PHYSICAL REVIEW D VOLUME 48, NUMBER 5 1 SEPTEMBER 1993

Quark mass matrices with full first-order perturbation

Dongsheng Du
Institute of High Energy Physics, P.O. Box 918(4), Beijing 100039, China

Zhi-zhong Xing*
China Center of Advanced Science and Technology (World Laboratory), P.O. Box 8730, Beijing 100080, China and Institute of High Energy Physics, P.O. Box 918(4), Beijing 100039, China

(Received 26 January 1993)

In view of current experimental constraints on the top-quark mass m_t and the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V_{ij}, we study a modified form of the Fritzsch quark mass matrices, in which two nonzero diagonal elements for the charm and strange quarks are introduced as the additional first-order perturbative terms. In a reasonable analytical approximation, this modification can yield the upper bound of m_t twice as much as that predicted by the Fritzsch *Ansatz*. The magnitudes of the CKM matrix elements and the parametrization-invariant measure of *CP* violation are restricted very well in terms of ratios of quark masses, and some interesting relations such as $|V_{ub}/V_{cb}|^2 \approx m_u/m_c$, $|V_{td}/V_{ts}|^2 \approx m_d/m_s$, and $|V_{ub}| \approx |V_{ts}|$ are obtained to better accuracy.

PACS number(s): 12.15.Ff

In the standard electroweak model, the family structure of Yukawa couplings is not constrained by gauge invariance so that the values of the fermion masses and flavor-mixing parameters are completely model undetermined. In trying to understand this puzzle, various forms of mass matrices have been proposed [1]. Among those suggested *Ansätze*, the most popular and economical one is the Fritzsch *Ansatz* [2]

$$M_F^{(q)} = \begin{pmatrix} 0 & x_q e^{i\alpha_q} & 0 \\ x_q e^{-i\alpha_q} & 0 & y_q e^{i\beta_q} \\ 0 & y_q e^{-i\beta_q} & z_q \end{pmatrix} , \tag{1}$$

where x_q, y_q, and z_q are real parameters and $q = u$ (d) denotes the up (down) charge sector of the quark mass matrices. Within this scheme the masses of the heavy quarks (t,b) are introduced initially as the "driving terms," while the masses of the light quarks (c,s) and (u,d) are generated through the first- and second-order perturbations, respectively. In other words, the masses of the ith family $(i=1,2)$ fully pick up values from mixing of the ith and $(i+1)$th families. Unfortunately, the prediction of the Fritzsch *Ansatz* to the top-quark mass [3],

$$m_t \le m_c[(m_s/m_b)^{1/2} - |V_{cb}|]^{-2} , \tag{2}$$

with $m_t^{\rm phys} \le 90$ GeV [4], is below or, at best, near to the lower bound of m_t ($m_t^{\rm phys} > 91$ GeV) indicated by the present measurements [5]. This implies that the form of the Fritzsch quark mass matrices should be either abandoned or modified to fit our better experimental data on the top-quark mass and flavor mixing. In Ref. [6] Albright has pursued a numerical approach to investigate the general form of the three-family mass matrices, where the 22, 13, and 31 matrix elements are nonzero for both the up and down charge sectors. He identified the crucial role played by the 22 elements in extending the allowed range for m_t but keeping a good fit to the mixing data. In contrast the off-diagonal 13 and 31 elements were expected to have little effect on the upper bound of m_t [6,7]. Recently three specific phenomenological approaches have been reattempted for the modification of the Fritzsch mass matrices $M_F^{(u)}$ and $M_F^{(d)}$ [8–10]. The first is to treat the Fritzsch *Ansatz* as the tree approximation resulting from a spontaneously broken symmetry of the Lagrangian, and to take into account the corresponding radiative corrections so as to make the *Ansatz* compatible with all the existing experimental data [8]. Another approach is to introduce two additional nonzero off-diagonal elements into $M_F^{(u)}$ and $M_F^{(d)}$ so that the generalized mass matrices can serve as a full realization of the idea that the lighter quarks in each charge sector get masses through mixing [9]. However, this treatment fails to lead to an appreciable increase in the upper bound of the top-quark mass, as pointed out by Albright and Lindner [6,7]. The third approach, examined by Gupta and Johnson [10], is to introduce a nonzero diagonal element for the charm quark into the Fritzsch matrix $M_F^{(u)}$ in view of the fact that m_c is much larger than m_u, m_d, and m_s, but comparable with m_b. They obtain $m_t^{\rm phys} \le 170$ GeV by comparing their theoretical results for the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements with the relevant experimental data [11].

In this Brief Report we start from a somewhat different point of view and take an analytical approach to modify the form of the Fritzsch quark mass matrices. Following the assumption that the matrices for both up and down sectors have parallel dynamical origins in a perturbative structure around a heavy quark [2,12], we introduce two nonzero diagonal elements for the charm and strange quarks into $M_F^{(u)}$ and $M_F^{(d)}$ as the additional first-order perturbation:

$$M_F^{(q)} \rightarrow M^{(q)} = \begin{pmatrix} 0 & x_q e^{i\alpha_q} & 0 \\ x_q e^{-i\alpha_q} & w_q & y_q e^{i\beta_q} \\ 0 & y_q e^{-i\beta_q} & z_q \end{pmatrix} , \tag{3}$$

*Mailing address after 1 November 1993: Sektion Physik der Universität München, W-8000 Munich 2, Germany.

0556-2821/93/48(5)/2349(4)/$06.00

where w_q is a real parameter. This modification can of course be understood as a generalization of the Gupta-Johnson's work [10] but it is based on the following nontrivial consideration: (1) the physical value of m_c is far above the QCD threshold scale $\Lambda_{\rm QCD}$ and has the same order as that of m_b; and (2) the ratio m_c/m_t is smaller than m_s/m_b in view of the existing lower bound of m_t indicated by experimental data [5,13]. As a matter of fact, a nonzero w_q means that the second family quarks c and s may not gain their masses fully via mixing of the second and third families. Note that the form of $M^{(q)}$ in Eq. (3) includes the same number of parameters as that with full off-diagonal mixing [7,9]; therefore, it is also worth looking at. Instead of trying to justify $M^{(q)}$ by dynamical principles or to construct a model giving rise to $M^{(q)}$ in a natural way, here we are going to explore its consequences on the top-quark mass and flavor mixing. A remarkable result of ours is that $M^{(q)}$ can yield the upper bound of m_t twice as much as that predicted by the Fritzsch *Ansatz* [comparing Eq. (2) with Eq. (21)]. We also find that the magnitudes of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements and the parametrization-invariant measure of *CP* violation may be restricted very well by ratios of quark masses, and some interesting relations such as $|V_{ub}/V_{cb}|^2 \approx m_u/m_c$, $V_{td}/V_{ts}|^2 \approx m_d/m_s$, and $|V_{cb}| \approx |V_{ts}|$ are obtained in better accuracy. Therefore, we hope that our generalization of the Fritzsch *Ansatz* might contain something reasonable and provide some clues to the origin of the quark mass matrices. This will be helpful in constructing models of the Higgs-boson couplings of quarks, which are currently of much experimental and theoretical interest.

In Eq. (3), the diagonal elements corresponding to the light quarks u and d are expected to be vanishingly small. Based on the assumption that the masses of the heavy quarks t and b set the scales of $M^{(u)}$ and $M^{(d)}$, respectively, we can require further that the diagonal elements corresponding to t and b are approximately equal to the observed values of their masses:

$$z_u \approx m_t \ , \quad z_d \approx m_b \ . \tag{4}$$

It should be noted that the mass matrices $M^{(q)}$ can yield $z_u \approx m_t$ and $z_d \approx m_b$ naturally only if $x_q \ll y_q \ll z_q$ and $w_q \ll z_q$. Considering that the intermediate masses m_c and m_s may not pick up their values fully through mixing of the second and third families, we treat w_u and w_d as the additional first-order perturbative variables which may be comparable with y_u and y_d, respectively [12]. As a result, we restrict the values of w_q in the range

$$0 \le w_u \le m_c \ , \quad 0 \le w_d \le m_s \ . \tag{5}$$

It can be seen later on that a nonzero w_q has little effect on the scale term z_q and the second-order perturbative term x_q in $M^{(q)}$, but can affect the first-order perturbative term y_q significantly.

In order to derive the CKM matrix, one first diagonalizes $M^{(q)}$ through the following unitary transformation:

$$M^{(q)} = U^{(q)} M^{(q)}_{\rm diag} U^{(q)\dagger} \tag{6}$$

with

$$M^{(q)}_{\rm diag} = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & -m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} . \tag{7}$$

Here m_1, m_2, and m_3 correspond to m_u, m_c, and m_t for $M^{(u)}_{\rm diag}$ and m_d, m_s, and m_b for $M^{(d)}_{\rm diag}$, respectively. In view of our knowledge of the quark masses [13], we retain only the leading powers of m_1/m_2, m_1/m_3, m_2/m_3, and w_q/m_3, and obtain

$$\begin{aligned} x_q &= \left[\frac{m_1 m_2 m_3}{m_1 - m_2 + m_3 - w_q} \right]^{1/2} \approx (m_1 m_2)^{1/2} \ , \\ y_q &= (-x_q^2 + z_q w_q + m_1 m_2 + m_2 m_3 - m_1 m_3)^{1/2} \\ &\approx [(m_2 + w_q) m_3]^{1/2} \ , \\ z_q &= m_1 - m_2 + m_3 - w_q \approx m_3 \ . \end{aligned} \tag{8}$$

Thus the matrix $U^{(q)}$ can be approximately given by

$$U^{(q)} = \begin{bmatrix} 1 & -\left[\dfrac{m_1}{m_2}\right]^{1/2} & \left[\dfrac{m_1 m_2 (m_2 + \omega_q)}{m_3^3}\right]^{1/2} \\ \left[\dfrac{m_1}{m_2}\right]^{1/2} e^{-i\alpha_q} & e^{-i\alpha_q} & \left[\dfrac{m_2 + w_q}{m_3}\right]^{1/2} e^{-i\alpha_q} \\ -\left[\dfrac{m_1(m_2 + w_q)}{m_2 m_3}\right]^{1/2} e^{-i(\alpha_q + \beta_q)} & -\left[\dfrac{m_2 + w_q}{m_3}\right]^{1/2} e^{-i(\alpha_q + \beta_q)} & e^{-i(\alpha_q + \beta_q)} \end{bmatrix} . \tag{9}$$

The flavor-mixing matrix

$$V_{\rm CKM} \equiv U^{(u)\dagger} U^{(d)} \tag{10}$$

is then obtained as

$$V_{\rm CKM} = \begin{bmatrix} 1 & -B + Ae^{i\phi_1} & Ae^{i\phi_1}(\xi D - \zeta C e^{i\phi_2}) \\ -A + Be^{i\phi_1} & e^{i\phi_1} & e^{i\phi_1}(\xi D - \zeta C e^{i\phi_2}) \\ Be^{i\phi_1}(\zeta C - \xi D e^{i\phi_2}) & e^{i\phi_1}(\zeta C - \xi D e^{i\phi_2}) & e^{i(\phi_1 + \phi_2)} \end{bmatrix} , \tag{11}$$

where

$$A=\left[\frac{m_u}{m_c}\right]^{1/2},\quad B=\left[\frac{m_d}{m_s}\right]^{1/2},\quad C=\left[\frac{m_c}{m_t}\right]^{1/2},\tag{12}$$

$$D=\left[\frac{m_s}{m_b}\right]^{1/2},\quad \zeta=\left[\frac{m_c+w_u}{m_c}\right]^{1/2},\quad \xi=\left[\frac{m_s+w_d}{m_s}\right]^{1/2};$$

$$\phi_1=\alpha_u-\alpha_d\ ,\quad \phi_2=\beta_u-\beta_d\ .\tag{13}$$

In obtaining Eq. (11), we have only kept the leading term of every matrix element [14]. To transform $V_{\rm CKM}$ into a simplier form, we make the following successive rotations of the quark fields:

$$\begin{aligned}&(1)\ c\to ce^{i\phi_1}\ ,\quad t\to te^{i(\phi_1+\phi_2)}\ ,\\&(2)\ u\to ue^{i(\phi_1-\phi)}\ ,\quad d\to de^{i(\phi_1-\phi)}\ ,\\&(3)\ t\to te^{i\theta}\ ,\quad b\to be^{i\theta}\ ,\end{aligned}\tag{14}$$

where

$$\begin{aligned}&\phi=\arctan[\sin\phi_1/(\cos\phi_1-A/B)]\ ,\\&\theta=\arctan[\sin\phi_2/(-\cos\phi_2+\xi D/\zeta C)]\ .\end{aligned}\tag{15}$$

Accordingly we obtain

$$V_{\rm CKM}=\begin{bmatrix}1-\frac{1}{2}\eta^2 & \eta & A\rho e^{i\phi}\\ -\eta & 1-\frac{1}{2}\eta^2 & \rho\\ -B\rho e^{i(\phi_1-\phi)} & -\rho & 1\end{bmatrix},\tag{16}$$

where

$$\begin{aligned}&\eta=(A^2+B^2-2AB\cos\phi_1)^{1/2}\ ,\\&\rho=(\xi^2C^2+\zeta^2D^2-2\zeta\xi CD\cos\phi_2)^{1/2}\ ,\end{aligned}\tag{17}$$

and we have added the next-to-leading terms in the diagonal elements V_{ud} and V_{cs} by using the unitary condition.

Clearly, the form of $V_{\rm CKM}$ in Eq. (16) is quite similar to that parametrized by Wolfenstein [15]. To some extent, this similarity might imply the reasonableness of the quark mass matrices $M^{(q)}$ given in Eq. (3), and one may be interested in speculating the underlying physics which gives rise to $M^{(q)}$. In the following paragraph we will explore some consequences of our modified form of the Fritzsch *Ansatz*.

Except for the top-quark mass, there still exist four unknown variables in the CKM matrix $V_{\rm CKM}$ [see Eqs. (15)–(17)]: ϕ_1, ϕ_2, w_u, and w_d, originating from the proposed mass matrices $M^{(u)}$ and $M^{(d)}$. Here we do not want to allow these unknown parameters to vary numerically so as to achieve close agreement between the theoretical results and the experimentally determined central values of the magnitudes of the CKM matrix elements [6,8,10]. Instead of quantitative evaluation, we use our analytical results to restrict the magnitudes of the nine CKM matrix elements, to give an upper bound of the top-quark mass, and to look at the rephasing-invariant measure of *CP* violation.

According to our argument that the diagonal element w_u (w_d) may vary from zero to m_c (m_s) in $M^{(u)}$ ($M^{(d)}$), we obtain from Eqs. (5) and (12) that

$$1\le\zeta\le\sqrt{2}\ ,\quad 1\le\xi\le\sqrt{2}\ .\tag{18}$$

For various possible values of the unknown phases ϕ_1 and ϕ_2, the ranges of η and ρ in Eq. (17) are limited by

$$\begin{aligned}&B-A\le\eta\le B+A\ ,\\&D-\sqrt{2}C\le\rho\le\sqrt{2}(D+C)\ .\end{aligned}\tag{19}$$

In obtaining the lower bound of ρ one needs to take $\zeta=\sqrt{2}$ and $\xi=1$, in view of the fact that $m_t^{\rm phys}>91$ GeV and $C<D$ [5,13], to ensure sufficient cancellation between C and D. Using Eqs. (16)–(19), the magnitudes of the CKM matrix elements can be approximately restricted by ratios of quark masses as follows:

$$1-\frac{m_u}{m_c}-\frac{m_d}{m_s}-2\left[\frac{m_um_d}{m_cm_s}\right]^{1/2}\le|V_{ud}|\approx|V_{cs}|\le1-\frac{m_u}{m_c}-\frac{m_d}{m_s}+2\left[\frac{m_um_d}{m_cm_s}\right]^{1/2},$$

$$\left[\frac{m_d}{m_s}\right]^{1/2}-\left[\frac{m_u}{m_c}\right]^{1/2}\le|V_{us}|\approx|V_{cd}|\le\left[\frac{m_d}{m_s}\right]^{1/2}+\left[\frac{m_u}{m_c}\right]^{1/2},$$

$$\left[\frac{m_s}{m_b}\right]^{1/2}-\sqrt{2}\left[\frac{m_c}{m_t}\right]^{1/2}\le|V_{cb}|\approx|V_{ts}|\le\sqrt{2}\left[\left[\frac{m_s}{m_b}\right]^{1/2}+\left[\frac{m_c}{m_t}\right]^{1/2}\right],\tag{20}$$

$$\frac{|V_{ub}|}{|V_{cb}|}\approx\left[\frac{m_u}{m_c}\right]^{1/2},\quad\frac{|V_{td}|}{|V_{ts}|}\approx\left[\frac{m_d}{m_s}\right]^{1/2},\quad|V_{tb}|\approx1\ .$$

Three features should be noted in the above expressions.

(1) The magnitudes of V_{us} and V_{cd} (V_{ud} and V_{cs}), which correspond to the flavor mixing between (within) the first and second quark families, are mainly limited by m_u, m_d, m_c, and m_s. They are approximately independent of the masses of the third quark family, m_t and m_b.

(2) The magnitudes of V_{cb} and V_{ts}, which correspond to the flavor mixing between the second and third quark families, are mainly determined by m_c, m_s, m_t, and m_b. They are approximately independent of the masses of the first quark family, m_u and m_d.

(3) The ratios $|V_{ub}/V_{cb}|^2$ and $|V_{td}/V_{ts}|^2$ are approximately equal to m_u/m_c and m_d/m_s, respectively [11,16]. They are almost irrelevant to the corresponding masses of the heavy quarks b and t.

It is remarkable that Eq. (20) can straightforwardly yield the upper bound of the top-quark mass:

$$m_t\le2m_c[(m_s/m_b)^{1/2}-|V_{cb}|]^{-2}\ .\tag{21}$$

This result shows that our mass matrices $M^{(q)}$ can make the upper bound of m_t twice as much as that predicted by the Fritzsch *Ansatz* [3,4]. In Eq. (21) the coefficient 2 comes out under the condition $w_u = m_c$ and $w_d = 0$. This means that in our generalized quark mass matrices the maximal value of the upper bound of m_t is obtained when the diagonal elements corresponding to the light quarks u, d, and s are vanishingly small, while those corresponding to the heavy quarks c, b, and t are approximately equal to the observed values of their masses. This special condition has just been chosen by Gupta and Johnston [10] so that they obtain $m_t^{\text{phys}} \leq 170$ GeV in their numerical calculation. Here we get it in a natural and analytical way. Using the current values $|V_{cb}| = 0.030-0.058$ [5], m_c (1 GeV) $= 1.35 \pm 0.05$ GeV, and $m_s/m_b = 0.033 \pm 0.011$ [13], and transforming $m_t(1\ \text{GeV})$ into $m_t(m_t)$ [17], we obtain the approximate value of the upper bound of the top-quark mass as

$$m_t^{\text{phys}} \leq 190\ \text{GeV} . \tag{22}$$

Of course, this result increases the values of the upper bound of m_t predicted in Refs. [3,4,7,9].

Finally, let us look at the rephasing-invariant measure of CP violation in the electroweak interactions [18]:

$$J \equiv |\text{Im}(V_{ij} V_{kl} V_{il}^* V_{kj}^*)| . \tag{23}$$

Using our flavor mixing matrix V_{CKM} with leading-order elements in Eq. (16), we obtain

$$J \approx AB\rho^2 \sin\phi_1 . \tag{24}$$

Note that our CP-violating phase factor ($e^{i\phi_1}$) is associated with the second-order perturbative terms in the quark mass matrices $M^{(u)}$ and $M^{(d)}$. Thus CP violation might be vanishing small if m_u and m_d tend to be zero or α_u is equal to α_d. A similar result has also been obtained in Ref. [11] with more detailed arguments. As a result, the maximal value of J in Eq. (24) is given by

$$J_{\max} \approx 2AB(C+D)^2 , \tag{25}$$

which is of order $10^{-4}-10^{-3}$ for current values of the quark masses [5,13].

We have followed a phenomenological approach to find a modified form of the Fritzsch quark mass matrices so as to deal with flavor mixing and CP violation in the electroweak interactions. Considering that the masses of the second family quarks are not too small compared with those of the third family quarks and might not pick up values fully from mixing these two families, we introduced two nonzero diagonal elements for c and s into the Fritzsch mass matrices as the additional first-order perturbation. Our treatment leads to an interesting Wolfenstein pattern of the flavor mixing matrix, and yields the upper bound of the top quark mass twice as much as that predicted by the Fritzsch *Ansatz*. In addition, the magnitudes of the CKM matrix elements and the rephasing-invariant measure of CP violation can be restricted very well by ratios of quark masses, and some interesting relations such as $|V_{ub}/V_{cb}|^2 \approx m_u/m_c$ and $|V_{td}/V_{ts}|^2 \approx m_d/m_s$ are obtained in better accuracy.

In conclusion, our modification of the Fritzsch *Ansatz* can fit current experimental data on the CKM matrix and the top-quark mass. Evidently any simple *Ansatz* (containing only a few free parameters), which can account for the observed systematics of fermion masses, is useful in order to find clues of the origin of the fermion mass matrices. We are looking forward to obtaining more precise experimental data to examine our phenomenological model.

We would like to thank Professor H. Fritzsch for his helpful comments on our manuscript. This work was supported in part by the National Natural Science Foundation of China.

[1] For an extensive list of references, see H. Fritzsch, Phys. Lett. B **184**, 391 (1987); G. C. Branco and D.-D. Wu, *ibid.* **205**, 253 (1988); E. A. Paschos and U. Turke, Phys. Rep. **178**, 173 (1989).

[2] H. Fritzsch, Phys. Lett. **73B**, 317 (1978); Nucl. Phys. **B155**, 189 (1979).

[3] H. Harari and Y. Nir, Phys. Lett. B **195**, 586 (1987); C. H. Albright, *ibid.* **227**, 171 (1989).

[4] Y. Nir, SLAC Report No. SLAC-PUB-5676 & WIS-91/73/Oct-PH, 1991 (unpublished).

[5] Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992).

[6] C. H. Albright, Phys. Lett. B **246**, 451 (1990).

[7] C. H. Albright and M. Lindner, Z. Phys. C **44**, 673 (1989).

[8] G. C. Branco and F. Mota, Phys. Lett. B **280**, 109 (1992).

[9] R. E. Shrock, Phys. Rev. D **45**, 10 (1992).

[10] S. N. Gupta and J. M. Johnson, Phys. Rev. D **44**, 2110 (1991).

[11] Note that X.-G. He and W.-S. Hou have modified the Fritzsch *Ansatz* in another way. In their quark mass matrices, the u-type matrix is of the Fritzsch form, while for the d-type it is diagonal except in the d-s plane, which generates the Cabibbo rotation. Their *Ansatz* can also increase the value of the upper bound for the top quark mass. See X.-G. He and W.-S. Hou, Phys. Rev. D **41**, 1517 (1990).

[12] P. M. Fishbane and P. Q. Hung, Phys. Rev. D **45**, 293 (1992).

[13] J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982).

[14] More precisely, we obtain $V_{ub} \approx \xi BD^3 + Ae^{i\phi_1}(\xi D - \zeta Ce^{i\phi_2})$ and $V_{td} \approx \zeta AC^3 + Be^{i\phi_1}(\zeta C - \xi Be^{i\phi_2})$. In view of our current knowledge of the quark masses (see Refs. [5,12]), $V_{td} \approx Be^{i\phi_1}(\zeta C - \xi De^{i\phi_2})$ is a safe approximation, while $V_{ub} \approx Ae^{i\phi_1}(\xi D - \zeta Ce^{i\phi_2})$ may deviate from its precise value for about 10% since m_b is not so large as m_t.

[15] L. Wolfenstein, Phys. Rev. Lett. **51**, 145 (1983).

[16] G. C. Branco and L. Lavoura, Phys. Rev. D **44**, 582 (1991).

[17] C. H. Albright, C. Jarlskog, and B.-A. Lindholm, Phys. Rev. D **38**, 872 (1988).

[18] C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985); D.-D. Wu, Phys. Rev. D **33**, 860 (1986).