Low-energy study of four-fermion theory with the CP^1 model in 2+1 dimensions

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We study the four-fermion theory with the CP^1 model in 2+1 dimensions. The low-energy effective action is derived and the low-lying spectrum of the model is obtained. Through the Landau-Ginzburg description, it is shown that, for the superconducting case, the model is identical to the anyonic superconductivity model.

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I. INTRODUCTION AND A MODEL

The copper-oxide high-T_c superconductor (HTCS) has a layered structure and is conceptually a doped antiferromagnet. In its theoretical consideration, the quasi-two-dimensional properties of the HTCS motivate us to study (2+1)-dimensional quantum field theory, and there have been many attempts to understand HTCS's [1–5].

Recently, Shankar [6] has proposed a new model, which describes hole dynamics in a quantum antiferromagnet. According to the model there are two types of holes (we denote them as A and B holes) in an antiferromagnet due to a bipartite spin lattice. In a continuum and low-momentum limit, it becomes a field-theoretical model describing two types of massless fermions, which correspond to the A and B holes, coupled minimally to the gauge field of the CP^1 model, the "spin gauge field," which describes the spin wave of an antiferromagnet in the undoped case. As an aside, the author has argued that there may be Cooper pairing as in BCS theory. However, the above results have been derived in 1+1 dimensions. Its extension to 2+1 dimensions has been done in Ref. [7] in which the continuum theory was established, the parity even mass was obtained by solving the Schwinger-Dyson equation, and eventually parity invariant low-energy effective action for the HTCS was obtained.

In this paper, we take the continuum Lagrangian of Ref. [7] obtained from the microscopic Hamiltonian constructed on a square lattice as a starting point. The Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_s + \mathcal{L}_h, \tag{1}$$

where $\mathcal{L}_s$ describes the antiferromagnetic spin wave, which we represent by the CP^1 model, and $\mathcal{L}_h$ describes the dynamics of holes, doped in the spin system, which are minimally coupled to the spin gauge field of the CP^1 model.

The Lagrangian $\mathcal{L}_s$ is given by

$$\mathcal{L}_s = (2/g^2)\left|(\partial_\mu - ia_\mu)\, z\right|^2, \tag{2}$$

where a_μ is the spin gauge field, g is a running coupling constant, and the z field describing the spin wave is composed of two complex scalar fields, z_1 and z_2, as

$$z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \tag{3}$$

and satisfies the constraint condition $z^\dagger z = 1$.

The Lagrangian $\mathcal{L}_h$ is the system of massless fermions coupled to the spin gauge field with additional attractive and repulsive four-fermion terms. The fermions have two flavors due to the lattice doubling [7,8] and represent the A and B holes simultaneously as

$$\psi^f = \begin{pmatrix} \chi_A^f \\ \chi_B^f \end{pmatrix} \quad (\, f = 1,\ 2\,), \tag{4}$$

where χ_A^f (χ_B^f) are two-component spinors describing A (B) holes. Here, we set the number of flavors to an arbitrary positive integer N_f, for a moment, not to 2 for later consideration, and omit the flavor index f. Then the $\mathcal{L}_h$ is given by

$$\mathcal{L}_h = \bar{\psi}\left(i\partial\!\!\!/ + a\!\!\!/\tau_3 + A\!\!\!/\right)\psi - \kappa\left[\left(\bar{\psi}\psi\right)^2 - \left(\bar{\psi}\tau_3\psi\right)^2\right], \tag{5}$$

where A_μ is the electromagnetic gauge field, κ is a coupling constant, and

$$\tau_3 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad (\, I : 2 \times 2 \text{ unit matrix }). \tag{6}$$

The matrix τ_3 implies that the A and B holes are coupled to the spin gauge field with opposite sign. The Dirac γ matrices and their algebra are chosen as

$$\gamma^0 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} i\sigma_1 & 0 \\ 0 & i\sigma_1 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix},$$
$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}, \qquad g^{\mu\nu} = \mathrm{diag}\,[1, -1, -1]\,, \tag{7}$$

where σ_i ($i = 1, 2, 3$) are Pauli matrices.

The aim of this paper is to analyze system (1) and compare it with the anyon superconductivity model. In Sec. II we derive the low-energy one-loop effective action of Eq. (1) for the external electromagnetic and spin gauge field. The low-energy spectrum of the model is obtained and the model is compared with the anyonic superconductivity model in Sec. III. In Sec. IV the Landau-Ginzburg description for the effective action is given. Finally, Sec. V contains the conclusion.

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II. EFFECTIVE ACTION

The partition function is given by

$$Z[A_\mu] = \int \mathcal{D}a_\mu \mathcal{D}z^\dagger \mathcal{D}z \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left(i\int d^3x \mathcal{L}\right). \tag{8}$$

Because the z field and the ψ field interact with each other indirectly through a spin gauge field a_μ, we can deal with both of them independently.

Let us first consider the partition function due to the fermionic part which we define as $Z_1[A_\mu, a_\mu]$:

$$\begin{aligned} Z_1[A_\mu, a_\mu] &\equiv \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left(i\int d^3x \mathcal{L}_h\right) \\ &= \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}\sigma\mathcal{D}\pi \exp\left\{i\int d^3x\left[\bar{\psi}\left(i\partial + a\tau_3 + A\right)\psi - \sigma\bar{\psi}\psi - i\pi\bar{\psi}\tau_3\psi + \frac{N_f}{2\kappa}\left(\sigma^2+\pi^2\right)\right]\right\} \\ &= \int \mathcal{D}\sigma\mathcal{D}\pi \exp\left[i\int d^3x \frac{N_f}{2\kappa}\left(\sigma^2+\pi^2\right)\right] \det{}^{N_f}\left(i\not{\partial} + \not{a}\tau_3 + \not{A} - \sigma - i\pi\tau_3\right), \end{aligned} \tag{9}$$

where we introduce two auxiliary fields, σ and π, to linearize the four-fermion terms. Before we proceed, it should be noted that it is possible for the fermion masses, that is, the saddle points of the σ and π fields, to be generated dynamically. It has been shown that the dynamical mass generation can occur in the large-N expansion [8,9]. Especially, for parity even mass, $\langle i\pi\rangle \neq 0$, it has been shown that it is possible even at $N_f = 2$ by solving the Schwinger-Dyson equation [7]. We would only follow these results and not give the detail analysis. In our case, only the fact that the dynamical mass generation may occur is important, and we fix N_f by 2.

Following the above arguments, we may evaluate the determinant of Eq. (9) around the saddle points $m_o \equiv \langle\sigma\rangle$ and $m_e \equiv \langle i\pi\rangle$, where m_o and m_e are parity odd and even masses, respectively. The determinant is evaluated in the quadratic approximation and in the low-momentum limit [4,7,9,10]. In this quadratic approximation, the couplings between the fields σ and π near the saddle points and the gauge fields cannot occur because of the gauge symmetry, and we can set σ ($i\pi$) field to m_o (m_e). Therefore, the path integration over the variables in (9) may be ignored and the determinant becomes just the partition function $Z_1[A_\mu, a_\nu]$. Then the partition function (9) is obtained as

$$Z_1[A_\mu, a_\nu] \simeq \exp\left\{i\int d^3x\left[-\frac{\Pi_e}{4}\left(f_{\mu\nu}^2 + F_{\mu\nu}^2\right) - \frac{\Pi}{2}f_{\mu\nu}F^{\mu\nu} - \frac{\theta_+}{2}\epsilon^{\mu\nu\lambda}\left(a_\mu\partial_\nu a_\lambda + A_\mu\partial_\nu A_\lambda\right) - \theta_-\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu A_\lambda\right]\right\} \tag{10}$$

where the following is defined:

$$\theta_\pm = \frac{1}{2\pi}\left[\mathrm{sgn}(m_+) \pm \mathrm{sgn}(m_-)\right], \quad m_\pm = m_o \pm m_e,$$

$$\Pi_e = \frac{1}{6\pi}\left(\frac{1}{|m_+|} + \frac{1}{|m_-|}\right), \quad \Pi = \frac{1}{6\pi}\left(\frac{1}{|m_+|} - \frac{1}{|m_-|}\right), \tag{11}$$

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

Next we turn to the z field part of the partition function in Eq. (8), which we define as $Z_2[a_\mu]$:

$$\begin{aligned} Z_2[a_\mu] &\equiv \int \mathcal{D}z^\dagger\mathcal{D}z \exp\left(i\int d^3x \mathcal{L}_s\right) \\ &= \int \mathcal{D}z^\dagger\mathcal{D}z\mathcal{D}\lambda \exp\left\{i\int d^3x\left[|(\partial_\mu - ia_\mu)z|^2 - \lambda(|z|^2 - 2/g^2)\right]\right\}, \end{aligned} \tag{12}$$

where an auxiliary field λ is introduced to implement the constraint $z^\dagger z = 1$ and the fields are rescaled as $z \to \sqrt{2/g^2}z$ and $\lambda \to (2/g^2)\lambda$. Since the calculation of $Z_2[a_\mu]$ in the low-energy limit is well known [10,11,15], we only give the result that is obtained as

$$Z_2[a_\mu] \simeq \exp\left(-\frac{i}{96\pi|m|}\int d^3x f_{\mu\nu}^2\right), \tag{13}$$

where m is the saddle point of the λ field. It should be noticed that Eq. (13) is evaluated in the disordered phase of the CP^1 model [11] because the HTCS is in the phase [12].

Assembling the results (10) and (13), we obtain the partition function (8) as

$$\begin{aligned} Z[A_\mu] &= \int \mathcal{D}a_\mu Z_1\left[A_\mu, a_\mu\right] Z_2\left[a_\mu\right] \\ &= \int \mathcal{D}a_\mu \exp\left(i\int d^3x \mathcal{L}_{\text{eff}}\right) \end{aligned} \tag{14}$$

with the effective Lagrangian given by

$$\begin{aligned} \mathcal{L}_{\text{eff}} = &-\frac{\Pi_s}{4}f_{\mu\nu}^2 - \frac{\Pi_e}{4}F_{\mu\nu}^2 - \frac{\Pi}{2}f_{\mu\nu}F^{\mu\nu} - \frac{\theta_+}{2}\epsilon^{\mu\nu\lambda} \\ &\times\left(a_\mu\partial_\nu a_\lambda + A_\mu\partial_\nu A_\lambda\right) - \theta_-\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu A_\lambda, \end{aligned} \tag{15}$$

where

$$\Pi_s = \frac{1}{6\pi}\left(\frac{1}{|m_+|} + \frac{1}{|m_-|} + \frac{1}{4|m|}\right).$$

From the definitions Eq. (11) for θ_+ and θ_-, one would realize that the Lagrangian (15) cannot be used in itself for further studies, but instead it has to be reexpressed for the two cases of $|m_o| > |m_e|$ and $|m_o| < |m_e|$. For $|m_o| > |m_e|$, $\theta_+ = \mathrm{sgn}(m_o)/\pi$ and $\theta_- = 0$, and, for $|m_o| < |m_e|$, $\theta_+ = 0$ and $\theta_- = \mathrm{sgn}(m_e)/\pi$. In the following sections, we would give the separate arguments associated with these two cases to the results to be obtained.

III. LOW-ENERGY SPECTRUM OF THE MODEL

With the effective Lagrangian (15), we may obtain the low-energy spectrum of the model by investigating the pole structure of the electromagnetic current-current correlation function given by

$$\langle j^\mu(x) j^\nu(y)\rangle = (-i)^2 \frac{1}{Z[A_\alpha]} \frac{\delta^2 Z[A_\alpha]}{\delta A_\mu(x)\delta A_\nu(y)}. \qquad (16)$$

Before performing the functional derivatives of Eq. (16), we decouple the A_μ field from the a_μ field in the effective Lagrangian because the gauge fields A_μ and a_μ are coupled to each other. In the momentum space, the effective Lagrangian (15) may be rewritten as

$$\mathcal{L}_{\text{eff}} = a_\mu M^{\mu\nu} a_\nu + A_\mu N^{\mu\nu} A_\nu + a_\mu I^{\mu\nu} A_\nu, \qquad (17)$$

where

$$\begin{aligned} M^{\mu\nu} &= \frac{\Pi_s}{2}\left(k^\mu k^\nu - g^{\mu\nu}k^2\right) + \frac{i\theta_+}{2}\epsilon^{\mu\nu\lambda}k_\lambda, \\ N^{\mu\nu} &= \frac{\Pi_e}{2}\left(k^\mu k^\nu - g^{\mu\nu}k^2\right) + \frac{i\theta_+}{2}\epsilon^{\mu\nu\lambda}k_\lambda, \\ I^{\mu\nu} &= \Pi\left(k^\mu k^\nu - g^{\mu\nu}k^2\right) + i\theta_-\epsilon^{\mu\nu\lambda}k_\lambda. \end{aligned} \qquad (18)$$

If we complete the square form in the A_μ field, the Lagrangian becomes

$$\begin{aligned} \mathcal{L}_{\text{eff}} = {} & (a_\mu + \tfrac{1}{2}A_\sigma I^{\sigma\rho} M^{-1}_{\rho\mu}) M^{\mu\nu} (a_\nu + \tfrac{1}{2} M^{-1}_{\nu\rho} I^{\rho\sigma} A_\sigma) \\ & + A_\mu (N^{\mu\nu} - \tfrac{1}{4} I^{\mu\rho} M^{-1}_{\rho\sigma} I^{\sigma\nu}) A_\nu. \end{aligned} \qquad (19)$$

Then, if we put this Lagrangian into the partition function (14) and shift the a_μ field, the variable of path integration, as

$$a_\mu \longrightarrow a_\mu - \tfrac{1}{2} M^{-1}_{\mu\rho} I^{\rho\sigma} A_\sigma,$$

we obtain

$$Z[A_\mu] = \exp\left[i\int \frac{d^3k}{(2\pi)^3} A_\mu(k)\left(N^{\mu\nu} - \frac{1}{4} I^{\mu\rho} M^{-1}_{\rho\sigma} I^{\sigma\nu}\right) A_\nu(-k)\right] \int \mathcal{D}a_\mu \exp\left[i\int \frac{d^3k}{(2\pi)^3} a_\mu(k) M^{\mu\nu} a_\nu(-k)\right], \qquad (20)$$

which is the variable-decomposed form. Here we should notice that the gauge fixing is needed to evaluate $M^{-1}_{\rho\sigma}$ in Eq. (19) due to the gauge symmetry. In our case, $\partial_\mu a^\mu = 0$ is chosen. In this gauge choice, $N^{\mu\nu} - (1/4) I^{\mu\rho} M^{-1}_{\rho\sigma} I^{\sigma\nu}$ of Eq. (20) is derived as

$$\begin{aligned} N^{\mu\nu} - \frac{1}{4} I^{\mu\rho} M^{-1}_{\rho\sigma} I^{\sigma\nu} = \frac{1}{\theta_+^2 - \Pi_s^2 k^2} \Bigg\{ & \left[\frac{\Pi_e}{2}(\theta_+^2 - \Pi_s^2 k^2) + \frac{\Pi_s}{2}(\theta_-^2 - \Pi^2 k^2) - \Pi\theta_+\theta_-\right](k^\mu k^\nu - g^{\mu\nu}k^2) \\ & + \frac{i}{2}\theta_+\left[\theta_+^2 - \theta_-^2 - \left(\Pi_s^2 + \Pi^2\right)k^2\right]\epsilon^{\mu\nu\lambda}k_\lambda \Bigg\}. \end{aligned} \qquad (21)$$

Now it is straightforward to obtain the current-current correlation function by acting the functional differentiations to the partition function (20) according to (16). As was noted in the last section, we give the results for the two cases of $|m_o| > |m_e|$ and $|m_o| < |m_e|$.

For $|m_o| > |m_e|$, in the $k \to 0$ limit,

$$\langle j^\mu(k) j^\nu(-k)\rangle|_{\theta_-=0} \overset{k\to 0}{\simeq} \frac{i\theta_+^2}{k^2\Pi_s^2 - \theta_+^2}\left[\Pi_e\left(k^\mu k^\nu - g^{\mu\nu}k^2\right) + i\theta_+\epsilon^{\mu\nu\lambda}k_\lambda\right]. \qquad (22)$$

In this case, the model has a massive excitation with mass $\sqrt{\theta_+^2/\Pi_s^2}$, which corresponds to the massive spin gauge boson. Here, if we look at the effective Lagrangian (15) with $\theta_- = 0$, there is the Chern-Simons term for the electromagnetic gauge field [13]. This implies that the A_μ field is topologically massive, which does not spoil gauge invariance. Thus the superconductivity does not appear [14]. So, the case of $|m_o| > |m_e|$ is beyond our interest.

For $|m_o| < |m_e|$,

$$\langle j^\mu(k) j^\nu(-k)\rangle|_{\theta_+=0} \overset{k\to 0}{\simeq} i\frac{\theta_-^2}{\Pi_s}\frac{k^\mu k^\nu}{k^2} - i\frac{\theta_-^2}{\Pi_s} g^{\mu\nu}. \qquad (23)$$

On the right-hand side of (23), the first term shows that the model has a gapless (massless) excitation, which corresponds to massless spin gauge boson and the second is the Meissner term, which gives the longitudinal mass to the electromagnetic gauge field. Now it is obvious that we would be interested in the case of $|m_o| < |m_e|$ to study the superconductivity.

IV. LANDAU-GINZBURG DESCRIPTION

Since we have obtained the Meissner term in Eq. (23), we would pay attention to the case of $|m_o| < |m_e|$. In this section, it is shown that the electromagnetic gauge field has not only a topological mass but also a longitudinal mass following the Landau-Ginzburg description of Lykken, Sonnenschein, and Weiss [4]. We will discuss the case of $|m_o| > |m_e|$ briefly.

Let us return to the effective Lagrangian (15). With $|m_o| < |m_e|$, then it becomes

$$\begin{aligned} \mathcal{L}_{\text{eff}}|_{\theta_+=0} = {} & -\frac{\Pi_s}{4} f_{\mu\nu}^2 - \frac{\Pi_e}{4} F_{\mu\nu}^2 \\ & - \frac{\Pi}{2} f_{\mu\nu} F^{\mu\nu} - \theta_- \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu A_\lambda. \end{aligned} \qquad (24)$$

Now we change the variables from a_μ to $\tilde{f}_\mu = (1/2)\epsilon_{\mu\nu\lambda} f^{\nu\lambda}$, the dual of $f^{\mu\nu}$, in the path integral (14). Since $\tilde{f}_\mu$ satisfies the Bianchi identity $\partial_\mu \tilde{f}^\mu = 0$, we introduce the Lagrangian multiplier field ϕ in the path integral to impose this constraint. Then Eq. (14) becomes

$$Z\Big[A_\mu\Big] = \int \mathcal{D}\tilde{f}_\mu \mathcal{D}\phi \exp\Big[i\int d^3x\Big(-\frac{\Pi_s}{2}\tilde{f}_\mu^2 - \frac{\Pi_e}{4}F_{\mu\nu}^2 - \frac{\Pi}{2}\epsilon_{\mu\nu\lambda}F^{\mu\nu}\tilde{f}^\lambda - \theta_-\tilde{f}_\mu A^\mu + \phi\partial_\mu\tilde{f}^\mu\Big)\Big]. \tag{25}$$

Integrating out the field $\tilde{f}_\mu$, we get the partition function as

$$Z[A_\mu] = \int \mathcal{D}\phi \exp\Big\{i\int d^3x\Big[-\frac{1}{4}\Big(\Pi_e - \frac{\Pi^2}{\Pi_s}\Big)F_{\mu\nu}^2 + \frac{\theta_-\Pi}{2\Pi_s}\epsilon_{\mu\nu\lambda}A^\mu F^{\nu\lambda} + \frac{1}{2\Pi_s}(\partial_\mu\phi + \theta_- A_\mu)^2\Big]\Big\}. \tag{26}$$

This final expression gives just the Landau-Ginzburg description of anyonic superconductivity. So the remaining arguments are parallel to those of anyonic case. The photon A_μ has a mass $\sqrt{\theta_-^2/\Pi_s}$, the square of which is just the coefficient of Meissner term in Eq. (23) and leads to the electromagnetic super current of the conventional BCS theory, in addition to the topological mass. The massless field ϕ that has been introduced as a Lagrangian multiplier in Eq. (25) may be identified with the one representing the massless spin gauge field [4,5,7,15]. It may be interesting that the Chern-Simons term, which is absent in Eq. (24), appears in Eq. (26). However, since we do not set the parity-odd mass m_o to zero, the presence of Chern-Simons term is natural. In fact, if m_o vanishes, the effect due to the Chern-Simons term does not exist at all.

For $|m_o| > |m_e|$, $\theta_- = 0$, we could not obtain the Lagrangian of the Landau-Ginzburg type as in the previous case. Formally, this is because of the Chern-Simons term for a_μ field. The gauge field A_μ has only a transverse (topological) mass as was shown in the preceding section. In this case, if we ignore the interaction term between the fields a_μ and A_μ by assuming $|m_o| \gg |m_e|$, $\Pi \sim 0$, the Lagrangian becomes a system of two distinct topologically massive gauge theories.

V. CONCLUSION

We have studied the four-fermion theory coupled to the CP^1 model in 2+1 dimensions, which is the (2+1)-dimensional continuum Lagrangian for the HTCS, which is an extension of the model proposed by Shankar. Assuming the parity even (m_e) and odd (m_o) masses which are generated dynamically, the low-energy effective action has been derived explicitly in the quadratic approximation. By investigating the pole structure of the electromagnetic current-current correlation function, we have obtained the low-energy spectrum of the model. For the case of $|m_o| > |m_e|$, the model has a finite gap of $\sqrt{\theta_+^2/\Pi_s^2}$ and the gauge fields a_μ and A_μ are both topologically massive. For the case of $|m_o| < |m_e|$, the current-current correlator gives the Meissner term and shows that the model has no gap, which is identified with the masslessness of the spin gauge field a_μ. In this case, the electromagnetic gauge field A_μ has not only a topological (transverse) mass but also a longitudinal mass due to the Meissner term. Therefore the model is superconducting for $|m_o| < |m_e|$. The Landau-Ginzburg description of the model for this superconducting case is identical to that of anyonic superconductivity model.

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[1] P. W. Anderson, G. Baskaran, Z. Zou, and T. Hsu, Phys. Rev. Lett. **58**, 2790 (1987); I. Affleck and J. B. Marston, Phys. Rev. B **37**, 3774 (1988); J. B. Marston and I. Affleck, *ibid.* **39**, 11 538 (1989).
[2] X. G. Wen, F. Wilczek, and A. Zee, Phys. Rev. B **39**, 11 413 (1989).
[3] R. B. Laughlin, Science **242**, 525 (1988).
[4] J. D. Lykken, J. Sonnenschein, and N. Weiss, Int. J. Mod. Phys. A **6**, 1335 (1991), and references therein.
[5] T. Banks and J. D. Lykken, Nucl. Phys. **B336**, 500 (1990).
[6] R. Shankar, Phys. Rev. Lett. **63**, 203 (1989); Nucl. Phys. **B330**, 433 (1990).
[7] N. Dorey and N. E. Mavromatos, Phys. Lett. B **250**, 107 (1990).
[8] G. W. Semenoff and L. C. R. Wijewardhana, Phys. Rev. Lett. **63**, 2633 (1989).
[9] G. W. Semenoff and L. C. R. Wijewardhana, Phys. Rev. D **45**, 1342 (1992); M. Carena, in *The Vancouver Meetings–Particles and Fields '91*, Proceedings of the Joint Meeting of the Division of Particles and Fields of the American Physical Society and the Particle Physics Division of the Canadian Association of Physicists, Vancouver, 1991, edited by D. Axen, D. Bryman, and M. Comyn (World Scientific, Singapore, 1992), p. 843.
[10] A. M. Polyakov, *Gauge Fields and Strings* (Harwood, New York, 1987).
[11] X. G. Wen and A. Zee, Nucl. Phys. **B326**, 619 (1989); B. Rosenstein and A. Kovner, *ibid.* **B346**, 576 (1990).
[12] See, for example, D. Vaknin *et al.*, Phys. Rev. Lett. **58**, 2802 (1987), and references therein.
[13] Y. H. Chen and F. Wilczek, Int. J. Mod. Phys. B **3**, 117 (1989).
[14] E. Fradkin, *Field Theories of Condensed Matter Systems* (Addison-Wesley, Reading, MA, 1991).
[15] A. M. J. Schakel, Phys. Rev. D **44**, 1198 (1991).