Breaking of conformal invariance and electromagnetic field generation in the Universe

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It is shown that the breaking of conformal invariance in quantum electrodynamics due to the trace anomaly results in the generation of long-wave electromagnetic fields during the inflationary stage of the evolution of the Universe. If the coefficient of the logarithmic charge renormalization is large (due to a large number of charged particles species), these primordial electromagnetic fields can be strong enough to create the observed galactic magnetic fields.

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It is well known that quantum fluctuations of massless scalar and tensor (gravitational) fields are very much amplified at the inflationary stage and create considerable density inhomogeneities [1–4] or relic gravitational waves [5–12]. This is closely related to the fact that these fields are not conformally invariant even though they are massless. Conformal invariance of a massless scalar field ϕ can be only achieved by the nonminimal coupling to the curvature scalar $R\phi^2/6$, and in this case quantum fluctuations are not amplified in the Robertson-Walker background. Note, however, that this condition is obligatory but not sufficient. For example, quantum fluctuations of a scalar field are not amplified in the case of conformally noninvariant coupling $\xi R\phi^2$ with positive and not too small ξ not equal to $1/6$. Gravitational waves are also conformally noninvariant in standard general relativity [13]. The amplification of the quantum fluctuations in cosmological conditions can be understood as particle production by an external gravitational field. Since the Robertson-Walker metric is known to be conformally flat, the background gravitational field does not produce particles if the underlying theory is conformally invariant [14]. In particular, electromagnetic waves are not produced in such conditions since the classical electrodynamics is conformally invariant in the limit of vanishing masses of fermions. Quantum corrections, however, are known to break conformal invariance. There are three possible sources of its breaking: nonzero masses of charged particles, the absence of conformal invariance for (even massless) scalar field as mentioned above, and the quantum conformal anomaly due to the celebrated triangle diagrams [15]. In what follows we consider the last case because the other two produce generically much smaller effects.

It was shown in Ref. [16] that production of photons in a conformally flat cosmological background due to the trace anomaly can be considerable and that the Maxwell equations are modified by the anomaly in the following way:

$$\partial_\mu F^\mu_\nu + \kappa \frac{\partial_\mu a}{a} F^\mu_\nu = 0, \tag{1}$$

where $a = a(\tau)$ is the scale factor, τ is the conformal time, the metric has the form $ds^2 = a^2(\tau)(d\tau^2 - dr^2)$, and the contraction of the indices is made with the metric tensor of the flat space-time. The numerical coefficient κ in SU(N)-gauge theory with N_f number of charged fermions is equal to

$$\kappa = \frac{\alpha}{\pi}\left(\frac{11N}{3} - \frac{2N_f}{3}\right). \tag{2}$$

Here α is the fine-structure constant which is to be taken at the momentum transfer p equal to the Hubble parameter during inflation, $p = H$. In the asymptotically free theory one would expect $\alpha \approx 0.02$.

It is convenient to choose the gauge condition $\partial_\mu A^\mu = 0$. In this gauge the time component of the vector potential A_τ satisfies the same equation as in the conformally invariant case while the space component is modified by the extra term in the equations of motion:

$$\left(\partial_\tau^2 - \frac{1}{a^2}\partial_l^2 + \kappa\frac{a'}{a}\partial_\tau\right) A_j(\tau, r) = 0, \tag{3}$$

where prime means differentiation with respect to conformal time τ and we put $A_\tau = 0$ since this component is not amplified by the Robertson-Walker background.

Quantization of the electromagnetic field in this background metric with the account of the conformal invariance breaking can be made with the usual decomposition

$$A_j(\tau, r) = \int \frac{d^3k}{(2\pi)^{3/2}(2\omega)^{1/2}}[c_j(k)A(\tau,k)e^{-ikr} + \text{H.c.}], \tag{4}$$

where operators c_j satisfy

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$$\langle c_j(k)c^{*l}\rangle_{\rm vac} = \delta(k'-k)(\delta_j^l - k_j k^l/k^2). \quad (5)$$

The C function $A(\tau,k)$ satisfies the equation

$$A'' + k^2 A + \kappa\frac{a'}{a}A' = 0. \quad (6)$$

Note that the amplitudes of massless minimally coupled scalar field and of gravitational wave satisfy the same equation with $\kappa = 2$.

At the inflationary stage when $a'/a = -1/\tau$, Eq. (6) is solved as

$$A(k,\tau) = (k\tau)^\nu C_1[J_\nu(k\tau) + C_2 J_{-\nu}(k\tau)], \quad (7)$$

where $\nu = (\kappa+1)/2$.

The coefficients C_1 and C_2 can be found from the condition that $A(k,\tau)$ tends to $\exp(i\omega t)$ in the limit of a vanishing Hubble parameter. In the de Sitter stage, $\tau \approx (1/H - t)$, as $H \to 0$, where t is the physical time. Using this equation and the asymptotic expressions for the Bessel functions (as $k\tau \to \infty$) we find

$$C_1 = \frac{\sqrt{2\pi}}{1-\exp(-2i\pi\nu)}\left(\frac{H}{k}\right)^{\kappa/2}\exp\left(\frac{ik}{H} - \frac{i\pi}{4} - \frac{i\pi\nu}{2}\right), \quad (8)$$

$$C_2 = -\exp(-i\pi\nu). \quad (9)$$

Near the end of inflation, that is, for $\tau \to 0$, $A(k,\tau)$ is given by the expression

$$|A(k,\tau)| \to \frac{\sqrt{2\pi}}{2^\nu|1-\exp(-2i\pi\nu)|\Gamma(\nu+1)}\left(\frac{H}{k}\right)^{\kappa/2}. \quad (10)$$

Here k is the conformal momentum related to the physical one by the scale factor, $p = a(t)k$.

The evolution of these waves, after inflation was over, is also given by Eq. (6) with $a'/a = 1/\tau$ at the radiation domination (RD) stage and with $a'/a = 2/\tau$ at the matter domination (MD) stage. Note that the conformal time τ has now a different behavior than during inflation. Namely, at the inflationary stage $\tau \to 0$, while at the RD or MD stages τ goes to infinity. We have to solve Eq. (6) with the initial condition (10), but the behavior of the solution is clear from the following simple arguments. When the wavelength is large in comparison with the horizon its amplitude remains constant until it reenters the horizon (it is seen, of course, from the formal solution). Note that only waves which were much larger than the horizon at the end of inflation are known and are given by

$$K(p,k;g(\mu),m(\mu),\xi(\mu);\mu) = K(p,k;\bar g(t),\bar m(t),\bar\xi(t);\mu e^t)\exp\left[-2\int_{g(\mu)}^{\bar g(t)} dx\frac{\gamma_F(x)}{\beta(x)}\right], \quad (11)$$

where

$$t = \int_{g(\mu)}^{\bar g(t)}\frac{dx}{\beta(x)}, \qquad \bar m(t) = m(\mu)\left[\int_{g(\mu)}^{\bar g(t)} dx\frac{\gamma_m(x)}{\beta(x)}\right], l^4. \quad (12)$$

If $\kappa \ll 1$, the corresponding field is negligibly small, but for $\kappa \sim 1$ the amplitude of the magnetic field can be large enough to seed the observed magnetic fields in galaxies. Indeed, it is known from observations that the strength of the galactic magnetic fields is of the order of $B = 10^{-6}$ G, and correspondingly its energy density is close to that of the electromagnetic background radiation. The field given by Eq. (10) is about 10^{40} below the necessary value if $(Hl)^\kappa \sim 1$. However, the factor Hl is extremely large. For $H = 10^{12}$ GeV and $l = L_{\rm gal}$ it is about 10^{46}. So for positive $\kappa \sim 1$ the magnetic field generated during the inflationary stage can be large enough to create the observed fields in galaxies even without the dynamo amplification [17,18].

Note that the mechanism discussed here is operative only for asymptotically free theories with $\kappa > 0$. If κ is dominated by fermions and thus is negative, the mechanism is not effective, although the conformal invariance is broken and massless particle production may be noticeable. In the simplest version of the grand unified SU(5) model with three generation of fermions $\kappa \approx 0.06$. This value is too small to get the necessary amplification of electromagnetic quantum fluctuations during inflation, and so a larger group with a larger number of charged bosons is required even if the dynamo mechanism is operative. The latter may amplify the seed magnetic field by about ten orders of magnitude, permitting a slightly smaller value of κ.

In contrast with the magnetic field the electric field generated during the inflationary stage not only is not amplified, but is dumped down due to the large conductivity of the primeval plasma. One may argue that the conductivity is nonzero and large already during the de Sitter stage because of the thermal background with a temperature $H/2\pi$ [19]. This background is associated with the horizon at $1/H$. However, the particle polarization which could damp the electric field cannot compete with the universe expansion and hence is not able to screen the field. The electric field should be screened during the Friedmann stage after the inflaton energy density is transformed into the energy density of real particles in the cosmic plasma. The characteristic time scale of this process is of the order of the horizon, and so one may expect generation of chaotic electric currents on cosmologically large scales. These chaotic electric currents may in turn generate cosmic magnetic fields.

Note that there are several other proposals in the literature for magnetic field generation in a theory with broken conformal invariance. In Ref. [20] it was assumed that there exists a nonminimal coupling of the curvature scalar to the electromagnetic field of the form $\xi R A_\mu^2$ so that not only conformal but also gauge invariance of the electromagnetism is broken. Another proposal [21] is the coupling of the electromagnetic field strength tensor to the dilaton field $\exp(\phi)F_{\mu\nu}^2$. In both these cases it was argued that it is possible to generate the seed magnetic field of the appropriate amplitude during inflation. Our model does not demand any extra coupling in addition to the standard electrodynamics, but to get a large enough field the number of charged bosons with $m < H$ should be about 30 (as we have already noted, the increase of the number of charged fermions acts in the opposite direction, reducing the field amplification). Reversing the arguments one can put a bound on κ from the absence of too strong cosmic electromagnetic fields.

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