Cosmological bound on the decay $\pi^0 \to \gamma X$

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(Received 23 February 1993)

Using the upper bound on the effective number of light neutrino species during primordial nucleosynthesis and the cosmological pion-pole mechanism $\gamma\gamma \to \pi^0 \to \gamma X$, we obtain an upper limit on the branching ratio for the decay $B(\pi^0 \to \gamma X) < 3 \times 10^{-13}$, where X is any long-lived weakly interacting neutral vector particle with a mass smaller than the neutral pion mass.

PACS number(s): 98.80.Cq, 13.40.Hq, 14.80.Er

Experimental searches for exotic neutral pion decays have been reported recently [1,2]. These experiments served not only to test the standard model, but also to hunt for new physics. So far, the experimental results are consistent with standard physics. As such, upper limits on the branching ratios for the rare decays $B(\pi^0 \to \nu\bar{\nu}) < 8.3 \times 10^{-7}$ [1] and $B(\pi^0 \to \gamma X) < 5 \times 10^{-4}$ [2] have been set, where X is a hypothetical long-lived weakly interacting neutral particle with mass $0 \le m_X < m_{\pi^0}$. From the conservation of angular momentum, the decay $\pi^0 \to \nu\bar{\nu}$ is allowed only if the right-handed (RH) neutrino exists. Also, observation of such a radiative decay of pion would indicate the unambiguous existence of a new vector particle, which might be a new U(1) gauged boson proposed in some extended standard models [3–5].

On the other hand, recently, we have considered the cosmological bound on the decay $\pi^0 \to \nu\bar{\nu}$ [6]. If RH neutrinos exist, they would be produced from the cosmic thermal background via the pion-pole mechanism (πPM), $\gamma\gamma \to \pi^0 \to \nu\bar{\nu}$, at the temperature of about the pion rest mass [7]. As long as the neutrino lives longer than the time scale for the freeze-out of weak interactions ($\simeq 1$ sec), in order that the amount of RH neutrinos thus produced would not exceed the primordial nucleosynthesis bound on the effective number of light neutrino species, $N_\nu \le 3.3$ [8], the branching ratio for the decay $B(\pi^0 \to \nu\bar{\nu})$ must be less than 3×10^{-13} [6], which is much better than the experimental limit. Here, in a similar way, by calculating the amount of X bosons produced from the cosmic photons via the π PM, we will set, from the nucleosynthesis bound, an upper bound on the decay $\pi^0 \to \gamma X$.

It was Fischbach *et al.* [7] who first applied the π PM to astrophysics and cosmology. The cross section for the reaction $\gamma\gamma \to \gamma X$ via a pion resonance can be approximated by

$$\sigma(s) = \frac{8\pi \Sigma(s) \Gamma(\pi^0 \to \gamma\gamma) \Gamma(\pi^0 \to \gamma X)}{(s - m_\pi^2)^2 + m_\pi^2 \Gamma_\pi^2} , \quad (1)$$

where $m_\pi \simeq 135$ MeV and $\Gamma_\pi \simeq 7.83$ eV are, respectively, the mass and width of pion, and $\sqrt{s}$ is the center-of-mass energy; $\Sigma(s)$ is generally a polynomial of s with $\Sigma(s = m_\pi^2) = 1$, which would take the functional form

$$\Sigma(s) = \left[\frac{s - m_X^2}{m_\pi^2 - m_X^2} \right]^3 , \quad (2)$$

if X is a vector boson directly coupled to quarks or leptons [3]. Because $\Gamma_\pi / m_\pi \simeq 5.8 \times 10^{-8}$ is quite small, we would have $\Sigma(s) \simeq 1$ at s near the pion pole (when $s = m_\pi^2$) unless m_X is very close to m_π. Thus, in general, the cross section $\sigma(s)$ is much smaller at s far from the pion pole than at s near the pole. Note that Eq. (1) gives the correct off-shell scattering cross section at large s. If the photons are in thermal equilibrium with a temperature T, the production rate of X bosons is given by

$$Q = \tfrac{1}{2} \langle \sigma v_{\rm rel} \rangle n_\gamma^2 , \quad (3)$$

where σ is the cross section given by Eq. (1), $v_{\rm rel}$ is the relative velocity of the incident photons, and $n_\gamma = [2\zeta(3)/\pi^2] T^3 \equiv AT^3$ [$\zeta(3) = 1.202$] is the number density of the thermal photons. In Eq. (3), the angular brackets imply a thermal average and the factor of $\frac{1}{2}$ appears due to identical initial photons. At temperature T, the average energy per photon is $\langle E_\gamma \rangle = [\pi^4/30\zeta(3)] T \equiv CT$. In the following, we shall use the approximation, $\langle \sigma v_{\rm rel} \rangle \simeq \sigma(s)$, where $s = 4\langle E_\gamma \rangle^2$.

Now we turn to the calculation of the amount of X bosons produced from the cosmic thermal background. Since the whole process takes place in a radiation-dominated universe, the number density n_X of X bosons at time t is governed by the Boltzmann equation

$$\frac{dn_X}{dt} = -3Hn_X + Q , \quad (4)$$

where Q is given by Eq. (3) and H is the Hubble parameter given by

$$H = \left[\frac{8\pi G}{3} \frac{\pi^2}{30} g_*(T) \right]^{1/2} T^2 \equiv BT^2 . \quad (5)$$

Here, G is Newton's constant, and $g_*(T)$ is the total number of degrees of freedom at temperature T. In Eq. (4), we have neglected the inverse scattering processes. Since $R \propto 1/T$, where R is the cosmic scale factor, by writing $n_X = f(T)T^3$, we obtain from Eqs. (1), (3), and (4) that

$$\frac{df}{dT}=-\frac{A^2}{2B}\sigma(s)\ , \tag{6}$$

where $s=4C^2T^2$. Equation (6) can be directly integrated and we obtain

$$f(T_1)-f(T_0)=\frac{2\pi A^2}{BC}\frac{\Gamma_\pi^2 R}{m_\pi^3}\int_{\tau_1}^{\tau_0}\frac{\Sigma(\tau)}{(\tau^2-1)^2+\gamma^2}d\tau\ , \tag{7}$$

where $\tau\equiv\sqrt{s}/m_\pi$, $\gamma\equiv\Gamma_\pi/m_\pi$, $R\equiv\Gamma(\pi^0\to\gamma X)/\Gamma(\pi^0\to\gamma\gamma)$, and we have taken $\Gamma(\pi^0\to\gamma\gamma)\simeq\Gamma_\pi$. In Eq. (7), $T_1\simeq1$ MeV is the temperature at which weak interactions freeze-out. Note that the upper limit T_0 of the integral cannot be arbitrarily large [the integral is divergent at large T_0 when $\Sigma(s)$ takes the form in Eq. (2)]. Here, we choose $T_0\simeq100$ MeV and assume $f(T_0)=0$, based on the fact that the X boson has been decoupled from the thermal background at a much higher temperature and then the number density n_X might be negligibly small right after the quark-hadron phase transition which occurs at a temperature of about 100 MeV. Since $\Sigma(\tau)\simeq1$ when $\tau\simeq1$, we can estimate the integral in Eq. (7) by

$$\int_{\tau_1}^{\tau_0}\frac{\Sigma(\tau)}{(\tau^2-1)^2+\gamma^2}d\tau\simeq\frac{k}{\gamma}\ , \tag{8}$$

where k is a number of the order of 1. By using $\Sigma(\tau)$ as given in Eq. (2), we find numerically that $k\simeq1.57$. In fact, the result is rather insensitive to different functional forms of $\Sigma(\tau)$, and also to both integration limits as long as the integration range has covered the pion pole reasonably well. Since the production of X bosons via the πPM proceeds at a temperature $T\simeq0.185m_\pi\simeq25$ MeV, we take $g_*(T)\simeq10.75$ and note that there would be no further heating of the thermal bath due to particle annihilations or phase transitions from $T\simeq25$ MeV down to $T\simeq1$ MeV. Finally, we obtain from Eqs. (7) and (8) that

$$f(T=1\ \text{MeV})=1.57\frac{2\pi A^2}{BC}\frac{\Gamma_\pi R}{m_\pi^2}\ , \tag{9}$$

provided that the lifetime of X boson is longer than 1 sec.

So far, we have not restricted the X boson mass m_X except $0\le m_X<m_\pi$. For $m_X\ll1$ MeV, at $T\ge1$ MeV, the X bosons are relativistic and their average energy per particle is $\langle E_X\rangle=CT$. Thus, their energy density ρ_X is given by

$$\rho_X=n_X\langle E_X\rangle=f(T)T^3CT=1.57\frac{2\pi A^2}{B}\frac{\Gamma_\pi R}{m_\pi^2}T^4\ . \tag{10}$$

Expressing ρ_X in terms of n_{eff}, the equivalent number of light neutrino species, i.e., $\rho_X=\frac{7}{8}(\pi^2/15)n_{\text{eff}}T^4$, we find

$$n_{\text{eff}}=9.8\times10^{11}R\ . \tag{11}$$

Therefore, in order that the contribution of the energy density of X bosons to the thermal background during the time of nucleosynthesis does not exceed the equivalent of 0.3 neutrino species, i.e., $n_{\text{eff}}<0.3$, the branching ratio R in Eq. (11) must be such that

$$R<3\times10^{-13}\ . \tag{12}$$

For $m_X\gg1$ MeV, at $T\ge1$ MeV, the X bosons are nonrelativistic and ρ_X is simply given by

$$\rho_X=n_Xm_X=f(T)T^3m_X=1.57\frac{2\pi A^2}{BC}\frac{m_X\Gamma_\pi R}{m_\pi^2}T^3\ . \tag{13}$$

Again, expressing ρ_X in terms of n_{eff}, we find

$$n_{\text{eff}}=4.9\times10^{13}\frac{m_X}{m_\pi}R\left[\frac{\text{MeV}}{T}\right]\ . \tag{14}$$

When $m_X\le m_\pi$, $n_{\text{eff}}<0.3$ implies

$$R<6\times10^{-15}\ . \tag{15}$$

In conclusion, we have applied the πPM to set a cosmological upper limit on the branching ratio for the decay of a neutral pion into a γ plus a weakly interacting neutral vector particle X with mass smaller than the pion mass and lifetime longer than 1 sec, $B(\pi^0\to\gamma X)<6\times10^{-15}-3\times10^{-13}$, depending on the mass of X. This limit is at least nine orders of magnitude better than the experimental limit.

This work was supported in part by the R.O.C. NSC Grant No. NSC82-0208-M-001-131-T.

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