Resuscitation of minimal SO(10) grand unification

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It has been claimed that the minimal (nonsupersymmetric) SO(10) grand unified theory, with a single breaking scale M_U, is ruled out by the measurements of the standard-model gauge couplings at the Z^0 mass, and by the nonobservation of proton decay. We argue here that, if the threshold effects are taken into account, that theory is still quite satisfactory. We present an example in which the masses of all superheavy particles differ from M_U by at most a factor of 10. Our picture of SO(10) breaking requires the rate of proton decay to be close to the present experimental limit.

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I. INTRODUCTION

The grand unified theory (GUT) SO(10) [1] is the simplest GUT that unifies all the quarks and leptons of one family in a single representation. The original minimal SO(10), breaking at a single scale M_U to the standard model, seemed very attractive, because it gave what appeared to be a good prediction for $\sin^2\theta_W$, as well as the correct ratio between the bottom-quark mass m_b and the τ-lepton mass m_τ. The SO(10) theory also provides a motivation for a small but nonzero neutrino mass. However, with the accurate determination of the gauge couplings at the Z^0 mass [2], it has been claimed [3] that the SO(10) theory does not really unify those couplings. Furthermore, it predicts too fast a rate for proton decay. Thus, the minimal SO(10) model has been declared dead. In this note we want to argue that this conclusion need not be correct.

Two solutions have been proposed to save SO(10). One was the observation that in supersymmetric SO(10) unification is easily achieved [3]. The other one [4] was the use of an intermediate scale M_I much different from M_U, with SO(10) breaking to one of its subgroups at M_U, and that subgroup afterwards breaking to the standard model (SM) at M_I. Typically, $M_U \sim 10^{16}$ GeV while $M_I \sim 10^8$–10^{13} GeV [5], depending on the specific subgroup. We wish to point out here that the minimal nonsupersymmetric SO(10) with a single breaking scale is a satisfactory theory, provided one allows some gauge- and Higgs-boson masses to be a factor between 10 and 30 below the highest scale M_U. Such a spread is, from a theoretical point of view, quite reasonable [6], and indeed the practical importance of allowing for such threshold effects has been previously emphasized by Dixit and Sher [7]. We next discuss the relation of this analysis to a quantitative theory of fermion masses [8].

In order to generate fermion masses we use the Higgs-multiplet structure consisting of a **10** and a **126** of SO(10). We label the relevant components of the Higgs representations by their $SU(2)_L \otimes SU(2)_R \otimes SU(4)_{PS}$ ($\equiv G_{224}$) dimensions [9, 10]. The (2, 2, 1) component of the **10** gets a vacuum expectation value (VEV) v_1, generates the major part of the quark and lepton mass matrices, and is responsible for the qualitatively good ratio between m_b and m_τ. The (1, 3, 10) component of the **126**, which acquires a VEV v_R, is needed in order to give a large mass to the right-handed neutrinos, so as to make the physical-neutrino mass small, by the Gell-Mann–Ramond–Slansky mechanism [11]. It is necessary to go beyond the **10** to get a quantitative fit of the fermion masses; here we follow Babu and Mohapatra (BM) [12] and use the (2, 2, 15) component of the **126**. As pointed out by BM, it is natural that the (2, 2, 15) have a nonzero VEV v_{15} smaller than v_1 provided that the mass m_{15} of the (2, 2, 15) is larger than v_R. To be specific,

$$v_{15} \sim v_1 (v_R^2/m_{15}^2)\,. \tag{1}$$

Since we want to maintain the qualitatively correct prediction for the ratio between m_b and m_τ, we consider the (2, 2, 15) as a relatively small correction to the dominant (2, 2, 1). This suggests that (v_{15}/v_1) lie between 0.1 and 0.01, so that

$$m_{15} = (3 \text{ to } 10)\, v_R\,. \tag{2}$$

The breaking of SO(10) to the standard model requires the presence of at least two VEV's. One of these may be chosen to be v_R; but v_R cannot be the only VEV, since by itself alone it only breaks SO(10) to SU(5) [13]. We concentrate on one possible choice for the second VEV, the VEV v_U of the (1, 1, 1) of the **210**, which by itself alone would break SO(10) to G_{224}. In order to achieve unification it is necessary that v_U be greater than v_R. We assume that all the gauge bosons in SO(10) that can cause proton decay have a mass $M_U \approx v_U$, so that M_U defines the unification scale. To satisfy the present experimental bound on proton decay $\Gamma(p \to e^+\pi) > (5\times 10^{32}\,\mathrm{yr})^{-1}$ [14], we require [15] $M_U > 1\times 10^{15}$ GeV, for a unified coupling constant $\omega_U \equiv 4\pi/\alpha_U^2 \approx 40$. In SO(10) there also are nine gauge bosons of G_{224} which are not gauge bosons of the standard model, but do not lead to proton decay. We assume these to have a mass $M_R \approx v_R$. Our basic assumption of a single unification scale with threshold effects leads to the requirement

$$M_U/M_R \lesssim 30\,. \tag{3}$$

We thus might say that we have an intermediate-energy range, from M_U to M_R, in which the gauge symmetry is G_{224}; but, as M_R is so close to M_U, we find it more appropriate to describe the process as a direct breaking of SO(10) to the SM, with threshold effects; and we will not bother to trade the gauge couplings of the SM by the ones of G_{224} at v_R. We require all the Higgs representations to have masses between M_R and M_U, except for the standard-model Higgs doublet, coming from the (2, 2, 1) of the **10**, which necessarily lies at the Fermi scale. By doing this we restrict all the threshold effects to the energy range from M_U to M_R, which is constrained by Eq. (3). No fine-tuning, other than the minimal one required to have one Higgs doublet at the Fermi scale, is needed in our picture of the breaking. The mass of the (2, 2, 15) of the **126** is restricted by Eq. (2). Finally, the mass of the (1, 3, 10) of the **126** is required not to be greater than $3v_R$, because v_R is precisely the VEV of one component of the (1, 3, 10).

II. THE THRESHOLD EFFECTS

We use the two-loop renormalization-group equations (RGE's) for the evolution of the gauge coupling constants of the SM from M_Z to M_U. We define $\omega_i \equiv 4\pi/g_i^2$, where the hypercharge gauge coupling g_1 is normalized in the GUT fashion. The two-loop RGE's are

$$\frac{d\,\omega_i(\mu)}{d\ln\mu} = -\frac{a_i}{2\pi} - \sum_j \frac{b_{ij}}{8\pi^2\omega_j}\,, \tag{4}$$

where ω_1 corresponds to $U(1)_Y$, ω_2 to $\mathrm{SU}(2)_L$, and ω_3 to $\mathrm{SU}(3)_C$. The matrix of the one-loop coefficients a and the matrix of the two-loop coefficients b are

$$a = \begin{pmatrix} \frac{41}{10} \\ -\frac{19}{6} \\ -7 \end{pmatrix}, \quad b = \begin{pmatrix} \frac{199}{50} & \frac{27}{10} & \frac{44}{5} \\ \frac{9}{10} & \frac{35}{6} & 12 \\ \frac{11}{10} & \frac{9}{2} & -26 \end{pmatrix}. \tag{5}$$

These coefficients include the effects of the standard-model gauge bosons, of three generations of fermions, and of one Higgs doublet. The threshold effects at M_U are taken into account, following the prescription of the "effective gauge theory" [16,17], by means of the boundary conditions at M_U:

$$\omega_G(M_U) = \omega_1(M_U) + \frac{\lambda_1}{12\pi} = \omega_2(M_U) + \frac{\lambda_2}{12\pi} = \omega_3(M_U) + \frac{\lambda_3}{12\pi}\,, \tag{6}$$

where the subscript G stands for SO(10), and the three constants λ_1, λ_2, and λ_3 include the effects of the superheavy gauge and Higgs bosons having masses which are not exactly equal to M_U.

We shall treat $\omega_1(M_Z)$ and $\omega_2(M_Z)$ as input, with fixed values (their error bars are very small) [3]

$$\omega_1(M_Z) = \frac{1}{0.016887}\,, \quad \omega_2(M_Z) = \frac{1}{0.03322} \quad (M_Z = 91.173\,\mathrm{GeV})\,. \tag{7}$$

Similarly, the λ's will be treated as input, and the output values will be M_U, which we require to be larger than 10^{15} GeV, $\omega_G(M_U)$, and $\omega_3(M_Z)$, which should agree with the experimental value [2]

$$\frac{1}{\omega_3(M_Z)} = 0.120 \pm 0.007\,. \tag{8}$$

At the one-loop level, the problem can be solved analytically, because in that case the three ω_i have constant slopes relatively to $\ln\mu$. The solution is

$$\ln\frac{M_U}{M_Z} = \frac{1}{218}\,[\,60\,\pi\,\omega_1(M_Z) - 60\,\pi\,\omega_2(M_Z) + 5(\lambda_1 - \lambda_2)]\,, \tag{9}$$

$$\omega_G(M_U) = \frac{1}{218}\left[\,95\,\omega_1(M_Z) + 123\,\omega_2(M_Z) + \frac{95\lambda_1 + 123\lambda_2}{12\pi}\right], \tag{10}$$

$$\omega_3(M_Z) = \frac{1}{218}\left[\,-115\,\omega_1(M_Z) + 333\,\omega_2(M_Z) + \frac{-115\lambda_1 + 333\lambda_2 - 218\lambda_3}{12\pi}\right]. \tag{11}$$

In practice, all the numerical results that we shall give were found numerically by integration of the two-loop RGE; nonetheless, the one-loop analytical solutions are useful in guiding us as to what behavior we should expect from the two-loop numerical ones. If we neglect the threshold effects, i.e., set all three λ's equal to zero, we obtain from Eqs. (9) and (11) the results $M_U = 7.8\times10^{12}$ GeV and $\omega_3(M_Z) = 14.7$. Clearly, M_U is too low and $\omega_3(M_Z)$ is too high, and these are the problems referred to in the Introduction.

The gauge bosons with mass $M_R < M_U$ help us solve this problem. It is important to realize that this is possible in the GUT SO(10) but not in the GUT SU(5) [13], because in SU(5) there are no gauge bosons such as the ones of G_{224}, which do not cause proton decay and therefore may have a mass somewhat lower than M_U. The gauge bosons give a large contribution to the λ's:

$$\lambda_i^V = \sum_m l_i^{V_m}\left(1 + 21\ln\frac{M_U}{M_{V_m}}\right), \tag{12}$$

where the index m runs over all the standard-model representations of superheavy gauge bosons, which are listed in Table I, and the l_i are the corresponding Dynkin indices relative to the standard-model gauge group i, multiplied by their dimensions relative to the other two

TABLE I. Standard-model representations of the superheavy gauge bosons, their Dynkin indices and their masses.

$\mathrm{SU}(2)_L \otimes \mathrm{SU}(3)_C[\mathrm{U}(1)_Y]$	l_1	l_2	l_3	P	Q	Mass
(1,1)[0]	0	0	0	0	0	M_R
$(1,1)[\sqrt{\frac{3}{5}}]$	$\frac{3}{5}$	0	0	3	- 69	M_R
$(1,1)[-\sqrt{\frac{3}{5}}]$	$\frac{3}{5}$	0	0	3	- 69	M_R
$(1,3)[\frac{2}{3}\sqrt{\frac{3}{5}}]$	$\frac{4}{5}$	0	$\frac{1}{2}$	4	- 201	M_R
$(1,\overline{3})[-\frac{2}{3}\sqrt{\frac{3}{5}}]$	$\frac{4}{5}$	0	$\frac{1}{2}$	4	- 201	M_R
$(2,3)[\frac{1}{6}\sqrt{\frac{3}{5}}]$	$\frac{1}{10}$	$\frac{3}{2}$	1	- 7	270	M_U
$(2,\overline{3})[-\frac{1}{6}\sqrt{\frac{3}{5}}]$	$\frac{1}{10}$	$\frac{3}{2}$	1	- 7	270	M_U
$(2,3)[-\frac{5}{6}\sqrt{\frac{3}{5}}]$	$\frac{5}{2}$	$\frac{3}{2}$	1	5	- 6	M_U
$(2,\overline{3})[\frac{5}{6}\sqrt{\frac{3}{5}}]$	$\frac{5}{2}$	$\frac{3}{2}$	1	5	- 6	M_U

standard-model gauge groups. From Eqs. (9) and (11) we see that we should be particularly interested in the quantities

$$P \equiv 5(l_1 - l_2)\,, \tag{13}$$

$$Q \equiv -115 l_1 + 333 l_2 - 218 l_3\,, \tag{14}$$

which therefore are also listed in Table I.

It is appropriate to note that the assumption that all the gauge bosons which may cause proton decay have a single mass M_U, while all the superheavy gauge bosons which do not lead to proton decay have a common mass M_R, is only a symplifying assumption. In principle, the nine representations of superheavy gauge bosons in Table I are allowed, from our point of view, to have nine different masses, in the neighborhood of M_U and M_R.

The fact that P is positive for the gauge bosons with mass $M_R < M_U$ indicates that these gauge bosons help render M_U larger; similarly, the fact that Q is negative for those gauge bosons indicates that they also render $\omega_3(M_Z)$ smaller. Both effects are desirable, and they are stronger the larger the ratio $M_U{:}M_R$ is. Notice that it is desirable that as many gauge bosons as possible have mass $M_R < M_U$; that is the reason why it is desirable, from our point of view, that the intermediate-energy symmetry group be G_{224} instead of one of its subgroups. (Compare for instance the cases 1b and 2b of Ref. [5].)

From the data in Table I and from Eq. (12) we conclude that

$$\begin{aligned} \lambda_1^V &= 8 + \frac{294}{5} \ln \frac{M_U}{M_R}\,, \\ \lambda_2^V &= 6\,, \\ \lambda_3^V &= 5 + 21 \ln \frac{M_U}{M_R}\,. \end{aligned} \tag{15}$$

Let us now turn to the threshold effects of the superheavy Higgs bosons. These are given by

$$\lambda_i^S = -\sum_n l_i^{S_n} \ln \frac{M_U}{M_{S_n}}\,, \tag{16}$$

where the sum runs over all the scalar representations which are not Goldstone bosons of the breaking of SO(10) to the SM. The meaning of the l_i is the same as before, being understood, however, that, if the scalar-boson representation is complex, the l_i must be multiplied by 2. Notice that the contributions of the Higgs bosons to the threshold effects are much smaller than the ones of the gauge bosons just because of the presence of the factor 21 in Eq. (12). But there are many more Higgs bosons than gauge bosons.

In order to simplify our study, we shall not decompose each representation of scalars of SO(10) into representations of the SM, and take into account that each SM representation may in principle have a different mass; rather, we shall make the simplifying assumption that all the SM

TABLE II. G_{224} representations of the superheavy scalars contained in the **210** of SO(10), and their Dynkin indices.

G_{224}	l_1	l_2	l_3	P	Q	Mass
(1, 1, 1)	0	0	0	0	0	
(2, 2, 6)	$\frac{26}{5}$	6	4	-4	528	
(1, 1, 15)	$\frac{8}{5}$	0	4	8	-1056	M_1
(2, 2, 10)	$\frac{54}{5}$	10	12	4	-528	M_1
(2, 2, $\overline{10}$)	$\frac{54}{5}$	10	12	4	-528	M_1
(1, 3, 15)	$\frac{114}{5}$	0	12	114	-5238	M_4
(3, 1, 15)	$\frac{24}{5}$	30	12	-126	6822	M_5

TABLE III. G_{224} representations of the superheavy scalars contained in the **126** of SO(10), and their Dynkin indices.

G_{224}	l_1	l_2	l_3	P	Q	Mass
(1, 1, 6)	$\frac{4}{5}$	0	2	4	−528	M_1
(2, 2, 15)	$\frac{154}{5}$	30	32	4	−528	M_1
(1, 3, 10)	$\frac{156}{5}$	0	18	156	−7512	M_2
(3, 1, $\overline{10}$)	$\frac{36}{5}$	40	18	−164	8568	M_3

representations contained in one representation of G_{224} have the same mass. Then, we need only give the branching of each SO(10) representation into representations of G_{224}.

We first consider the scalars in the **210** of SO(10). The branching of that representation in representations of G_{224} is given in Table II. We need not take into account the (1, 1, 1) of G_{224}, because it does not contribute to the λ's. We must also discard the (2, 2, 6) of G_{224}, because it consists of the Goldstone bosons of the breaking of SO(10) to G_{224}. Considering the values of P and Q of the other representations given in Table II, we conclude that most G_{224} representations in the **210** have very small effects on M_U and on $\omega_3(M_Z)$. The exceptions are the (1, 3, 15), which is strongly detrimental (if its mass is lower than M_U) for unification, and the (3, 1, 15), which is strongly beneficial.

We now consider the scalars of the **126** of SO(10). The branching of this representation into representations of G_{224} is given in Table III. However, implicitly included in Table III are also the three SM representations of Table IV, which are contained in the (1, 3, 10) of G_{224} and should not be taken into account, because they are the Goldstone bosons of the breaking of G_{224} to the standard model. In both Tables III and IV, the values of l_1, l_2, and l_3 have been multiplied by 2, because the **126** is complex. Considering the values of P and Q of each scalar representation in Table III we conclude that the (1, 1, 6) and the (2, 2, 15) of G_{224} have relatively small effects on M_U and on $\omega_3(M_Z)$, such that they would not be able to contribute significantly to correct the bad values of these parameters, except if their masses were very far from M_U; the (1, 3, 10) has bad effects on M_U (it makes it decrease) and on $\omega_3(M_Z)$ (it makes it increase) if it has a mass smaller than M_U; the (3, 1, $\overline{10}$), on the other hand, contributes to increase M_U and to decrease $\omega_3(M_Z)$, if it has a mass smaller than M_U.

For our numerical examples of the next section, we group the Higgs bosons together in five sets with different masses M_1–M_5, as indicated in Tables II and III. On considering the multiplet (1, 3, 10) with mass M_2, we naturally exclude the Goldstone bosons indicated in Table IV. Then, from Tables II–IV and from Eq. (16), we find the scalar contributions to the λ's:

$$\begin{aligned} \lambda_1^S &= -\frac{274}{5}\ln\frac{M_U}{M_1} - \frac{142}{5}\ln\frac{M_U}{M_2} - \frac{36}{5}\ln\frac{M_U}{M_3} \\ &\quad -\frac{114}{5}\ln\frac{M_U}{M_4}, \\ \lambda_2^S &= -50\ln\frac{M_U}{M_1} - 40\ln\frac{M_U}{M_3} - 30\ln\frac{M_U}{M_5}, \\ \lambda_3^S &= -62\ln\frac{M_U}{M_1} - 17\ln\frac{M_U}{M_2} - 18\ln\frac{M_U}{M_3} \\ &\quad -12\ln\frac{M_U}{M_4} - 12\ln\frac{M_U}{M_5}. \end{aligned} \tag{17}$$

Together with the threshold effects from the superheavy gauge bosons, see Eqs. (15), we then obtain from Eqs. (9) and (11), and from the input data in Eq. (7), the one-loop-level results

$$\ln\frac{M_U}{M_Z} = 25.220233 + \frac{1}{109}\left(147\ln\frac{M_U}{M_R} - 12\ln\frac{M_U}{M_1} - 71\ln\frac{M_U}{M_2} + 82\ln\frac{M_U}{M_3} - 57\ln\frac{M_U}{M_4} + 63\ln\frac{M_U}{M_5}\right), \tag{18}$$

$$\omega_3(M_Z) = 14.742162 + \frac{1}{218\pi}\left(-945\ln\frac{M_U}{M_R} + 264\ln\frac{M_U}{M_1} + 581\ln\frac{M_U}{M_2} - 714\ln\frac{M_U}{M_3} + 436.5\ln\frac{M_U}{M_4} - 568.5\ln\frac{M_U}{M_5}\right), \tag{19}$$

TABLE IV. Goldstone-boson representations contained in the (1, 3, 10) of G_{224}, and their Dynkin indices.

$SU(2)_L \otimes SU(3)_C[U(1)_Y]$	l_1	l_2	l_3	P	Q
$(1,3)[\frac{2}{3}\sqrt{\frac{3}{5}}]$	$\frac{8}{5}$	0	1	8	- 402
(1,1)[0]	0	0	0	0	0
$(1,1)[-\sqrt{\frac{3}{5}}]$	$\frac{6}{5}$	0	0	6	- 138

TABLE V. Numerical examples, obtained using the two-loop RGE, for how the threshold effects can lead to correct values for M_U and $\alpha_S(M_Z)$.

$\frac{M_U}{M_R}$	$\frac{M_U}{M_1}$	$\frac{M_U}{M_2}$	$\frac{M_U}{M_3}$	$\frac{M_U}{M_4}$	$\frac{M_U}{M_5}$	$M_U/(10^{15}\text{GeV})$	ω_G	$\alpha_S(M_Z)$
30	8	20	12	5	25	1.727	37.427	0.119
25	3	23	18	2	10	1.773	39.037	0.123
20	4	10	16	2	7	1.628	39.023	0.121
15	2	7	9	1.5	14	1.695	40.017	0.126
10	2.5	3.5	9	1	10	1.529	39.938	0.125

which may be compared with the two-loop numerical examples presented in the next section.

III. NUMERICAL EXAMPLES

We have numerically integrated the two-loop RGE, for values of M_U/M_R between 10 and 30 and for random choices of M_1–M_5 lying between M_U and M_R, subject to the following constraints: (1) M_1 = 3–10 times M_R, in order to satisfy Eq. (2), and (2) $M_2 \leq 3M_R$. Using Eqs. (7), we have solved for $\alpha_s(M_Z)$, M_U, and ω_G. The solutions in general do not differ very much from those obtained using the one-loop RGE. It is easy to find solutions that give satisfactory values for $\alpha_s(M_Z)$ with $M_U > 10^{15}$ GeV as required from proton decay. A few examples are given in Table V. For all the examples given and for nearly all our solutions the minimum expected rate of proton decay is within a factor of 5–10 of the present experimental limit. The values of $\alpha_s(M_Z)$ for our solutions are generally 0.12 or higher.

It is interesting to visualize what the threshold effects are really doing to the coupling constants. This can be seen, in one particular example (the third example of Table V), in Fig. 1. In that figure, the values of

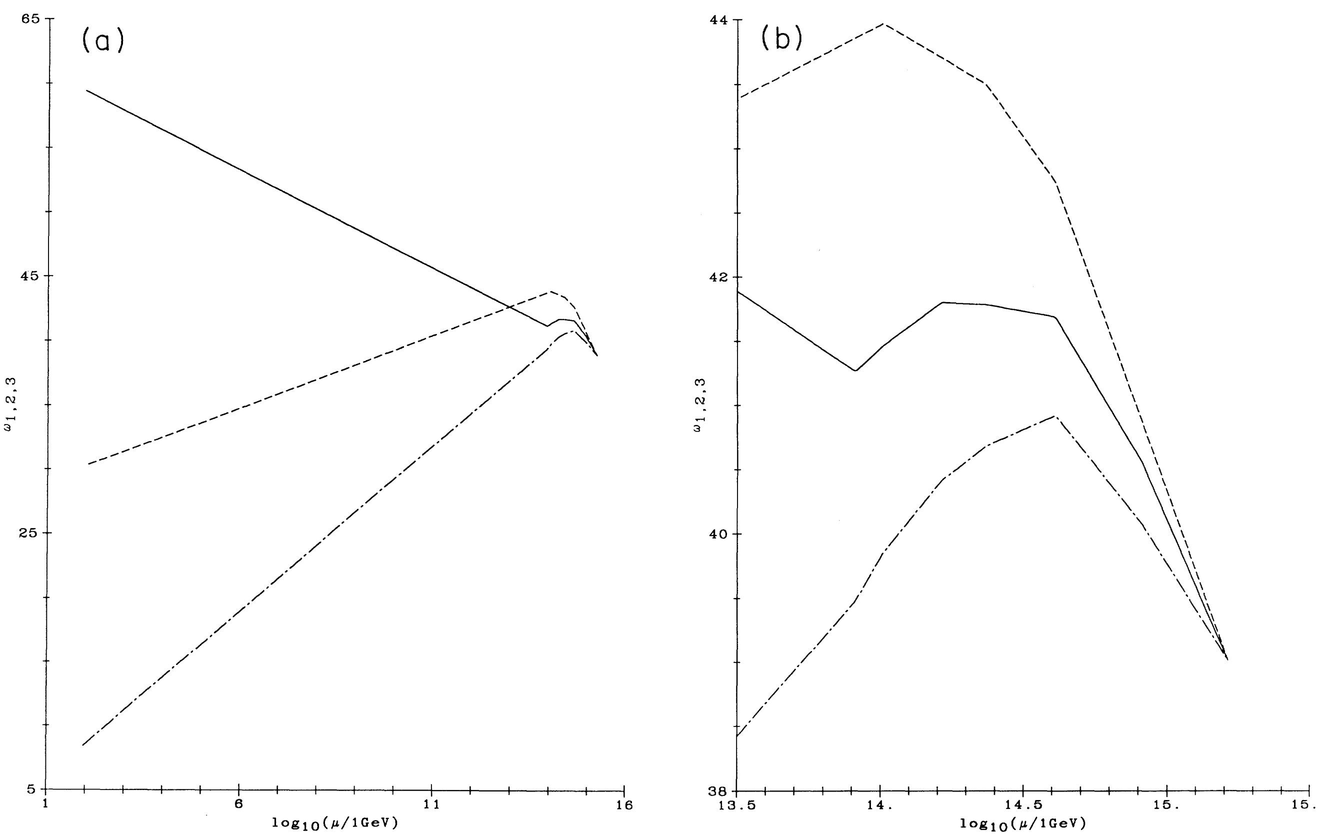

FIG. 1. (a) Graphs of $\omega_1(\mu)$ (full line), $\omega_2(\mu)$ (dashed line), and $\omega_3(\mu)$ (dashed-dotted line) as functions of μ, for the third example of Table V, and with the threshold effects taken into account as explained in the last paragraph of Sec. III. (b) Same as (a), but displaying only the region $\mu > 10^{13.5}$ GeV, so that the individual threshold effects may be recognized.

$$\overline{\omega_1}(\mu) \equiv \omega_1(\mu) + \frac{8}{12\pi} + \frac{294}{5}\frac{\theta(\mu - M_R)}{12\pi}\ln\frac{\mu}{M_R} - \frac{274}{5}\frac{\theta(\mu - M_1)}{12\pi}\ln\frac{\mu}{M_1} - \cdots ,$$

$$\overline{\omega_2}(\mu) \equiv \omega_2(\mu) + \frac{6}{12\pi} - 50\frac{\theta(\mu - M_1)}{12\pi}\ln\frac{\mu}{M_1} - \cdots , \quad (20)$$

$$\overline{\omega_3}(\mu) \equiv \omega_3(\mu) + \frac{5}{12\pi} + 21\frac{\theta(\mu - M_R)}{12\pi}\ln\frac{\mu}{M_R} - 62\frac{\theta(\mu - M_1)}{12\pi}\ln\frac{\mu}{M_1} - \cdots ,$$

(where Heaviside θ functions have been used to simplify the notation) were plotted against $\log_{10}\mu$. For $\mu = M_U$ we obtain $\overline{\omega_1}(\mu) = \overline{\omega_2}(\mu) = \overline{\omega_3}(\mu)$, as in Eq. (6). In Fig. 1(a) one sees that ω_1 and ω_2 meet before M_R and then diverge from each other, but before they are too far away the threshold effects set in and unify them again, this time, however, also with ω_3, at the energy scale M_U. In Fig. 1(b) one observes in detail the various thresholds: at M_R the gauge bosons of G_{224} deflect the β functions upwards, and after that, at the various scalar thresholds, the scalars have the effect of deflecting the β functions downwards, till finally the gauge couplings meet.

IV. CONCLUSIONS

In this work we have speculated on the possibility that threshold effects may resuscitate SO(10) and restore grand unification. While other popular ideas restore unification by "populating the desert" at low energies, as with supersymmetry [3], or at medium energies, as in the intermediate-scales approach [4, 5], we do it at high energies, by assuming that some superheavy particles have masses $\sim$ 10 times smaller than the ones of the gauge bosons responsible for proton decay. We find that this is indeed possible in the case of the GUT SO(10), because that model contains some superheavy gauge bosons which do not lead to proton decay, and some large and left-right-asymmetric representations of superheavy scalars, which have a powerful effect on the β functions, such that they may unify the coupling constants even if they are given only one order of magnitude in the energy scale to act. Our picture of symmetry breaking is, however, quite constrained, such that it only works if $\alpha_s(M_Z)$ is not below the present experimental value, and such that it predicts the rate of proton decay not to be very far from the present experimental bound.

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