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Finiteness of Chern-Simons theory for noncovariant gauges

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Existing all order proofs of the finiteness of the quantized Chern-Simons theory in the Landau gauge are extended to certain more general noncovariant gauges. The relevant additional supersymmetry also holds in that case with the antighost equation playing the usual role. The solution of the consistency problem of possible quantum corrections is quite involved in the noncovariant case, yielding nevertheless a very simple solution.

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I. INTRODUCTION

The pure Chern-Simons (CS) theory [1,2] uses the CS density

$$\mathcal{L}_{\rm inv}=-\tfrac{1}{2}\epsilon^{\mu\nu\rho}{\rm Tr}(A_\mu\partial_\nu A_\rho-ig\tfrac{2}{3}A_\mu A_\nu A_\rho) \tag{1.1}$$

as a Lagrangian in the action

$$\hat{L}=-\frac{k}{2\pi}\int_{\mathcal{M}}d^3x\mathcal{L}_{\rm inv}\ , \tag{1.2}$$

defined on a three-dimensional manifold $\mathcal{M}$. In addition to its diffeomorphism invariance, this action is invariant under non-Abelian gauge transformations [e.g., SU(N)] of the gauge fields A_μ which are connected to unity. In addition, the quantized theory derived from the path integral with $\exp(i\hat{L})$ exhibits an invariance under not-connected continuous maps $\mathcal{M}\rightarrow G$, provided k obeys a "quantization condition" [2], e.g., $k\in\mathbb{Z}$ for SU(N). Witten's beautiful results [3] on knot theory, derived from the quantized version of (1.2) thus imply the insensitivity of $\exp(i\hat{L})$ with respect to general quantum corrections to k.

The quantization of (2+1)-dimensional gravity has been reduced to the one of (1.1) with ISO(2,1) as the corresponding gauge group [4]. Relations of (1.2) also exist to other fields such as conformal field theory [1].

Although the tractability of (1.2) with (1.1) in the problems mentioned above is intimately related to the fact that only a finite number of degrees of freedom remains in a reduced canonical phase space, in order to tackle quantization and renormalization also methods of covariant quantum field theory have been employed widely. The vanishing of quantum corrections was found in this way perturbatively to one loop [5] and higher [6], making use of the (covariant) Landau gauge $\partial^\mu A_\mu=0$. A common difficulty to all such calculations represents a regularization: introducing any scale as in the Pauli-Villars regularization automatically breaks scale invariance, whose survival in the quantized theory is crucial for proving finiteness. In addition, dimensional regularization may not be trusted because of the ϵ tensor, and the analytic regularization may break gauge invariance.

Using the Becchi-Rouet-Stora (BRS) formalism, the gauge-fixed action including the Faddeev-Popov (FP) term, in addition possesses, other than BRS invariance, further (global) supersymmetries when the Landau gauge is chosen [7]. Together with BRS and anti-BRS transformations, the symmetries form an algebra involving the translation operator. In an all order proof of finiteness based upon a sector of those symmetries, also, a global relation derived from the equation of motion of the FP ghost c ("antighost equation") plays a pivotal role [8].

The Landau gauge has been known for a long time to assume a special position among all possible linear and nonlinear gauge conditions. Hence generalizations are desirable. In view of the general covariance of (1.1), $\partial^\mu A_\mu=g^{\mu\nu}\partial_\nu A_\mu$ implies fixing the metric $g^{\mu\nu}$ globally. As shown in [9], $g^{\mu\nu}$ may be generalized to a local form $g^{\mu\nu}(x)$. In [9] the finiteness proof could be extended to this case, because local generalizations of the supersymmetries mentioned above are available.

Noncovariant gauge conditions are characterized by a fixed vector as in the axial gauge $n^\mu A_\mu=0$, breaking global Lorentz covariance. They have been used in perturbation theory [10] and in the presence of boundaries [11] also for CS theories. Recently, also, an even larger supersymmetric algebra has been associated with such gauges [12]. Although a special case of that gauge, the temporal gauge $A_0=0$ has been important in the Hamiltonian formalism on general manifolds $\mathcal{M}$, one must be careful to fix a residual gauge freedom [11], a problem related to the regularization of ill-defined propagator poles in momentum space [13]. Our present work considers yet a different type of noncovariant generalizations of the Landau gauge:

$$F^\mu A_\mu=H^{\mu\nu}\partial_\nu A_\mu=0\ , \tag{1.3}$$

where the constant metric $H^{\mu\nu}$ is *not* symmetric. It describes a "gauge family" related by GL(3,$\mathbb{R}$) redefinitions. For a general $H^{\mu\nu}$ this family of gauges does not necessarily lead to the Landau gauge, because a complete diagonalization is not always possible. However $H^{\mu\nu}$ may be separated uniquely as

$$H^{\mu\nu}=\epsilon^{\mu\nu\rho}n_\rho+\eta^{\mu\nu} \tag{1.4}$$

with a symmetric $\eta^{\mu\nu}=\eta^{\nu\mu}$. By appropriate O(3) transformations and rescalings, invoking the $GL(3,\mathbb{R})$ invariance of (1.1), $\eta^{\mu\nu}$ can be brought always in diagonal form:

$$\eta^{\mu\nu}=\mathrm{diag}(\epsilon_1,\epsilon_2,\epsilon_3)\ , \tag{1.5}$$

where $\epsilon_i=0,\pm 1$. Thus (1.3) certainly generalizes the Landau gauge ($\epsilon_i=\epsilon_j\neq 0$) in the sense that among the general $H^{\mu\nu}$ in (1.3) there is a subset of metrices $H^{(\mu\nu)}$ connected smoothly by global $GL(3,\mathbb{R})$ transformations. On the other hand, the vector n_ρ determines yet another antisymmetric part.

We show in Sec. II that the pole structure of the propagators only depends on $\eta^{\mu\nu}$, whereas n_ρ also appears in the couplings to the FP ghosts. This, of course, limits admissible gauges to $\eta^{\mu\nu}\neq 0$. In view of later considerations we discuss at this place power counting renormalizability for special "degenerate" gauges with $\eta^{\mu\nu}n_\nu=0$.

Section III A is devoted to the supersymmetries of the action with the gauge fixing (1.3). Not surprisingly, those symmetries and the antighost (AG) equation are very similar to the Landau case. In Sec. III B the translation into Ward identities of the effective action is accomplished. We also list the consistency conditions of the possible quantum corrections and we find just straightforward generalizations of the identities in the Landau gauge [8]. However, the basic difference to previous work is that making *Ansätze* for the quantum corrections as local polynomials in the fields for a general gauge (1.3) we allow *all* tensorial factors, not only $\eta^{\mu\nu}$, δ^μ_ρ, $\epsilon^{\mu\nu\rho}$ and $\epsilon_{\mu\nu\rho}$ as in the covariant case. Because of the complexity of the computation we only indicate the main steps in Sec. IV, relegating some technical explanations to Appendices A and B. Since we work in full generality many of our results may be useful for other noncovariant situations as well. We have in mind, especially, the use of covariant gauge with (noncovariant) boundary conditions [11]. Fortunately the final result, summarized in Sec. V, is quite simple. It essentially implies that the finiteness proof of [8], employing the present techniques, may be extended to degenerate gauges (1.4) with $\eta^{\mu\nu}n_\nu=0$ or to gauges linearly related to the Landau gauge [$\det(\eta^{\mu\nu})\neq 0$, $n_\mu=0$].

II. GAUGE-FIXED ACTION

A. Generalities

The gauge fields in (1.1) are Lie algebra valued with an infinitesimal gauge invariance

$$\delta A_\mu=[D_\mu,\delta\omega]=\partial_\mu\delta\omega-ig[A_\mu,\delta\omega] \tag{2.1}$$

determined by the gauge group G. From (2.1) the action acquires a gauge-fixing part corresponding to (1.3):

$$\begin{aligned}L_{\mathrm{GF}}&=\mathrm{Tr}\int_x B(F^\mu A_\mu)-\bar{b}(F^\mu[D_\mu,c])\\&=\mathrm{Tr}\int_x s[\bar{b}(F^\mu A_\mu)]\ ,\end{aligned} \tag{2.2}$$

where B is a Lagrange-multiplier field and $\bar{b}$ and c represent the FP ghosts. In the last expression the BRS operation s is used

$$sA_\mu=[D_\mu,c]\ ,\quad sc=igcc\ ,\quad s\bar{b}=B\ ,\quad sB=0\ , \tag{2.3}$$

which leaves $L_{\mathrm{inv}}+L_{\mathrm{GF}}$ invariant, because $s^2=0$ on all fields. Occasionally we shall use A^a_μ defined as $A_\mu=A^a_\mu T_a$ so that the gauge-covariant derivative reads explicitly

$$D^a_{c\mu}:=\partial_\mu\delta^a_c-igf_{bc}{}^aA^b_\mu \tag{2.4}$$

with the structure constants f of G. In the course of our analysis we shall be forced to specify the gauge group, restricting ourselves to SU(2) and SU(3). We believe, though, our results to be more general; however, the construction of covariant tensors for larger groups seems to be a nontrivial problem (cf. below), becoming important for *noncovariant* quantum corrections.

The A_μ propagator and the mixed A_μ-B propagator are simply obtained by inverting the quadratic part of the field equations:

$$\begin{aligned}\Delta^{(A^a-A^b)}_{\mu\nu}(x,y)&=(\partial H\partial)^{-1}\epsilon_{\mu\nu\rho}F^\rho(x)\delta(x-y)\delta^{ab}\ ,\\\Delta^{(A^a-B^b)}_{\mu}(x,y)&=(\partial H\partial)^{-1}\partial^x_\mu\delta(x-y)\delta^{ab}\ ,\end{aligned} \tag{2.5}$$

with the inverse operator of

$$(\partial H\partial)=H^{\alpha\beta}\partial_\alpha\partial_\beta=\eta^{\alpha\beta}\partial_\alpha\partial_\beta \tag{2.6}$$

to be understood in momentum space. Equation (2.5) as well as the ghost propagator depend on $\eta^{\mu\nu}$ alone:

$$\Delta^{\bar{b}^a c^b}(x,y)=-(\partial H\partial)^{-1}\delta(x-y)\delta^{ab}\ . \tag{2.7}$$

B. Power counting

As long as $\det(\eta^{\mu\nu})\neq 0$ the situation with respect to power counting is precisely as in the Landau case with (renormalizably) UV-divergent one particle irreducible (1PI) vertex functions [6,7]. Degenerate gauges $\det(\eta^{\mu\nu})=0$ require a special analysis, because the propagators do not provide damping in (at least) one direction. We first take the case with $n_\rho=0$, $\mathrm{rank}(\eta^{\mu\nu})=1$, i.e., $\eta^{\mu\nu}=\mathrm{diag}(1,0,0)$ in a suitable coordinate system. Clearly x^2 and x^3 are not affected by the "dynamics" of the system as far as the local properties of the theory are concerned. With the propagator in (2.5), written here as $\Delta_{\mu\nu}$ for the gauge field and Δ for the ghosts,

$$\begin{aligned}&\Delta_{23}(z)=\partial_1^{-1}\delta^3(z)\ ,\\&\Delta_{12}=\Delta_{13}=0\ ,\\&\Delta(z)=-(\epsilon_1\partial_1^2)^{-1}\delta^3(z)\ ,\end{aligned} \tag{2.8}$$

and with the ghost vertex also coupled to A_1 only with one derivative ∂_1, any 1PI graph may be written as $[\delta(0)]^2$ $(\mathrm{1PI})_{\mathrm{reduced}}$ with a regularization, e.g., by two cutoffs in k_2 and k_3 in momentum space yielding $[\delta(0)]^2_{\mathrm{reg}}=\Lambda_2\Lambda_3/(2\pi)^2$. Apart from this overall factor the remaining possible quantum corrections comprise only UV-finite graphs. Such 1PI vertices with a "factored" divergence are not unusual in low-dimensional theories. A similar phenomenon appears, e.g., in non-Einsteinian gravity in the light cone gauge [14]. Of course, the infrared (IR) divergence must be regularized as well, say, e.g., for $\epsilon_1=+1$ by the usual regularizator mass μ in $\Delta_{23}|_{\text{reg}}=(\partial_1^2+\mu^2-i\varepsilon)^{-1}\partial_1\delta(z^1)$.

Next we consider the situation of $\text{rank}(\eta^{\mu\nu})=1$ but $n_\rho\neq 0$. Then $\eta^{\mu\nu}$ and n_ρ can be brought to the form $\eta^{\mu\nu}=\text{diag}(\epsilon_1,0,0)$, $n_\mu=(0,0,1)$ using an O(3) rotation to make $n_2=0$, $n_3=1$. Here the propagator contains $(\partial\eta\partial)\propto\partial_1^2$ only, and the x^3 direction decouples as before. However one component of $\Delta_{\mu\nu}$,

$$\Delta_{23}(z)=(\epsilon_1\partial_1^2)^{-1}(\epsilon_1\partial_1-\partial_2)\delta^3(z) , \tag{2.9}$$

as well as the ghost vertex mix the 1 and 2 directions with no damping provided for the latter. E.g., the graph with $2n$ external A_1 lines at one-loop order consisting of propagators (2.9) in momentum space will contain k_2 integrations up to $\int dk_2 k_2^{2n}\propto\delta^{(2n)}(0)$. Thus no factorization as in the x^3 direction can be expected. Of course, UV regularization by a cutoff

$$\delta^{(2n)}(0)|_{\text{reg}}=(-1)^n\frac{\Lambda^{2n+1}}{2n+1}$$

is possible. Clearly this gauge is not a renormalizable one. However, in view of the rather trivial structure of possible Feynman graphs, the (potentially different) cutoffs from $\delta(0)$, $\delta''(0)$, . . . provide increasing mass dimension. According to the action principle [15] the polynominal quantum corrections for Ward-type identities remain limited in number—in contrast with usual nonrenormalizable theories, where this number may be increasing with loop order.

If $\text{rank}(\eta^{\mu\nu})=2$ [i.e., $\eta^{\mu\nu}=\text{diag}(\epsilon_1,\epsilon_2,0)$] both cases $n_\mu=0$ and $n_\mu\neq 0$ [i.e., $n_\mu=(0,0,1)$] may be discussed together. Here the nonvanishing components of $\Delta_{\mu\nu}$ are

$$\begin{aligned}\Delta_{13}(z)&=(\epsilon_1\partial_1^2+\epsilon_2\partial_2^2)^{-1}(\partial_1-\epsilon_2\partial_2)\delta^3(z) ,\\ \Delta_{23}(z)&=(\epsilon_1\partial_1^2+\epsilon_2\partial_2^2)^{-1}(\epsilon_1\partial_1-\partial_2)\delta^3(z) ,\end{aligned} \tag{2.10}$$

whereas the ghost A vertex may be read off from $(\bar{b}_{,i}=\partial_i\bar{b})$

$$L_{\bar{b}Ac}=-ig\,\text{Tr}\int_x\{(\epsilon_1\bar{b}_{,1}+\bar{b}_{,2})[A_1,c]+(\epsilon_2\bar{b}_{,2}-\bar{b}_{,1})[A_2,c]\} . \tag{2.11}$$

Here only the x^3 direction is not involved in the Feynman rules, yielding an overall factor $\delta(0)|_{\text{reg}}=\Lambda/2\pi$ in front of all 1PI vertices. Residual power counting refers only to the x^1-x^2 plane. The corresponding UV divergence is given by

$$\omega=2-\tfrac{1}{2}(n_{A^3}+n_{\bar{b}Ac}+a_A) , \tag{2.12}$$

where n_{A^3} and $n_{\bar{b}Ac}$ are the number of vertices A^3 and ghost vertices; a_A counts external A lines. (2.12) signals a superrenormalizable theory. A simple analysis shows that logarithmic divergencies occur for the AA self-energy to at most one loop, for the ghost self-energy to at most two loops. The ghost-A vertex represents the third primitively divergent graph with the divergence restricted to one loop only. The ghost propagator needs an "intrinsic" IR cutoff.

To summarize this section, in noncovariant gauges of the type (1.3), including the degenerate ones with $\det(\eta^{\mu\nu})=0$, graphs in perturbation theory can be defined by simple regularization, although in one case the criterion of power-counting renormalizability is not satisfied. This is, though, sufficient for the application of the action principle [15] to each fixed order of perturbation theory. It says that the terms breaking symmetry relations of the quantized, but regularized theory must be integrals of local expressions in the fields. For our solution of the consistency problem of quantum corrections the polynomial nature, i.e., the presence of a *finite* number of such terms at each fixed order of perturbation theory, is necessary. Then the *Ansätze* are restricted, among other things, by simple dimensional arguments. Such arguments usually break down in nonrenormalizable theories by power counting. One of our "degenerate" gauges [$\eta=\text{diag}(\epsilon_1,0,0)$, $n_\mu=(0,0,1)$] was nonrenormalizable in this sense, but it exhibited still a very simple structure of possible 1PI graphs so that the dimensional argument may still be applied.

III. SYMMETRIES AND IDENTITIES

A. Supersymmetry algebra of the action

Also, for the gauge (1.3) the supersymmetries of [7] may be formulated. The anticommuting operation with ghost number $+1$, corresponding to a global transformation

$$\bar{v}_\rho A_\mu=\epsilon_{\mu\rho\nu}F^\nu\bar{b} , \quad \bar{v}_\rho c=A_\rho , \quad \bar{v}_\rho\bar{b}=0 , \quad \bar{v}_\rho B=\partial_\rho\bar{b} , \tag{3.1}$$

as well as another one with ghost number -1,

$$\begin{aligned}&v_\rho A_\mu=\epsilon_{\mu\rho\nu}F^\nu c , \quad v_\rho c=0 ,\\ &v_\rho\bar{b}=-A_\rho , \quad v_\rho B=[D_\rho,c] ,\end{aligned} \tag{3.2}$$

leaves the gauge-fixed action invariant. This is not only true in the Landau gauge $F^\nu=\partial^\nu$, but also in our more general case. The algebra of anticommutators for v_ρ, $\bar{v}_\rho$, and s,

$$\begin{aligned}&s^2=0 , \quad \{v_\rho,s\}=0 , \quad \{v_\rho,v_\sigma\}=\{\bar{v}_\rho,\bar{v}_\sigma\}=0 ,\\ &\{\bar{v}_\rho,s\}=\partial_\rho+\text{equation of motion} ,\\ &\{\bar{v}_\rho,v_\sigma\}=\epsilon_{\rho\sigma\alpha}F^\alpha+\text{equation of motion}\end{aligned} \tag{3.3}$$

is closed on shell, together with the translation operator which is hidden in $F^\mu=H^{\mu\alpha}\partial_\alpha$. Thanks to this fact also the ghost operation

$$g_a A_\mu=g_a\bar{b}=0 , \quad g_a c=T_a , \quad g_a B=ig[T_a,\bar{b}] \tag{3.4}$$

represents a global invariance of $L_{\text{inv}}+L_{\text{GB}}$. The pivotal role of this invariance has been widely exploited in finiteness proofs of CS theories [8] and elsewhere [9, 16]. Equation (3.4) may be included in the algebra (3.3), because (now Φ represents any field)

$$\{g_a,s\}\Phi_A=ig[T_a,\Phi_A]\ ,\qquad \{g_a,g_b\}=\{g_a,v_\rho\}=\{g_a,\bar v_\rho\}=0\ , \tag{3.5}$$

provided the operator $\mathcal{T}_a$ generating global gauge symmetries $\mathcal{T}_a\Phi=ig[T_a,\Phi]$ is included as well. $\mathcal{T}_a$ commutes with all previously introduced symmetries.

B. Identities for the quantum theory

The symmetries of (1.1) with (2.2) may be immediately transcribed in symmetry relations (Slavnov-Taylor identities) of the generating functional

$$W(j,k)=\int(d\Phi)\exp(i\tilde L)\ ,\qquad \tilde L=L_{\rm inv}+L_{\rm GF}+\int_x k^A(s\Phi_A)+\int_x j^A\Phi_A\ , \tag{3.6}$$

where Φ_A represents the set $(A_\mu,c,\bar b,B)$, $k_A=(k^\mu,l,0,0)$, $j^A=(j^\mu,\bar\xi,\xi,C)$. Before implementing the symmetries of Sec. III A we translate the fields in functional space $\delta\Phi_A=\zeta_A(x)$ to obtain the respective quantized "field equations":

$$\int(d\Phi)(-\tfrac{1}{2}\epsilon^{\mu\nu\rho}F_{\nu\rho}-F^\mu B+ig\{\eta^\mu,c\}+j^\mu)\exp(i\tilde L)=0\ , \tag{3.7}$$

$$\int(d\Phi)([D_\mu,\eta^\mu]+ig[c,l]-\bar\xi)\exp(i\tilde L)=0\ , \tag{3.8}$$

$$\int(d\Phi)(-F^\mu[D_\mu,c]-\xi)\exp(i\tilde L)=0\ , \tag{3.9}$$

$$\int(d\Phi)(F^\mu A_\mu+C)\exp(i\tilde L)=0\ . \tag{3.10}$$

The replacements $\Phi_A\to\delta/i\delta j^A$, $[D_\mu,c]=sA_\mu\to\delta/i\delta k^\mu$ yield first-order functional differential equations for W, except in the term

$$F_{\nu\rho}=\partial_\nu A_\rho-\partial_\rho A_\nu-ig[A_\nu,A_\rho] \tag{3.11}$$

of (3.7). Since translations in functional space are allowed by almost any definition of the measure $(d\Phi)$, (3.7)–(3.10) are not expected to develop correction terms on the right-hand side (RHS). As usual, translating (3.9) and (3.10) into identities of the effective action

$$\Gamma^{\rm eff}(\varphi)=-i\ln W-\int_z J^A\Phi_A\ ,\quad \Phi_A=\frac{-i}{W}\frac{\delta W}{\delta j^A} \tag{3.12}$$

one obtains

$$\Gamma^{\rm eff}(\varphi)=\Gamma(\varphi_i,k^i)+{\rm Tr}\int_x BF^\mu A_\mu\ , \tag{3.13}$$

where for simplicity the same symbols are used for the fields $\varphi_A=(\varphi_i,\bar b,B)$, $\varphi_i=(A_\mu,c)$ and where $\bar b$ only appears together with the source k^μ:

$$k^i=(k^\mu+F^\mu\bar b,l)=(\eta^\mu,l)\ . \tag{3.14}$$

A peculiar feature of a general derivative gauge of which the Landau gauge represents just one special case, is the antighost (AG) equation, derived from (3.8) with (3.10) [9,16] by integrating (3.8). Using (3.13) and (3.14) it implies

$$G_a\Gamma:={\rm Tr}\int_x T_a\frac{\delta\Gamma}{\delta c}=\Delta_a\ , \tag{3.15}$$

where the local expression

$$\Delta_a={\rm Tr}\int_x T_a(ig[\eta^\mu,A_\mu]-ig[l,c]) \tag{3.16}$$

is only linear in the fields φ_i. Equation (3.15) with (3.16) is nothing else but the symmetry equation g_a of (3.4), transformed in an identity for Γ.

We now turn to the other symmetries of the gauge-fixed action, starting with the BRS symmetry (2.3). Transforming $\delta\Phi_A=\delta\lambda s\Phi_A$ with an anticommuting constant $\delta\lambda$, in (3.6) only the term $j^A\Phi_A$ contributes. The resulting formal identity $\int_x j^A(\delta W)/(\delta k^A)=0$ may develop a symmetry-breaking term. After the step (3.12) and (3.13) we end up with the well-known Lee identity

$$\mathcal{B}(\Gamma)={\rm Tr}\int_x\frac{\delta\Gamma}{\delta\varphi_i}\frac{\delta\Gamma}{\delta k^i}=\mathcal{A}\ , \tag{3.17}$$

including the integral of a local polynomial $\hat{\mathcal{A}}$ of $O(\hbar)$ on the RHS. $\mathcal{B}$ and the linearized operator

$$\mathcal{B}_\Gamma={\rm Tr}\int_x\frac{\delta\Gamma}{\delta A_\mu}\frac{\delta}{\delta\eta^\mu}+\frac{\delta\Gamma}{\delta\eta^\mu}\frac{\delta}{\delta A_\mu}+\frac{\delta\Gamma}{\delta c}\frac{\delta}{\delta l}+\frac{\delta\Gamma}{\delta l}\frac{\delta}{\delta c} \tag{3.18}$$

satisfy the identity

$$\mathcal{B}_\Gamma\mathcal{B}(\Gamma)=0\ . \tag{3.19}$$

In contrast with (3.18), the other symmetries of Sec. II A only provide linear identities for Γ. Again quantum corrections as in (3.17) must be expected, except for the translation identity

$$T_\rho\Gamma={\rm Tr}\int_x\left[\varphi_{i,\rho}\frac{\delta}{\delta\varphi_i}+k^i_{,\rho}\frac{\delta}{\delta k^i}\right]\Gamma=0 \tag{3.20}$$

and for the global symmetry transformation

$$\mathcal{T}_a\Gamma=ig\,{\rm Tr}\int_x\left[[T_a,\varphi_i]\frac{\delta}{\delta\varphi_i}+[T_a,k^i]\frac{\delta}{\delta k^i}\right]\Gamma=0\ , \tag{3.21}$$

because our gauge choice, as well as any "reasonable" regularization scheme (Pauli-Villars, analytic, etc.), preserves these symmetries.

For v_ρ in (3.1) and $\bar v_\rho$ in (3.2) analogous steps as in the BRS case lead to

$$\mathcal{V}_\rho\Gamma=\mathcal{A}_\rho\ , \tag{3.22}$$

$$\bar{\mathcal{V}}_\rho\Gamma=\bar\Delta_\rho+\bar{\mathcal{A}}_\rho\ , \tag{3.23}$$

where

$$\mathcal{V}_\rho=\epsilon_{\rho\mu\nu}{\rm Tr}\int_x\left[(F^\mu c)\frac{\delta}{\delta A_\nu}+(F^\nu\eta^\mu)\frac{\delta}{\delta l}+\epsilon^{\mu\alpha\beta}(F^\nu A_\alpha)\frac{\delta}{\delta\eta^\beta}\right]\ , \tag{3.24}$$

$$\bar{\mathcal{V}}_\rho={\rm Tr}\int_x\left[\epsilon_{\rho\mu\nu}\eta^\mu\frac{\delta}{\delta A_\nu}+A_\rho\frac{\delta}{\delta c}+l\frac{\delta}{\delta\eta^\rho}\right]\ , \tag{3.25}$$

$$\bar\Delta_\rho={\rm Tr}\int_x(\eta^\mu_{,\rho}A_\mu-l_{,\rho}c)\ . \tag{3.26}$$

In the derivation of (3.23) the field equation (3.7) must be used before transforming the identity in W into one in Γ.

From (3.15), (3.18), (3.19), (3.24), and (3.25) the algebra of $\mathcal{B}$, G_a, $\mathcal{V}_\rho$, $\overline{\mathcal{V}}_\rho$, relevant for the effective action may be derived:

$$\mathcal{B}_\Gamma \mathcal{B}(\Gamma)=0 , \tag{3.27}$$

$$\{G_a,G_b\}=\{\mathcal{V}_\rho,\mathcal{V}_\sigma\}=\{\overline{\mathcal{V}}_\rho,\overline{\mathcal{V}}_\sigma\}=0 , \tag{3.28}$$

$$\{G_a,\mathcal{V}_\rho\}=\{G_a,\overline{\mathcal{V}}_\rho\}=0 , \tag{3.29}$$

$$G_a\mathcal{B}(\Gamma)+\mathcal{B}_\Gamma G_a\Gamma=\mathcal{T}_a\Gamma , \tag{3.30}$$

$$\mathcal{V}_\rho\mathcal{B}(\Gamma)+\mathcal{B}_\Gamma\mathcal{V}_\rho\Gamma=0 , \tag{3.31}$$

$$\overline{\mathcal{V}}_\rho\mathcal{B}(\Gamma)+\mathcal{B}_\Gamma\overline{\mathcal{V}}_\rho\Gamma=T_\rho\Gamma , \tag{3.32}$$

$$\{\mathcal{V}_\rho,\overline{\mathcal{V}}_\sigma\}\Gamma=-\epsilon_{\rho\sigma\lambda}F^\lambda\Gamma . \tag{3.33}$$

We note that $\overline{\mathcal{V}}_\rho$, $\mathcal{B}$, and G_a do not explicitly depend on the gauge condition. This sector alone was used in the seminal work of [8] for the Landau gauge. We enlarged this algebra somewhat by including the gauge-dependent $\mathcal{V}_\rho$ [cf. (3.24)] [7], but we shall see below that also for our more general gauge no further restrictions for the quantum corrections are obtained from the latter sector.

IV. QUANTUM CORRECTIONS

Equations (3.27)–(3.33) imply consistency conditions for the quantum corrections $\mathcal{A}$, $\mathcal{A}_\rho$, and $\overline{\mathcal{A}}_\rho$ in (3.17), (3.22), and (3.23). In order to be even more general we also admit now a quantum correction to the AG equation (3.15): $G_a\Gamma=\Delta_a+\mathcal{A}_a$.

Inserting the latter equation together with (3.17), (3.21), and (3.23) into (3.27)–(3.33) we arrive at the consistency conditions for the quantum corrections. With $\Gamma=L+O(\hbar)$ and L defined like Γ in (3.13), to $O(\hbar)$ all relations become linear:

$$\begin{aligned}
&BB:\ \mathcal{B}_L\mathcal{A}=0 ,\\
&GG:\ G_a\mathcal{A}_b+G_b\mathcal{A}_a=0 ,\\
&VV:\ \mathcal{V}_\rho\mathcal{A}_\sigma+\mathcal{V}_\sigma\mathcal{A}_\rho=0 ,\\
&\overline{V}\overline{V}:\ \overline{\mathcal{V}}_\rho\overline{\mathcal{A}}_\sigma+\overline{\mathcal{V}}_\sigma\overline{\mathcal{A}}_\rho=0 ,\\
&GV:\ G_a\mathcal{A}_\rho+\mathcal{V}_\rho\mathcal{A}_a=0 ,\\
&G\overline{V}:\ G_a\overline{\mathcal{A}}_\rho+\overline{\mathcal{V}}_\rho\mathcal{A}_a=0 ,\\
&GB:\ G_a\mathcal{A}+\mathcal{B}_\Gamma\mathcal{A}_a=0 ,\\
&V\overline{V}:\ \mathcal{V}_\rho\overline{\mathcal{A}}_\sigma+\overline{\mathcal{V}}_\sigma\mathcal{A}_\rho=0 ,\\
&VB:\ \mathcal{V}_\rho\mathcal{A}+\mathcal{B}_\Gamma\mathcal{A}_\rho=0 ,\\
&\overline{V}B:\ \overline{\mathcal{V}}_\rho\mathcal{A}+\mathcal{B}_\Gamma\overline{\mathcal{A}}_\rho=0 .
\end{aligned} \tag{4.1}$$

Possible *Ansätze* for the $\mathcal{A}$'s are determined by the ghost number and by the dimensions of the symmetry operators. They contain a limited number of terms if no inverse powers of another dimensional quantity (regularization mass) can occur. Since we are interested in the local behavior of the theory, only UV cutoffs are relevant in this context. As shown above, even for a nonrenormalizable degenerate gauge such inverse powers do not appear in perturbation theory. The worst UV behavior in this sense occurs in the nondegenerate case $\det(\eta^{\mu\nu})\neq 0$ which thus automatically comprises the other ones. With the mass dimension $[A]=[\partial]=1$, $[\eta]=2$, $[l]=3$, $[c]=0$ the integrands in $\mathcal{A}=\int_x a(x)$, etc. are to be constructed so as to have respective dimensions $[a]=[a_a]=[a_\rho]=3$, $[\bar{a}_\rho]=4$, and respective ghost numbers $(1,-1,1,-1)$:

$$\begin{aligned}\mathcal{A}=&R^{(1)}(c,A^3)+R^{(2)}(c,A^2,\partial)+R^{(3)}(c,A,\partial^2)\\ &+R^{(4)}(c^2,\eta,A)+R^{(5)}(c^2,\eta,\partial)+R^{(6)}(c^3,l) ,\end{aligned} \tag{4.2}$$

$$\mathcal{A}_a^G=S_a^{(1)}(\eta,A)+S_a^{(2)}(l,c) , \tag{4.3}$$

$$\mathcal{A}_\rho=\mathcal{A}(R^{(i)}\to W_\rho^{(i)}) , \tag{4.4}$$

$$\begin{aligned}\overline{\mathcal{A}}_\rho=&T_\rho^{(1)}(\eta,A^2)+T_\rho^{(2)}(\eta,A,\partial)+T_\rho^{(3)}(\eta^2,c)\\ &+T_\rho^{(4)}(l,c,A)+T_\rho^{(5)}(l,c,\partial) .\end{aligned} \tag{4.5}$$

Each generic expression in (4.2)–(4.5) consists of between two and four terms with different coefficients. As an example we show $R^{(1)}$ in (4.2):

$$R^{(1)}(c,A^3)=\sum_j T^j_{abcd}r_j^{(1)\mu\nu\rho}\int_x c^a A^b_\mu A^c_\nu A^d_\rho . \tag{4.6}$$

In the noncovariant case the coefficients $r_j^{(1)}$ need not consist of the ϵ tensor alone. Therefore the covariant tensor T_{abcd} in the adjoint representation is completely general. E.g., for SU(2) the only possibility is a combination of δ_{ab} and ϵ_{abc}. But already for SU(3) a more careful analysis is required. Tensors T_{ab} and T_{abc} are easily constructed from the Killing metric, from the structure constants f_{abc} and from the symmetric coefficients d_{abc} which also form the building blocks of higher terms. T_{abcd} at first may contain 15 terms ($\delta\delta$, ff, dd, df with different combinations of indices) which can be reduced by identities (cf. Appendix A) to 8 terms. A suitable basis is given by $J_{abcd}=\delta_{ac}\delta_{bd}$, $K_{abcd}=\delta_{ab}\delta_{cd}+\delta_{ad}\delta_{bc}$, and the six tensors $[I_{abcd}=\mathrm{Tr}(\lambda_a\lambda_b\lambda_c\lambda_d)]$

$$\tilde{I}_{abdc}=I_{abcd}-a_1K_{abcd}-a_2J_{abcd} , \tag{4.7}$$

which are invariant under cyclic permutations of the indices and where the trace terms J and K are taken out ($\tilde{I}_{aacd}=0$, etc.). The coefficients a_1 and a_2 for SU(3) are given in Appendix A. In the evaluation of the consistency condition (4.1) also terms with five fields appear. Here the traceless part (in pairs of indices ab etc.) of the tensor $I_{abcde}=\mathrm{Tr}(\lambda_a\lambda_b\lambda_c\lambda_d\lambda_e)$ is needed. Together with four trace terms (Appendix A) the independent tensors with five indices can be isolated in this manner.

In order to limit the computational effort for the solution of (4.1) as much as possible we have found it essential to start at the relations GG and $G\overline{V}$ in (4.1) and to proceed to $\overline{V}\overline{V}$ and $\overline{V}B$. Many relations are redundant, but very useful in order to check the calculation. Previous treatments of the covariant case [8] found this sector of (4.1) sufficient for the proof of finiteness. In our more general noncovariant case the analysis is very lengthy. Some intermediate steps are given in Appendix B. The final result turns out to be

$$\mathcal{A}_a = G_a(\alpha L) , \tag{4.8}$$

$$\overline{\mathcal{A}}_\rho = \overline{\mathcal{V}}_\sigma(\beta\delta^\sigma_\rho L_{\text{inv}} + \gamma^\sigma_\rho L) , \tag{4.9}$$

$$\mathcal{A} = 0 , \tag{4.10}$$

where α, β and γ^σ_ρ are the only free constants surviving the set of restrictions (4.1) without considering the ones involving $\mathcal{V}_\rho$. It can be shown, though, that the remaining relations with $\mathcal{V}_\rho$ cannot restrict (4.8)–(4.10) any further, especially in view of the term γ^σ_ρ in (4.9). The reason is that $\mathcal{V}_\rho$ annihilates L. As indicated already in connection with the AG identity (3.15) no quantum correction is expected in any case here. However, unfortunately, α is not related to β and γ^σ_ρ by the consistency conditions. Thus no further restrictions are provided by $\alpha=0$ in (4.8) for (4.9) and (4.10).

We now discuss the effect of different gauge choices on γ^σ_ρ. In the most general nondegenerate gauge of (1.4) with $\det(\eta^{\mu\nu})\neq 0$ and $n_\mu \neq 0$ it is evidently possible to have, e.g., $\gamma^\sigma_\rho = \eta^{\sigma\alpha} n_\alpha n_\rho \neq \delta^\sigma_\rho$ with no counterterm to the covariant action able to compensate for such an anomalous addition. This (potential) anomaly refers to $\bar{v}_\rho$, a symmetry which according to (3.3) does not (anti) commute with s off shell. Hence theorems proving the gauge independence (not gauge invariance) of such an anomaly [17] cannot be invoked to guarantee the absence of such a term. Thus the crucial first step in the proof of finiteness breaks down. Of course, this does not exclude the possibility that the theory is finite after all. In any case it would be desirable to check this fact perturbatively before attempting to find another proof.

As the next case we consider $\det(\eta^{\mu\nu})\neq 0$ and $n_\mu = 0$. Here no combination of the ϵ tensors and the $\eta^{\mu\nu}$ [including $\epsilon_{\mu\nu\lambda}\epsilon_{\alpha\beta\rho}\eta^{\mu\alpha}\eta^{\nu\beta}\eta^{\lambda\sigma} = 2\epsilon_1\epsilon_2\epsilon_3\delta^\sigma_\rho$ in the frame (1.5)] is able to produce a tensor $\gamma^\sigma_\rho \neq \delta^\sigma_\rho$. Hence the quantum corrections allowed can be expressed as counterterms Δ ($\Gamma^{\text{ren}} = \Gamma + \Delta$ where $\mathcal{B}_L\Delta = 0$). The renormalization procedure, however, is ambiguous anyhow with respect to terms of precisely this type ("stability" [8]), i.e.,

$$\mathcal{B}_L\Delta = \overline{\mathcal{V}}_\rho \Delta = G_a \Delta = 0 . \tag{4.11}$$

Thus because of the special form of the RHS in (4.8) and (4.9) the terms with β and $\gamma^\sigma_\rho = \gamma\delta^\sigma_\rho$ can be considered to be absorbed in Δ. But the latter must vanish identically because of (4.11): the second relation essentially requires terms with ghosts (as in L), and the last relation essentially forbids such terms. It is not surprising that we recover in this way the result of [8] for the Landau gauge. The gauge condition $\partial^\mu A_\mu = 0$ can be interpreted always as $\eta^{\mu\nu}\partial_\nu A_\mu = 0$ with the "metric" $\eta^{\mu\nu} = \eta^{\nu\mu}$ without having to specify that this "metric" is really proportional to $\delta^{\mu\nu}$. On the other hand, a symmetric metric $\eta^{\mu\nu}$ represents a general type of noncovariant gauge. Orthogonal transformations and rescalings of that metric parametrize a family of gauges in which finiteness of the CS theory can thus be proved. Of course, the symmetric constant metric $\eta^{\mu\nu}$ represents just one special case of a local metric $\sqrt{g}g^{\mu\nu}(x)$. Finiteness of that "local" gauge was proved in [8]. Our present approach shows that diffeomorphism with respect to $g^{\mu\nu}(x)$ [8] need not be involved for a constant "background metric" $\eta^{\mu\nu}$. In another sense, our result is more general since it involves also the "degenerate" metrics $\det(\eta^{\mu\nu})=0$, because according to the discussion in Sec. II B the present result may be carried over to the case of $\text{rank}(\eta^{\mu\nu})\neq 3$.

We now include the antisymmetric part $H^{[\mu\nu]}$ in (1.4), represented by the vector $n_\rho \neq 0$ in the degenerate case. Still no γ^σ_ρ different from δ^σ_ρ can be constructed as long as $\eta^{\mu\nu}n_\nu = 0$. Those were just the remaining cases discussed in Sec. II B for which finiteness is thus proved as well by the present argument.

V. SUMMARY

Previous proofs of all order finiteness of CS theories were restricted to a Landau-type gauge $\partial^\mu A_\mu = 0$ which may be interpreted as $\eta^{\mu\nu}\partial_\nu A_\mu = 0$ with a metric $\eta^{\mu\nu} = \eta^{\nu\mu}$, $\det(\eta^{\mu\nu})\neq 0$. In our present work we generalize $\eta^{\mu\nu}$ to contain also an antisymmetric part $\epsilon^{\mu\nu\rho}n_\rho$. The main effort of the present paper was the solution of the consistency equations of the basic symmetries (BRS invariance, supersymmetries v_ρ and $\bar{v}_\rho$, AG equation) for the most general noncovariant case, i.e., admitting arbitrary tensors in the *Ansätze* of possible quantum corrections. For technical reasons we restricted the gauge group to SU(N), $N \leq 3$. The final result of this cohomology were the simple formulas (4.8)–(4.10). The finiteness proof was found to be applicable especially also to "degenerate" gauges with $\det(\eta^{\mu\nu})=0$, $n_\mu \neq 0$ and to gauges $\eta^{\mu\nu}n_\nu = 0$. Our investigation was motivated by studies of CS theories with boundaries, but especially also by the fact that these degenerate gauges with successively decreasing $\text{rank}(\eta^{\mu\nu})$ bridge the gap to pure axial gauge [3,12] where the CS theory becomes a manifestly free one. For all such degenerate cases the usual difficulties of axial gauges appear, which may be expressed either in terms of the regularization problem of $(k\cdot n)^{-1}$ terms in the propagator or by the difficulty to impose global boundary conditions in x space. The latter problem is intimately related to global questions and thus to the study of topological properties of the manifold on which the gauge fields can be defined according to (1.1) and (1.2). Of course, our strictly local results are not able to say anything about that.

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APPENDIX A: COVARIANT TENSORS OF SU(3)

As mentioned after (4.6), the most general covariant tensor T_{abcd} may be formed from products such as $\delta\delta$, ff, dd, and fd. Together with all independent permutations of indices this yields altogether 15 terms. The Jacobi identities

$$[[A,B],C]+[[B,C],A]+[[C,A],B]=0 , \tag{A1}$$

$$[\{A,B\},C]+[\{B,C\},A]+[\{C,A\},B]=0 \tag{A2}$$

for $A=\lambda_a$, $B=\lambda_b$, etc., imply two conditions:

$$f_{abe}f_{ecd}-f_{ace}f_{ebd}+f_{ade}f_{ebc}=0\ , \tag{A3}$$

$$f_{ade}d_{ebc}+d_{ace}f_{ebd}+d_{abe}f_{ecd}=0\ . \tag{A4}$$

A third one follows from the cyclic property of the traces:

$$\mathrm{Tr}(ABCD)-\mathrm{Tr}(BCDA)=0\ , \tag{A5}$$

$$\mathrm{Tr}([A,B][C,D])+\mathrm{Tr}([A,C][B,D]) =\mathrm{Tr}(\{A,B\}\{C,D\})-2\mathrm{Tr}(ACDB+ABDC)\ , \tag{A6}$$

i.e.,

$$d_{abe}d_{ecd}-d_{bce}d_{ead}-f_{ace}f_{ebd}-\tfrac{2}{3}(\delta_{ab}\delta_{cd}-\delta_{bc}\delta_{da})=0\ , \tag{A7}$$

$$f_{abe}f_{ecd}+f_{ace}f_{ebd}+3d_{ade}d_{bce}-\delta_{ab}\delta_{cd}-\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc}=0\ . \tag{A8}$$

A fifth independent relation may be obtained by evaluating the identity $\ln\det(A)=\mathrm{Tr}\ln A$ for $A=1+x^a\lambda_a$, isolating the term $O(x^4)$ [18]:

$$d_{abe}d_{cde}+d_{ace}d_{bde}+d_{ade}d_{bce}-\tfrac{1}{3}(\delta_{ab}\delta_{cd}+\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc})=0\ . \tag{A9}$$

Separating the 15 possible terms for T_{abcd} in two sets $\{A\}$ with nine tensors $\{\delta\delta,\ ff,\ dd\}$, and $\{B\}$ with six tensors $\{fd\}$, one notices that the five relations (A3), (A4), (A7), (A8), and (A9) transform each set into itself. By different choices of the indices in (A4) four equations can be obtained, three of which are independent. Hence from $\{B\}$ three independent tensors remain. Writing down the remaining conditions (A3) and (A7)–(A9) also for different choices of indices, four independent equations are obtained which suffice to reduce the set $\{A\}$ to five independent elements. The total number of remaining independent tensors (eight), of course, coincides with number of unitary representations in the product of four octets. A suitable basis is given by, e.g.,

$$\{\delta_{ab}\delta_{cd},\delta_{ac}\delta_{bd},\delta_{ad}\delta_{bc},f_{abe}f_{ecd},d_{abe}d_{ecd},d_{abe}f_{ecd},d_{ace}f_{ebd},d_{ade}f_{ebc}\} \tag{A10}$$

or, in accordance with a more compact form of writing the possible quantum corrections (cf. Appendix B), by

$$\{I_{ac}I_{bd},(I_{ab}I_{cd}+I_{ad}I_{bc}),\tilde{J}_{abcd},\tilde{J}_{abdc},\tilde{J}_{acbd},\tilde{J}_{acdb},\tilde{J}_{adbc},\tilde{J}_{adcb}\}\ , \tag{A11}$$

with $I_{ab}=\delta_{ab}\propto\mathrm{Tr}(\lambda_a\lambda_b)$ and with the traceless $\tilde{J}_{abcd}$ of (4.7) and $a_1=\frac{26}{11}$, $a_2=-\frac{10}{21}$ for SU(3).

For the covariant tensor J_{abcde} of Sec. IV the traceless part reads

$$\begin{aligned}\tilde{J}_{abcde}:=&\mathrm{Tr}(\lambda_a\lambda_b\lambda_c\lambda_d\lambda_e)-b_1(\delta_{ab}J_{cde}+\delta_{bc}J_{ade}+\delta_{cd}J_{abe}+\delta_{de}J_{abc}+\delta_{ae}J_{bcd})\\&-b_2(\delta_{ab}J_{ced}+\delta_{bc}J_{aed}+\delta_{cd}J_{aeb}+\delta_{de}J_{acb}+\delta_{ae}J_{bdc})\\&-b_3(\delta_{ac}J_{bde}+\delta_{bd}J_{ace}+\delta_{ce}J_{abd}+\delta_{ad}J_{bce}+\delta_{be}J_{acd})\\&-b_4(\delta_{ac}J_{bed}+\delta_{bd}J_{aec}+\delta_{ce}J_{adb}+\delta_{ad}J_{bec}+\delta_{be}J_{adc})\end{aligned} \tag{A12}$$

for SU(3) with

$$15b_i=(9,1,-3,-2)\ . \tag{A13}$$

APPENDIX B: STEPS LEADING TO EQS. (4.8)–(4.10)

Taking into account the group theoretical results of Appendix A, the respective independent terms in (4.2) may be written as

$$\begin{aligned}&R^{(1)}(c,A^3)=\int_x r_1^{(1)\mu(\nu\rho)}\mathrm{Tr}(cA_\mu)\mathrm{Tr}(A_\nu A_\rho)+\tilde{\mathrm{T}}\mathrm{r}\int_x r_2^{(1)\mu\nu\rho}(cA_\mu A_\nu A_\rho)\ ,\\&R^{(2)}(c,A^2,\partial)=\mathrm{Tr}\int_x(r_1^{(2)\mu\nu\rho}cA_\mu A_{\nu,\rho}+r_2^{(2)\mu\nu\rho}cA_{\nu,\rho}A_\mu)\ ,\\&R^{(3)}(c,A,\partial^2)=\mathrm{Tr}\int_x r^{(3)\mu(\nu\rho)}c_{,\nu}A_{\mu,\rho}\ ,\\&R^{(4)}(c^2,\eta,A)=\int_x r_4^{(4)}{}^{\alpha}_{\mu}\mathrm{Tr}(c\eta^\mu)\mathrm{Tr}(cA_\alpha)+\tilde{\mathrm{T}}\mathrm{r}\int_x[r_1^{(4)}{}^{\alpha}_{\mu}(cc\eta^\mu A_\alpha)+r_2^{(4)}{}^{\alpha}_{\mu}(ccA_\alpha\eta^\mu)+r_3^{(4)}{}^{\alpha}_{\mu}(c\eta^\mu cA_\alpha)]\ ,\\&R^{(5)}(c^2,\eta,\partial)=\mathrm{Tr}\int_x(r_1^{(5)}{}^{\alpha}_{\mu}c\eta^\mu c_{,\alpha}+r_2^{(5)}{}^{\alpha}_{\mu}cc_{,\alpha}\eta^\mu)\ ,\\&R^{(6)}(c^3,l)=\tilde{\mathrm{T}}\mathrm{r}\int_x r^{(6)}(cccl)\ ,\end{aligned} \tag{B1}$$

where $\widetilde{\mathrm{Tr}}$ indicates the traceless part of the tensor J_{abcd} in the sense of (4.7). Similarly we have, in (4.3),

$$\begin{aligned} S_a^{(1)}(\eta, A) &= \mathrm{Tr}\int_x (s_1^{(1)\nu}{}_\mu T_a \eta^\mu A_\nu + s_2^{(1)\nu}{}_\mu T_a A_\nu \eta^\mu) ,\\ S_a^{(2)}(l,c) &= \mathrm{Tr}\int_x (s_1^{(2)} T_a lc + s_2^{(2)} T_a cl) , \end{aligned} \tag{B2}$$

and, in (4.5),

$$\begin{aligned} T_\rho^{(1)}(\eta, A^2) &=: \mathrm{Tr}\int_x t^{(1)\alpha\beta}_{\mu\rho}\eta^\mu A_\alpha A_\beta ,\\ T_\rho^{(2)}(\eta, A, \partial) &= \mathrm{Tr}\int_x t^{(2)\beta\alpha}_{\mu\rho}\eta^\mu A_{\beta,\alpha} ,\\ T_\rho^{(3)}(\eta^2, c) &=: \mathrm{Tr}\int_x t^{(3)}{}_{\mu\nu\rho} c\,\eta^\mu \eta^\nu ,\\ T_\rho^{(4)}(l,c,\partial) &= \mathrm{Tr}\int_x t^{(4)\alpha}{}_\rho l c_{,\alpha} ,\\ T_\rho^{(5)}(l,c,A) &= \mathrm{Tr}\int_x (t_1^{(5)\alpha}{}_\rho c l A_\alpha + t_2^{(5)\alpha}{}_\rho c A_\alpha l) . \end{aligned} \tag{B3}$$

In the evaluation of (4.1) the symmetry properties of each term in the coordinate indices must be checked carefully before setting the corresponding factor equal to zero. It is also essential to isolate the independent group theoretical tensors whenever nonlinear transformations [as in $\mathcal{B}(L)$] are involved. The result for GG is simple

$$s_1^{(2)} + s_2^{(2)} = 0 , \tag{B4}$$

but already for $G\bar{V}$ there are three equations from which we just write down one for purpose of illustration:

$$t^{(3)}_{\mu\kappa\rho} = \epsilon_{\rho\kappa\nu} s_1^{(1)\nu}{}_\mu - \epsilon_{\rho\mu\nu} s_2^{(1)\nu}{}_\kappa . \tag{B5}$$

Solving GB yields already 10 (tensor) equations between the $r_j^{(i)}$ and the $s_j^{(i)}$ and the BB equations are so complicated that the results from the GG and the GB equations are inserted first. After these steps

$$s_2^{(1)\nu}{}_\mu = -s_1^{(1)\nu}{}_\mu = s_1^{(2)} \delta^\nu_\mu \tag{B6}$$

and all $r_j^{(i)}$ vanish, except

$$r_2^{(2)\mu\nu\rho} = r_1^{(2)(\mu\nu)\rho} . \tag{B7}$$

As a next step $\bar{V}\bar{V}$ and $\bar{V}B$ are used in a lengthy analysis leading finally to (4.8)–(4.10). A complete presentation of all steps is contained in [19].

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