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On the renormalization-group analysis of gauge groups containing U(1) ⊗ U(1) factors

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I point out the existence of misconceptions of a similar nature in the analyses of two different cases in which a grand unified theory—SO(10) in the first case, SU(8) in the second one—breaks to the standard model via an intermediate-energy gauge group containing a factor U(1) ⊗ U(1). In the second case, false conclusions have been drawn concerning the possible unification of the gauge couplings. The erroneous procedure has been to use two U(1) factors which, though orthogonal relative to complete representations of the grand unification group, are not orthogonal when considering only the low-mass fields.

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I. INTRODUCTION

The purpose of this Brief Report is to correct the inadequate renormalization-group (RG) treatments of two different cases in which a grand unified theory (GUT) breaks to the standard model (SM) through an intermediate-energy gauge group containing two U(1) factors. In Secs. II and III, I present those treatments as they were given in the literature, and show that they lead to a paradox.[1] In Sec. IV, I outline the correct way of analyzing such cases. It is possible that there exist in the literature other instances of the same inadequate treatment, beyond the ones that I present in Secs. II and III.

The correct treatment of cases in which multiple U(1) factors are present differs, for each particular case, according to the set of fields which is considered to be light. The RG treatment implies the calculation of loop diagrams. At the tree level, the U(1) factors may be arbitrarily chosen, any orthogonal rotation of one choice yielding another choice which is just as good. But at the one- or two-loop levels, this is no longer true. Loops of light particles lead to divergent off-diagonal self-energies, which means that the gauge bosons of an arbitrary choice of U(1) factors will not in general be the eigenstates of propagation. One must diagonalize the matrix of self-energies in order to identify the correct choice of U(1) factors, and only afterwards can one proceed with the RG analysis. This must be done order-by-order in perturbation theory; in this Brief Report I shall remain at the one-loop level.

II. FIRST EXAMPLE

The first example [1,2] of the paradox is in a breaking chain of SO(10):

$$\begin{aligned} \mathrm{SO}(10) &\to \mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_R \otimes \mathrm{U}(1)_X \\ &\to \mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y , \end{aligned} \tag{1}$$

where X is proportional to the baryon number minus the lepton number, and Y is the standard-model hypercharge. The first symmetry breaking occurs at the energy scale M_U; the second one occurs at the energy scale M_I. R and X are two orthogonal and correctly normalized generators of SO(10); Y and another U(1) charge Z are given by an orthogonal rotation of R and X:

$$Y = \sqrt{3/5}R - \sqrt{2/5}X, \quad Z = \sqrt{2/5}R + \sqrt{3/5}X . \tag{2}$$

All U(1) charges in this Brief Report are normalized in the GUT fashion.

There are in the theory a large number of scalars with masses of order M_U, which are not relevant in this context, because they decouple. The RG evolution from M_U to M_I is governed by the low-mass particles only, which, in addition to the SM gauge bosons, consist in three families of fermions, one scalar doublet, and one scalar singlet S. The values of R and X, and also of Y and Z, for these fermion and scalar fields, are given in Table I.

The RG equation at the one-loop level for the evolution of the gauge coupling g_i of the gauge group i is [3]

$$dg_i/d\ln\mu = (1/16\pi^2)\, a_i g_i^3 \tag{3}$$

(no summation in i), which becomes linear if one uses instead of g_i the variable $\omega_i \equiv 4\pi/g_i^2$:

$$d\omega_i/d\ln\mu = -a_i/2\pi . \tag{4}$$

The coefficient a_i is given, in the case of a general U(1) charge K, by the trace of K^2 over all the low-mass

TABLE I. R, X, Y, and Z values of the low-mass fermions and scalars in the SO(10)-breaking chain of Eq. (1).

Field	R	$\sqrt{8/3}X$	$\sqrt{60}Y$	$\sqrt{40}Z$
$\begin{pmatrix}\nu_L \\ e_L\end{pmatrix}$	0	1	−3	3
$\bar{\nu}_L$	−1/2	−1	0	−5
$\bar{e}_L$	1/2	−1	6	−1
$\begin{pmatrix}u_L \\ d_L\end{pmatrix}$	0	−1/3	1	−1
$\bar{u}_L$	−1/2	1/3	−4	−1
$\bar{d}_L$	1/2	1/3	2	3
$\begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix}$	1/2	0	3	2
S	1	2	0	10

[1] Section III may be skipped without inconvenience.

fermions and scalars (the high-mass ones decouple). In this paper I use a definition of the trace as being 2/3 times the sum over all light fermions, plus 1/3 times the sum over all light scalars. Using this prescription and the values in Table I one finds $a_R = a_X = 9/2$. It is important to keep in mind that the origin of Eq. (3) is the vertex-correction diagram of Fig. 1(a), which is essentially a vacuum-polarization diagram (Fig. 2); the other three vertex-correction diagrams [in Fig. 1(b)] have divergences which cancel due to the Ward identity, and which moreover are individually zero if one computes them in the Landau gauge [4]. If the diagram in Fig. 2 would be nonvanishing and mix R and X, then the diagram in Fig. 1(a) would also yield a term proportional to $g_X^2 g_R$ to the equation for $dg_R/d\ln\mu$.

At M_I the group $\mathrm{U}(1)_R \otimes U(1)_X$ breaks to $\mathrm{U}(1)_Y$ upon S acquiring a vacuum expectation value. As a consequence of the first of Eqs. (2), ω_Y at M_I is given as a function of ω_R and ω_X at the same mass scale by

$$\omega_Y(M_I) = \tfrac{3}{5}\omega_R(M_I) + \tfrac{2}{5}\omega_X(M_I)\,. \tag{5}$$

At M_U all gauge couplings unify, in particular $\omega_R(M_U) = \omega_X(M_U) \equiv \omega_U$. We therefore obtain

$$\omega_Y(M_I) = \omega_U + (\tfrac{9}{2}/2\pi)\ln(M_U/M_I)\,. \tag{6}$$

I now observe that there is a paradox. It is clear that, between M_U and M_I, we might have worked with the gauge group $\mathrm{U}(1)_Y \otimes \mathrm{U}(1)_Z$ [$\otimes\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L$] instead of working with $\mathrm{U}(1)_R \otimes \mathrm{U}(1)_X$. This is because Y and Z, being orthogonal combinations of R and X, have the same properties of orthogonality [between themselves and also to $\mathrm{SU}(3)_C$ and to $\mathrm{SU}(2)_L$] and normalization as R and X. The two sets of U(1) charges are equally legitimate choices. Now, if one computes the trace of Y^2 one obtains $a_Y = 41/10$. Therefore, the procedure of using $\mathrm{U}(1)_Y \otimes \mathrm{U}(1)_Z$ between M_U and M_I, and then dropping $\mathrm{U}(1)_Z$ below M_I, leads to

$$\omega_Y(M_I) = \omega_U + (\tfrac{41}{10}/2\pi)\ln(M_U/M_I)\,, \tag{7}$$

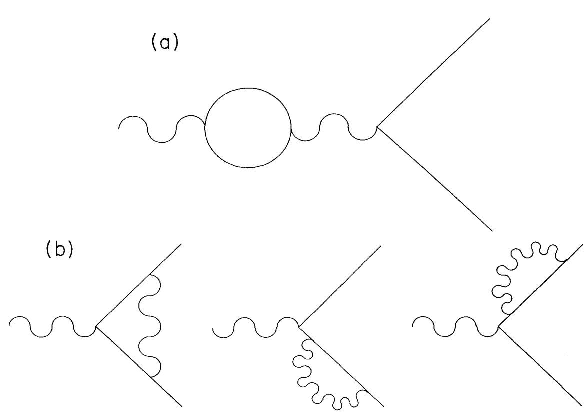

FIG. 1. Feynman diagrams relevant for the computation of the one-loop RG equation for a U(1) gauge coupling. The diagram in (a) is in fact the only relevant one, because the divergences cancel among the diagrams in (b), due to the Ward identity. Similar diagrams with scalars instead of fermions must also be taken into account.

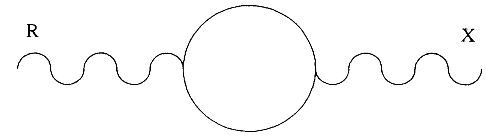

FIG. 2. The vacuum-polarization diagram mixing the gauge bosons of $\mathrm{U}(1)_R$ and $\mathrm{U}(1)_X$. A similar diagram exists with scalars instead of fermions. The sum of the diagrams is proportional to the trace of the product of R and X.

which contradicts Eq. (6).

The difference between Eqs. (6) and (7) is neither very large (9/2 not being too different from 41/10) nor very important, because, in the end, one anyway concludes that the breaking chain of Eq. (1) does not work, because with the new very precise values obtained at the CERN e^+e^- collider LEP for the gauge couplings at M_Z one finds that there is no M_U at which they unify.[2] But it is clear that there is an error of principle somewhere. The error might have had more dramatic consequences if instead of $\mathrm{U}(1)_R \otimes \mathrm{U}(1)_X$ one had used another orthogonal combination of those two U(1) factors. This is illustrated by the following case.

III. SECOND EXAMPLE

The second example [7] is a GUT with a gauge group SU(8) which contains as a subgroup the product of the Georgi-Glashow SU(5) [8] by a gauged horizontal-symmetry group $\mathrm{SU}(3)_H$. The breaking chain considered is

$$\begin{aligned}\mathrm{SU}(8) &\to \mathrm{SU}(3)_H \otimes \mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_A \otimes \mathrm{U}(1)_B \\ &\to \mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y\,.\end{aligned} \tag{8}$$

The Higgs fields used to break $\mathrm{SU}(3)_H$, and to break $\mathrm{U}(1)_A \otimes \mathrm{U}(1)_B$ to $\mathrm{U}(1)_Y$, at M_I, have not been considered in connection with the RG analysis in the original proposal of this theory. I shall also neglect them here, because they are irrelevant for my aim. The low-mass fermions are the usual three families, and the low-mass scalars are two doublets of $\mathrm{SU}(2)_L$. They and their U(1) charges are listed in Table II. The SM hypercharge Y and another U(1) charge W are orthogonal combinations of A and B given by

$$Y = -\tfrac{4}{5}A + \tfrac{3}{5}B\,, \quad W = \tfrac{3}{5}A + \tfrac{4}{5}B\,. \tag{9}$$

It should be noted that in this model there is a host of superheavy fermions, which are needed in order to obtain complete representations of SU(8) and to cancel the anomalies, but this needs not concern us here.

Now, if we compute the traces of A^2 and of B^2 we obtain the results 63/20 and 13/5, respectively. Using then the first of Eqs. (9), we get

[2] The numerical difference between the two procedures may however slightly affect analyses which use the gauge group $\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_R \otimes \mathrm{U}(1)_X$ as part of a three-step breaking chain of SO(10) [5,6].

TABLE II. A, B, Y, and W values of the low-mass fermions and scalars in the SU(8) breaking chain of Eq. (8).

Field	$\sqrt{60}A$	$\sqrt{60}B$	$\sqrt{60}Y$	$\sqrt{60}W$
$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	3	−1	−3	1
$\bar{e}_L$	−6	2	6	−2
$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	−2	−1	1	−2
$\bar{u}_L$	2	−4	−4	−2
$\bar{d}_L$	−1	2	2	1
$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	−3/2	3	3	3/2

$$\begin{aligned}\omega_Y(M_I) &= \omega_U + \frac{1}{2\pi}\left(\frac{16}{25}\frac{63}{20} + \frac{9}{25}\frac{13}{5}\right)\ln\frac{M_U}{M_I} \\ &= \omega_U + \frac{1}{2\pi}\frac{369}{125}\ln\frac{M_U}{M_I}\,. \end{aligned} \tag{10}$$

This result was used in Ref. [7] to obtain unification. It is very different from the one obtained in the standard model with two Higgs doublets:

$$\omega_Y(M_I) = \omega_U + \frac{1}{2\pi}\frac{21}{5}\ln\frac{M_U}{M_I}\,. \tag{11}$$

While unification is impossible (the GUT triangle does not close) using Eq. (11), it is possible (after taking into account the experimental error bars and the two-loop RG effects) using Eq. (10), provided M_I is very close to the Fermi scale. However, the result in Eq. (11) is also obtained if one runs $U(1)_Y \otimes U(1)_W$ between M_U and M_I, instead of running $U(1)_A \otimes U(1)_B$. As the two procedures are equally reasonable, one has here the same paradox as in the preceding section, now with more dramatic consequences. Using the coefficient 369/125 in Eq. (10), one is able to achieve unification, without having to populate the GUT desert with fermions or scalars which deflect the RG evolution, as is done for instance in the supersymmetric standard model [9]. This possibility looks suspect. Is it that by simply changing the gauge group, without populating the desert with any particles, one is able to restore unification?

IV. SOLUTION OF THE PROBLEM

The solution of the paradoxes of the preceding two sections lies in the realization that the vacuum-polarization diagram in Fig. 2 may lead to a divergent mixing of two U(1) gauge bosons when only the low-energy particles run in the loop. Even though the two U(1) charges (say, R and X from the example of Sec. I) are orthogonal in the context of the whole GUT, which means that the trace of (RX) over any representation (of fermions or of scalars) of the grand unification group vanishes, they are in general not orthogonal when only the low mass particles (which include both fermions and scalars, the latter usually not constituting complete representations of the grand unification group) are taken into account. It does not make sense to use the charges R and X if they are not orthogonal relative to the low-mass particles: the vacuum-polarization diagram of Fig. 2, which is proportional to the trace of (RX), leads to a divergent mixing of their gauge fields, which therefore are not the eigenstates of propagation. Under these conditions, it is meaningless to talk about the gauge couplings of $U(1)_R$ and $U(1)_X$ and their RG evolution. When there are two (or more) U(1) factors present, out of all the possible choices of pairs of U(1) factors, related among themselves by orthogonal transformations, one must choose the one in which the trace of the product of the two U(1) charges over the low-mass fields vanishes.[3] This is done by a simple diagonalization of the matrix of the traces.

Let us work out this prescription for the example of Sec. II. In that example, the trace of (YZ) is $1/\sqrt{150}$ and the trace of (RX) is $1/\sqrt{6}$, and therefore neither $U(1)_R \otimes U(1)_X$ nor $U(1)_Y \otimes U(1)_Z$ are the correct choice. The correct choice is $U(1)_M \otimes U(1)_N$:

$$M = Y\cos\theta - Z\sin\theta\,, \quad N = Y\sin\theta + Z\cos\theta\,, \tag{12}$$

with

$$\tan(2\theta) = 2\,\mathrm{tr}\,(YZ)/(\mathrm{tr}\,Z^2 - \mathrm{tr}\,Y^2)\,. \tag{13}$$

This gives

$$\begin{aligned}\cos\theta &= (\sqrt{3}+\sqrt{2})/\sqrt{10}\,, \\ \sin\theta &= (\sqrt{3}-\sqrt{2})/\sqrt{10}\,. \end{aligned} \tag{14}$$

The trace of (MN) is then zero as required.[4] The one-loop RG equations for the gauge couplings of $U(1)_M$ and $U(1)_N$ are

$$\begin{aligned}\frac{d\omega_M}{d\ln\mu} &= -\frac{1}{2\pi}\left(\frac{9}{2} - \frac{\sqrt{6}}{6}\right)\,, \\ \frac{d\omega_N}{d\ln\mu} &= -\frac{1}{2\pi}\left(\frac{9}{2} + \frac{\sqrt{6}}{6}\right)\,. \end{aligned} \tag{15}$$

The correct choice of U(1) factors depends on which fields (fermions and scalars) one considers to have low mass. This is usually decided by means of the minimal fine-tuning hypothesis, but that hypothesis does not necessarily hold. Suppose for instance that, in the example of Sec. II, one wanted to have two low-mass scalar doublets instead of only one. Then, the trace of Y^2 would be 21/5 instead of 41/10, the trace of Z^2 would be 149/30 instead of 49/10, and the trace of (YZ) would be $2/\sqrt{150}$ instead of $1/\sqrt{150}$. Equation (13) would then yield

[3]At the two-loop level one must use the more general requirement that the two-loop self-energies of the U(1) gauge bosons be diagonal, i.e., that the gauge bosons be the eigenstates of propagation at the two-loop level. Only afterwards can one proceed with the two-loop RG analysis.

[4]Indeed, the trace of (MN) is separately zero for the fermion and scalar sectors, a consequence of the fact that the low-energy fermions are in a complete representation of SO(10). But this is not so in general; for instance, it is not so in the SU(8) theory of Sec. III.

$$\cos\theta = 2\sqrt{6}/5\,, \quad \sin\theta = 1/5\,, \tag{16}$$

instead of Eq. (14), and the RG equations for ω_M and ω_N would be

$$\frac{d\omega_M}{d\ln\mu} = -\frac{1}{2\pi}\frac{25}{6}\,, \quad \frac{d\omega_N}{d\ln\mu} = -\frac{5}{2\pi}\,, \tag{17}$$

instead of Eqs. (15). Upon changing the low-mass fields present in the theory one should also change the U(1) factors used in the RG evolution, even if the gauge group does not change.

This also applies at a particle threshold. If one would make the (admittedly rather artificial) assumption that some fermion or scalar fields would have their masses somewhere in the middle of the GUT desert, instead of being either at M_U or at M_I, then at the corresponding thresholds one would have to switch U(1) factors, satisfying the following prescription. As discussed previously, in the energy range just below M_U (and above the first threshold) the U(1) factors must be orthonormal relative to the GUT group, because we want their gauge couplings to unify at M_U, and they must also be orthogonal relative to the particles with masses smaller than that energy range [which means that the trace over those particles of the product of two U(1) charges should vanish]. On the other hand, at all other (lower) intermediate-energy ranges, the U(1) factors must be orthogonal relative to the particles with mass smaller than the particular energy range, but not orthonormal relative to the whole GUT group — nor do they need to be, because in that case the gauge couplings do not unify at the upper boundary of the energy range. At each threshold one has to switch the U(1) factors used in the RG analysis according to these principles.

One now asks what will be the result for the low-energy gauge coupling of the standard-model hypercharge Y if one uses in the running, as one should, the gauge couplings of orthogonal U(1) groups. The answer to this question is given by the following reasoning. Let (for whatever GUT with whatever low-mass fields) $\mathrm{U}(1)_P$ and $\mathrm{U}(1)_Q$ be orthogonal with respect to the low-mass fields, the breaking chain being, as a generalization of Eqs. (1) and (8),

$$\begin{aligned}\mathrm{GUT} &\to G \otimes \mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_P \otimes \mathrm{U}(1)_Q \\ &\to \mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y\,.\end{aligned} \tag{18}$$

The standard-model hypercharge Y will be given by

$$Y = P\cos\psi + Q\sin\psi\,, \tag{19}$$

for some angle ψ. Then

$$\begin{aligned}\omega_Y(M_I) &= \omega_P(M_I)\cos^2\psi + \omega_Q(M_I)\sin^2\psi \\ &= \omega_U + (1/2\pi)[(\mathrm{tr}\,P^2)\cos^2\psi \\ &\qquad + (\mathrm{tr}\,Q^2)\sin^2\psi]\ln(M_U/M_I) \\ &= \omega_U + (1/2\pi)(\mathrm{tr}\,Y^2)\ln(M_U/M_I)\,,\end{aligned} \tag{20}$$

where in the last step use was made of Eq. (19) and of the fact that the trace of (PQ) is zero by definition of orthogonality. Equation (20) means that one does not gain anything, in what concerns unification, by using an intermediate-energy gauge group $\mathrm{U}(1)_P \otimes \mathrm{U}(1)_Q$ instead of simply using $\mathrm{U}(1)_Y$. The final result for the $\mathrm{U}(1)_Y$ gauge coupling is the same, just as if that gauge coupling had been evolved from M_U to M_I with an a_Y coefficient [see Eq. (3)] given, as usual, by the trace of Y^2. This shows that Eqs. (7) and (11) are correct, though they have been obtained by an incorrect reasoning, and that the claim made in Ref. [7] is not correct: unless one populates the GUT desert with some particles (fermions, scalars, or gauge bosons [10]), unification cannot be achieved by just formally using an extended gauge group. This is what one should have expected.

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