Environment-induced decoherence, classicality, and consistency of quantum histories

Juan Pablo Paz and Wojciech Hubert Zurek
Theoretical Astrophysics, T6, MS-B288, Los Alamos National Laboratory, Los Alamos, New Mexico 87545
(Received 19 April 1993)

We prove that for an open system, in the Markovian regime, it is always possible to construct an infinite number of nontrivial sets of histories that exactly satisfy the probability sum rules. In spite of being perfectly consistent, these sets manifest a very nonclassical behavior: they are quite unstable under the addition of an extra instant to the list of times defining the history. To eliminate this feature—the implications of which we discuss—and to achieve the stability that characterizes the quasiclassical domain, it is necessary to separate the instants which define the history by time intervals significantly larger than the typical decoherence time. In this case *environment induced superselection* is very effective. The quasiclassical domain is defined by predictably evolving perferred states, "pointer projectors," which give rise to consistent *preferred sets of histories.*

PACS number(s): 03.65.Bz, 03.65.Ca, 03.65.Db, 98.80.Hw

I. INTRODUCTION

Environment-induced superselection [1] explains why macroscopic objects are not observed in the majority of the quantum states admissible in their Hilbert spaces. The basic idea is that these objects are impossible to isolate from their surroundings [2]. The continuous interaction with this environment results in a process of *decoherence* [3] which destroys, on a very short *decoherence time scale,* the purity of nearly all of the initial superpositions thus erasing quantum coherence between states which result in a different evolution of their quantum environment. Only the preferred set of relatively stable states (or the associated sets of observables) will exhibit the key attribute of "classical reality," which is characterized by the predictive power of the associated records [4,5]. In order to study decoherence, the analysis of the evolution of the "reduced density matrix" for the system (obtained from the full density matrix by tracing out the environment variables) is often the most convenient strategy [3,6]. Nevertheless, the role of the *records* accessible to the observers as well as the *correlations* between these records and the rest of the Universe must be recognized in the discussion of the *existential interpretation* of quantum mechanics suggested by the decoherence process [6,7].

The *consistent histories* formulation (proposed by Griffiths [8] and developed by Omnès [9] and by Gell-Mann and Hartle [10]) is based in the study of properties of sets of quantum histories, which are represented by time-ordered sequences of projection operators. Although quantum mechanics prevents one from assigning probabilities to arbitrary sets of quantum histories, it is still possible to do so in certain cases. The validity of the probability sum rules, the absence of quantum interference, for sets of histories, which is one of the properties of the classical domain, can be ascertained by analyzing if consistency conditions are satisfied [8]. These conditions, which were originally expressed as commutation relations between the projectors which define historical events [8,9], can be related to the properties of an object called a "decoherence functional" [10].

The aim of this paper is to make contact between these two approaches and study also some interesting aspects of the consistent histories formulation. For this purpose, we will apply it to a situation which is usually considered when discussing the environment-induced superselection approach. The Universe is divided into a "system of interest" $\mathcal{S}$ and the "environment" $\mathcal{E}$. The histories of interest refer to the system $\mathcal{S}$ only (i.e., histories will be made of projectors $\mathbf{P}=I_{\mathcal{E}}\otimes P_{\mathcal{S}}$, where $I_{\mathcal{E}}$ is the identity in the Hilbert space of the environment and $P_{\mathcal{S}}$ is a projector in the Hilbert space of the system).

As the first step in our study we will address a technical point and examine under what conditions the decoherence functional for the above histories can be constructed *entirely* from the reduced density matrix of the system. The conclusion of our analysis is not unexpected but is still worth mentioning: this construction is possible only in the Markovian regime in which the reduced density matrix satisfies a master equation which is local in time. It is, of course, of interest to inquire how accurately one can assess consistency of histories by investigating the evolution of the density matrix when the evolution is, for example, approximately Markovian. We shall not discuss this issue here: in the rest of the paper we will restrict ourselves to consider situations in which the evolution is exactly Markovian.

As a second step, we will show how to construct sets of histories that exactly satisfy the probability sum rules. In particular, we will analyze histories for which the projectors are associated with the eigenstates of the reduced density matrix. As these states form the so-called "Schmidt basis" [11], the above histories are going to be called "Schmidt histories" (although this may be an extrapolation of the existing terminology). We will show that, in the Markovian regime, *sets of Schmidt histories are always consistent.* We will also analyze some of their intriguing and, from the point of view of devising an interpretation, somewhat worrisome properties. In particular, we will show that the events that form Schmidt histories are themselves history dependent: the set of projectors defining Schmidt events in the "next time" t_n depends on which projectors were applied in the past. These projectors are imposed "from the outside" of the Universe, that is, they do not depend just on the initial conditions or on the Hamiltonian but primarily on arbitrary selections of, for example, the time sequence. This implies that histories are influenced by an "unphysical"—and subjective—act of deciding what questions (which sets of projectors) are going to be posed and when will they be posed. (This is not to be confused with the events which occur within the Universe and may decide, for example, the existence or nonexistence of certain systems, thus making certain questions interesting or natural.)

In the original proposal of Griffiths [8], quantum histories were defined as chains of branch-independent projectors $P^{(k)}_{\alpha_k}(t_k)$ [the superindex (k) labels the set of projectors and the subindex α_k enumerates the different events within the complete set]. A complete set of histories was therefore characterized by a fixed choice of a set of projectors $P^{(k)}(t_k)$ at every time. By contrast, branch-dependent histories are chains of the form

$$C_\alpha = P^{(1)}_{\alpha_1}(t_1) \cdots P^{(n,\alpha_1,\ldots,\alpha_{n-1})}_{\alpha_n}(t_n) ,$$

where *the set* of projectors used at time t_k depends on the alternatives $\alpha_1, \ldots, \alpha_{k-1}$ that were realized in the past. Such branch-dependent histories were first considered by Omnès [9] (who called them histories of type II). Their use has been recently advocated by Gell-Mann and Hartle [12].

We will show that the "branch dependence" of Schmidt histories is a consequence of an inevitable property of the reduced dynamics which we shall call, in the absence of a more concise description, "environment-induced noncommutativity" (EINC). Two initially commuting density matrices describing an open system (and corresponding, for example, to two alternative events) will in general evolve into two noncommuting operators as a result of the interaction with the environment. Therefore, at some later time they will not be simultaneously diagonalizable and will give rise to distinct sets of Schmidt states.

The existence of environment-induced noncommutativity, which can be shown to be a very generic consequence of the openness of the system [13], does not seem to have been widely appreciated (some of its implications for consistent histories were discussed in Ref. [6]). As we are going to restrict our analysis to the Markovian regime, we will specifically address the importance of EINC in this context. We will show that this property of the evolution of the reduced density matrix is in fact predicted by generic Markovian master equations. Moreover, we will show that using such a master equation one can study the consequences of EINC in physically relevant situations. In particular, we will consider a system with a finite-dimensional Hilbert space interacting with a large environment. For concreteness, the system can be thought of as described by a set of atomic levels and the environment as the quantized electromagnetic field. The interaction between the system and the environment generates spontaneous transitions between the levels of the system. In the Markovian approximation, the evolution of the reduced density matrix can be modeled using the Bloch equations [14] which are widely used in quantum optics [15]. We will show that these equations predict the existence of environment-induced noncommutativity and that this effect can intervene on a physically interesting time scale.

Finally, we will discuss some of the consequences of environment-induced noncommutativity for the relationship between the process of decoherence and consistent histories approach. We will conclude that, when EINC is strong, Schmidt sets are not good candidates to describe a quasiclassical domain. This is because they are highly unstable since the Schmidt projectors that form a perfectly consistent set at times $t_1, t_2, \ldots, t_n$ are generally quite different from the ones that have to be used if the time sequence is $t_2, \ldots, t_n$. This rather quantum-mechanical property (which could be crudely described as "instability under observation") disappears if one considers special sets of histories for which the minimal temporal separation $t_i - t_{i-1}$ is larger than a certain quantity. This turns out to be crucial in achieving classicality. Thus, Schmidt histories are defined by a sequence of stable projectors if the differences $(t_i - t_{i-1})$ are sufficiently larger than the typical decoherence time of the system (which is the time needed for an arbitrary initial state to become approximately diagonal in a fixed "pointer" basis). In this way, we conclude that sets of histories associated with "pointer states" [4] may be the basis for the description of our quasiclassical domain. In brief, histories expressed in terms of pointer states are stable; preferred states are selected for the predictability of their evolution. Moreover, when a large quantum intervention (such as would be encountered by Schrödinger's cat) forces the system into a superposition of preferred states, consistency of *preferred histories* will be approximately restored on a decoherence time scale.

This paper is organized as follows: In Sec. II we briefly review both the environment induced superselection and the consistent histories approaches. In Sec. III we analyze the conditions under which it is possible to write the decoherence functional in terms of the reduced density matrix of the system and discuss the origin of environment-induced noncommutativity. In Sec. IV we analyze the properties of the sets of Schmidt histories. Our conclusions are stated in Sec. V.

II. ENVIRONMENT-INDUCED SUPERSELECTION AND CONSISTENT HISTORIES

Here we shall provide a brief overview of these two approaches and of the relation between quantum and classical they suggest. More detailed reviews of the environment-induced superselection and the decoherence process are available elsewhere [3,6]. The paper of Griffiths [8] is still an excellent introduction to the consistent histories approach. In more recent publications, Gell-Mann and Hartle [10,12] introduce and discuss the decoherence functional also using the sum over histories formulation and discussing the relation with consistency. Omnès gives a useful overview in his most recent review article [9].

A. Environment-induced superselection and the existential interpretation

The quantum formalism allows for the existence of many more states of the objects described by it than we seem to encounter. In particular, macroscopic objects appear to us in a small classical subset of a much larger quantum selection available in the Hilbert space. The purpose of environment-induced superselection is to "outlaw" the vast majority of such states by appealing to their instability in the presence of the environment. The key point of this approach is simple: It starts with the realization that there is a basic difference between the consequences of quantum evolution for systems which are *closed* (isolated from their environments) and *open* (interacting with the "rest of the Universe").

Especially important is the fact that evolution of open quantum systems violates the "equal rights amendment" guaranteed for each and every state in the Hilbert space of a closed system by the quantum superposition principle. The process of decoherence affects different states differently. Some such states evolve essentially unperturbed by the coupling to the outside. They form a "preferred set of states" in the Hilbert space of the system, known as the "pointer basis" which can be, in principle, found by using the recently proposed predictability sieve [6,16]. By contrast, superpositions of such pointer states rapidly decay into density matrices which turn out to be, after the characteristic decoherence time has elapsed, given by mixtures approximately diagonal in the pointer basis.

Thus, decoherence results in a negative selection process which dynamically eliminates most of the superpositions. In course of a dynamical evolution the remaining preferred states evolve predictably, with the least possible entropy production, which can be regarded as a measure of the rate of decoherence. This rate would be especially large when a system is prepared in a state unstable to the monitoring by the environment, as would be typically the case for the apparatus pointer in course of quantum measurement. However, even in such instances—when monitored on a time scale larger than the decoherence time—the system would obey an effective environment-induced superselection rule which will prevent it from existing in the vast majority of the states.

The distinguishing feature of classical observables, the essence of "classical reality," is the persistence of properties of classical systems, which can exist in predictably evolving states and follow a trajectory which appears to be deterministic. Relative stability—or, more precisely, relative predictability of the evolution of the states of open quantum systems—emerges as a useful criterion which can be employed to distinguish states which can persist (or deterministically evolve into other predictably evolving states). This emphasis on predictable existence gives rise to an *existential interpretation of quantum mechanics*. Only the states which can *exist* on a time scale accessible to the observers will be regarded by them as a part of the classical domain. Moreover, observers are also a part of the quantum Universe, and their perceptions are formed from their memories—records of the past measurements. These records are states of physical (and, generally, very open systems), which rapidly decohere and are subject to environment-induced superselection. Thus, the observers accessing their own records will be restricted to perceiving their own memory in terms of sets of preferred "pointer states" which can *exist* for a long time. In this sense the dynamics of the observables of the open system "strikes twice": on the one hand limiting the set of external observables to these which have the predictability characterizing the classical domain, while on the other hand constraining the states of internal records accessible to the observer.

The ultimate focus of this approach is then the persistence of correlations between the states of two systems—the states of memory (the records) and the states of the measured system. Interaction with the environment is used to turn a nonseparable quantum correlation between them, represented by a state of the form

$$|\Psi_{AS}\rangle=\sum_i \gamma_i |A_i\rangle|\sigma_i\rangle \ , \tag{1}$$

where $|A_i\rangle$ are the record states and $|\sigma_i\rangle$ are the corresponding states of the system, into a classical correlation present in the density matrix:

$$\rho_{AS}=\sum_i |\gamma_i|^2 |A_i\rangle\langle A_i||\sigma_i\rangle\langle\sigma_i| \ . \tag{2}$$

The environment contributes to this transition by becoming correlated with the preferred sets of states:

$$|\Psi_{AS}\rangle\otimes|\mathcal{E}_0\rangle\rightarrow\sum_i \gamma_i |A_i\rangle|\sigma_i\rangle|\mathcal{E}_i\rangle \ , \tag{3}$$

where $|\mathcal{E}_i\rangle$ are orthonormal states of the environment. Tracing out of the environment results in a reduced density matrix ρ_{AS} given by (2).

While the study of the stability of correlations is the point of departure for the discussion of the interpretation [3,6], much can be learned by studying the reduced dynamics of individual open quantum systems such as an exactly solvable quantum harmonic oscillator immersed in a heat bath of other oscillators. Such studies demonstrate that the decoherence time scales are indeed very short for macroscopic quantum objects and that the form of the interaction between the system and the environment has a crucial influence on the selection of the preferred set of states [16–18].

The division of the Universe into subsystems is, in addition to quantum theory, the only crucial input. While this assumption is far from trivial, one can argue that it is needed to formulate the very problem of the emergence of classicality which is being addressed by this approach [6]. In particular, the measurement problem cannot be stated without dividing the Universe into an apparatus and a measured system. The addition of an environment, if anything, makes the discussion more realistic.

B. Consistent histories

The basic concept used in the consistent histories approach is obviously that of a "history" which is a sequence of events defined at various moments of time. An event in quantum mechanics can be thought of as corresponding to a projection operator. Therefore, histories are represented by time-ordered sequences of projection operators. In the ordinary formulation of quantum mechanics it is always possible to assign probabilities to single events defined by projectors, and representing (for example) the alternative outcomes of a measurement performed at an arbitrary time. Quantum interference prevents us from assigning probabilities to arbitrary histories but probabilities can be consistently assigned to special sets of histories. The basic idea in the consistent histories approach is to analyze sets of histories and establish the mathematical conditions that must be satisfied in order to be able to define a probability measure in such sets.

A history is formally represented by a chain of Heisenberg projectors of the form

$$C_\alpha=\{P^{(1)}_{\alpha_1}(t_1),\ldots,P^{(n)}_{\alpha_n}(t_n)\}$$

(where we assume $t_1<\cdots<t_n$). The superscript (k) labels the set of projectors used at time t_k and α_k denotes the particular alternative. We will consider the possibility that the set used at time t_k depends on the previous alternatives α_j, $t_j<t_k$. When necessary, we will make this dependence explicit by writing

$$P^{(k,\alpha_1,\ldots,\alpha_{k-1})}_{\alpha_k}(t_k)$$

but we will try to avoid using this cumbersome notation when there is no danger of confusion. A set of histories is said to be exhaustive if it covers all possible alternatives at all of the different times. Technically this is expressed by the identify $\sum_\alpha C_\alpha=I$ (where α denotes the set of alternatives $\alpha=\{\alpha_1,\ldots,\alpha_n\}$).

Sums of chains $C_\alpha+C_{\alpha'}$ are also histories which provide a coarser description than the one given by the individual chains C_α and $C_{\alpha'}$. Thus, the sum of the operators corresponds to the logical operation "or" ($\wedge$), i.e., $C_{\alpha\wedge\alpha'}=C_\alpha+C_{\alpha'}$. The history $C_{\alpha\wedge\alpha'}$ is not necessarily a chain itself. (It is called a class operator in [19]. It might only be a chain if the histories are branch independent and the alternatives α and α' differ at a single time.) In what follows C_α will denote a generic history (which may or may not be a chain of projectors).

In standard quantum mechanics the probability of a given event, associated with the projector $P_i(t)$, is computed using the formula

$$p(i)=\mathrm{Tr}[P_i^\dagger(t)\rho(t_0)P_i(t)]$$

[where $\rho(t_0)$ is the initial density matrix]. If we generalize this formula to histories, the natural candidate for the probability of the history C_α is

$$p(C_\alpha)=\mathrm{Tr}[C^\dagger_\alpha\rho(t_0)C_\alpha]\ . \tag{4}$$

The failure of the probability sum rules can be easily seen if we apply formula (4) to the "coarse-grained" history $C_\alpha\wedge C_{\alpha'}$:

$$p(C_\alpha\wedge C_{\alpha'})=p(C_\alpha)+p(C_{\alpha'})+2\,\mathrm{Re}\{\mathrm{Tr}[C^\dagger_\alpha\rho(t_0)C_{\alpha'}]\}\ . \tag{5}$$

The last term in (5) clearly violates the probability sum rules. Thus, these rules are satisfied in a complete set of histories if and only if the last term vanishes for every pair of histories in the set. To express and analyze the validity of this condition it is convenient to define the decoherence functional $D(\alpha,\alpha')$ as

$$\begin{aligned}D(\alpha,\alpha')&\equiv\mathrm{Tr}[C^\dagger_\alpha\rho(t_0)C_{\alpha'}]\\&=\mathrm{Tr}[P^n_{\alpha_n}(t_n)\cdots P^1_{\alpha_1}(t_1)\rho(t_0)\\&\quad\times P^1_{\alpha'_1}(t_1)\cdots P^n_{\alpha'_n}(t_n)]\ .\end{aligned} \tag{6}$$

The necessary and sufficient condition to define the probability measure in the set of histories is now easily written as [9]

$$\mathrm{Re}[D(\alpha,\alpha')]=0 \quad \text{for all } \alpha\neq\alpha'\ . \tag{7}$$

When this condition is satisfied, the set is a *consistent set of histories* and the probabilities are given in terms of the diagonal elements of the decoherence functional. A more restrictive condition than (7) has been proposed by Gell-Mann and Hartle [10] who call for the cancellation of all the nondiagonal elements of the decoherence functional and not only of its real part:

$$D(\alpha,\alpha')=0 \quad \text{for all } \alpha\neq\alpha' \tag{8}$$

(this is obviously a sufficient condition for consistency). A brief remark about terminology is in order here. The above consistency conditions are referred to as "weak decoherence," (7), and "medium decoherence," (8), by Gell-Mann and Hartle [10,12,19]. They also presented and discussed these conditions in the most general context of histories which are not necessarily chains of projectors [12,19] (both Omnès and Griffiths concentrated their analysis in special histories represented by chains of projectors and special coarse grainings assuring that sums of chains are chains themselves [8,9]). We prefer to reserve the word "decoherence" for the physical process outlined in the previous subsection. Therefore, following Griffiths [8] and Omnès [9], we will refer to the various conditions which assures the validity of the probability sum rules as "consistency conditions." In practice, we will always use condition (8) ("medium decoherence" in the terminology of Gell-Mann and Hartle).

Before closing this subsection let us make a remark on the definition of the decoherence functional given in (6). In that expression we are obviously using the Heisenberg picture and the projectors $P^k_{\alpha_k}(t_k)$ are defined in terms of the Schrödinger picture projectors as

$$P^k_{\alpha_k}(t_k)=U^\dagger(t_0,t_k)P^k_{\alpha_k}(t_0)U(t_0,t_k)\ , \tag{9}$$

where $U(t_0,t_k)$ is the evolution operator that propagates the state vector from t_0 to t_1. Introducing (9) into (6) we can obtain the following well-known formula where the projectors are constant (Schrödinger) operators:

$$D(\alpha,\alpha')=\mathrm{Tr}[P^n_{\alpha_n}U(t_{n-1},t_n)\cdots P^1_{\alpha_1}\rho(t_1)\times P^1_{\alpha'_1}\cdots U^\dagger(t_{n-1},t_n)P^n_{\alpha'_n}] . \quad (10)$$

In the forthcoming discussion, and for reasons that will become evident later, it will be more convenient to use a different expression for $D(\alpha,\alpha')$ that can be easily derived from Eq. (10). Introducing the propagator of the density matrix (a superoperator acting in the space of operators [20]), which we denote as $K^{t_f}_{t_i}$ and is defined as

$$K^{t_f}_{t_i}[\rho(t_i)]=U(t_i,t_f)\rho(t_i)U^\dagger(t_i,t_f)=\rho(t_f) , \quad (11)$$

the decoherence functional (10) can be rewritten as

$$D(\alpha,\alpha')=\mathrm{Tr}(P^n_{\alpha_n}K^{t_n}_{t_{n-1}}\{\cdots P^1_{\alpha_1}K^{t_1}_{t_0}[\rho(t_0)]\times P^1_{\alpha'_1}\cdots\}P^n_{\alpha'_n}) . \quad (12)$$

C. Decoherence, consistency, the quantum, and the classical

The manner in which the decoherence process and the resulting environment-induced superselection explain the emergence of the classical from the quantum substrate is quite clear. It has been briefly described in Sec. II A. When considered in the context of Everett's "many worlds" point of view, decoherence defines branches. Its focus on the stable existence of the records allows one to understand Bohr's "Copenhagen interpretation" as, in effect, an observer's memory of one of Everett's branches, with the apparent collapses induced by the effective superselection rules [3].

The stated goals of the consistent histories approach were initially somewhat different: consistency was invoked to discuss sequences of events in a closed evolving quantum system without the danger of logical contradictions [8]. However, this goal as well as the validity of the probability sum rules are also a precondition for classical behavior. Thus, at least some of the aspects of the classical domain should be related to consistency.

It was since realized that consistency alone does not suffice to define classical behavior [10]. For example, given a closed system, it is always possible to find a consistent set of histories which are defined simply by the projectors constructed from the evolved eigenstates of the initial density matrix. Thus, when

$$\rho_i=\sum_n p_n|n\rangle\langle n| ,$$

events represented by projectors

$$\Pi_i(t_k)=|i\rangle\langle i|$$

or by their direct sums can be always used to construct consistent histories. However, when this simple algebraic algorithm is applied to the classic test cases (such as the measurement problem or a Schrödinger cat) it will result in extremely nonclassical consistent histories with the events corresponding to superpositions of various outcomes of measurements, dead and alive cats, etc.

At the very least, consistency would need to be supplemented by extra ingredients (which, in the context of consistent histories interpretation, are yet to be identified) in order to become an effective tool in studying classicality. Moreover, it appears likely that exact consistency may be too strong a requirement, and will have to be relaxed in order to be relevant for the study of "classicality."

On the other hand, as we will explicitly show in the next section, the decoherence process and the resulting environment-induced superselection rules enforce the approximate validity of the probability sum rules for the *preferred histories* of the system of interest. (From the perspective of the consistent histories approach, the process of tracing over the environment can be naturally related to a coarse-grained class of histories.)

The relation between the process of decoherence and the various ways of stating the requirement of consistency is then, at least in the usual context in which classicality is sought, quite straightforward: Decoherence is essential in eradicating elements of the density matrix which are off diagonal in the basis in which a classical history is always expressed. Consistency is obtained as a byproduct of the environment-induced superselection, of the continuous monitoring by the environment which maintains a "running record" of the evolution of the system. Indeed, it is tempting to paraphrase John Weelers [21] paraphrase of Niels Bohr [22] "No phenomenon is a phonomenon until it is a recorded phenomenon," and say that "No history is a classical history until it is a monitored history." Or, perhaps, to say it even more succinctly "A classical history is a chain of events recorded by the environment."

In what follows, we will investigate the connection between the two formalisms and focus our attention on the possibility of constructing perfectly consistent histories for the system out of the eigenstates of the reduced density matrix. In this respect, it is worth remembering here that the reduced density matrix can always be instantaneously diagonalized. Its eigenstates, which are sometimes called Schmidt states [11] are not necessarily identical (or even approximately the same) as the pointer states: The two sets of states can be expected to coincide only when the decoherence process has been effective which, in turn, implies restrictions on the time scales. Thus, the time at which the Schmidt states are calculated must be larger than the typical decoherence time scale of the problem. In that case the Schmidt states becomes independent of the details of the initial condition and coincide with the pointer projectors.

III. CONSISTENT HISTORIES FOR AN OPEN SYSTEM

In this section we will first establish the conditions under which the decoherence functional can be constructed entirely in terms of the reduced density matrix of the system. This will be shown to be possible in the Markovian regime of the reduced evolution. In this case, we will analyze the importance of the "environment induced noncommutativity" in determining the properties of consistent histories. We should remark that, while our analysis of induced noncommutativity will be restricted to the Markovian regime, EINC is a generic property of the evolution of an open system.

A. Decoherence functional for an open system

When we evaluate the decoherence functional in the histories of the system, we have to use projectors of the form $I_{\mathcal{E}}\otimes P^k_{\alpha_k}(t_k)$ but we must remember that the evolution between intermediate times is entirely unitary, that ρ_U is the full density matrix and that the final trace is over the whole Hilbert space. Thus, the decoherence functional is obtained by tracing over the environment at the final time while the reduced density matrix is defined by tracing over the environment at every moment of time [23]. This indicates that the decoherence functional could be written in terms of the reduced density matrix only when taking the trace over the environment at the end is not very different from doing it at every time [23]. This will be the case whenever the time evolution of the reduced density matrix does not depend upon the correlations that are created dynamically between the system and the environment. The demonstration of this simple observation can be easily done if we decompose the final trace

$$\mathrm{Tr}_{\{\mathcal{S},\mathcal{E}\}}=\mathrm{Tr}_{\mathcal{S}}\mathrm{Tr}_{\mathcal{E}}$$

and try to move the trace over the environment to the inside of the expression for the decoherence functional. Using Eq. (10) we obtain

$$D(\alpha,\alpha')=\mathrm{Tr}_{\mathcal{S}}\{P^n_{\alpha_n}\mathrm{Tr}_{\mathcal{E}}[\mathcal{O}_n(t_n)]P^n_{\alpha'_n}\}\ , \tag{13}$$

where we defined $\mathcal{O}_0(t_0)=\rho_U(t_0)$ and

$$\mathcal{O}_{k+1}(t_{k+1})\equiv U(t_k,t_{k+1})(I_{\mathcal{E}}\otimes P^k_{\alpha_k})\mathcal{O}_k(t_k)(I_{\mathcal{E}}\otimes P^k_{\alpha'_k})\times U^\dagger(t_k,t_{k+1}),\quad 1\le k\le n\ . \tag{14}$$

From this expression we can now notice that the trace can be moved one more step towards the center only if $\mathrm{Tr}_{\mathcal{E}}[\mathcal{O}_n(t_n)]$ is a function of $\mathrm{Tr}_{\mathcal{E}}[\mathcal{O}_{n-1}(t_{n-1})]$, which is the trace of an operator defined at t_{n-1}. This is not possible in general since $\mathrm{Tr}_{\mathcal{E}}[\mathcal{O}_n(t_n)]$ may depend on the full operator $\mathcal{O}_{n-1}(t_{n-1})$ and not only upon its partial traces (this is precisely what happens when the correlations between the system and its environment affect the reduced dynamics of the system and produce non-Markovian effects). We will restrict our future considerations to those cases where this is true, or equivalently, when there is a well-defined reduced evolution operator, denoted as $\hat{K}^{t_n}_{t_{n-1}}$, acting in the following way:

$$\mathrm{Tr}_{\mathcal{E}}[\mathcal{O}_n(t_n)]=\hat{K}^{t_n}_{t_{n-1}}\{P^{n-1}_{\alpha_{n-1}}\mathrm{Tr}_{\mathcal{E}}[\mathcal{O}_{n-1}(t_{n-1})]P^{n-1}_{\alpha'_{n-1}}\}\ .$$

If this operator exists the decoherence functional can be written as

$$D(\alpha,\alpha')=\mathrm{Tr}_{\mathcal{S}}(P^n_{\alpha_n}\hat{K}^{t_n}_{t_{n-1}}\{\cdots P^1_{\alpha_1}\hat{K}^{t_1}_{t_0}[\rho_r(t_0)]\times P^1_{\alpha'_1}\cdots\}P^n_{\alpha'_n})\ . \tag{15}$$

This is the main equation we will use in the next section. It is worth noting the similarity between the expression (12) (which is the decoherence functional for a closed system) and (15), in which all the operators are members of the "reduced theory." In the previous section we showed that the decoherence functional for a closed system can be written in three equivalent ways given by Eqs. (6), (10), and (12). However, we can prove that it is not possible to write the "reduced" decoherence functional in a way resembling Eqs. (6) or (10). In this sense, Eq. (15) is unique. In fact, only if the evolution is unitary (as it is for the full density matrix) is it true that to propagate the density matrix ρ we just have to multiply it from left and right with two operators that act on the same Hilbert space as ρ. This is the crucial property [see Eq. (11)] allowing us to show the equivalence between Eqs. (6), (10), and (12). In the case of the reduced density matrix this property is no longer valid since two operators A and B satisfying

$$\hat{K}^{t_f}_{t_i}[\rho_r(t_i)]=A\rho_r(t_i)B=\rho_r(t_f) \tag{16}$$

do not exist: Existence of such operators would imply that initial pure states would remain pure forever, which is in contradiction with well-known properties of the reduced dynamics.

Summarizing, we conclude that when the reduced evolution operator $\hat{K}$ exists, the decoherence functional can be written as (15) in terms of "reduced objects." Although this operator does not exist, in general, it is also clear that there are very important cases for which the existence of $\hat{K}$ is guaranteed. We will discuss those cases in the next subsection.

B. Reduced dynamics and environment-induced noncommutativity

The existence of the reduced evolution operator $\hat{K}$ is a strong requirement. For example, it implies that the reduced density matrix satisfies a purely differential equation (the evolution cannot have memory). Such a Markovian equation does not exist, in general, since the exact master equation (the equation for the reduced density matrix) is typically nonlocal in time. Moreover, the existence of a local master equation is a necessary but not a sufficient condition for the existence of $\hat{K}$. In fact, there are cases for which a local but explicitly time-dependent master equation exists and the evolution is still (weakly) non-Markovian. In those cases the reduced evolution operator may not exist because the correlations still play some role in the reduced dynamics. Technically, the condition that guarantees the existence of $\hat{K}$ is the *locality of the Feynman-Vernon influence functional* [24] which enters in the path-integral representation of the decoherence functional. This implies that the master equation is local in time and, for most realistic examples, also has time-independent coefficients. Thus, we will restrict ourselves to consider cases for which the influence functional is local and the master equation is time independent.

This strong assumption will still allow us to study realistic and relevant situations. Let us now mention two important physical examples in which the existence of $\hat{K}$ constitute a sound approximation.

The first example is the well-known linear quantum Brownian motion (QBM). In this case the system is a particle which interacts with an environment formed by a collection of harmonic oscillators. Assuming that the initial state does not contain correlations between the particle and its environment, the model can be fully characterized by the spectral distribution and by the initial state of the environment (usually taken as thermal equilibrium at some temperature T). In such a rather general case, the existence of a local master equation was recently proved [25]. It was shown that the master equation for the linear QBM is always of second order in partial derivatives and has time-dependent coefficients that vary with temperature and with the spectral density of the environment (time dependence in the coefficients is responsible for all the non-Markovian effects). A particularly important case is that of an Ohmic environment (linear spectral density) at high temperatures ($k_B T \gg \hbar\Lambda \gg \hbar\Omega_R$, where Λ is the high-frequency cutoff of the environment and Ω_R is the renormalized frequency of the system). In that case, after a short transient whose duration is determined by Λ, the master equation for the reduced density matrix ρ reads

$$\dot{\rho} = -\frac{i}{\hbar}[H_R,\rho] - \frac{\gamma}{2\hbar}[\{p,x\},\rho] - \frac{i\gamma}{\hbar}([x,\rho p]-[p,\rho x]) - \frac{2m\gamma k_B T}{\hbar^2}[x,[x,\rho]] . \quad (17)$$

Above, H_R denotes the renormalized Hamiltonian of the system and γ is a constant that fixes the relaxation rate. Although this equation is not valid for low temperatures, it has been also shown [17,18] that in that regime (i.e., $k_B T < \hbar\Lambda$) the high-temperature approximation remains rather accurate since the coefficients approach their asymptotic values very fast.

A second example in which the use of a local master equation is a reasonable approximation can be found in the domain of atomic physics and quantum optics. In that case we consider the system to be an atom and the environment to be formed by the infinite number of modes of the quantized electromagnetic field. When the interaction between the system and the environment is taken into account, a local master equation, known as Bloch equation, can be derived under a number of approximations. The essential ones are the following: absence of initial correlations between the system and the environment, Markovian behavior (very short lifetime of correlations in the environment), weak coupling (the equations are valid to second order in an expansion in the coupling constant), and rotating wave approximation (by which rapidly varying terms, counterrotating, are supposed to average to zero). If we denote with $|n\rangle$ the eigenstates of H_0, the Hamiltonian of the isolated atom, the Bloch equations (in the interaction picture associated with H_0) read

$$\dot{\rho}_{nm} = -\frac{i}{\hbar}[H_d,\rho]_{nm} + \delta_{nm}\sum_k w_{nk}\rho_{kk} - \Gamma_{nm}\rho_{nm} . \quad (18)$$

Here, the driving Hamiltonian H_d accounts for the coherent effects associated with the interaction between the atom and the electromagnetic field (such as coherent driving producing Rabi oscillations). The constants w_{nk} are transition rates that, in the absence of driving, determine the evolution of the diagonal elements of ρ (populations) while Γ_{nm} are related to decay rates that affect the evolution of the nondiagonal elements (coherences). These constants can be, in principle, expressed in terms of some microscopic model and cannot be thought of as being independent of each other because of the fluctuation-dissipation relations (and the conservation of probability which implies that $\Gamma_{nn} = \sum_k w_{kn}$).

Let us now discuss one of the most remarkable features of the reduced dynamic associated with the above master equations: the existence of "environment-induced noncommutativity" (EINC): The existence of EINC is a consequence of the nonunitarity of the reduced dynamics which does not necessarily preserve the commutation relations. Two initial states that satisfy

$$[\rho_a(0),\rho_b(0)]=0$$

may evolve in such a way that

$$[\rho_a(t),\rho_b(t)]\neq 0 .$$

This is EINC, a property with important consequences in determining the qualitative nature of some interesting sets of consistent histories that we will consider in the next section. It is worth noting that this effect takes place on a rather special time scale. Commutativity, rather than noncommutativity, is induced on the decoherence time scale which is very much shorter than the time needed to approach equilibrium. Thus, on that time scale every initial state will become approximately diagonal in the same pointer basis (and therefore the final states will always commute). Therefore, the time scale on which EINC is most important is shorter than the decoherence time scale.

The existence of EINC is a prediction of both master equations (17) and (18). In particular, we can show that in the linear QBM the commutator changes as

$$\frac{d}{dt}[\rho_a(t),\rho_b(t)] \propto \frac{2m\gamma k_B T}{\hbar^2}\{[\rho_a(t),x],[\rho_b(t),x]\} . \quad (19)$$

Similarly, it is simple to show that Bloch equations also predict the existence of EINC with the only restriction that the dimension of the system's Hilbert space be greater than two (no EINC for a spin-$\frac{1}{2}$ system). Finally, we should stress that in order to be really sure about the physical nature of the EINC predicted from the master equations (17) and (18), we still need to show that the effect occurs on the decoherence time scale which is compatible with the ones used to obtain those equations. We will illustrate that this is indeed the case using a specific example in the next section.

IV. CONSISTENCY, DECOHERENCE, AND CLASSICALITY

A. Schmidt histories are consistent

Using Eq. (15) for the reduced decoherence functional, it is very simple to find a systematic way of constructing an infinite number of consistent sets of histories. The method give a clear prescription for choosing the sets of projectors $P^k_{\alpha_k}$ that guarantee perfect consistency. Given a time sequence $t_1, t_2, \ldots, t_n$, the decoherence functional for the histories of the system

$$D(\alpha,\alpha')=\mathrm{Tr}_{\mathcal{S}}(P^n_{\alpha_n}\hat{K}^{t_n}_{t_{n-1}}\{\cdots P^1_{\alpha_1}\hat{K}^{t_1}_{t_0}[\rho_r(t_0)]P^1_{\alpha'_1}\cdots\}P^n_{\alpha'_n}) \tag{20}$$

is automatically diagonal in its first index if we choose the projectors $P^1_{\alpha_1}$ in such a way that they commute with the reduced density matrix at time t_1. These projectors (if chosen to be one dimensional) are associated with the instantaneous eigenstates of the reduced density matrix (the so-called Schmidt basis). Doing so, the decoherence functional reads

$$D(\alpha,\alpha')=\delta_{\alpha_1,\alpha'_1}\mathrm{Tr}_{\mathcal{S}}(P^n_{\alpha_n}\hat{K}^{t_n}_{t_{n-1}}\{\cdots P^2_{\alpha_2}\hat{K}^{t_2}_{t_1}[P^1_{\alpha_1}\rho_r(t_1)P^1_{\alpha_1}]P^2_{\alpha'_2}\cdots\}P^n_{\alpha'_n}) \ . \tag{21}$$

Analogously, to achieve diagonality in the second index of the decoherence functional, we should choose the projectors $P^2_{\alpha_2}$ in such a way that they commute with the path projected reduced density matrix

$$\hat{K}^{t_2}_{t_1}[P^1_{\alpha_1}\rho_r(t_1)P^1_{\alpha_1}] \ .$$

However, because of the existence of environment-induced noncommutativity, the eigenstates of the path projected density matrix will generally depend on the alternative α_1. Therefore, the set of projectors chosen at time t_2 will generally depend on the previous alternatives and the history will be branch dependent.

This procedure can be implemented recursively for arbitrarily many steps. It will produce a set of branch-dependent histories for which the decoherence functional is automatically diagonal. At time t_k, the projectors are associated with the eigenvectors of the path projected reduced density matrix

$$\hat{K}^{t_k}_{t_{k-1}}\{\cdots\hat{K}^{t_2}_{t_1}[P^1_{\alpha_1}\rho_r(t_1)P^1_{\alpha_1}]\cdots\} \ .$$

These projectors always exist and, in general, due to EINC, depend upon the alternatives $\alpha_1, \ldots, \alpha_{k-1}$. As we mentioned above, we will refer to them as "Schmidt histories." It is important to realize that by following the above procedure we can construct an infinite number of different sets of histories all of which are exactly consistent. Thus, we can obtain a different set just by choosing a different sequence of historical instants $\{t_k\}$. Moreover, by changing the time sequence we may drastically change the sets of projectors. In this sense, these sets are highly unstable.

To illustrate this point and clarify the nature of the instability let us imagine that we follow the above procedure and construct a consistent set of histories specifying projectors at times $t_1, t_2, \ldots, t_n$. The histories belonging to this set are strings of the form

$$P^{(1)}_{\alpha_1}(t_1)P^{(\alpha_1,2)}_{\alpha_2}(t_2)\cdots P^{(n,\alpha_1,\ldots,\alpha_{n-1})}_{\alpha_n}(t_n) \ .$$

Let us now construct another consistent set by using the same method but specifying histories at times $t_2, \ldots, t_n$. In this way we obtain histories which are strings of the form

$$\tilde{P}^{(2)}_{\alpha_2}(t_2)\cdots\tilde{P}^{(n,\alpha_2,\ldots,\alpha_{n-1})}_{\alpha_n}(t_n) \ .$$

The sets are unstable because, due to the environment-induced noncommutativity, the projectors $P^k_{\alpha_k}$ are different from the projectors $\tilde{P}^k_{\alpha_k}$. In fact, in the second case the set of projectors we must use at t_2 depends only upon the initial density matrix while in the first case may strongly depend on the alternatives α_1. In the next subsection we will illustrate this fact with an example that demonstrates that the effect is real and can be rather large.

The natural question to ask is if there is some situation in which the above diagonalization procedure generates a unique (and stable) output. This is going to be the case only when the eigenbasis of the "path projected reduced density matrix" at time t_k (i.e.,

$$\hat{K}^{t_k}_{t_{k-1}}\{\cdots\hat{K}^{t_2}_{t_1}[P^1_{\alpha_1}\rho_r(t_1)P^1_{\alpha_1}]\cdots\})$$

is independent of the path projected reduced density matrix at time t_{k-1}. This requirement is satisfied when there exists a stable pointer basis [6]: We need the environment to help select a preferred (and stable) set of states. However, this can only happen if we wait long enough between the intermediate times for which we specify the history. Roughly speaking, the difference $\Delta t = t_i - t_{i-1}$ must be larger than the typical decoherence time scale τ_{dec} of the problem.

B. The importance of environment-induced noncommutativity: An example

We will analyze here a particular example that illustrates the importance of EINC in producing consistent histories which may be highly unstable. For simplicity, we will use Bloch equations and consider a system with a low-dimensional Hilbert space. As we have mentioned above, Bloch equations cannot result in EINC if the dimension of the system's Hilbert space is equal to two.

Thus, we need at least three dimensions. However, we are also interested in showing an example in which EINC takes place in a time scale for which Bloch equations are valid. This implies, roughly speaking, that the interesting effect should take place in a time scale longer than the lifetimes Γ_{nn}^{-1}. It is simple to show that this cannot be done in a three-dimensional example. Thus, we will take our system to have a Hilbert space with four dimensions. We will consider the simplest situation in which all the levels are stable except one (for example, $|4\rangle$) which can decay only to the ground state (for example, $|1\rangle$). Thus, in this case we see, neglecting induced emission and absorption processes, that all the coefficients entering in the Bloch equation are either identically zero or given by

$$\Gamma_{44}=2\Gamma_{14}=2\Gamma_{24}=2\Gamma_{34}=w_{14}\equiv\Gamma \ . \tag{22}$$

It is interesting to note that the decoherence time scale of this system can be controlled in a very simple way. In the absence of external driving (i.e., $H_d=0$) the nondiagonal elements ρ_{k4} will disappear in a time scale related to Γ^{-1}. However, as there is only one dissipative channel, the density matrix will not become diagonal in the three-dimensional subspace generated by $\{|k\rangle, k=1,2,3\}$. This situation can be changed if one introduces a coherent driving by coupling the system to intense laser fields which are in resonance with the transitions between state $|4\rangle$ and other states. The intensities of the different laser fields control the frequencies of the Rabi oscillations and these frequencies control the decoherence time scale of the system. In our example, we use this idea to make the decoherence time scale rather large (this allows us to observe EINC on a reasonable time scale). In particular, we consider the simple driving terms

$$H_d=\Omega_{14}|1\rangle\langle 4|+\Omega_{13}|1\rangle\langle 3|+\Omega_{12}|1\rangle\langle 2|+\text{H.c.} \tag{23}$$

Using this Hamiltonian in Bloch equations (which were integrated numerically), we demonstrated the existence of EINC for a rather robust set of parameters and for relevant time scales. An example is displayed in Fig. 1 in which approximately 40% of the probability is in branches that show significant degree of instability. We remark that the parameters we used are rather reasonable from the point of view of the systems for which Bloch equations are typically used in atomic physics.

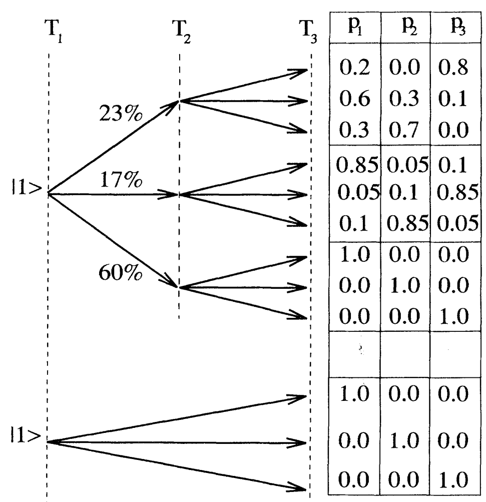

FIG. 1. Consistent Schmidt histories for a system with a four-dimensional Hilbert space are represented by a branching diagram. This example corresponds to a system described by Bloch equations (18). The coefficients defining the driving Hamiltonian (23) are $\Omega_{12}=5$, $\Omega_{13}=10$, and $\Omega_{14}=50$ and the decay rate (22) is $\Gamma=3000$ (measured in units in which the time separation $T_2-T_1=T_3-T_2$ is set to unity). The origin of the diagram corresponds to the state $|1\rangle$, which is the state of the system at time T_1 (we do not draw the fourth branch corresponding to the unstable state $|4\rangle$ since it has zero probability in our example). The tabulated p_i's are the projection of the state defining each branch onto the basis $|i\rangle$, i.e., $p_i=|\langle i|\Psi\rangle|^2$. When the histories are constructed at times T_1, T_3, there are three consistent branches corresponding to the states $|i\rangle$, $i=1,2,3$ (T_3-T_1 is larger than the decoherence time and the states $|i\rangle$ form a pointer basis). On the contrary, when the histories are constructed at times T_1, T_2, T_3, there are nine consistent branches. In the first six (which carry 40% of the total probability) the effect of environment induced noncommutativity is important: the states associated with the different consistent branches form a basis of Hilbert space which is different from the one formed by the $|i\rangle$ vectors. The quantum instability of the Schmidt histories is easily noticeable.

V. CONCLUSIONS

Let us summarize our results. We analyzed first the conditions under which the decoherence functional can be written entirely in terms of "reduced" quantities and showed that this can be done when the correlations dynamically created between the system and the environment do not affect the future evolution of the reduced density matrix. As can be explicitly shown, this is the case for the high-temperature limit of the Caldeira-Leggett model of Ohmic dissipation and in any other case for which the Feynman-Vernon influence functional is local in time. Expressing the decoherence functional in terms of elements of the reduced dynamics and using the properties of the reduced evolution operator we proved that it is always possible to construct an infinite number of sets of perfectly consistent histories. We also showed that these sets are generally rather unstable under "observations" since by deleting one of the times at which the histories are defined, the projection operators defining the consistent histories may be substantially changed.

In discussing the classical limit using the consistent histories approach various concepts are usually brought together. The first one is consistency, which in this formulation is the primary criterion. Coarse graining is usually invoked as a necessary way of achieving consistency. This is even the case when a separation of relevant and irrelevant degrees of freedom is made. In fact, in discussions of Caldeira-Leggett type of models it is usually argued that to achieve consistency for histories of the Brownian particle one needs to introduce some degree of spatial coarse graining [26] and consider the histories defined by a sequence of "slits" characterized by some widths. In this case, the width of the slits is associated with the degree of coarse graining. However, as we explicitly showed here, such coarse grainings are *not necessary* to achieve consistency. In fact, the Schmidt projectors can be one dimensional and still define consistent histories. The problem with the usual argument in favor of coarse graining as a way to achieve consistency is that it uses an essential extra ingredient that remains *hidden* for the most part. Thus, a spatial coarse graining is needed *only* if one restricts to considering very special histories constructed with special classes of projection operators: position projections, for example. But there is nothing in the consistent histories approach telling us that we must like a set of projectors better than another. The extra ingredient that makes us think that is "natural" to describe the world around us using position projectors (or any other set of projectors we happen to like) has nothing to do with consistency. This extra ingredient is the crucial one in defining the quasiclassical domain.

As we discussed in Sec. II C, there are very simple ways of achieving consistency for a closed system. The sets one constructs in this way are based on the use of projection operators that are blatantly nonclassical. Our results show that for an open system (which interacts with an external environment) the situation is much the same. Consistency is achieved easily by means of the Schmidt histories. However, these histories have no relation with the one describing a sensible quasiclassical domain.

The Schmidt histories we discussed provide an interesting example that may help us to disentangle the many concepts that enter in the definition of a quasiclassical domain. On the one hand, they are consistent but do not require any (spatial) coarse graining. On the other hand, although we cannot prove it rigorously, it is likely that this set will also satisfy other criteria that have been advanced so far (and in a less rigorous manner) to characterize the quasiclassical domain [10,27]. In fact, the set of Schmidt histories is likely to be "full" since to every history of the system there should exist a projection operator in the complete Hilbert space. Schmidt histories are therefore an example of a set which is consistent, rather fine grained (for the system) and most likely full. Despite all these properties the set is still very nonclassical. This is, of course, unless we require *predictability* (or stability of the set under the addition of extra intermediate times) in which case we need to require the separation between time slices to be larger than the typical decoherence time. In that case, the set becomes stable and the projectors are determined (by the environment and not by us) to be the ones associated with the pointer basis.

A further conclusion one can draw from our paper is the following. In working within the framework of the consistent histories approach one may be tempted to think that by looking for sets of histories that satisfy the consistency conditions (or other stronger versions like those invoked by Gell-Mann and Hartle) one is trying to find "real" histories that are "out there" in some vague way. In saying this, we do not claim that the original proponents of the formalism really made this assertion: we are just discussing what we believe is part of the informal folklore of the field. In some sense, consistent histories would be the "natural" way of looking at the system: finding the right projectors that, when used to describe the system, do not "damage it" in any way. However, our example proves that this is certainly not the case. Consistent histories are not "real" in any sense. The projectors used in constructing them must be chosen from the outside, by us the physicists, and have a decisive effect on the consistent histories of the quantum system. In our example, we could decide to add an extra instant to the list of times defining the stroboscopic history and by this simple act we would have to change completely the description of the system. Reality is a subtle concept that has been debated over the years in many physics texts and does not have a clear definition. In Einstein's view, an essential ingredient characterizing it is *predictability*. In that sense, histories could be considered to be real if, apart from the consistency condition, they are predictable behaving in a stable way. As we showed, this is the case if histories are constructed with pointer states.

Consistent histories interpretation was introduced by Griffiths [8] and pursued by others to allow for a discussion of the quantum evolutions without reference to "measurements" or "collapses" of the wave function. The difficulties we have pointed out in our discussion appear to stem not from attempting to achieve this goal, but from retaining the key elements of the formal machinery of "measurements" (such as projection operators acting at well-defined instants of time) which then tend to influence histories in a distinctly nonclassical manner. Instead of introducing such formal constructs "from the outside" of the investigated quantum Universe, one might search for the equivalents "on the inside," in the structure of the correlations between the quantum systems. Instead of projection operators, one would then have "records"—relatively stable states, which, owing to the nature of the quantum dynamics, retain their correlations with the observables of other quantum systems. This program is, of course, embodied in the environment-induced superselection approach.

ACKNOWLEDGMENTS

Part of this work was done while we were participating in the workshop on Decoherence and the Physics of Information held at the Aspen Center for Physics during June of 1992. We would like to thank the Center for the hospitality and all the participants of the workshop for all their many comments. In particular, we would like to thank Andy Albrecht, Fay Dowker, Murray Gell-Mann, Robert Griffiths, Salman Habib, Jonathan Halliwell, Jim Hartle, Raymond Laflamme, Seth Lloyd, and Bill Unruh for many interesting discussions.

[1] W. H. Zurek, Phys. Rev. D **26** 1862 (1982).
[2] H. D. Zeh, Found. Phys. **1**, 69 (1971); E. Joos and H. D. Zeh, Phys. B **59**, 223 (1985).
[3] W. H. Zurek, Phys. Today **44**(10), 36 (1991).
[4] W. H. Zurek, Phys. Rev. D **24**, 1516 (1981).
[5] Predictability is indeed a key in the notion of physical reality used in the classical EPR paper: A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935). There the "elements of reality" are defined as follows: "If, without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."
[6] W. H. Zurek, Prog. Theor. Phys. **89**, 281 (1993); in *Physical Origins of Time Asymmetry,* edited by J. Halliwell *et al.* (Cambridge University Press, Cambridge, England, 1993).
[7] W. H. Zurek, Phys. Today **46**(4), 84 (1993).
[8] R. Griffiths, J. Stat. Phys. **36**, 219 (1984).
[9] R. Omnès, J. Stat. Phys. **53**, 893 (1988); **53**, 933 (1988); **53**, 957 (1988); Ann. Phys. (N.Y.) **201**, 354 (1990); Rev. Mod. Phys. **64**, 339 (1992).
[10] M. Gell-Mann and J. B. Hartle, in *Complexity, Entropy and the Physics of Information,* Santa Fe Institute Studies in the Sciences of Complexity, Vol. VIII edited by W. Zurek, (Addision-Wesley, Reading, 1990).
[11] A. Albrecht, Phys. Rev. D **46**, 5504 (1992); Phys. Rev. D (to be published).
[12] M. Gell-Mann and J. B. Hartle, Phys. Rev. D **47**, 3345 (1993).
[13] J. P. Paz and W. Zurek (unpublished).
[14] W. H. Louisell, *Quantum Statistical Properties of Radiation* (Wiley, New York, 1973).
[15] C. Cohen Tannoudji *et al.*, *Atom Photon Interactions* (Wiley, New York, 1992).
[16] W. H. Zurek, S. Habib, and J. P. Paz, Phys. Rev. Lett. **70**, 1187 (1993).
[17] W. G. Unruh and W. H. Zurek, Phys. Rev. D **40**, 1071 (1989).
[18] J. P. Paz, S. Habib, and W. H. Zurek, Phys. Rev. D **47**, 488 (1993).
[19] J. B. Hartle, in *Gravitation and Quantizations,* Proceedings of the Les Houches Summer School, Les Houches, France, 1992, edited by B. Julia and J. Zinn-Justin, Les Houches Summer School Proceddings Vol. LVII (North-Holland, Amsterdam, 1993).
[20] R. Balescu, *Equilibrium and Nonequilibrium Statistical Mechanics* (Wiley, New York, 1975).
[21] J. A. Wheeler, in *The American Phylosophical Society and The Royal Society: Papers Read at a Meeting,* (American Phylosophical Society, Philadelphia, 1981).
[22] N. Bohr, *Atomic Physics and Human Knowledge* (Wiley, New York, 1960).
[23] This situation has also been discussed in J. Finkelstein, Phys. Rev. D **47**, 5930 (1993).
[24] R. P. Feynman and F. L. Vernon, Ann. Phys. (N.Y.) **24**, 118 (1963).
[25] B. L. Hu, J. P. Paz, and Y. Zhang, Phys. Rev. D **45**, 2843, (1992); **47**, 1576 (1993). For a simpler derivation of this master equation, see J. P. Paz, in *Physical Origins of Time Asymmetry,* edited by J. Halliwell *et al.* (Cambridge University Press, Cambridge, England, 1993).
[26] F. Dowker and J. J. Halliwell, Phys. Rev. D **46**, 1580 (1992).
[27] M. Gell-Mann and J. B. Hartle, in *Proceedings of the 25th International Conference on High Energy Physics,* Singapore, 1990, edited by K. K. Phua and Y. Yamaguchi (World Scientific, Singapore, 1991).