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Breathing mode in the extended Skyrme model

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We study an extended Skyrme model which includes fourth- and sixth-order terms. We explore some static properties such as the Δ-nucleon mass splitting and investigate the Skyrmion breathing mode in the framework of the linear response theory. We find that the monopole response function has a pronounced peak located at $\sim$ 400 MeV, which we identify as the Roper resonance $N(1440)$. As compared to the standard one, the extended Skyrme model provides a more accurate description of baryon properties.

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I. INTRODUCTION

At low energy the QCD running coupling constant α_s becomes large which renders the standard perturbation theory inapplicable. In order to describe hadronic physics at low energy, effective theories including the main features of QCD (e.g., chiral symmetry) have therefore been proposed. These theories carry the label "effective" because their degrees of freedom are the hadronic observable instead of the fundamental constituents, quarks and gluons, which are confined. Ten years before the advent of QCD, Skyrme proposed a model [1] for hadronic physics which involves only meson fields (pions), and where baryons emerge as topological solitons. This model is now recognized as the simplest chiral realization of QCD at low energy and large N_c [2]. In the following we will refer to this model as the minimal Skyrme model.

The minimal Skyrme model has been studied extensively to describe static properties of baryons [3–7] as well as dynamical properties, in particular the simplest vibrational excitation of the Skyrmion: the breathing mode [8–15]. Concerning the static properties, the predictions of the model are generally within 30% of experimental values when the parameters are adjusted to fit the nucleon and the Δ masses. In this paper we investigate whether this discrepancy is due to the use of the minimal Skyrme model. One motivation for this study is that it is easy to show that the second- and fourth-order terms in the pion field derivatives have the same contribution to the soliton mass so that further terms should be added. Moreover, in taking into account the Casimir energy of the Skyrmion, the authors of Refs. [16,17] have found that the nucleon mass is lowered to a value between 0 and 400 MeV which is much too small. They also proved that the $O(N_c^1)$ and $O(N_c^0)$ mass contributions are of same order. Thus the large N_c expansion is a poor approximation within this minimal Skyrme model.

In this article we mainly focus on the breathing mode. One problem regarding this mode in the minimal Skyrme model is that it does not show up in analysis based on phase shifts [10,12], whereas when other calculations found it [8,9,11,13–15], its excitation energy stands between 200 and 300 MeV which is too small compared to its experimental value of 500 MeV.

One possible way to circumvent these difficulties is to improve this model by adding higher order terms in derivatives in the corresponding chiral Lagrangian as the sixth-order term generated by ω-meson exchange [17] (first proposed in Ref. [18]).

This extended model has already been used to describe the Roper resonance. Kaulfuss and Meissner [19] used both scaling approximation and semiclassical quantization. They found a resonance at an excitation energy of $\sim$ 480 MeV. However, these authors used values for the parameters of the model which are in conflict with those determined by chiral perturbation theory [20,21] and, moreover, changed the sign of the so-called Skyrme term. Schwesinger and Weigel [22] have also used phase shifts analysis within this extended Skyrme model. However, they did not find the Roper resonance.

In this work, we describe this low-lying monopole resonance within the same extended Skyrme model of Refs. [17,22] using the linear response theory. This method is more transparent and has already been shown to be powerful in different domains such as giant resonances in nuclear physics [23] or nucleon polarizabilities in hedgehog models in hadronic physic [24]. In a previous article [15] we demonstrated the practicability of this approach. However, this calculation was limited to the minimal Skyrme model only.

The present article is organized as follows. In Sec. II we introduce the extended Skyrme model and define our notation. In Sec. III we review the linear response approach of Ref. [15] and specialize to the extended model considered here. We finally discuss our results concerning some static properties of the soliton and the Roper resonance in Sec. IV.

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II. EXTENDED SKYRME MODEL

The simplest chiral effective Lagrangian proposed by Skyrme [1] reads

$$\mathcal{L}_{\rm Sk} = \left[\frac{F_\pi^2}{16}\right] \ {\rm Tr}\ (\partial_\mu U \partial^\mu U^\dagger) + \left[\frac{1}{32e^2}\right] \ {\rm Tr}\ \{[(\partial_\mu U)U^\dagger, (\partial_\nu U)U^\dagger]^2\}, \quad (2.1)$$

where U is an SU(2) matrix parametrized by the (Goldstone) pion fields π_a, normalized to the pion decay constant F_π :

$$U = \exp[2i\boldsymbol{\pi}\cdot\boldsymbol{\tau}/F_\pi] \ , \quad (2.2)$$

the τ_a's being the usual Pauli matrices. The first term in the Lagrangian (2.1) corresponds to the well known nonlinear σ model and the second one, which is of fourth order in powers of the derivatives of the pion field, was introduced by Skyrme to stabilize the soliton. It is generally referred to as the Skyrme term. It can be derived from a local approximation of an effective model with ρ mesons [25]. Similarly, a term of order six can be generated from ω-meson exchange [18,26]. It reads

$$\mathcal{L}_6 = -\frac{1}{2}\frac{\beta_\omega^2}{m_\omega^2} B_\mu B^\mu, \quad (2.3)$$

where the anomalous baryon current B_μ is given by

$$B^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}{\rm Tr}\left\{(\partial_\nu U)U^\dagger(\partial_\alpha U)U^\dagger(\partial_\beta U)U^\dagger\right\}. \quad (2.4)$$

The two new constants appearing in Eq. (2.3) are the ω-meson mass m_ω and the parameter β_ω which can be related to the $\omega \to \pi\gamma$ width [27].

In Ref. [17] it was shown that the effect of the Casimir energy, including the term $\mathcal{L}_6$ in the Lagrangian (2.1), is to lower the nucleon mass by about 500 MeV, so that one obtains a more satisfactory value of ~1 GeV. Anticipating on the last section, let us observe that the axial-vector coupling constant g_A is found to be 1.24 instead of the Skyrme model prediction 0.34. So there are good reasons to think that the extension of the Skyrme model which consist in adding the sixth-order term (2.3) to the Lagrangian (2.1) is a good approximation to describe the baryonic sector.

There are of course other sixth-order terms in the pion field derivatives such as the contributions of the ρ and scalar mesons. Nevertheless, a Lagrangian containing all possible sixth-order terms leads in general to an Euler-Lagrange differential equation of order higher than two. Consequently, one is not sure to find a soliton-type solution. This question is still open up to now since the parameters which correspond to these terms have not yet been determined. Fortunately, the term $\mathcal{L}_6$ in Eq. (2.3) has the noteworthy property (as the Skyrme term) of being quadratic in the time derivatives and to lead to an equation of motion of second order as we will see below [see Eq. (2.7)]. For this reason we consider only this term in this work.

Our starting point is then the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm Sk} + \mathcal{L}_6 + \tfrac{1}{16}F_\pi^2 m_\pi^2 {\rm Tr}\left(U + U^\dagger - 2\right). \quad (2.5)$$

The first and second terms in Eq. (2.5) have been discussed above. The last term which is proportional to the square of the pion mass m_π implements a small explicit breaking of chiral symmetry. Assuming the hedgehog ansatz for the pion field $\boldsymbol{\pi}(\mathbf{r},t) = F_\pi F(r,t)\ \hat{\mathbf{r}}/2$ [see Eq. (2.2)], the Lagrangian density (2.5) becomes

$$\mathcal{L} = \left[\frac{F_\pi^2}{8} + \frac{\sin^2 F}{e^2 r^2} + \frac{\beta_\omega^2}{8m_\omega^2\pi^4}\left(\frac{\sin F}{r}\right)^4\right]\partial_\mu F \partial^\mu F - \left(\frac{\sin F}{2r}\right)^2\left[F_\pi^2 + 2\frac{\sin^2 F}{e^2 r^2}\right] + \frac{1}{4}F_\pi^2 m_\pi^2(\cos F - 1) \ . \quad (2.6)$$

The corresponding classical Euler-Lagrange equation reads

$$g^2\ddot{F} + \alpha\dot{F}^2 = \left(g^2 F'\right)' - \left(\theta + \alpha F'^2\right) \ , \quad (2.7)$$

where the time-dependent functions g, α, and θ are, respectively,

$$g(r,t) = \left[\frac{(eF_\pi)^2}{4}r^2 + 2\sin^2 F + \frac{\beta_\omega^2\ e^2}{4m_\omega^2\ \pi^4}\frac{\sin^4 F}{r^2}\right]^{\frac{1}{2}} \ ,$$

$$\alpha(r,t) = \left[1 + \frac{\beta_\omega^2\ e^2}{4m_\omega^2\ \pi^4}\left(\frac{\sin F}{r}\right)^2\right]\sin(2F) \ , \quad (2.8)$$

$$\theta(r,t) = \left[\frac{(eF_\pi)^2}{4}r^2 + \sin^2 F\right]\frac{\sin(2F)}{r^2} + \frac{1}{4}(eF_\pi)^2 m_\pi^2\ r^2 \sin F \ .$$

Primes and dots in Eq. (2.7) indicate radial coordinate differentiations and time differentiations respectively. In order to ensure that the baryon number is equal to one, the equation of motion (2.7) has to be solved with the conditions

$$F(0,t) \ = \ \pi \ , \quad F(\infty,t) \ = \ 0 \ ; \quad (2.9)$$

which are sufficient for a differential equation of second order.

III. LINEAR RESPONSE ANALYSIS

In order to describe the low-lying monopole vibrations of the Skyrmion within the extended Skyrme model (2.5) we explore the response of the Skyrmion to an external infinitesimal monopole field with a frequency Ω. The response function is determined from the evolution of the isoscalar mean square radius of the Skyrmion with respect to the frequency Ω.

An external time-dependent monopole field corresponds to the addition of the following term to the Lagrangian density (2.6):

$$\mathcal{L}_{\text{int}} = eF_\pi^3 r^2 B^0(r,t)\ \epsilon \sin(\Omega t)\exp(\eta t), \tag{3.1}$$

where $B^0(r,t)$ is the time component of the baryon current (2.4),

$$B^0(r,t) = -\frac{1}{2\pi^2}\frac{\sin^2(F)}{r^2}\frac{\partial F}{\partial r},$$

and η a vanishingly small positive number. Adding this interaction term (3.1) to the Lagrangian density (2.6), the new corresponding Euler-Lagrange equation reads

$$g^2\ddot{F} + \alpha\dot{F}^2 = \left(g^2 F'\right)' - \left(\theta + \alpha F'^2\right) + \epsilon\frac{(eF_\pi)^3 r}{\pi^2}\sin^2 F \sin(\Omega t)\exp(\eta t), \tag{3.2}$$

where g, α, and θ have been defined in the previous section. This last equation is to be solved with the boundary conditions $F(t=-\infty, r) = F_s(r)$ and $\dot{F}(t=-\infty, r) = 0$ where $F_s(r)$ is the static solution of Eq. (2.7).

Because the term (3.1) is weak (in the domain $t \in [-\infty, 0]$), it introduces small changes of the classical solution. We thus can treat this solution in a linear approximation. To first order in ϵ, $F(r,t)$ has the form

$$F(r,t) = F_s(r) + \delta F(r,t) \ + \delta F^*(r,t)\ ,$$

where $\delta F(r,t)$ is linear in the field strength ϵ. Moreover, its time dependence reads

$$\delta F(r,t) = -i\frac{\epsilon}{2}R(r)\exp[i(\Omega - i\eta)t].$$

Thus, Eq. (3.2) becomes

$$[(\Omega - i\eta)^2 - \mathcal{A}_s](g_s R) = -\frac{(eF_\pi)^3}{\pi^2}\frac{r\sin^2 F_s}{g_s}, \tag{3.3}$$

where the function g_s and the operator $\mathcal{A}_s$ are, respectively,

$$g_s(r) = \left[\frac{(eF_\pi)^2}{4}r^2 + 2\sin^2 F_s + \frac{\beta_\omega^2\ e^2}{4m_\omega^2\ \pi^4}\frac{\sin^4 F_s}{r^2}\right]^{\frac{1}{2}}, \tag{3.4}$$

$$\mathcal{A}_s \equiv -\frac{d^2}{dr^2} + \frac{g_s''}{g_s} - \frac{2}{g_s^2}\Bigg\{\sin(2F_s)\left[F_s'' + \frac{\beta_\omega^2\ e^2}{4m_\omega^2\ \pi^4}\left(\frac{\sin^2 F_s}{r^2}F_s'\right)'\ \right] + \cos(2F_s)\left[F_s'^2 - \frac{(eF_\pi)^2}{4} - \frac{2}{r^2}\sin^2 F_s\right]$$
$$-\frac{\sin^2 F_s}{r^2}\left[1 + \frac{\beta_\omega^2\ e^2}{4m_\omega^2\ \pi^4}\ F_s'^2\right] - \frac{(eF_\pi)^2}{8}m_\pi^2\ r^2\cos F_s\Bigg\}. \tag{3.5}$$

The isoscalar mean square radius [3] is given by

$$\langle r^2\rangle = -\frac{2}{\pi}\int_0^\infty r^2\sin^2(F)F'\ dr.$$

Up to first order in ϵ it reads

$$\langle r^2\rangle = \langle r^2\rangle_s - i\frac{\epsilon}{2}\{f(\Omega)\exp[i(\Omega - i\eta)t] - f^*(\Omega)\exp[-i(\Omega + i\eta)t]\},$$

where $\langle r^2\rangle_s$ is the static mean square radius [3] and f the response function

$$f(\Omega) = \frac{4}{\pi}\int_0^\infty r\frac{\sin^2 F_s}{g_s}(g_s R)\ dr. \tag{3.6}$$

By using Eq. (3.3), we can extract the following spectral representation of the response function (3.6):

$$f(\Omega) = -\frac{1}{\pi}\sum_n \frac{|\langle\phi|\phi_n\rangle|^2}{(\Omega - i\eta)^2 - \omega_n^2}\ , \tag{3.7}$$

where the state ϕ is defined by

$$\langle r|\phi\rangle = \frac{2eF_\pi}{\pi}\frac{r\sin^2 F_s}{g_s}, \tag{3.8}$$

and the ϕ_n are the eigenstates of the operator $\mathcal{A}_s$ [see Eq. (3.5)], with the eigenvalues ω_n^2, normalized according to

$$\int_0^\infty \phi_n(r)\phi_m(r)eF_\pi dr = \delta_{nm}\quad .$$

In Eq. (3.7) the limit $\eta \to 0^+$ is, as usual [23], implicit and corresponds to the boundary condition specified above. The quantity of interest here is the imaginary part of the response function, which is directly related to the distribution of collective strength (see, e.g., [23]). The energy at which the imaginary part of the response function (3.7) exhibits an unbound peak is identified with the excitation energy of the Roper resonance.

IV. RESULTS AND SUMMARY

In this section we present our results for some static properties and for the energy of the Roper resonance within the extended Skyrme model (2.5) and compare them to those of the minimal one. First of all, because it is a meson theory, the parameters of the model have to be fixed by fitting the low-energy meson observables. Concerning the pion decay constant, the pion, and the

ω-mesons masses, the experiment yields 186 MeV, 139.5 MeV, and 782 MeV, respectively. For the dimensionless parameter e, Riggenbach *et al.* [21], in analyzing the K_{l4} decays, reduce by a factor 2 the error bars on the chiral low-energy constants [20]. As a consequence, one finds $e = 7.1 \pm 1.2$ [17] which is surprisingly very close to the value $e = 2\pi$ proposed by Skyrme a long time ago [1]. The last parameter β_ω is obtained by fitting the $\omega \to \pi\gamma$ width, yielding $\beta_\omega = 9.3$ [27]. In order to compare the minimal and the extended Skyrme models we will consider two sets of parameters: [i] $F_\pi = 186$ MeV, $e = 7.1$, and $\beta_\omega = 0$ (minimal Skyrme model); [ii] $F_\pi = 186$ MeV, $e = 7.1$, and $\beta_\omega = 9.3$ (extended Skyrme model).

In Table I we report the soliton mass, the isoscalar root mean square radius, the axial-vector coupling constant, and the Δ-nucleon mass splitting for the different sets of parameters. In the third row we report the experimental values [28].

Concerning the soliton mass we find 972 MeV and 1563 MeV in the case [i] and [ii], respectively. We have to subtract $\sim$ 1 GeV (Casimir energy) in the case of the minimal model (case [i]) which leads to a value of $\sim$ 0 MeV [17]. In the extended Skyrme model (case [ii]) we must subtract $\sim$ 500 MeV [17] which leads to a reasonable value of $\sim$ 1 GeV. Regarding the Δ-nucleon mass splitting, which is not affected by the Casimir effect, we find 1192 MeV in the case [i] which is not a realistic value compared to the experimental one 290 MeV. However, in the case [ii] we obtain 227 MeV which is more acceptable. Furthermore, the axial-vector coupling constant g_A is found to be 0.34, in the case [i], instead of the experimental value 1.23 which is very close to the value predicted by the improved Skyrme model (case [ii]). Of course, one has to account for the Casimir effect in the calculation of g_A. However, the corrections seem to be small within this extended Skyrme model [17]. For reference we plot in Fig. 1 the chiral function F_s solution of the static Euler-Lagrange equation (2.7) for the minimal and extended model.

For the breathing mode of the Skyrmion, we display in Fig. 2 the imaginary part of the response function (3.7) for the two sets of parameters. In both cases we find an unbound peak. In case [i] we find this peak to be broad and located at 500 MeV. Nevertheless, this result cannot be reliable since the other predicted properties are unrealistic (see Table I). On the contrary, in the case of the extended Skyrme model, the response function exhibits a pronounced sharp peak located at $\sim$ 400 MeV which we identify to the Roper resonance. This value corresponds to the Roper-nucleon mass splitting and, consequently, is not sensitive to the Casimir effect.

As mentioned in the Introduction, the authors of Ref. [22] found no trace of the Roper resonance within the same extended Skyrme model. So it is natural to check if this result is due to the parameters they chose. When we take their parameters (F_π = 142.4 MeV, e = 9.92,

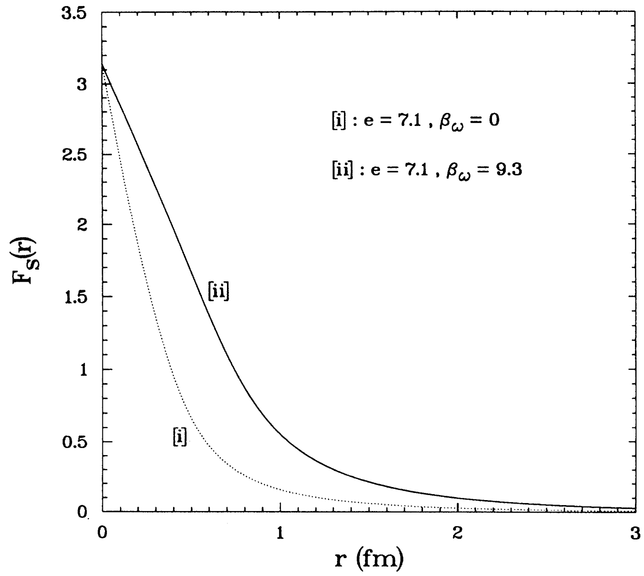

FIG. 1. Static hedgehog solution $F_s(r)$ with experimental values of F_π, m_π, and m_ω (see Sec. IV).

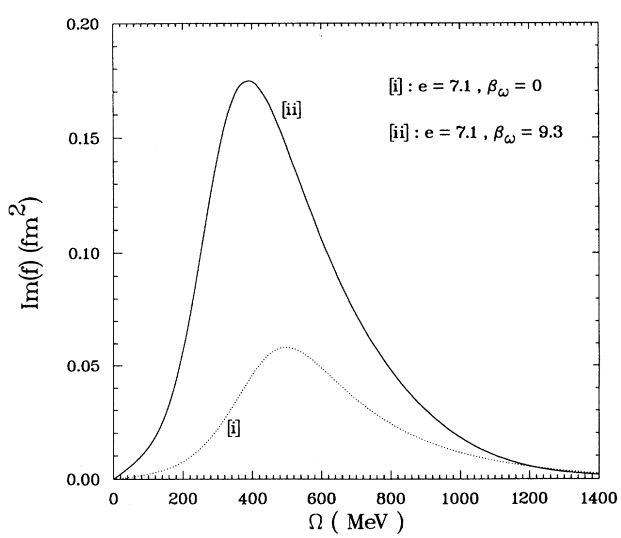

FIG. 2. Imaginary part of the response function f (fm^2) versus the energy Ω in MeV with experimental values of F_π, m_π, and m_ω (see Sec. IV).

TABLE I. Here we report, for the two different combinations of the parameters, the soliton mass M_S, the isoscalar root mean square radius r_0, the axial-vector coupling constant g_A, the Δ-nucleon mass splitting, and the Roper-nucleon mass splitting $\hbar\Omega_{\rm Roper}$. The last line is derived from the data [28]. F_π, m_π, and m_ω are taken to their experimental values (see Sec. IV).

e	β_ω	M_S (MeV)	r_0 (fm)	g_A	$M_\Delta - M_N$ (MeV)	$\hbar\Omega_{\rm Roper}$ (MeV)
7.1[a]	0	972	0.30	0.34	1192	500
7.1[a]	9.3[b]	1563	0.61	1.24	227	395
		939	0.72	1.23	290	500

[a]From Ref. [21].
[b]From Ref. [27].

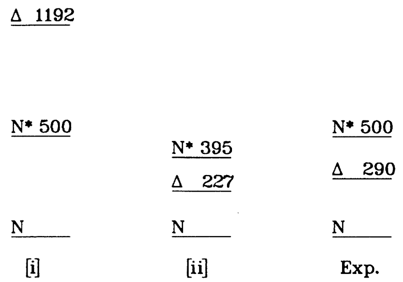

FIG. 3. Energies splitting of the Δ and Roper N^* resonances with respect to the nucleon N according to the cases [i] and [ii] of the parameters (see Sec. IV) and to the experiment [28]. All the energies are in MeV.

and $\beta_\omega = 13.6$) to investigate the Roper resonance with our approach, we find a sharp peak located at 320 MeV. This leads us to think that the difficulty in finding the breathing mode resonance in Ref. [22] is due to the implementation of the phase shifts method.

To summarize the main results of our calculations, we plot in Fig. 3 the energy spectrum of the Δ and Roper (N^*) resonances for the different values of the parameters and compare them to the experimental one. We see obviously that the case [ii], which corresponds to the extended model, is the closest to the experimental situation. In order to obtain a better agreement one has to take into account higher order terms in addition to $\mathcal{L}_6$ [see Eq. (2.3)] in the chiral Lagrangian.

Finally, the message that we want to transmit through this work is that one should not restrict oneself to the standard Skyrme model [1] [see Eq. (2.1)] for the description of low-energy hadron physics, but consider extensions of this model including higher order terms in powers of the derivatives of the pion field. In this sense, the model considered here can be considered as a minimal extension of the Skyrme Lagrangian. This claim confirms the conclusions of Refs. [17,29]. A more realistic improvement consists of considering effective Lagrangians which incorporate low mass mesons with finite mass [27,30].

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