -m_t (A_t + \mu \cot \beta) & m_{\bar{t}_R}^2 + m_t^2 + 0.16 D_Z \end{bmatrix}, \\ \mathcal{M}_{\bar{b}}^2 &= \begin{bmatrix} m_{\bar{b}_L}^2 + m_b^2 - 0.42 D_Z & -m_b (A_b + \mu \tan \beta) \\ -m_b (A_b + \mu \tan \beta) & m_{\bar{b}_R}^2 + m_b^2 - 0.08 D_Z \end{bmatrix}, \end{aligned} \quad (5a) \quad (5b)$$

where  $D_Z = M_Z^2 \cos 2\beta$ ,  $\tan \beta$  being the ratio of the vacuum expectation values of the two neutral Higgs fields of the MSSM [1].  $m_{\bar{t}_L, \bar{t}_R, \bar{b}_R}$  are soft breaking masses,  $A_{b,t}$  are parameters describing the strength of nonsupersymmetric trilinear scalar interactions, and  $\mu$  is the supersymmetric Higgs boson (Higgsino) mass, which also enters trilinear scalar vertices. Notice that the off-diagonal elements of these squark mass matrices are proportional to the quark mass. In the case of the supersymmetric partners of the light quarks mixing between the current eigenstates can therefore be neglected. However, mixing between  $\bar{t}$  squarks can be sizable and allows one of the mass eigenstates to be much lighter than the top quark. Sbottom mixing can also be significant if  $\tan \beta \gg 1$ ; even in supergravity models with radiative symmetry breaking  $\tan \beta$  can be as large as  $m_t/m_b$  [13].

The interactions of the photon and the  $Z$  boson with squark current eigenstates are described by the Lagrangian [1]

$$\begin{aligned} \mathcal{L}_{\bar{q}qV} &= -ie A^\mu \sum_{i=L,R} e_{q_i} \bar{q}_i^* \vec{\partial}_\mu \bar{q}_i \\ &\quad - \frac{ie}{s_W c_W} Z_\mu \sum_{i=L,R} (I_q^{3i} - 2e_{q_i} s_W^2) \bar{q}_i^* \vec{\partial}_\mu \bar{q}_i. \end{aligned} \quad (6)$$

After the introduction of nontrivial squark mixing, this becomes [14]

<sup>2</sup>We ignore generation mixing between squarks, which in the case of the MSSM is only induced radiatively by weak interactions.

where  $g_s$  is the strong-coupling constant and  $T^a$  are  $SU(3)_C$  generators. Note that in Eq. (8) we have assumed  $M_{\tilde{g}} > 0$ .

Including the corrections due to the squark-gluino vertex diagram shown in Fig. 1(a), and taking into account the mixing between the left- and right-handed squarks as well as the finite mass of the external quarks, the photon couplings to quarks are shifted by

$$(\delta V_q^\gamma)_{ve} = \frac{4}{3} \frac{\alpha_s}{\pi} \frac{e_q}{2} (C_3^{11} + C_3^{22}), \quad (9a)$$

$$(\delta A_q^\gamma)_{ve} = \frac{4}{3} \frac{\alpha_s}{\pi} \frac{e_q}{2} \cos 2\tilde{\theta} (C_3^{11} - C_3^{22}), \quad (9b)$$

$$(S_q^\gamma)_{ve} = -\frac{4}{3} \frac{\alpha_s}{\pi} \frac{e_q}{2} [m_q^2 (C_2^{11} + C_2^{22}) - m_q M_{\tilde{g}} \sin 2\tilde{\theta} (C_1^{11} - C_1^{22})]. \quad (9c)$$

The corresponding shifts of the  $Z$  boson couplings to quarks are

$$(\delta V_q^Z)_{ve} = \frac{4}{3} \frac{\alpha_s}{\pi} [(2I_q^{3L} \cos^2 \tilde{\theta} - 2s_{\tilde{W}}^2 e_q) C_3^{11} + (2I_q^{3L} \sin^2 \tilde{\theta} - 2s_{\tilde{W}}^2 e_q) C_3^{22}], \quad (10a)$$

$$(\delta A_q^Z)_{ve} = \frac{4}{3} \frac{\alpha_s}{\pi} [(2I_q^{3L} \cos^2 \tilde{\theta} - 2s_{\tilde{W}}^2 e_q) \cos 2\tilde{\theta} C_3^{11} - (2I_q^{3L} \sin^2 \tilde{\theta} - 2s_{\tilde{W}}^2 e_q) \cos 2\tilde{\theta} C_3^{22} + I_q^{3L} \sin^2 2\tilde{\theta} (C_3^{12} + C_3^{21})], \quad (10b)$$

$$(S_q^Z)_{ve} = -\frac{4}{3} \frac{\alpha_s}{\pi} [(2I_q^{3L} \cos^2 \tilde{\theta} - 2s_{\tilde{W}}^2 e_q) (m_q^2 C_2^{11} - m_q M_{\tilde{g}} \sin 2\tilde{\theta} C_1^{11}) + (2I_q^{3L} \sin^2 \tilde{\theta} - 2s_{\tilde{W}}^2 e_q) (m_q^2 C_2^{22} + m_q M_{\tilde{g}} \sin 2\tilde{\theta} C_1^{22}) + I_q^{3L} \sin 2\tilde{\theta} \cos 2\tilde{\theta} m_q M_{\tilde{g}} (C_1^{12} + C_1^{21})]. \quad (10c)$$

For  $i, j = 1, 2$  the  $C_k^{ij}$  are defined as  $C_k^{ij} \equiv C_k(s, m_q, m_{\tilde{q}_i}, m_{\tilde{q}_j}, M_{\tilde{g}})$ . We use the Passarino-Veltman reduction to scalar integrals [15], and the intermediate function  $C_{\pm}^{ij}$  and  $C_{\pm}^{ij}$  defined as (note that we use a slightly different notation than in Ref. [16])

$$C_{\pm}^{ij} = \frac{-1}{2s\beta_q^2} [2B_0(s, m_{\tilde{q}_i}, m_{\tilde{q}_j}) - B_0(m_q^2, M_{\tilde{g}}, m_{\tilde{q}_i}) - B_0(m_q^2, M_{\tilde{g}}, m_{\tilde{q}_j}) + (2M_{\tilde{g}}^2 + 2m_q^2 - m_{\tilde{q}_i}^2 - m_{\tilde{q}_j}^2) C_0^{ij}], \quad (11)$$

$$C_{\pm}^{ij} = \frac{1}{2s} [B_0(m_q^2, M_{\tilde{g}}, m_{\tilde{q}_i}) - B_0(m_q^2, M_{\tilde{g}}, m_{\tilde{q}_j}) + (m_{\tilde{q}_i}^2 - m_{\tilde{q}_j}^2) C_0^{ij}].$$

In terms of these functions, the  $C_k^{ij}$  are given by

$$C_1^{ij} = C_0^{ij} - 2C_{\pm}^{ij}, \quad (12a)$$

$$C_2^{ij} = 2C_{\pm}^{ij} - \frac{4}{s\beta_q^2} [C_3^{ij} + \frac{1}{2}(m_{\tilde{q}_i}^2 + m_{\tilde{q}_j}^2 - 2M_{\tilde{g}}^2 - 2m_q^2) C_{\pm}^{ij} - \frac{1}{2} B_0(s, m_{\tilde{q}_i}, m_{\tilde{q}_j}) + \frac{1}{4} B_1(m_q^2, M_{\tilde{g}}, m_{\tilde{q}_i}) + \frac{1}{4} B_1(m_q^2, M_{\tilde{g}}, m_{\tilde{q}_j})], \quad (12b)$$

$$C_3^{ij} = \frac{1}{4} [2M_{\tilde{g}}^2 C_0^{ij} + 1 + B_0(s, m_{\tilde{q}_i}, m_{\tilde{q}_j}) + (m_{\tilde{q}_i}^2 + m_{\tilde{q}_j}^2 - 2m_q^2 - 2M_{\tilde{g}}^2) C_{\pm}^{ij} + (m_{\tilde{q}_i}^2 - m_{\tilde{q}_j}^2) C_{\pm}^{ij}]. \quad (12c)$$

Here,  $B_1$  is given by

$$B_1(s, m_1, m_2) = \frac{1}{2s} [(s + m_1^2 - m_2^2) B_0(s, m_1, m_2) + A_0(m_2) - A_0(m_1)], \quad (13)$$

and the functions  $A_0$ ,  $B_0$ , and  $C_0$  correspond to the scalar one-, two-, and three-point functions [17], respectively, and are given in the Appendix.

The renormalized vertices are derived by adding the

$$\delta Z_V = -\frac{1}{3} \frac{\alpha_s}{\pi} \{ B_1(m_q^2, M_{\tilde{g}}, m_{\tilde{q}_1}) + B_1(m_q^2, M_{\tilde{g}}, m_{\tilde{q}_2}) + 2m_q^2 [(B_1'(m_q^2, M_{\tilde{g}}, m_{\tilde{q}_1}) + B_1'(m_q^2, M_{\tilde{g}}, m_{\tilde{q}_2})] - 2m_q M_{\tilde{g}} \sin 2\tilde{\theta} [B_0'(m_q^2, M_{\tilde{g}}, m_{\tilde{q}_1}) - B_0'(m_q^2, M_{\tilde{g}}, m_{\tilde{q}_2})] \}, \quad (15a)$$

$$\delta Z_A = -\frac{1}{3} \frac{\alpha_s}{\pi} \cos 2\tilde{\theta} [B_1(m_q^2, M_{\tilde{g}}, m_{\tilde{q}_1}) - B_1(m_q^2, M_{\tilde{g}}, m_{\tilde{q}_2})]. \quad (15b)$$

counterterm originating from the on-shell self-energies of the external quarks [Fig. 1(b)]. Following the procedure outlined in Refs. [16, 18], one obtains

$$\begin{aligned} (\delta V_q^\gamma)_{ct} &= e_q \delta Z_V, & (\delta A_q^\gamma)_{ct} &= e_q \delta Z_A, \\ (\delta V_q^Z)_{ct} &= v_q \delta Z_V + a_q \delta Z_A, & & \\ (\delta A_q^Z)_{ct} &= a_q \delta Z_V + v_q \delta Z_A. & & \end{aligned} \quad (14)$$

Here,  $\delta Z_V$  and  $\delta Z_A$  are given by

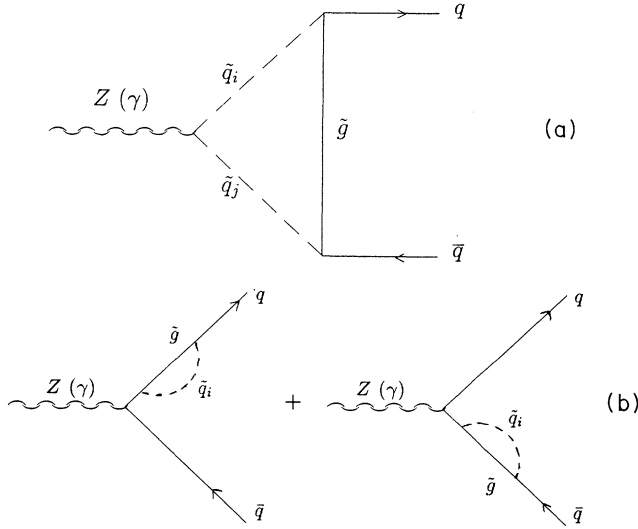


FIG. 1. (a) Vertex and (b) self-energy corrections to  $e^+e^- \rightarrow q\bar{q}$  from supersymmetric QCD.

The full SUSY-QCD correction to the vectorial and axial-vector couplings is just the sum of the unrenormalized vertex correction and the quark self-energy counterterms<sup>3</sup>:

$$\delta V_q^{\gamma,Z} = (\delta V_q^{\gamma,Z})_{\text{ve}} + (\delta V_q^{\gamma,Z})_{\text{ct}}, \quad (16a)$$

$$\delta A_q^{\gamma,Z} = (\delta A_q^{\gamma,Z})_{\text{ve}} + (\delta A_q^{\gamma,Z})_{\text{ct}}. \quad (16b)$$

The expressions (9)–(15) are rather cumbersome. For many applications squark mixing can be neglected. If, in addition, one assumes approximate degeneracy for the squarks,  $m_{\tilde{q}_1} = m_{\tilde{q}_2} \equiv m_{\tilde{q}}$ , the corrections simplify considerably, and one finds

$$\delta V_q^{\gamma,Z} = \frac{4}{3} \frac{\alpha_s}{\pi} (V_q^{\gamma,Z})^0 C, \quad \delta A_q^{\gamma,Z} = \frac{4}{3} \frac{\alpha_s}{\pi} (A_q^{\gamma,Z})^0 C, \quad (17)$$

$$S_q^{\gamma,Z} = -\frac{4}{3} \frac{\alpha_s}{\pi} m_{\tilde{q}}^2 (V_q^{\gamma,Z})^0 C_2(s, m_q, m_{\tilde{q}}, m_{\tilde{q}}, M_{\tilde{g}}),$$

where  $C$  is given by

$$C \equiv C_3(s, m_q, m_{\tilde{q}}, m_{\tilde{q}}, M_{\tilde{g}}) - \frac{1}{2} B_1(m_{\tilde{q}}^2, m_{\tilde{q}}, M_{\tilde{g}}) - \frac{1}{2} m_{\tilde{q}}^2 B'_1(m_{\tilde{q}}^2, m_{\tilde{q}}, M_{\tilde{g}}). \quad (18)$$

Furthermore, for massless final-state quarks the  $S_q$  term vanishes and the correction to the axial-vector and vector couplings can be expressed by a single two-dimensional integral as

$$C \simeq C_3 - \frac{1}{2} B_1 = \frac{1}{2} \int_0^1 x dx \int_0^1 dy \ln \frac{x(m_{\tilde{q}}^2 - M_{\tilde{g}}^2) + M_{\tilde{g}}^2}{-sx^2y(1-y) + x(m_{\tilde{q}}^2 - M_{\tilde{g}}^2) + M_{\tilde{g}}^2}, \quad (19)$$

in agreement with Ref. [10]. For large squark and gluino masses,  $m_{\tilde{q}}, M_{\tilde{g}} \gg s$ , the correction is just

$$C \rightarrow \frac{s}{12} \frac{1}{(m_{\tilde{q}}^2 - M_{\tilde{g}}^2)^4} \left[ \frac{1}{3} (m_{\tilde{q}}^2 - M_{\tilde{g}}^2)^3 - \frac{1}{2} M_{\tilde{g}}^2 (m_{\tilde{q}}^2 - M_{\tilde{g}}^2)^2 + M_{\tilde{g}}^4 (m_{\tilde{q}}^2 - M_{\tilde{g}}^2) - M_{\tilde{g}}^6 \ln \frac{m_{\tilde{q}}^2}{M_{\tilde{g}}^2} \right]. \quad (20)$$

If, in addition, the gluino mass can be neglected compared to the squark mass, one simply obtains  $C \simeq s/(36m_{\tilde{q}}^2)$ .

In terms of the vertices (1), the differential cross section  $d\sigma(e^+e^- \rightarrow q\bar{q})/d\cos\theta$  reads (we define  $\theta$  as the angle between the quark and the incoming positron)

$$\begin{aligned} \frac{d\sigma}{d\cos\theta} = & \frac{3}{8} N_c \beta_q \{ D_{\gamma\gamma} e_e^2 [(2 - \beta_q^2 \sin^2\theta)(V_q^\gamma)^2 + \beta_q^2(1 + \cos^2\theta)(A_q^\gamma)^2 - 2\beta_q^2 \sin^2\theta V_q^\gamma S_q^\gamma] \\ & + D_{Z\gamma} e_e v_e [(2 - \beta_q^2 \sin^2\theta) V_q^\gamma V_q^Z + \beta_q^2(1 + \cos^2\theta) A_q^\gamma A_q^Z - \beta_q^2 \sin^2\theta (V_q^\gamma S_q^Z + S_q^\gamma V_q^Z)] \\ & + D_{ZZ} (v_e^2 + a_e^2) [(2 - \beta_q^2 \sin^2\theta)(V_q^Z)^2 + \beta_q^2(1 + \cos^2\theta)(A_q^Z)^2 - 2\beta_q^2 \sin^2\theta V_q^Z S_q^Z] \\ & + 2D_{Z\gamma} e_e a_e \beta_q \cos\theta (V_q^\gamma A_q^Z + V_q^Z A_q^\gamma) + 8D_{ZZ} a_e v_e \beta_q \cos\theta V_q^Z A_q^Z \}. \end{aligned} \quad (21)$$

Here  $N_c = 3$  is the color factor, and  $\beta_q = (1 - 4m_q^2/s)^{1/2}$  the velocity of the final quarks. In Eq. (21) the leading electroweak radiative corrections have been included by introducing the quantities  $D_{\alpha\beta}$ ,  $\alpha, \beta = \gamma, Z$ , which are defined in terms of the Fermi coupling constant  $G_F$  and the running QED coupling  $\alpha(s)$ :

<sup>3</sup>Note that in this convention the vector couplings  $V_q$  and scalar couplings  $S_q$  do not reduce to their SM values even in the limit of infinite squark masses; of course, very heavy squarks and gluinos do decouple from physical observables such as cross sections. Physically equivalent results can be obtained by ignoring the diagrams of Fig. 1(b), and performing the renormalization by simply subtracting the corrections at zero-momentum transfer,  $s=0$ ; in this scheme separate counterterms for  $V_q$  and  $S_q$  can be defined, so that each coupling by itself reduces to its SM value in the limit of large sparticle masses.

$$D_{\gamma\gamma} = \frac{4\pi\alpha^2(s)}{3s}, \quad D_{ZZ} = \frac{G_F^2}{96\pi} \frac{M_Z^4 s}{(s-M_Z^2)^2 + (s\Gamma_Z/M_Z)^2},$$

$$D_{Z\gamma} = \frac{G_F\alpha(s)}{3\sqrt{2}} \frac{M_Z^2(s-M_Z^2)}{(s-M_Z^2)^2 + (s\Gamma_Z/M_Z)^2}. \quad (22)$$

At  $O(\alpha_s)$ , the deviations of the total cross section and the forward-backward asymmetry from the tree level values,  $\delta\sigma = \sigma - \sigma^0$  and  $\delta A_{\text{FB}} = A_{\text{FB}} - A_{\text{FB}}^0$ , are then

$$\delta\sigma = N_c\beta_q \left[ D_{\gamma\gamma} e_e^2 [(3-\beta_q^2)e_q\delta V_q^\gamma - \beta_q^2 e_q S_q^\gamma] + D_{ZZ}(v_e^2 + a_e^2)[(3-\beta_q^2)v_q\delta V_q^Z + 2\beta_q^2 a_q\delta A_q^Z - \beta_q^2 v_q S_q^Z] \right. \\ \left. + D_{Z\gamma} e_e v_e \left[ \frac{3-\beta_q^2}{2} (v_q\delta V_q^\gamma + e_q\delta V_q^Z) + \beta_q^2 a_q\delta A_q^\gamma - \frac{\beta_q^2}{2} (e_q S_q^Z + v_q S_q^\gamma) \right] \right], \quad (23a)$$

$$\delta A_{\text{FB}} = \frac{D_{Z\gamma} e_e a_e (e_q\delta A_q^Z + a_q\delta V_q^\gamma + v_q\delta A_q^\gamma) + 4D_{ZZ} a_e v_e (\delta V_q^Z a_q + \delta A_q^Z v_q)}{D_{Z\gamma} e_e a_e e_q a_q + 4D_{ZZ} a_e v_e a_q v_q} - \frac{\delta\sigma}{\sigma^0}. \quad (23b)$$

These expressions have to be supplemented by including the standard QCD corrections; the formulas for the cross section and the forward-backward asymmetry in the massive case can be found in Ref. [19]. In the case of the cross section, one can, however, use the Schwinger formulas [20], which provide a very good approximation to the exact result; this is done by performing the following substitution ( $\alpha, \beta = \gamma, Z$ ):

$$(V_q^\alpha)^0 (V_q^\beta)^0 \rightarrow (V_q^\alpha)^0 (V_q^\beta)^0 \left\{ 1 + \frac{4}{3} \frac{\alpha_s}{\pi} \left[ \frac{\pi^2}{2\beta_q} - \frac{3+\beta_q^2}{4} \left[ \frac{\pi^2}{2} - \frac{3}{4} \right] \right] \right\}, \quad (24a)$$

$$(A_q^\alpha)^0 (A_q^\beta)^0 \rightarrow (A_q^\alpha)^0 (A_q^\beta)^0 \left\{ 1 + \frac{4}{3} \frac{\alpha_s}{\pi} \left[ \frac{\pi^2}{2\beta_q} - \left[ \frac{19}{10} - \frac{22}{5}\beta_q + \frac{7}{2}\beta_q^2 \right] \left[ \frac{\pi^2}{2} - \frac{3}{4} \right] \right] \right\}. \quad (24b)$$

A similarly simple yet very accurate substitution also exists for standard QCD corrections to the forward-backward asymmetry [21]:

$$(V_q^\alpha)^0 (A_q^\beta)^0 \rightarrow (V_q^\alpha)^0 (A_q^\beta)^0 \left[ 1 + \frac{\alpha_s}{\pi} \frac{2}{\beta_q} (3-\beta_q^2) \sqrt{1-\beta_q^2} \right]. \quad (25)$$

Finally, on top of the  $Z$  resonance these expressions simplify considerably. In addition to the fact that only the  $Z$  exchange contribution has to be taken into account, one can neglect to a good approximation the quark masses (except possibly in the  $\bar{b}$  mass matrix; see below) since top decays of the  $Z$  boson are kinematically forbidden (to achieve a better precision one can eventually include the leading mass effects in the Born term as well as the QCD corrections in the case of the bottom quark; see Ref. [22]). In this case, the  $S_q$  terms vanish and the deviation of the decay width  $\Gamma_q = \Gamma(Z \rightarrow q\bar{q})$  and the forward-backward asymmetry  $A_{\text{FB}}$  from their tree-level values are simply given by

$$\frac{\delta\Gamma_q}{\Gamma_q^0} = 2 \frac{v_q\delta V_q^Z + a_q\delta A_q^Z}{v_q^2 + a_q^2},$$

$$\frac{\delta A_{\text{FB}}}{A_{\text{FB}}^0} = \frac{v_q\delta A_q^Z + a_q\delta V_q^Z}{a_q v_q} - 2 \frac{v_q\delta V_q^Z + a_q\delta A_q^Z}{v_q^2 + a_q^2}. \quad (26)$$

### III. RESULTS

We are now in a position to present some numerical examples. In Fig. 2 we show SUSY QCD corrections to the hadronic decay width of the  $Z$  boson; the solid (dashed)

curves are for  $b\bar{b}$  ( $c\bar{c}$ ) final states. We have set the  $A$  parameters in the squark mass matrices (5) to zero, and have assumed equal SUSY-breaking masses for all squarks, denoted by  $\langle m_q \rangle$ . Moreover, in this figure we have assumed that all parameters entering the squark mass matrices, as well as the gluino mass, can be varied independently (“global SUSY” scenario). The four upper curves are for negligible mixing between  $L$  and  $R$  squarks. Even in this case the “ $D$  terms” [ $D_Z$  in Eqs. (5)] lead to a non-negligible mass splitting between squarks of different flavor, if  $\tan\beta \neq 1$ . In particular, for  $\tan\beta > 1$  (which is favored by supergravity models [3]),  $\tilde{u}$ -type squarks are lighter than  $\tilde{d}$ -type squarks; as a result, for a given value of  $\langle m_q \rangle$  the corrections to  $c\bar{c}$  production are larger than those to  $b\bar{b}$  production.

The uppermost curves in Fig. 2 have been obtained by choosing a very small gluino mass,  $M_{\tilde{g}} = 3$  GeV. A gluino of this mass could have escaped all experimental searches, provided squarks are heavier than 100 GeV or so [23]. Although a careful study showed [24] that a GeV gluino does not reduce the slight discrepancy between values of  $\alpha_s$  extracted from low-energy experiments and those derived from event shape variables measured at  $\sqrt{s} \simeq M_Z$ , present measurements cannot exclude its existence, either. It should also be noted that squark mass

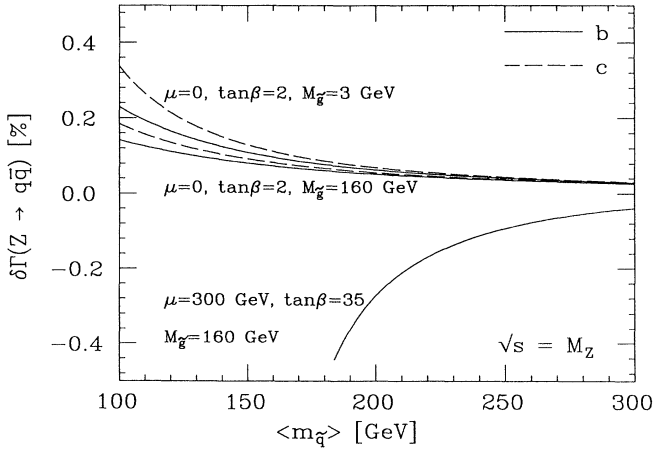


FIG. 2. Supersymmetric QCD corrections to the decay width of the  $Z$  boson into  $c\bar{c}$  (dashed) and  $b\bar{b}$  pairs (solid). We have assumed a “global SUSY” scenario, where all parameters of the squark mass matrices can be varied independently. For  $\tilde{c}$  and  $\tilde{b}$  squarks, the  $A$  terms are always negligible; the other parameters are as indicated in the figure. Notice that  $\langle m_{\tilde{q}} \rangle$  is the common SUSY-breaking diagonal squark mass, which is also the average first generation squark mass, since  $D$ -term contributions cancel after summing over a complete generation. The lowest curve ends at  $\langle m_{\tilde{q}} \rangle = 180$  GeV since for even smaller values,  $m_{\tilde{b}_1} < 45$  GeV.

bounds from hadron colliders [8] might be invalidated by the presence of such a light gluino. This is because in this scenario, squarks predominantly decay into gluinos, which lose a considerable fraction of their energy in QCD radiation prior to their decay, thereby leading to a rather soft missing  $p_T$  spectrum [25]. From Fig. 2 we conclude that one-loop SUSY QCD corrections to the hadronic width of the  $Z$  boson could amount to about 0.3%, or about 8% of the standard QCD correction. For very light gluinos, two-loop SUSY QCD corrections are also not entirely negligible [10]; they amount to about  $-2\%$  of the standard QCD corrections [24]. Altogether, SUSY QCD corrections to  $\Gamma_{\text{had}}$  therefore amount to at most  $+6\%$  of the standard QCD corrections, for a gluino mass of a few GeV and squark masses around 100 GeV. In this scenario the value of  $\alpha_S$  extracted from the measurement of  $\Gamma_{\text{had}}$  would therefore have to be reduced by about 6%. At the same time, in the presence of light gluinos the value of  $\alpha_S$  derived from event shape variables has to be increased by about 8% [24]. The net result is that the present small discrepancy between these two determinations of  $\alpha_S$  is diminished.<sup>4</sup>

If we chose gluino and squark masses above the region excluded by hadron collider searches [8] the maximal size

of the corrections to  $Z$  partial widths drops by about a factor of 2, as illustrated by the curves for  $\mu=0$  and  $M_g=160$  GeV in Fig. 2. Moreover, squark mass splitting due to  $D$  terms becomes less important, so that to good approximation the simplified expressions (17) and (20) can be used.

Finally, the lowest curve in Fig. 2 demonstrates that squark mixing can have sizable effects already for  $\tilde{b}$  squarks. The off-diagonal elements of the  $\tilde{b}$  squark mass matrix (5b) can be substantial if  $\tan\beta \gg 1$  and  $\mu$  is not too small. Indeed, for the parameters chosen in Fig. 2, the lighter  $\tilde{b}$  eigenstate would be lighter than 45 GeV, in violation of LEP bounds [7], unless  $\langle m_{\tilde{q}} \rangle \geq 180$  GeV.<sup>5</sup> In this scenario the correction to the partial width into  $b\bar{b}$  pairs is negative. The corrections to the  $u$ ,  $d$ ,  $s$ , and  $c$  partial widths for the same set of parameters are still positive, however, leading to a very small correction to the total hadronic width of the  $Z$  boson. In order to test this scenario experimentally one would thus have to measure the  $b\bar{b}$  cross section with a precision of a fraction of 1%, which appears to be quite difficult.

In Fig. 3 we plot the correction to the total cross section for the production of light quarks at  $\sqrt{s} = 500$  GeV. In this figure we have switched off both squark mixing (by setting  $A_q = \mu = 0$ ) and squark mass splitting through  $D$  terms (by setting  $\tan\beta = 1$ ); however, the previous figure showed that results for nonzero  $\mu$  and  $\tan\beta \neq 1$  are quite similar unless the gluino is very light or  $\tan\beta$  is very large. We see that the corrections reach a maximum at  $m_{\tilde{q}} \simeq 0.4\sqrt{s} \simeq 200$  GeV, almost independently of the value of  $M_g$ . If both  $m_{\tilde{q}}$  and  $M_g$  are much smaller than  $\sqrt{s}$  the corrections become negative; in the limit of exact SUSY ( $m_{\tilde{q}} \rightarrow m_q, M_g \rightarrow 0$ ) one encounters logarithmic infrared divergencies. For  $m_{\tilde{q}} > 200$  GeV the size of the corrections decreases rapidly. However, even if squarks are not accessible to the accelerator we study, i.e., for  $m_{\tilde{q}} > \sqrt{s}/2$ , the correction can be as large as  $+1\%$ , or about one third the standard QCD correction. We also observe that the correction depends less sensitively on the gluino mass than on the squark mass; this has also been found in Ref. [10].

In Figs. 4(a) and 4(b) we present results for SUSY QCD corrections to  $t\bar{t}$  production at  $\sqrt{s} = 500$  GeV, for  $m_t = 150$  GeV. We have fixed the gluino mass to 250 GeV and chosen  $\tan\beta = 2$ . The dashed curves are again valid for a “global SUSY” model, with  $m_{\tilde{t}_L} = m_{\tilde{t}_R} = A_t$  and  $\mu = 500$  GeV. In contrast, the solid curves are for a supergravity scenario, where scalar masses are assumed to be equal to each other, and also equal to the scalar trilinear interaction parameters  $A_q$ , at the scale of grand unification,  $M_X \simeq 10^{16}$  GeV. The parameters at the weak scale have then been computed by solving a set of coupled renormalization-group equations [4]; for simplicity we have treated them using the analytical approximations

<sup>4</sup>Strictly speaking the analysis of Ref. [24] is valid only for very heavy squarks; however, we expect it to hold also for squark masses around 100 GeV, since squark exchange diagrams contributing to  $q\bar{q}g\bar{g}$  production are not enhanced by large logarithms, unlike diagrams where a gluino pair is produced from a gluon.

<sup>5</sup>Note that hadron collider data do not exclude the existence of a single light squark species, if the mass of the lightest neutralino exceeds about 15 GeV [26].

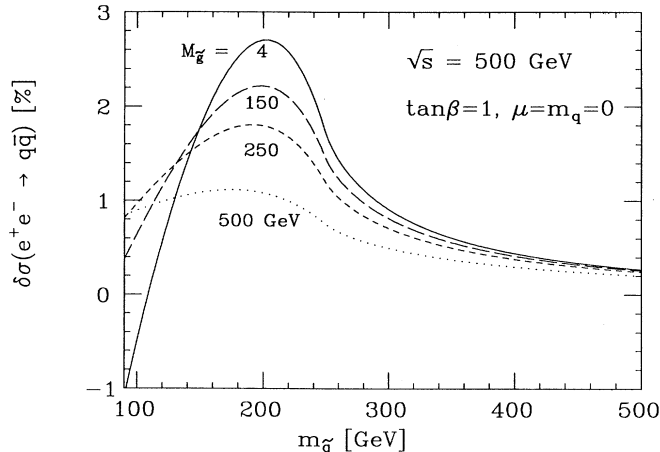


FIG. 3. Supersymmetric QCD corrections to the total cross section for the production of light  $q\bar{q}$  pairs at an  $e^+e^-$  collider with  $\sqrt{s} = 500$  GeV. In this figure we have switched off squark mixing and chosen  $\tan\beta=1$ , so that the superpartners of all five light quarks have the same mass  $m_{\tilde{q}}$ .

given in Ref. [27]. Notice that in this model  $\mu$  is no longer a free parameter, but determined by the requirement of correct  $SU(2)\times U(1)$  symmetry breaking,  $M_Z=91.1$  GeV. Moreover,  $m_{\tilde{t}_R}$  is considerably smaller than  $m_{\tilde{t}_L}$  at the weak scale, due to quantum corrections involving the  $t$  quark Yukawa coupling.

The results of Fig. 4 are presented as a function of the mass of the lighter  $\tilde{t}$  eigenstate. The mass of the heavier eigenstate varies between 390 and 620 GeV in the global SUSY model, and between 750 and 1100 GeV in the SUGRA scenario we are considering. Moreover, in the former case the  $\tilde{t}$  mixing angle is close to  $45^\circ$ , since the diagonal elements of the  $\tilde{t}$  mass matrix (5a) are almost equal, while in the latter case the angle is considerably larger than  $45^\circ$ , so that the light eigenstate is predominantly  $\tilde{t}_R$ . We see that the correction to the total cross section, shown in Fig. 4(a), is not very sensitive to the differences between the two models we are studying. The reason is that the total cross section is dominated by the photon exchange contribution, which does not depend on  $\tilde{t}$  mixing, see Eqs. (9a) and (23a). The corrections are smaller than for the production of light quarks (with  $m_{\tilde{q}}=m_{\tilde{t}_1}$ ) since in case of  $t\bar{t}$  production practically only one squark contributes in the loop, the heavier  $\tilde{t}$  eigenstate being much more massive.

In contrast, the forward-backward asymmetry (23b) is sensitive to the  $Z$  exchange contribution, and hence to  $\tilde{t}$  mixing; Fig. 4(b) shows that for small  $m_{\tilde{t}_1}$  even the sign of the correction differs for the two models. Unfortunately, the absolute value of this correction is always less than 0.5%; one would probably need a dedicated “top factory” to achieve this level of precision. SUSY QCD corrections to the forward-backward asymmetries of light quarks are always well below 0.1%, and can therefore safely be neglected.

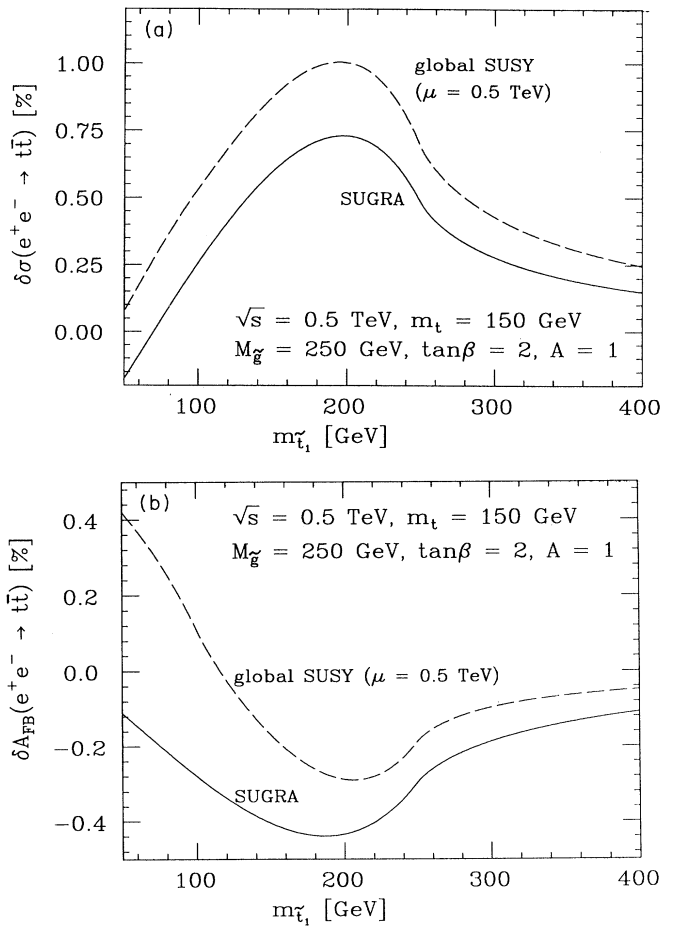


FIG. 4. Supersymmetric QCD corrections to the (a) total cross section and (b) forward-backward asymmetry for  $t\bar{t}$  pair production of a 500 GeV  $e^+e^-$  collider, as a function of the mass of the lighter  $\tilde{t}$  eigenstate. The dashed curves are for a “global SUSY” model with  $\mu=500$  GeV, while the solid curves are for a supergravity scenario with radiative symmetry breaking, where  $\mu$  is a derived quantity, as described in the text. In the former case we have assumed  $m_{\tilde{t}_L}=m_{\tilde{t}_R}=A$ , at the weak scale, while in the latter case these relations are valid only at the grand unified scale.

#### IV. SUMMARY AND CONCLUSIONS

In this paper we have presented explicit expressions for the  $\gamma q\bar{q}$  and  $Zq\bar{q}$  vertices, allowing for mixing between the superpartners of left- and right-handed quarks as well as for unequal squark masses in the loop. We have found corrections to the total cross section (or, on the  $Z$  pole, to the hadronic decay width of the  $Z$ ) to be usually positive, unless both squark and gluino masses are much smaller than the center-of-mass energy  $\sqrt{s}$ . In the limit of no squark mixing and degenerate squark masses we reproduce the results of Ref. [10]. For massless quarks, corrections are largest if  $m_{\tilde{q}} \simeq 0.4\sqrt{s}$ , i.e., just above the threshold for open squark production, where they can reach +2%; they fall below 1% at  $m_{\tilde{q}} \simeq 0.6\sqrt{s}$ , the exact value depending on the gluino mass. If the gluino mass is just a

few GeV and squark masses are around 100 GeV, which still appears to be allowed experimentally, supersymmetric QCD corrections might help to improve the agreement between values of  $\alpha_S$  derived from event shape variables at  $\sqrt{s} \simeq M_Z$  and from the total hadronic decay width of the  $Z$  boson. We also found that  $\tilde{b}$  mixing can be important, and can even flip the sign of the correction.

The corrections to the total  $t\bar{t}$  production cross section are usually smaller than for the case of light squarks, for a given mass of the lightest squark eigenstate of a given flavor. The reason is that  $\tilde{t}$  mixing pushes the mass of the heavier  $\tilde{t}$  eigenstate to such large values that its contribution is essentially negligible. We also computed corrections to the forward-backward asymmetry, and found them to be well below 0.1% for light quarks. In case of  $t$  quarks these corrections are sensitive to the details of  $\tilde{t}$  mixing, unlike the total  $t\bar{t}$  cross section; however, even for  $t$  quarks the forward-backward asymmetry is changed by less than 0.5%.

We conclude that, barring the existence of a very light

gluino, supersymmetric QCD corrections to the production of  $q\bar{q}$  pairs in  $e^+e^-$  annihilation are probably only observable at energies above the open squark threshold. They will therefore not be useful as a tool to search for supersymmetry; however, after the discovery of a “new physics” signal they might allow us to confirm its interpretation in terms of supersymmetry.

#### ACKNOWLEDGMENTS

One of us (A.D.) thanks Vernon Barger and the members of the Phenomenology group at the Physics Department of the University of Wisconsin for providing a hospitable working environment during his stay. This work was supported in part by the U.S. Department of Energy under Contract No. DE-AC02-76ER00881, and in part by the Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation. The work of M.D. was supported by a grant from the Deutsche Forschungsgemeinschaft under the Heisenberg program.

#### APPENDIX: SCALAR LOOP INTEGRALS

In this appendix we collect expressions that allow to evaluate the loop functions that appear in Sec. II. The scalar one-, two-, and three-point functions  $A_0$ ,  $B_0$ , and  $C_0$  are defined as [18]

$$\begin{aligned} A_0(m_0) &= \frac{(2\pi\mu)^{n-4}}{i\pi^2} \int \frac{d^n k}{k^2 - m_0^2 + i\epsilon}, \\ B_0(s, m_1, m_2) &= \frac{(2\pi\mu)^{n-4}}{i\pi^2} \int \frac{d^n k}{(k^2 - m_1^2 + i\epsilon)[(k-q)^2 - m_2^2 + i\epsilon]}, \\ C_0(s, m_1, m_2, m_3) &= \frac{(2\pi\mu)^{n-4}}{i\pi^2} \int \frac{d^n k}{[(k-p_1)^2 - m_1^2 + i\epsilon][(k-p_2)^2 - m_2^2 + i\epsilon](k^2 - m_3^2 + i\epsilon)}. \end{aligned} \quad (\text{A1})$$

Here  $n$  is the space-time dimension and  $\mu$  the renormalization scale.

After integration over the internal momentum  $k$ , the function  $A_0$  is given by

$$A_0(m_0) = m_0^2(1 + \Delta_0), \quad \Delta_i = \frac{2}{4-n} - \gamma_E + \ln(4\pi) + \ln \frac{\mu^2}{m_i^2}, \quad (\text{A2})$$

where  $\gamma_E$  is Euler's constant. The function  $B_0$  and its derivative with respect to  $s$ ,  $B'_0$ , are given by

$$\begin{aligned} B_0(s, m_1, m_2) &= \frac{1}{2}(\Delta_1 + \Delta_2) + 2 + \frac{m_1^2 - m_2^2}{2s} \ln \frac{m_2^2}{m_1^2} + \frac{x_+ - x_-}{4s} \ln \frac{x_-}{x_+}, \\ B'_0(s, m_1, m_2) &= -\frac{1}{2s} \left[ 2 + \frac{m_2^2 - m_1^2}{s} \ln \frac{m_1^2}{m_2^2} + \frac{2(m_1^2 - m_2^2)^2 - s(m_1^2 + m_2^2)}{x_+ - x_-} \ln \frac{x_-}{x_+} \right], \end{aligned} \quad (\text{A3})$$

with

$$x_{\pm} = s - m_1^2 - m_2^2 \pm \sqrt{s^2 - 2s(m_1^2 + m_2^2) + (m_1^2 - m_2^2)^2}. \quad (\text{A4})$$

Note that the  $x_{\pm}$  can be complex. For  $(m_1 - m_2)^2 < s < (m_1 + m_2)^2$ , the logarithms appearing in Eqs. (A3) can be expressed in terms of an arctan of a real argument. When writing these equations we have ignored the imaginary parts of  $B_0$  and  $B'_0$ ; they are not relevant for us, since to next-to-leading order we are only interested in the interference between the (real) tree-level and one-loop amplitudes.

In this paper we need the three-point scalar function  $C_0$  only for  $p_1^2 = p_2^2 = m_q^2$ ; in this case it can be written in integral form as

$$C_0(s, m_q, m_1, m_2, m_3) = -\int_0^1 dy \int_0^y dx (ay^2 + bx^2 + cxy + dy + ex + f)^{-1}, \quad (\text{A5})$$

where



$$a = m_q^2, \quad b = s, \quad c = -s, \quad d = m_2^2 - m_3^2 - m_q^2, \quad e = m_1^2 - m_2^2, \quad f = m_3^2 - i\epsilon. \quad (\text{A6})$$

$C_0$  can be expressed in terms of a sum of Spence functions

$$\text{Li}_2(x) = - \int_0^1 dt \ln(1-xt)/t :$$

$$C_0(s, m_q, m_1, m_2, m_3) = - \frac{1}{s\beta_q} \sum_{i=1}^3 \sum_{j=+,-} (-1)^j \left[ \text{Li}_2 \left[ \frac{x_i}{x_i - y_{ij}} \right] - \text{Li}_2 \left[ \frac{x_i - 1}{x_i - y_{ij}} \right] \right], \quad (\text{A7})$$

where we have defined

$$x_1 = \frac{2d + e(1 - \beta_q)}{2s\beta_q} + \frac{1}{2}(1 - \beta_q), \quad y_{1\pm} = \frac{-c - e \pm \sqrt{(c+e)^2 - 4b(a+d+f)}}{2b},$$

$$x_2 = \frac{2d + e(1 - \beta_q)}{s\beta_q(1 + \beta_q)}, \quad y_{2\pm} = \frac{-d - e \pm \sqrt{(d+e)^2 - 4f(a+b+c)}}{2(a+b+c)}, \quad (\text{A8})$$

$$x_3 = - \frac{2d + e(1 - \beta_q)}{s\beta_q(1 - \beta_q)}, \quad y_{3\pm} = \frac{-d \pm \sqrt{d^2 - 4af}}{2a}.$$

The  $x_i$  and  $y_{i\pm}$  can again be complex. Equation (A7) is only valid above the  $q\bar{q}$  threshold, i.e., for  $s > 4m_q^2$ . Below the threshold analytical continuation of complex logarithms requires the introduction of additional terms; see Ref. [15].

- 
- [1] For reviews, see H. E. Haber and G. L. Kane, *Phys. Rep.* **117**, 75 (1985); X. R. Tata, in *The Standard Model and Beyond*, Proceedings of the 9th Mt. Sorak Symposium on Theoretical Physics, Mt. Sorak, Korea, 1990, edited by J. E. Kim (World Scientific, Singapore, 1991).
- [2] E. Witten, *Nucl. Phys.* **B188**, 513 (1981); N. Sakai, *Z. Phys. C* **11**, 153 (1981); S. Dimopoulos and H. Georgi, *Nucl. Phys.* **B193**, 150 (1981).
- [3] For reviews, see H. P. Nilles, *Phys. Rep.* **110**, 1 (1984); P. Nath, R. Arnowitt, and A. Chamseddine, *Applied N=1 Supergravity*, ITCP Series in Theoretical Physics (World Scientific, Singapore, 1984).
- [4] For a recent review, see L. E. Ibáñez and G. G. Ross, in *Perspectives in Higgs Physics*, edited by G. L. Kane (unpublished).
- [5] See, e.g., M. Kamionkowski, in *High Energy Neutrino Astrophysics*, Proceedings of the Workshop, Honolulu, Hawaii, 1992, edited by V. J. Stenger, J. G. Learned, S. Pakvasa, and X. R. Tata (World Scientific, Singapore, 1992).
- [6] U. Amaldi, W. de Boer, and H. Fürstenau, *Phys. Lett. B* **260**, 447 (1991); P. Langacker and M. Luo, *Phys. Rev. D* **44**, 817 (1991); J. Ellis, S. Kelley, and D. V. Nanopoulos, *Phys. Lett. B* **260**, 131 (1991).
- [7] M. Davier, in *Proceedings of the Joint International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics*, Geneva, Switzerland, 1991, edited by S. Hegarty, K. Potter, and E. Quercigh (World Scientific, Singapore, 1992).
- [8] CDF Collaboration, F. Abe *et al.*, *Phys. Rev. Lett.* **69**, 3439 (1992).
- [9] S. Bertolini, F. Borzumati, A. Masiero, and G. Ridolfi, *Nucl. Phys.* **B353**, 591 (1991), and references therein.
- [10] K. Hagiwara and H. Murayama, *Phys. Lett. B* **246**, 533 (1990).
- [11] G. Bhattacharyya and A. Raychaudhuri, *Phys. Rev. D* **47**, 2014 (1993).
- [12] J. Ellis and S. Rudaz, *Phys. Lett.* **128B**, 248 (1983).
- [13] G. F. Giudice and G. Ridolfi, *Z. Phys. C* **41**, 447 (1988); M. Olechowski and S. Pokorski, *Phys. Lett. B* **214**, 239 (1988).
- [14] K.-I. Hikasa and M. Kobayashi, *Phys. Rev. D* **36**, 724 (1987).
- [15] G. Passarino and M. Veltman, *Nucl. Phys.* **B160**, 151 (1979).
- [16] W. Beenakker, W. Hollik, and S. C. van de Marck, *Nucl. Phys.* **B365**, 24 (1991).
- [17] G. 't Hooft and M. Veltman, *Nucl. Phys.* **B153**, 365 (1979).
- [18] W. Hollik, *Fortschr. Phys.* **38**, 165 (1991).
- [19] J. Jerzak, E. Laermann, and P. M. Zerwas, *Phys. Rev. D* **25**, 1218 (1980); A. Djouadi, *Z. Phys. C* **39**, 561 (1988).
- [20] J. Schwinger, *Particles, Sources and Fields*, Vol. II (Addison-Wesley, New York, 1973).
- [21] E. Laermann, Diploma thesis, RWTH Aachen, 1986 (unpublished).
- [22] A. Djouadi, J. H. Kühn, and P. M. Zerwas, *Z. Phys. C* **46**, 411 (1990).
- [23] See, e.g., HELIOS Collaboration, T. Akesson *et al.*, *Z. Phys. C* **52**, 219 (1991), and references therein.
- [24] J. Ellis, D. V. Nanopoulos, and D. A. Ross, *Phys. Lett. B* **305**, 375 (1993).
- [25] J. Ellis and H. Kowalski, *Phys. Lett.* **157B**, 437 (1985); G. Altarelli, B. Mele, and S. Petrarca, *ibid.* **160B**, 317 (1985); V. Barger, S. Jacobs, J. Woodside, and K. Hagiwara, *Phys. Rev. D* **33**, 57 (1986).
- [26] H. A. Baer, M. Drees, R. M. Godbole, J. F. Gunion, and X. R. Tata, *Phys. Rev. D* **44**, 725 (1991).
- [27] M. Drees and M. M. Nojiri, *Nucl. Phys.* **B369**, 54 (1992); *Phys. Rev. D* **47**, 376 (1993).