Small momentum evolution of the extended Drell-Hearn-Gerasimov sum rule

Véronique Bernard
Centre de Recherches Nucléaires et Université Louis Pasteur de Strasbourg, Physique Théorique, Bat 40A, BP 20 Cr, 67037 Strasbourg Cedex 2, France

Norbert Kaiser
Physik Department T30, Technische Universität München, James Franck Strasse, D-8046 Garching, Germany

Ulf-G. Meissner
Universität Bern, Institut für Theoretische Physik, Sidlerstrasse 5, CH-3012 Bern, Switzerland

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We investigate the momentum dependence of the extended Drell-Hearn-Gerasimov (DHG) sum rule. An economical formalism is developed which allows one to express the extended DHG sum rule in terms of a single virtual Compton amplitude in the forward direction. Rigorous results for the small momentum evolution are derived from chiral perturbation theory within the one-loop approximation. Furthermore, we evaluate some higher-order contributions arising from $\Delta(1232)$ intermediate states and relativistic corrections. We also discuss the limitations of our approach.

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I. INTRODUCTION

Many years ago, Drell and Hearn [1] and Gerasimov [2] (DHG) suggested a sum rule for spin-dependent Compton scattering. It expresses the squared anomalous magnetic moment of the nucleon in terms of a dispersive integral over the difference of the total photonucleon absorption cross sections $\sigma_{1/2}(\omega)$ and $\sigma_{3/2}(\omega)$ for the scattering of circular polarized photons on polarized nucleons. The subscripts $\lambda=\frac{1}{2}$ and $\frac{3}{2}$ denote the total γN helicity, corresponding to states with photon and nucleon spins antiparallel or parallel. Experimentally, this sum rule has never been tested directly since up to now no measurements of the helicity cross sections have been performed. However, models for the photoabsorption cross sections [3–5] do indicate its approximate validity (on a qualitative level). One can now extend this sum rule to virtual photons with $k^2<0$, the four-momentum transfer of the virtual photon,[1] since the corresponding helicity cross sections can be parametrized in terms of the spin-dependent nucleon structure functions. The recent data of the European Muon Collaboration [6] taken in the scaling region of large $|k^2|\simeq 10$ GeV^2 suggest not only that the pertinent sum rule behaves as $1/k^2$ for large $|k^2|$, but also that the sign is opposite to the DHG sum rule for real photons (which in standard notation is negative). Therefore the integral

$$I(k^2)=\int_{\omega_{\rm thr}}^{\infty}\frac{d\omega}{\omega}[\sigma_{1/2}(\omega,k^2)-\sigma_{3/2}(\omega,k^2)]\ , \qquad (1.1)$$

with ω the virtual photon energy in the nucleon rest frame, must change its sign between the photon point ($k^2=0$) and the European Muon Collaboration (EMC) region, $k^2\simeq-10$ GeV^2. A recent model predicts this turnover to happen at $k^2\simeq-0.8$ GeV^2 [7], and it explains this value mainly in terms of the low-energy contribution of the $\Delta(1232)$ resonance to the pertinent photoabsorption cross sections. Note that the model of Ref. [7], as well as the phenomenological analysis of Ref. [5], seems to indicate a positive slope of $I_p(k^2)$ in the vicinity of the photon point, $k^2\simeq 0$, whereas the older analysis of Anselmino, Ioffe, and Leader [8] gives a negative slope (as a function of k^2).

Here we wish to add some new insight into the momentum dependence of the integral $I(k^2)$ in the region of small k^2 where small means that $\sqrt{-k^2}$ does not exceed a few pion masses. Our model-independent analysis is based on the fact that at low energies the interactions of hadrons are governed by chiral symmetry and gauge invariance (when external photons are involved). One can systematically solve the chiral Ward-Takahashi identities of QCD via an expansion in external momenta and quark masses, which are considered small against the scale of chiral symmetry breaking, $\Lambda_\chi\simeq 1$ GeV. This method is called chiral perturbation theory. It uses the framework of an effective Lagrangian of the asymptotically observed fields. The low-energy expansion corresponds to an expansion in pion loops. In the presence of baryons, a complication arises. The nucleon (baryon) mass in the chiral limit is comparable to the chiral scale Λ_χ, and thus only baryon three-momenta can be considered small [9]. One can, however, restore the exact one-to-one correspondence between the loop and low-energy expansion using a nonrelativistic formulation of baryon chiral perturbation theory (ChPT) [10]. The nucleon is considered as a very heavy (static) source, and in that case, all momenta involved are small, therefore restoring the consistent power

[1]It is customary to set $k^2=-Q^2$ and only use Q^2. We will not do this in the following.

counting. In what follows we will use the nonrelativistic version of baryon ChPT which was systematically investigated in Ref. [11] as well as the relativistic formulation as spelled out in detail in Ref. [9]. This will allow us to extract the leading term in the chiral expansion of $I(k^2)$ and to calculate the derivative of $I(k^2)$ around $k^2 \simeq 0$. This is the region where ChPT applies. Furthermore, following the suggestion of Jenkins and Manohar [12], we will also add the $\Delta(1232)$ resonance to nonrelativistic baryon ChPT. The $\Delta(1232)$ is the lowest nucleon excitation, and its closeness to the nucleon mass $m_\Delta - m \simeq 2.1 M_\pi$ might indicate substantial contributions from it (this is also supported by phenomenological models). In fact, using these various approximation schemes, we will get a band of values for the slope of $I(k^2)$. Our most important result, however, is that independent of the scheme we are using, we find that $I(k^2)$ increases as $|k^2|$ increases (around $k^2 \simeq 0$). This new result should serve as a constraint for all model builders and should eventually be seen in refined phenomenological analyses or directly from the data (when they will become available). For a review on baryon ChPT, see Ref. [13].

The paper is organized as follows. In Sec. II, we spell out an economical formalism to calculate $I(k^2)$ in terms of a single function which possesses a right-hand cut starting at the single-pion production threshold. In Sec. III we use ChPT to calculate $I(k^2)$ for the proton and neutron at small k^2, in the extreme nonrelativistic and the fully relativistic formulation. The contribution of loops involving the $\Delta(1232)$ isobar in the nonrelativistic approach is also discussed. The numerical results and conclusions are presented in Sec. IV.

II. SPIN-DEPENDENT COMPTON SCATTERING: FORMALISM

In this section we outline the formalism necessary to describe the scattering of polarized (virtual) photons on polarized nucleons (protons and neutrons). Denote by p and k the four-momenta of the nucleon and photon, respectively. It is convenient to work with the two Lorentz invariants k^2 and $\omega = p\cdot k/m$, with m the nucleon mass. The spin of the photon and nucleon can couple to the values $\frac{1}{2}$ and $\frac{3}{2}$ with the corresponding photoabsorption cross sections denoted by $\sigma_{1/2}(\omega,k^2)$ and $\sigma_{3/2}(\omega,k^2)$, in order.[2] In what follows we are interested in the extended Drell-Hearn-Gerasimov sum rule; i.e., the integral

$$I(k^2)=\int_{\omega_{\rm thr}}^{\infty}\frac{d\omega}{\omega}[\sigma_{1/2}(\omega,k^2)-\sigma_{3/2}(\omega,k^2)]\ , \qquad (2.1)$$

with $k^2 \le 0$ and the threshold photon energy $\omega_{\rm thr}$ due to single-pion electroproduction, is given by

$$\omega_{\rm thr}=M_\pi+\frac{M_\pi^2-k^2}{2m}\ , \qquad (2.2)$$

where M_π denotes the pion mass. For real photons the expression (2.1) becomes the celebrated DHG sum rule

$$I(0)=\int_{\omega_{\rm thr}}^{\infty}\frac{d\omega}{\omega}[\sigma_{1/2}(\omega,0)-\sigma_{3/2}(\omega,0)] = -\frac{\pi e^2\kappa^2}{2m^2}\ . \qquad (2.3)$$

Here κ is the anomalous magnetic moment of the proton or neutron and we use standard units $e^2/4\pi = 1/137.036$. The DHG sum rule is derived under the assumption that the spin-dependent forward Compton amplitude for real photons $f_2(\omega^2)$ satisfies an unsubtracted dispersion relation which guarantees that the right-hand side of Eq. (2.3) converges. In what follows we will make use of the same assumption for virtual photons. To set the scale for $I(k^2)$, let us give the numerical values for the proton and neutron:

$$I_p(0)=-0.526\ {\rm GeV}^{-2}\ , \quad I_n(0)=-0.597\ {\rm GeV}^{-2}\ . \qquad (2.4)$$

Our main concern will be the k^2 evolution of the extended DHG sum rule, in particular around the origin $k^2 \simeq 0$. The interest in that comes from the relation of the helicity cross sections to the spin-dependent nucleon structure functions $G_1(\omega,k^2)$ and $G_2(\omega,k^2)$. Following the notation of Ioffe, Khoze, and Lipatov [15],[3] one can show that

$$\sigma_{1/2}(\omega,k^2)-\sigma_{3/2}(\omega,k^2) = \frac{4\pi e^2}{2m\omega+k^2}\frac{\omega}{m}\left[G_1(\omega,k^2)+\frac{k^2}{m\omega}G_2(\omega,k^2)\right]\ . \qquad (2.5)$$

The relation of these structure functions to the spin-dependent virtual Compton amplitudes in the forward direction $S_{1,2}(\omega,k^2)$ is standard:

$$2\pi G_i(\omega,k^2)={\rm Im}S_i(\omega,k^2)\quad (i=1,2)\ , \qquad (2.6)$$

which follows from the optical theorem. Furthermore, crossing symmetry implies that $S_1(\omega,k^2)$ and $G_2(\omega,k^2)$ are even functions under $(\omega\to-\omega)$ whereas $S_2(\omega,k^2)$ and $G_1(\omega,k^2)$ are odd. In fact, for our purpose one does not need information on both amplitudes $S_1(\omega,k^2)$ and $S_2(\omega,k^2)$ but only the particular combination entering Eq. (2.5). In order to isolate this relevant combination, one contracts the antisymmetric (in $\mu\leftrightarrow\nu$) part of the virtual Compton tensor in the forward direction with polarization vectors ϵ'_μ and ϵ_ν for the outgoing and incoming virtual photons, respectively. If we choose the gauge conditions

$$\epsilon\cdot p=\epsilon'\cdot p=\epsilon\cdot k=\epsilon'\cdot k=0$$

for the polarization vectors and work in the nucleon rest frame $p_\mu=(m,0,0,0)$, we obtain

[2]For the definition of these cross sections, see Ref. [15] (Chap. 2). We omit the tilde over the symbol σ used in that book.

[3]We use a different normalization for the nucleon spinors, $\bar{u}u=1$ instead of $\bar{u}u=2m$.

$$\epsilon'_\mu T^{\mu\nu}_{(a)}\epsilon_\nu = \frac{i}{2m^2}\chi^\dagger\left\{\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}'\times\boldsymbol{\epsilon})\left[\omega S_1(\omega,k^2)+\frac{\omega^2}{m}S_2(\omega,k^2)\right]-\boldsymbol{\sigma}\cdot\mathbf{k}\mathbf{k}\cdot(\boldsymbol{\epsilon}'\times\boldsymbol{\epsilon})\frac{S_2(\omega,k^2)}{m}\right\}\chi$$

$$=\frac{i\omega}{2m^2}\chi^\dagger\boldsymbol{\sigma}\cdot(\boldsymbol{\epsilon}'\times\boldsymbol{\epsilon})\chi\left[S_1(\omega,k^2)+\frac{k^2}{m\omega}S_2(\omega,k^2)\right], \quad (2.7)$$

where χ is a conventional two-component (Pauli) spinor. In Eq. (2.7) we have exploited the fact that under the chosen gauge $\boldsymbol{\epsilon}'\times\boldsymbol{\epsilon}$ is parallel to $\mathbf{k}$ and $\mathbf{k}^2=\omega^2-k^2$. Obviously, we are projecting out the particular combination of $S_1(\omega,k^2)$ and $S_2(\omega,k^2)$ whose imaginary part enters the extended DHG sum rule $I(k^2)$. In analogy to the real photon case, we call this combination

$$f_2(\omega^2,k^2)=\frac{e^2}{8\pi m^2}\left[S_1(\omega,k^2)+\frac{k^2}{m\omega}S_2(\omega,k^2)\right]. \quad (2.8)$$

Here we indicated already that $f_2(\omega^2,k^2)$ is an even function of ω which follows from the $(\omega\to-\omega)$ crossing properties of $S_{1,2}(\omega,k^2)$ [15]. The odd amplitude $\omega f_2(\omega^2,k^2)$ can now be expressed in terms of a single function $A(s,k^2)$ as

$$2\pi(s-m^2-k^2)f_2(\omega^2,k^2)$$
$$=e^2[A(s,k^2)-A(2m^2+2k^2-s,k^2)]. \quad (2.9)$$

Here we introduced the Mandelstam variable $s=(p+k)^2$ which is related to ω via $\omega=(s-m^2-k^2)/2m$. The function $A(s,k^2)$ appearing in Eq. (2.9) can always be chosen such that it has only a right-hand cut starting at the single-pion production threshold $s=(m+M_\pi)^2$. A similar construction of an amplitude with only a right-hand cut has been used by Meyer [14] who tried to relate the extended DHG sum rule to half-off-shell nucleon form factors. Under the assumption that $f_2(\omega^2,k^2)$ satisfies an unsubtracted dispersion relation (in ω) or equivalently that $A(s,k^2)$ satisfies a once-subtracted dispersion relation (in s, subtracted at an arbitrary point s_0), we can make use of the previous equations and calculate the extended DHG sum rule $I(k^2)$ as

$$I(k^2)=8\pi\int_{(m+M_\pi)^2}^\infty ds\frac{\mathrm{Im}f_2(\omega^2,k^2)}{s-m^2}$$
$$=4e^2\int_{(m+M_\pi)^2}^\infty ds\frac{\mathrm{Im}A(s,k^2)}{(s-m^2)(s-m^2-k^2)}$$
$$=\frac{4\pi e^2}{k^2}\left[A(m^2+k^2,k^2)-A(m^2,k^2)\right]. \quad (2.10)$$

This equation is our basic result. It is completely general and allows one to calculate the extended DHG sum rule $I(k^2)$ from a single function $A(s,k^2)$ which can be easily computed from the virtual Compton tensor in the forward direction. To repeat it, Eq. (2.10) was derived under the assumption that $A(s,k^2)$ obeys a once-subtracted dispersion relation. That this is not a too strong assumption, e.g., can be seen from the fact that in the relativistic formulation of baryon ChPT to one loop $A(s,k^2)$ indeed has this analytical property. Furthermore, as argued in Ref. [13] the asymptotic behavior of $S_{1,2}(\omega,k^2)$ following from Regge theory supports this assumption, i.e., $f_2(\omega^2,k^2)\sim\omega^{-1}$. However, a general proof for this is not yet available. In this sense the situation is analogous to $f_2(\omega^2,0)$ where the validity of an unsubtracted dispersion relation cannot yet be proven in general. In the following section, we will use ChPT (in the one-loop approximation) to evaluate $A(s,k^2)$ and to calculate $I(k^2)$ for k^2 in the vicinity of zero (this is where ChPT applies).

III. CHIRAL EXPANSION

At low energies, any QCD Green function can be systematically expanded in powers of small momenta and quark (pion) masses. This is done within the framework of an effective chiral Lagrangian of the asymptotically observed fields, here, the nucleons, pions, and photons. The low-energy expansion amounts to an expansion in (pion) loops of the effective theory. In the presence of baryons, a complication arises due to the baryon mass which is nonvanishing in the chiral limit and therefore adds a new scale to the theory. In that case there is in general no strict one-to-one correspondence between the low-energy and loop expansions. Stated differently, there is no guarantee that all next-to-leading-order corrections at order q^3 (with q denoting a generic small momentum) are given completely by the one-loop graphs. All calculations performed so far, however, indicate that the leading nonanalytic terms (in the quark masses) which arise due to infrared singularities in the chiral limit of vanishing pion mass are indeed produced. Furthermore, one also gets in the one-loop approximation an infinite tower of higher-order terms [9] which spoil the one-to-one mapping between low-energy and loop expansions. To overcome these difficulties, it was recently proposed to use a heavy fermion effective field theory, i.e., considering the baryons as very heavy [10] and to expand the theory in inverse powers of the baryon mass. In that case the n-loop contributions are suppressed by relative powers of q^{2n} (with q a genuine small momentum) and a consistent counting scheme emerges. Furthermore, in this framework one can easily couple in the $\Delta(1232)$ resonance since one does not encounter the usual problems with the relativistic spin-$\frac{3}{2}$ particle [12]. Nevertheless, we have to stress that the baryon mass m comparable to the chiral-symmetry-breaking scale Λ_χ is not very large. Therefore an expansion in powers of M_π/m is *a priori* not to be expected to converge very fast. Such M_π/m suppressed contributions are partly resummed in the relativistic approach. Of course, the evaluation of all M_π/m corrections is necessary to judge the quality of the chiral expansion. Furthermore, once the spin-$\frac{3}{2}$ decuplet is included, one has an extra nonvanishing scale in the chiral limit (the average octet-decuplet mass splitting) which again complicates the low-energy structure.

The basic $\pi N\gamma$ Lagrangian in the relativistic formulation of baryon ChPT to leading order $[O(q)]$ reads

$$\mathcal{L}=\mathcal{L}_{\pi N}^{(1)}+\mathcal{L}_{\pi\pi}^{(2)} ,$$
$$\mathcal{L}_{\pi N}^{(1)}=\bar{\Psi}\left[i\not{D}-m+\frac{g_A}{2}\not{u}\gamma_5\right]\Psi , \tag{3.1}$$
$$\mathcal{L}_{\pi\pi}^{(2)}=\frac{F^2}{4}\mathrm{Tr}[\nabla_\mu U\nabla^\mu U^\dagger+M_\pi^2(U+U^\dagger)] ,$$

where $U=\exp[i\tau\cdot\pi/F]$ embodies the Goldstone bosons, $u=\sqrt{U}$, and $u_\mu=iu^\dagger\nabla_\mu Uu^\dagger$, with ∇_μ the pertinent covariant derivative. The isospinor Ψ contains the proton and neutron fields. The superscript (i) denotes the chiral power of the corresponding terms; it counts derivatives and meson masses. The construction of this effective Lagrangian is unique. Let us point out that it contains four parameters. These are the pion decay constant F, the axial-vector coupling g_A, the nucleon mass (all in the chiral limit), and the leading term in the quark mass expansion of the pion mass, $M_\pi=\sqrt{2\hat{m}B}$. Here $\hat{m}=\frac{1}{2}(m_u+m_d)$ is the average light quark mass and $B=-\langle 0|\bar{u}u|0\rangle/F^2$ is the order parameter of the spontaneous chiral symmetry breaking. Calculating tree diagrams with this effective Lagrangian, one reproduces the well-known current algebra results. To restore unitarity one has to consider pion loops in addition. To give all corrections at next-to-leading order in the chiral expansion, one has to work out all one-loop diagrams constructed from the vertices in $\mathcal{L}$ and furthermore one has to add the tree graph contribution from the most general chirally symmetric counterterm Lagrangian $\mathcal{L}_{\pi N}^{(2)}+\mathcal{L}_{\pi N}^{(3)}+\mathcal{L}_{\pi\pi}^{(4)}$. For the (spin-dependent) Compton tensor under consideration here, however, no such counterterm can contribute. As stressed in Ref. [11], we are dealing with a pure loop effect (within the one-loop approximation).

As already noted, in Eq. (3.1) the troublesome nucleon mass term appears. In the extreme nonrelativistic limit, it can be eliminated in the following way. Decompose the baryon four-momentum as $p_\mu=mv_\mu+l_\mu$ with v_μ the four-velocity $(v^2=1)$ and l_μ a small off-shell momentum $(v\cdot l\ll m,)$, and write Ψ in terms of eigenstates of the velocity projection operator,

$$\Psi=e^{-imv\cdot x}(H+h) , \tag{3.2}$$

with $\not{v}H=H$ and $\not{v}h=-h$. Eliminating now the "small" component h via its equation of motion, one ends up with

$$\mathcal{L}_{\pi N}^{(1)}=\bar{H}(iv\cdot D+g_A S\cdot u)H+O(1/m) . \tag{3.3}$$

Here $S_\mu=(i/2)\gamma_5\sigma_{\mu\nu}v^\nu$ is the covariant spin operator which obeys $S\cdot v=0$. The nucleon mass term has disappeared, allowing for a consistent chiral power counting scheme. All one-loop contributions are of order q^3. Furthermore, one has to expand the tree contributions from the vertices of Eq. (3.1) in $1/m$ appropriately to collect all terms up to and including order q^3. For a more detailed discussion of these topics, see Ref. [11]. One can furthermore add the $\Delta(1232)$, which is a spin-$\frac{3}{2}$ field, very easily in the extreme nonrelativistic limit. For details on the couplings of the $\Delta(1232)$, see the Appendix. Here we just note that the mass splitting $m_\Delta-m$ stays finite in the chiral limit. Therefore loops with intermediate $\Delta(1232)$ states will count as order q^4 and higher (since the counterterm contributions start only at order q^5).

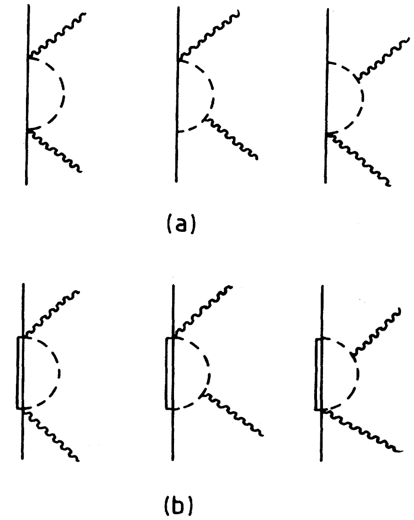

FIG. 1. (a) One-loop diagrams contributing to the spin-dependent Compton tensor in the heavy mass formulation of ChPT. Dashed lines denote pions. (b) One-loop Compton graphs including the $\Delta(1232)$ resonance in the heavy mass approach (denoted by a thick line).

Let us now turn to the calculation of $I(k^2)$ for small k^2. In Fig. 1(a) we show the pertinent Feynman diagrams which contribute in the heavy mass limit (with intermediate nucleons only). We work in the Coulomb gauge $\epsilon'\cdot v=\epsilon\cdot v=0$, which is very economical in the calculation of photon-nucleon processes since most diagrams (those with an isolated photon-nucleon vertex) are then identical to zero. The integral $I(k^2)$ takes the form

$$I(k^2)=I(0)+\tilde{I}(k^2) , \tag{3.4}$$

with $I(0)=-\pi e^2\kappa^2/2m^2$ the DHG sum rule value for real photons. In the heavy mass formulation of baryon ChPT, the leading term of the chiral expansion of $\tilde{I}(k^2)$ is given completely by the one-loop graphs in Fig. 1(a). All higher-order corrections to $\tilde{I}(k^2)$ are suppressed by further powers of the pion mass M_π and k^2. Some (but not all) of these corrections will be generated from loop diagrams with $\Delta(1232)$ intermediate states or in the relativistic version of baryon ChPT. The leading term of the chiral expansion of $\tilde{I}(k^2)$ can be given in closed form:

$$\tilde{I}(k^2)=\frac{e^2g_A^2}{4\pi F^2}\left\{-1+\left[1+\frac{4}{\rho}\right]^{1/2}\ln\left[\left[1+\frac{\rho}{4}\right]^{1/2}+\frac{\sqrt{\rho}}{2}\right]\right\}=\frac{e^2g_A^2}{48\pi F^2}\rho+O(\rho^2) , \tag{3.5}$$

with $\rho=-k^2/M_\pi^2>0$. We see that the slope of $I(k^2)$ at $k^2=0$ is negative and singular in the chiral limit; i.e., it diverges like $1/M_\pi^2$. This behavior is a direct consequence of the chiral structure of QCD which governs the low-energy strong-interaction phenomena. Furthermore, $\tilde{I}(k^2)$ is equal for both proton and neutron [within the $O(q^3)$ approximation to the virtual Compton tensor]. We should also add here that presently the usual DHG sum rule value $I(0)$ for real photons cannot be obtained through a dispersive integral such as Eq. (2.10) within the one-loop approximation of ChPT. In the heavy mass formulation, this term arises from real $1/m^2$ suppressed tree graphs involving the anomalous magnetic moment κ (in the chiral limit). In the relativistic version of baryon ChPT, the anomalous magnetic moment of the nucleon is generated from one-loop diagrams and it is nonvanishing in the chiral limit. In order to obtain a term proportional to κ^2 such as $I(0)$, one necessarily has to go to the level of two-loop graphs. This problem of how $I(0)$ can be obtained from a dispersion relation for loop amplitudes does, however, not affect our discussion of the k^2 dependence of $I(k^2)$. Extending the effective Lagrangian to the $\Delta(1232)$ resonance as spelled out in the Appendix, we have to calculate the diagrams of Fig. 1(b). These amount to some higher-order (q^n, $n\geq 1$) corrections to Eq. (3.5) which we include because of the phenomenological importance of this resonance [a complete evaluation of all $O(q)$ corrections to $I(k^2)$ corresponding to $O(q^4)$ for the virtual Compton tensor goes beyond the scope of this paper]. A straightforward calculation gives, for the sum of the nucleon and $\Delta(1232)$ one-loop diagrams,

$$\tilde{I}(k^2)=\frac{e^2g_A^2}{4\pi F^2}\left\{\frac{r}{\sqrt{r^2-1}}\ln(r+\sqrt{r^2-1})-\int_0^1 dx\frac{r}{\sqrt{r^2-1-\rho x(1-x)}}\ln\left[\frac{r}{\sqrt{1+\rho x(1-x)}}+\left[\frac{r^2}{1+\rho x(1-x)}-1\right]^{1/2}\right]\right\}, \tag{3.6}$$

with $r=(m_\Delta-m)/M_\pi\simeq 2.1$. Obviously, $\tilde{I}(0)=0$, in agreement with the celebrated low-energy theorem of Low [16] and Gell-Mann and Goldberger [16]. As a check, one can show that in the limit $m_\Delta-m\rightarrow\infty$ one recovers the result of Eq. (3.5). Again, there is no splitting between proton and neutron sum rules, i.e., $\tilde{I}(k^2)=\tilde{I}_p(k^2)=\tilde{I}_n(k^2)$. The slope of the extended DHG sum rule at the photon point is given as

$$I'(0)=-\frac{e^2g_A^2}{48\pi F^2M_\pi^2}\frac{r^2\sqrt{r^2-1}-r\ln(r+\sqrt{r^2-1})}{(r^2-1)^{3/2}}. \tag{3.7}$$

In the relativistic formulation, matters are different. First, one has to calculate many more Feynman diagrams. These generate some of the M_π/m suppressed higher-order corrections and naturally lead to a splitting between proton and neutron for the momentum dependence of the extended DHG sum rule, i.e., $\tilde{I}_p(k^2)\neq\tilde{I}_n(k^2)$. What is conceptually most important is that in the relativistic version of baryon ChPT one can indeed show that the amplitude function $A(s,k^2)$ obeys a once-subtracted dispersion relation. Using now the definitions of the various loop functions as given in Ref. [17] extended to $k^2\leq 0$, the following expressions can be deduced for $\tilde{I}_p(k^2)$ and $\tilde{I}_n(k^2)$:

$$\begin{aligned}\tilde{I}_p(k^2)=\frac{e^2g_A^2m^2}{4\pi F^2k^2}\int_0^1dx\int_0^1dy\Bigg\{&\left[1-\frac{3m^2}{k^2}y\right]\ln\frac{M_\pi^2(1-y)+m^2y^2+k^2y(y-1)}{M_\pi^2(1-y)+m^2y^2}\\&-2y\ln\frac{M_\pi^2(1-y)+m^2y^2+k^2xy(xy-1)}{M_\pi^2(1-y)+m^2y^2+k^2x(x-1)y^2}+\frac{\frac{3}{2}y(1-y)[k^2-(2m^2+k^2)y]}{M_\pi^2(1-y)+m^2y^2}\\&-\frac{2k^2xy(1-y)^2}{M_\pi^2(1-y)+m^2y^2+k^2x(x-1)(1-y)^2}+\frac{y^2[k^2(xy+x^2y-\frac{1}{2})+m^2(y-1)]}{M_\pi^2(1-y)+m^2y^2+k^2x(x-1)y^2}\\&+\frac{y^2[m^2(1-y)-k^2x^2y]}{M_\pi^2(1-y)+m^2y^2+k^2xy(xy-1)}+\frac{2m^2k^2(1-x)xy^4}{[M_\pi^2(1-y)+m^2y^2+k^2xy(xy-1)]^2}\\&+\frac{y^4[k^4x^2(1-x)(y-\frac{1}{2})+m^2k^2(\frac{1}{2}-\frac{3}{2}x+2x^2-xy)]}{[M_\pi^2(1-y)+m^2y^2+k^2x(x-1)y^2]^2}-\frac{\frac{3}{2}k^2m^2y^3(1-y)^2}{[M_\pi^2(1-y)+m^2y^2]^2}\Bigg\},\end{aligned} \tag{3.8}$$

$$\tilde{I}_n(k^2)=\frac{e^2g_A^2m^2}{4\pi F^2k^2}\int_0^1 dx\int_0^1 dy\left\{2(1-y)\ln\frac{M_\pi^2(1-y)+m^2y^2+k^2x(y-1)(1-x+xy)}{M_\pi^2(1-y)+m^2y^2+k^2x(x-1)(1-y)^2}\right.$$
$$+2y\ln\frac{M_\pi^2(1-y)+m^2y^2+k^2xy(xy-1)}{M_\pi^2(1-y)+m^2y^2+k^2x(x-1)y^2}+\frac{y^2[-2m^2+k^2(2x-1)]}{M_\pi^2(1-y)+m^2y^2+k^2x(x-1)y^2}$$
$$+\frac{2m^2y^2}{M_\pi^2(1-y)+m^2y^2+k^2xy(xy-1)}+\frac{k^2(1-x)y^4[k^2x^2+m^2(1-4x)]}{[M_\pi^2(1-y)+m^2y^2+k^2x(x-1)y^2]^2}$$
$$\left.+\frac{4m^2k^2x(1-x)y^4}{[M_\pi^2(1-y)+m^2y^2+k^2xy(xy-1)]^2}\right\}. \qquad (3.9)$$

As an important analytical check, we can again verify that $\tilde{I}_p(0)=\tilde{I}_n(0)=0$ and one can show that in the limit $m\to\infty$ both $\tilde{I}_p(k^2)$ and $\tilde{I}_n(k^2)$ tend to $\tilde{I}(k^2)$ as given in Eq. (3.5). With this we have collected all formulas necessary to study $I(k^2)$ for both the proton and neutron.

IV. RESULTS AND DISCUSSION

First, we must fix the parameters. Throughout, we use $F=93$ MeV, $M_\pi=139.57$ MeV, $m=938.27$ MeV, and $g_A=1.26$. In the case of the $\Delta(1232)$ resonance, we use the SU(4) relation among coupling constants, $g_{\pi N\Delta}=3g_{\pi N}/\sqrt{2}$, with $g_{\pi N}=g_A m/F$ given by the Goldberger-Treiman relation. The mass splitting between nucleon and $\Delta(1232)$ has a value of $m_\Delta-m=293$ MeV.

Consider now the proton. We will first discuss the slope of $I_p(k^2)$ at the photon point $k^2=0$. In the heavy mass limit with only intermediate nucleon states, we find

$$\left.\frac{dI_p(k^2)}{dk^2}\right|_{k^2=0}=-\frac{e^2g_A^2}{48\pi F^2M_\pi^2}=-5.7\ \mathrm{GeV}^{-4}. \qquad (4.1)$$

This value is decreased by 16% when the $\Delta(1232)$ resonance is included in the one-loop graphs as inspection of Eq. (3.7) reveals. Therefore the $\Delta(1232)$ does not play a major role in determining the slope of $I_p(k^2)$ in our approach. Much more drastic is the effect of the relativistic M_π/m suppressed terms. In the fully relativistic calculation where many (but not all) of such terms are included, we find $I_p'(0)=-2.2$ GeV^{-4} for the proton and $I_n'(0)=-1.7$ GeV^{-4} for the neutron. It is instructive to compare these numbers with some phenomenological analyses and models of $I(k^2)$. In the work of Anselmino, Ioffe, and Leader [8], the k^2 dependence of the extended DHG sum rule is modeled via vector meson exchange at low momentum transfer. One finds

$$I_p'(0)=-4\pi e^2\frac{\Gamma_p}{M_\rho^4}\left[1+\frac{1}{4}\left[\frac{M_\rho}{m_p}\right]^2\frac{\kappa_p^2}{\Gamma_p}\right]=-2.2\ \mathrm{GeV}^{-4} \qquad (4.2)$$

for $\Gamma_p=0.126$, the experimental value of the integrated spin-structure function $g_1(x)$ of the proton [6]. In the more recent model of Burkert and Ioffe [7], one finds, however, a positive slope due to the Δ resonance and similarly in the phenomenological analysis of Burkert and Li [5]. According to this work, $I_p(k^2)$ first decreases in the region $0\le -k^2\le 0.1$ GeV^2 and then a rapid rise in k^2 sets in. This kink structure comes from the N-$\Delta(1232)$ magnetic transition form factor. Furthermore, it was observed in the phenomenological analysis of Ref. [5] that the DHG sum rule at $k^2=0$ is almost completely saturated by the $\Delta(1232)$ contribution. Indeed, both features seem to be connected with each other. If the $\Delta(1232)$ contribution to the helicity cross sections $\sigma_\lambda(\omega,k^2)$ is removed, $I_p(0)$ almost vanishes and the kink around $k^2=0$ disappears. As already mentioned, the DHG sum rule $I_p(0)=-\pi e^2\kappa_p^2/2m^2$ can be different from zero first within a full two-loop calculation in ChPT, since κ_p itself is a one-loop effect. Therefore we regard the kink around $k^2=0$ [if it is really present in the experimental[4] $I_p(k^2)$] as a two- and higher-loop effect. If we ignore the kink in $I_p(k^2)$ of Ref. [5] and only consider the rise in k^2, then the global behavior of $I_p(k^2)$ for 0.05 GeV$^2\le -k^2\le 0.5$ GeV^2 suggests an average negative slope of about -2 GeV^{-4}. This value is comparable to our relativistic ChPT result. It is important to stress the difference in the underlying physical picture of the ChPT and the phenomenological approach. The cross sections $\sigma_{1/2}$ and $\sigma_{3/2}$ originate from the sum of all inelastic (electroproduction) channels. Our calculation shows how the one-pion channel is constrained by the chiral structure of QCD. In contrast, in Ref. [5] the one-pion and one-η channels are represented by resonance production. The role of the Δ is completely different in the two cases. In the ChPT case, the loops with intermediate isobars contribute to the two-pion inelastic channels; i.e., they are corrections to the dominant one-pion channel. In the phenomenological approach, the Δ is used to fit the one-pion channel. A direct comparison of the Δ contribution is therefore misleading.

In Fig. 2 we show $\tilde{I}_p(k^2)$ for $-k^2\le 0.25$ GeV^2. In the

[4]Presently known are only the cross sections σ_λ at the resonance position (i.e., $M_R^2=m^2+2m\omega+k^2$). The evaluation of the extended DHG sum rule furthermore requires knowledge of the full energy dependence of the resonance cross sections and resonance transition form factors. Relaxing the assumptions of Ref. [5] on these parameters may well make the kink around $k^2=0$ disappear.

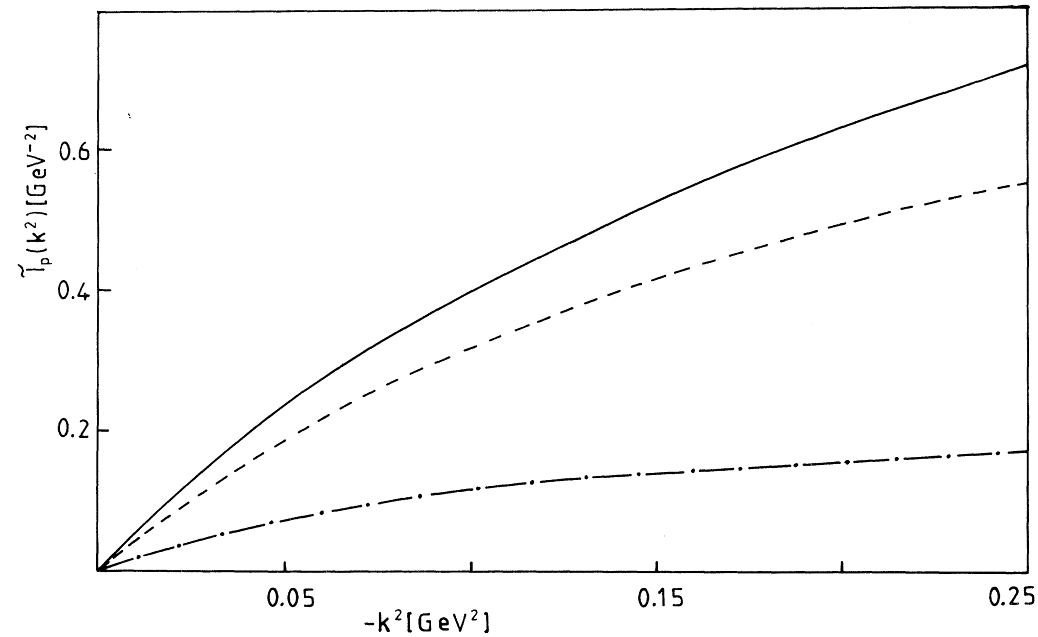

FIG. 2. Momentum dependence of the extended DHG sum rule $\tilde{I}_p(k^2)$. The solid line gives the one-loop result in the heavy mass limit of baryon ChPT. The dashed line is obtained from one-loop graphs involving nucleons as well as $\Delta(1232)$ resonances. The dot-dashed line gives the result of the relativistic version of baryon ChPT to one loop.

heavy mass limit, half of the value of $I_p(0)$ (in magnitude) is reached at $k^2 \simeq 0.06$ GeV^2. The crossover where $I_p(k^2)$ goes from negative to positive values takes place at $k^2 \simeq -0.15$ GeV^2. This is a very low value compared to previous phenomenological analysis, but compared to the pion mass scale M_π^2 it is already quite large, $k^2 \simeq -7.7 M_\pi^2$. Therefore one can no longer trust the one-loop approximation in that region of k^2 where a sign change of $I_p(k^2)$ takes place. Including some higher-order chiral corrections through loops with $\Delta(1232)$ resonances, the momentum dependence of $\tilde{I}_p(k^2)$ becomes softer and the corresponding numbers decrease by roughly 30%. The zero of $I_p(k^2)$ is now shifted to a higher value of $k^2 \simeq -0.23$ GeV^2. In the relativistic formulation of ChPT where in addition to the leading terms also many higher-order corrections are included, $\tilde{I}_p(k^2)$ is much smaller than in the case of infinite nucleon mass. This phenomenon, that higher-order relativistic corrections are quite large, was also observed in previous calculations of the nucleon electromagnetic polarizabilities [17]. However, since the M_π/m corrections generated in the one-loop approximation of relativistic baryon ChPT are by no means complete, one cannot draw any conclusions about the convergence of the chiral expansion at the moment.

In summary, we have presented a novel formalism to calculate the momentum dependence of the extended DHG sum rule at finite $k^2 \leq 0$. A single amplitude function $A(s,k^2)$ which enters the spin-dependent virtual Compton tensor in the forward direction is sufficient to evaluate $I(k^2)$, as long as $A(s,k^2)$ satisfies a once-subtracted dispersion relation. We have used baryon chiral perturbation theory to investigate the behavior of the extended DHG sum rule $I(k^2)$ in the vicinity of $k^2=0$. We could give a (rather wide) range of values for the slope $I_p'(0)$. Eventually, this prediction will be tested experimentally; at present, we consider it as a constraint following from the chiral structure of QCD which will be useful for phenomenological analysis and model building. Furthermore, a higher-loop calculation has to be performed to find out the stability of the results presented here.

Note added. Recently, Soffer and Teryaev [19] proposed a different model for $I_p(k^2)$. They combine a Schwinger sum rule (based on the assumption of double spectral representations) with a simple parametrization of $I_{1+2}(k^2)$. In this work, $I_p(k^2)$ shows no kink and the slope is essentially determined by the electromagnetic charge radius of the proton. One obtains $I_p'(0) = -5.0$ GeV^{-4}, close to our nonrelativistic result.

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APPENDIX: Δ(1232) IN THE HEAVY MASS FORMULATION

Here we discuss briefly the description of the $\Delta(1232)$ resonance in the heavy mass formulation following Ref. [11]. To leading order [up to $O(q)$], the relevant effective Lagrangian reads (we write down only those terms which are actually needed for our purpose)

$$\mathcal{L}^{(1)}_{\pi N\Delta} = -i\bar{T}^{\mu a} v\cdot D^{ab} T^b_\mu + \delta m \bar{T}^{\mu a} T^a_\mu + \frac{3g_A}{2\sqrt{2}} (\bar{T}^{\mu a} u^a_\mu H + \bar{H} u^a_\mu T^{\mu a}) \ . \tag{A1}$$

The Rarita-Schwinger spinor T^a_μ with a an isospin index and μ a Lorentz index incorporates the four charge states of the $\Delta(1232)$ as follows:

$$T^1_\mu = \frac{1}{\sqrt{2}} \begin{bmatrix} \Delta^{++} - \Delta^0/\sqrt{3} \\ \Delta^+/\sqrt{3} - \Delta^- \end{bmatrix}_\mu ,$$
$$T^2_\mu = \frac{i}{\sqrt{2}} \begin{bmatrix} \Delta^{++} + \Delta^0/\sqrt{3} \\ \Delta^+/\sqrt{3} + \Delta^- \end{bmatrix}_\mu , \tag{A2}$$
$$T^3_\mu = -\left[\frac{2}{3}\right]^{1/2} \begin{bmatrix} \Delta^+ \\ \Delta^0 \end{bmatrix}_\mu .$$

Furthermore, in the heavy mass limit this field is subject to the constraint $v_\mu T^{\mu a} = 0$. In (A1), $\delta m = m_\Delta - m$ stands for the mass splitting of nucleon and $\Delta(1232)$ and

$$u^a_\mu = \frac{i}{2} \mathrm{Tr}(\tau^a u^\dagger \nabla_\mu U u^\dagger) = -\partial_\mu \pi^a / F - e\epsilon^{a3b} A_\mu \pi^b / F + \cdots$$

gives rise to the chiral couplings of pions and photons to the $N\Delta$ system. We already exploited the SU(4) relation $g_{\pi N\Delta} = 3g_{\pi N}/\sqrt{2}$, with $g_{\pi N} = g_A m/F$ between the $\pi N\Delta$ and πNN coupling constants. The empirical information on the $\Delta \to \pi N$ decay width confirms that this relation holds very well within a few percent. In the heavy mass limit, the propagator of the $\Delta(1232)$ reads

$$P^{\mu\nu} = \frac{i}{v\cdot l - \delta m} [v^\mu v^\nu - g^{\mu\nu} - \tfrac{4}{3} S^\mu S^\nu] \ , \tag{A3}$$

where S_μ is the covariant spin operator of heavy mass approach satisfying $v\cdot S = 0$. Let us finally remark that this formulation of $\Delta(1232)$ couplings is completely equivalent to the usual isobar model as discussed in Ref. [18] for the special choice $v_\mu = (1,0,0,0)$. This corresponds to the standard nonrelativistic description.

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