Isospin violation in QCD sum rules for baryons

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We thoroughly analyze isospin-violating effects in QCD sum rules for the masses of nucleons, Σ, and Ξ hyperons. After comparing with experimental mass splittings in isotopic multiplets, we obtain for the isospin breaking in the quark condensate $\langle 0|\bar{u}u-\bar{d}d|0\rangle/\langle 0|\bar{u}u|0\rangle=(2\pm1)\times10^{-3}$, a value significantly smaller than the one usually adopted. We present arguments in favor of our result and critically analyze previous estimates. The value of the quark mass difference $m_d-m_u=3.0\pm1.0$ MeV (at normalization point $\bar{\mu}=0.5$ GeV) is also determined.

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I. INTRODUCTION

The pioneering work of Gasser and Leutwyler [1] has made it clear that the difference of u and d quark masses is nonzero even in the absence of electromagnetic interactions, and is of the order of the quark masses themselves. Weinberg [2] in his famous paper demonstrated that the values of u and d quark masses can be determined from the masses in the pseudoscalar nonet in a model-independent way and found $m_u=4.2$ MeV, $m_d=7.5$ MeV, and

$$\mu=m_d-m_u=3.3\text{ MeV}\ . \tag{1.1}$$

The nonzero value of μ causes the difference between the values of the QCD condensate of u and d quarks. The parameter

$$\gamma=\frac{\langle 0|\bar{d}d|0\rangle}{\langle 0|\bar{u}u|0\rangle}-1 \tag{1.2}$$

characterizes the isospin violation in quark condensates.

The knowledge of the numerical value of γ is important as it enters along with the mass difference μ in the determination of the value of isospin splitting in hadronic multiplets, the violation of isospin in various decays, etc. The magnitude of γ is also interesting from the viewpoint of nuclear physics. Indeed, it enters in recent attempts to explain the discrepancy between the theoretical and experimental results on the difference of mirror nuclei masses, known as the Nolen-Schiffer (NS) anomaly [3]. The idea behind the explanations put forward recently [4,5] is based on the reasonable assumption that quark condensates in nuclei are suppressed compared to their vacuum values and as a consequence the neutron-proton mass difference *in nuclei,* entering in the formula for the mass difference of mirror nuclei, is smaller than that for free protons and neutrons.

The parameter γ was calculated in a number of papers using different approaches. Gasser and Leutwyler [6] carried out the calculations in the framework of chiral perturbation theory. Paver, Riazzudin, and Scadron [7] considered the constituent quark model, whereas the Nambu–Jona-Lasinio model was used in Refs. [4,5]. In several papers γ was obtained from the mass splittings in the framework of QCD sum rules [8–13], with results ranging from -3×10^{-3} to -1×10^{-2}.

We see certain shortcomings in at least part of the above-mentioned calculations. For this reason we made a new attempt at extracting the parameter γ from the values of the mass splittings in the baryonic octet based on the QCD sum-rule technique (for a discussion of previous calculations and a comparison with ours, see Sec. V).

From our point of view, this technique appears to be the most promising to extract the γ parameter. The reasons are the following. Experimentally, the isospin mass splitting in the baryon octet is known with good accuracy. The electromagnetic contributions to the mass splittings are reliably estimated [14] and are rather small, especially for hyperons. The QCD sum-rule method of mass determination works well in the case of the baryonic octet: three terms of the operator product expansion (OPE) are calculated and all the self-consistency checks are satisfied. Using this method the baryonic masses [15–17], magnetic moments [18,19], and other static parameters were calculated, all in good agreement with experiment. In the baryon octet there are three values of isospin mass splittings which can be used for the determination of γ: $n-p$, $\Sigma^--\Sigma^+$ and $\Xi^--\Xi^+$. (The $\Sigma^--\Sigma^0$ splitting is not suitable for this goal due to the mixing of the Σ^0 with the Λ via isospin-violating interactions.) In the QCD sum-rule approach there are two equations for each mass splitting, corresponding to chirality-conserving and chirality-violating parts of the polarization operator. Therefore, there are six equations in which γ enters and many checks for self-consistency can be made. An essential feature of these equations is that γ appears with opposite signs in the $n-p$ (or $\Sigma^- - \Sigma^+$) and $\Xi^- - \Xi^+$ splitting, while the $n-p$ splitting is more sensitive to μ than to γ. This permits us to obtain reliable upper and lower bounds on γ, while determining μ in an independent way and allowing for a check of Weinberg's prediction (1.1).

II. THE METHOD

In the QCD sum-rule method for baryons we consider the polarization operator

$$\Pi(q)=i\int d^4x\langle 0|T\{\eta(x)\bar{\eta}(0)\}|0\rangle , \tag{2.1}$$

where $\eta(x)$ is the current with the baryon quantum numbers. For the proton

$$\eta(x)=\epsilon^{abc}u^a(x)C\gamma_\mu u^b(x)\gamma_5\gamma_\mu d^c(x) . \tag{2.2}$$

Here, $u^a(x)$ and $d^c(x)$ stand for the u and d quark fields, C is the charge conjugation matrix, and a,b,c are color indices. In order to obtain the hyperon currents, the following substitutions must be done in (2.2):

Σ^-: $u\to d, d\to s$,

Σ^+: $d\to s$,

Ξ^-: $u\to s$,

Ξ^0: $u\to s, d\to u$.

As discussed in Refs. [15,20], the current as defined in (2.2) seems to be the most suitable one for the calculation of baryon masses. It results in a relatively small contribution of higher excited states in both chiral structures Π_1 and Π_2 of the polarization operator

$$\Pi(q)=\not{q}\,\Pi_1(q^2)+\Pi_2(q^2) \tag{2.3}$$

and in good convergence of the OPE.

For each structure $\Pi_{1,2}$ we can write the dispersion relation

$$\Pi_{1,2}=\frac{1}{\pi}\int_0^\infty \frac{\rho_{1,2}(s)}{s-q^2}ds . \tag{2.4}$$

The left-hand side (LHS) of Eq. (2.4) is calculated in the framework of the OPE at large negative values of q^2, i.e., $|q^2|\gg R_c^{-2}$, where R_c denotes the confinement radius. In the OPE we keep terms up to dimension $d=7$. As was shown in Refs. [16,17] (see also Appendix B of Ref. [18]), operators of higher dimension ($d=8$ for Π_1, $d=9$ for Π_2) give small contributions to the sum rules. We also neglect perturbative corrections of the order α_s (as can be shown using the results of Ref. [21] they mainly affect the residue at the baryon pole, but not the baryon masses).

The RHS of Eq. (2.4) is represented in terms of physical states, and modeled in such a way that the lowest-energy baryon state is singled out while higher-energy states are approximated by a continuum

$$\rho_{1,2}=\lambda^2(1,m)\delta(s-m^2)+\phi_{1,2}(s)\theta(s-W_{1,2}^2) . \tag{2.5}$$

Here, λ denotes the overlap

$$\langle 0|\eta|B\rangle=\lambda_B v_B \tag{2.6}$$

between the vacuum and the respective baryon, while v_B is the baryon spinor. The functions $\phi_{1,2}$ in the second term of (2.5) are determined as the discontinuities of $\Pi_{1,2}$ at large s:

$$\phi_{1,2}(s)=\frac{1}{2i}[\Pi_{1,2}(s+i\epsilon)-\Pi_{1,2}(s-i\epsilon)] . \tag{2.7}$$

The continuum thresholds $W_{1,2}^2$ (which may be unequal in the general case), the pole position m, and the overlap λ^2 will be the variables to be determined from the sum rules.

We apply the Borel transform with Borel mass M to both sides of Eq. (2.4). This procedure is useful for several reasons. It removes the subtraction terms from the dispersion relation and suppresses the contribution of excited states in the RHS of (2.4). It furthermore suppresses the contribution of the next to leading terms in the OPE of the LHS of (2.4), thus improving convergence of the series. After the Borel transform, the sum rules appear as equations that hold for a range of values of M, the confidence interval, where the contributions of higher-order terms in the OPE are small and the impact of the parametrization of the excited states on the RHS, which is model dependent, is minimal and does not exceed the contribution of the pole term. This method was used in Refs. [15–17] to determine octet baryon masses in the absence of isospin violation. The parameters m, λ, and W were obtained with an accuracy of about 10–15 %. There, it was shown that in the nucleon case the quark condensate $\langle 0|\bar{u}u|0\rangle=\langle 0|\bar{d}d|0\rangle$ plays the dominant role. When considering hyperons we must include the strange quark mass m_s which breaks the SU(3) flavor symmetry, as well as the flavor symmetry breaking in the strange condensate

$$\beta=\frac{\langle 0|\bar{s}s|0\rangle}{\langle 0|\bar{u}u|0\rangle}-1 . \tag{2.8}$$

The best fit of hyperon masses to the QCD sum-rule calculations is provided by the values $m_s=150$ MeV and $\beta=-0.2$ (see Ref. [19]).

III. SUM RULES FOR ISOSPIN SPLITTING IN THE BARYON OCTET

In order to include isospin-violating effects, we need to take into account the nonzero values of the quark masses m_u, m_d. In order to extract the isotopic mass differences, it appears reasonable to consider the difference of the sum rules for baryons that differ by isospin projection only. Thus, we shall arrive at equations for the parameters δm, $\delta\lambda^2$, δW^2, as well as γ. Since we neglect electromagnetic effects, δm represents a subtracted mass difference

$$\delta m=(\delta m)_{\rm phys}-(\delta m)_{\rm el} , \tag{3.1}$$

where $(\delta m)_{\rm phys}$ denotes the physical (experimental) value of the mass splitting, and $(\delta m)_{\rm el}$ is the contribution due to electromagnetic interactions. For the latter we use the values given in [14]:

$$(m_n - m_p)_{\rm el} = -0.76 \pm 0.30 \text{ MeV} , \tag{3.2}$$

$$(m_{\Sigma^-} - m_{\Sigma^+})_{\rm el} = 0.17 \pm 0.30 \text{ MeV} , \tag{3.3}$$

$$(m_{\Xi^-} - m_{\Xi^0})_{\rm el} = 0.86 \pm 0.30 \text{ MeV} . \tag{3.4}$$

Taking the experimental mass differences from Ref. [22],

$$(m_n - m_p)_{\rm phys} = 1.29 \text{ MeV} , \tag{3.5}$$

$$(m_{\Sigma^-} - m_{\Sigma^+})_{\rm phys} = 8.09 \pm 0.09 \text{ MeV} , \tag{3.6}$$

$$(m_{\Xi^-} - m_{\Xi^0})_{\rm phys} = 6.4 \pm 0.6 \text{ MeV} , \tag{3.7}$$

we arrive at

$$\delta m_N = 2.05 \pm 0.30 \text{ MeV} , \tag{3.8}$$

$$\delta m_\Sigma = 7.9 \pm 0.33 \text{ MeV} , \tag{3.9}$$

$$\delta m_\Xi = 5.54 \pm 0.67 \text{ MeV} . \tag{3.10}$$

We shall take our calculations to linear order in the isospin symmetry violating quantities, i.e., μ, γ, and m_s. The polarization operators for Σ^+ and Ξ^0 were calculated in Ref. [17] to linear order in the strange quark mass. It is then trivial to obtain the proton polarization operator including the contribution from the light quark masses $m_{d,u}$ by simply replacing m_s by $m_d(m_u)$ in the polarization operator for $\Sigma^+(\Xi^0)$. The neutron result is then arrived at by further substituting $m_u \leftrightarrow m_d$. The appearance of the γ factor is also easily understood. In the lowest-order diagram for the OPE of the polarization operator of the proton (neutron), it is the u (d) quarks that form a loop. Therefore, for the chosen form of the source current (2.2) for the proton (neutron), the u (d) condensate appears in the chirality-conserving structure while the d (u) condensate appears in the chirality-violating one (see also Ref. [5]). The polarization operators for Σ and Ξ can be obtained from the corresponding formulas in Ref. [17] in a similar manner.

Thus, using Eqs. (14) and (17) of Ref. [17][1] we obtain the sum rules for the nucleon:

$$\{2\mu[aM^2E_0(W_N^2/M^2)L^{-1} - \tfrac{1}{6}m_0^2 aL^{-2}] + \tfrac{8}{3}\gamma a^2 L\}e^{m_N^2/M^2}$$
$$= \delta\tilde{\lambda}_N^2 - 2\tilde{\lambda}_N^2 \delta m_N \frac{m_N}{M^2} - \frac{1}{2}\exp\left[-\frac{W_N^2 - m_N^2}{M^2}\right] L^{-1}(W_N^4 + \tfrac{1}{2}b)\delta W_{1N}^2 , \tag{3.11}$$

$$\{2\mu[M^6E_2(W_N^2/M^2)L^{-2} - \tfrac{4}{3}a^2] + 2\gamma a M^4 E_1(W_N^2/M^2)\}e^{m_N^2/M^2}$$
$$= \delta m_N \left[2\frac{m_N^2}{M^2} - 1\right]\tilde{\lambda}_N^2 - \delta\tilde{\lambda}_N^2 m_N + 2a\exp\left[-\frac{W_N^2 - m_N^2}{M^2}\right] W_N^2 \delta W_{2N}^2 , \tag{3.12}$$

where $\delta f = f(n) - f(p)$.

The functions

$$E_0(x) = 1 - e^{-x} , \tag{3.13}$$

$$E_1(x) = 1 - (1+x)e^{-x} , \tag{3.14}$$

$$E_2(x) = 1 - \left[1 + x + \frac{x^2}{2}\right]e^{-x} \tag{3.15}$$

take into account the continuum. The parameters a, b, and m_0^2 are connected with the condensates

$$a = -(2\pi)^2\langle 0|\bar{u}u|0\rangle_{\bar{\mu}} = 0.55 \text{ GeV}^3 , \tag{3.16}$$

$$b = (2\pi)^2\left\langle 0\left|\frac{\alpha_s}{\pi}G_{\mu\nu}^a G_a^{\mu\nu}\right|0\right\rangle = 0.5 \text{ GeV}^4 , \tag{3.17}$$

$$-g\left\langle 0\left|\bar{u}\sigma_{\mu\nu}\frac{\lambda^a}{2}G_{\mu\nu}^a u\right|0\right\rangle_{\bar{\mu}} = m_0^2\langle 0|\bar{u}u|0\rangle , \tag{3.18}$$

with $m_0^2 = 0.8 \text{ GeV}^2$. (For a discussion of the numerical values used here see Ref. [23].)

The factor

$$L = \left[\frac{\ln(M/\Lambda)}{\ln(\bar{\mu}/\Lambda)}\right]^{4/9} \tag{3.19}$$

accounts for the anomalous dimensions ($\bar{\mu}$ is the normalization point). In what follows we use the numerical values $\Lambda = 150$ MeV and $\bar{\mu} = 0.5$ GeV. Also, we take the value for the residue at the nucleon pole and the continuum threshold W^2 obtained from the best fit for the sum rule in the nucleon channel (isospin symmetric case, see Appendix B of Ref. [18]):

$$\tilde{\lambda}_N^2 = 32\pi^4\lambda_N^2 = 2.1 \text{ GeV}^6 , \tag{3.20}$$

$$W_N^2 = 2.3 \text{ GeV}^2 . \tag{3.21}$$

[1]We take this opportunity to correct a misprint in Ref. [17]. The factor $\frac{1}{2}$ in front of the first term of Eq. (17) for $\delta_{2\Xi}$ should be replaced by $\frac{2}{3}$.

For the sake of generality we have assumed that the values for the continuum threshold differences $\delta W^2_{1,2N}$ in Eqs. (3.11) and (3.12) may be different although it is a simple and plausible assumption to take them to be equal. The sum rules (3.11) and (3.12) must hold in the Borel confidence interval [16]

$$0.8 \le M^2 \le 1.4 \text{ GeV}^2 . \tag{3.22}$$

For the Σ hyperons we find, in a similar way,

$$(-\tfrac{2}{3}\mu m_0^2 aL^{-1}+\tfrac{8}{3}\gamma a^2 L)e^{m_\Sigma^2/M^2}=\delta\tilde{\lambda}_\Sigma^2-2\tilde{\lambda}_\Sigma^2\delta m_\Sigma\frac{m_\Sigma}{M^2}-\frac{1}{2}\exp\left[-\frac{W_\Sigma^2-m_\Sigma^2}{M^2}\right]L^{-1}\left[W_\Sigma^4+\frac{b}{2}-4m_s a(1+\beta)\right]\delta W_{1\Sigma}^2 , \tag{3.23}$$

$$\tfrac{16}{3}a^2(\mu+\gamma m_s)e^{m_\Sigma^2/M^2}=-\delta m_\Sigma\left[\frac{2m_\Sigma^2}{M^2}-1\right]\tilde{\lambda}_\Sigma^2+\delta\tilde{\lambda}_\Sigma^2 m_\Sigma-\frac{1}{2}\exp\left[-\frac{W_\Sigma^2-m_\Sigma^2}{M^2}\right]W_\Sigma^2[4a(1+\beta)+m_s W_\Sigma^2]\delta W_{2\Sigma}^2 , \tag{3.24}$$

where $\delta f = f(\Sigma^-)-f(\Sigma^+)$.

The constants $\tilde{\lambda}_\Sigma^2$ and W_Σ^2 were determined in Ref. [19]:

$$\tilde{\lambda}_\Sigma^2 = 3.7 \text{ GeV}^6 , \tag{3.25}$$

$$W_\Sigma^2 = 3.2 \text{ GeV}^2 . \tag{3.26}$$

Equations (3.23) and (3.24) must be satisfied in the interval

$$1.2 \le M^2 \le 1.8 \text{ GeV}^2 . \tag{3.27}$$

Note that in this case the isospin-violating effects manifest themselves only in the higher-order terms of the OPE; they disappear at high q^2 and do not contribute to the discontinuity; i.e. [cf. Eq. (2.7)],

$$\delta W_{1\Sigma}^2 = \delta W_{2\Sigma}^2 = 0 . \tag{3.28}$$

The sum rules for the Ξ hyperons read

$$-2\mu a[M^2 E_0(W_\Xi^2/M^2)L^{-1}+\tfrac{1}{6}m_0^2 L^{-2}]e^{m_\Xi^2/M^2}=\delta\tilde{\lambda}_\Xi^2-2\tilde{\lambda}_\Xi^2\delta m_\Xi\frac{m_\Xi}{M^2}-\frac{1}{2}\exp\left[-\frac{W_\Xi^2-m_\Xi^2}{M^2}\right]L^{-1}(W_\Xi^4+\tfrac{1}{2}b)\delta W_{1\Xi}^2 , \tag{3.29}$$

$$\{2\mu[M^6E_2(W_\Xi^2/M^2)L^{-2}+\tfrac{4}{3}a^2(1+\beta)^2]+2\gamma a[M^4E_1(W_\Xi^2/M^2)+\tfrac{4}{3}am_s(1+\beta)]\}e^{m_\Xi^2/M^2}$$
$$=\delta\tilde{\lambda}_\Xi^2 m_\Xi-\delta m_\Xi\left[\frac{2m_\Xi^2}{M^2}-1\right]\tilde{\lambda}_\Xi^2-2a\exp\left[-\frac{W_\Xi^2-m_\Xi^2}{M^2}\right]W_\Xi^2\delta W_{2\Xi}^2 , \tag{3.30}$$

where $\delta f = f(\Xi^-)-f(\Xi^0)$ and (see Ref. [19])

$$\tilde{\lambda}_\Xi^2 = 5.0 \text{ GeV}^6 , \tag{3.31}$$

$$W_\Xi^2 = 3.6 \text{ GeV}^2 . \tag{3.32}$$

The sum rules (3.29) and (3.30) are expected to be satisfied for Borel masses:

$$1.2 \le M^2 \le 1.8 \text{ GeV}^2 . \tag{3.33}$$

It is instructive to also consider the linear combinations of Eqs. (3.11) and (3.12), (3.23) and (3.24), and (3.29) and (3.30) which do not contain the unknown constants $\delta\tilde{\lambda}^2$. We present them here under the assumption $\delta W_1^2=\delta W_2^2$ for each baryon. Putting the baryon mass splittings on the LHS of the sum-rule equations, we obtain, for the nucleon,

$$\delta m_N = e^{m_N^2/M^2}\tilde{\lambda}_N^{-2}\{\mu[-2M^6E_2L^{-2}+\tfrac{8}{3}a^2-2m_N E_0 aM^2L^{-1}+\tfrac{1}{3}m_N m_0^2 aL^{-2}]-2\gamma a(M^4E_1+\tfrac{4}{3}am_N L)$$
$$-\delta W_N^2\exp(-W_N^2/M^2)[\tfrac{1}{2}W_N^4 m_N L^{-1}+\tfrac{1}{4}bm_N L^{-1}-2aW_N^2]\} . \tag{3.34}$$

For Σ hyperons (assuming again that we can neglect the isospin-violating effects in the continuum) we have

$$\delta m_\Sigma = e^{m_\Sigma^2/M^2}\tilde{\lambda}_\Sigma^{-2}\{\tfrac{16}{3}\mu a[a+\tfrac{1}{8}m_0^2 m_\Sigma L^{-1}]-\tfrac{8}{3}\gamma a^2(m_\Sigma L-2m_s)\} . \tag{3.35}$$

The analogous sum rule for Ξ-hyperons takes the form

$$\delta m_{\Xi}=e^{m_{\Xi}^2/M^2}\tilde{\lambda}_{\Xi}^{-2}\{2\mu[M^6E_2L^{-2}+\tfrac{4}{3}a^2(1+\beta)^2+am_{\Xi}(M^2E_0L^{-1}+\tfrac{1}{6}m_0^2L^{-2})] +2\gamma a[M^4E_1+\tfrac{4}{3}m_s a(1+\beta)]-\delta W_{\Xi}^2\exp(-W_{\Xi}^2/M^2)[\tfrac{1}{2}m_{\Xi}(W_{\Xi}^4+\tfrac{1}{2}b)L^{-1}-2aW_{\Xi}^2]\} . \quad (3.36)$$

IV. ANALYSIS OF THE SUM RULES

Each of the equations (3.34)–(3.36) can be written in the form

$$\delta m = A(M^2)\mu+B(M^2)\gamma+C(M^2)\delta W^2 . \quad (4.1)$$

Of these equations, (3.35) is the simplest as $\delta W_{\Sigma}^2=0$—see Eq. (3.28). For this case straightforward evaluation yields $A_{\Sigma}=1.4(1.7)$ and $B_{\Sigma}=-0.85$ GeV (-0.75 GeV) for a Borel mass $M^2=1.2$ GeV^2 (1.4 GeV^2). Substituting the values for the quark mass difference and the isotopic mass difference $\mu=3.3$ MeV [Eq. (1.1)] and δm_{Σ} [see (3.9)] we obtain, from (4.1),

$$\gamma=-2.7\times10^{-3}\ (-4.2\times10^{-3}) . \quad (4.2)$$

However, because of the fact that only one term of the OPE contributes to the Σ sum rules in this case, we cannot attach too much significance to this result. Including the uncertainties in (3.9) and an uncertainty of, say, 1 MeV to the quark mass difference, the only safe conclusion we can reach from the Σ-hyperon sum rules is that γ is negative, and lies somewhere in the interval $-6\times10^{-3}\le\gamma\le0$.

Let us now turn to the nucleons and Ξ hyperons. In this case we need an estimate for the difference in the continuum thresholds for the particles differing only in isospin projection. In the nucleon case, for example, we expect a difference in the continuum threshold for the neutron and the proton. It seems *a priori* reasonable to assume that δW^2 is positive, and that for each baryon

$$\frac{\delta W^2}{W^2}\approx2\frac{\delta m}{m} . \quad (4.3)$$

The equation for the Ξ hyperons (3.36) yields an important piece of information as γ enters with a positive sign while $C(M^2)$ is negative. Thus, we are able to obtain an upper bound on $|\gamma|$ by putting $\delta W_{\Xi}^2=0$. The numerical values for the coefficients $A-C$ are

$$A_{\Xi}=2.64\ (2.75) ,$$
$$B_{\Xi}=1.15\ (1.16)\ \text{GeV} , \quad (4.4)$$
$$C_{\Xi}=-0.174\ (-0.277)\ \text{GeV}^{-1}$$

for $M^2=1.6$ (1.2) GeV^2. The weak dependence of the coefficients on the Borel parameter indicates a certain amount of self-consistency in the sum rules. Using the numerical values of μ and δm_{Ξ} as given by Eqs. (1.1) and (3.10) we obtain

$$\gamma\gtrsim-3.5\times10^{-3} . \quad (4.5)$$

Larger values of $|\gamma|$ are only possible at the expense of larger values of the quark mass difference; e.g., $\gamma=-5\times10^{-3}$ requires $\mu\ge4.5$ MeV.

For the nucleon, Eq. (4.1) is satisfied with the coefficients

$$A_N=-0.46\ (-0.61) ,$$
$$B_N=-1.92\ (-1.84)\ \text{GeV} , \quad (4.6)$$
$$C_N=0.046\ (0.065)\ \text{GeV}^{-1}$$

at $M^2=1.0$ (1.2) GeV^2. From (4.3) we can estimate $\delta W_N^2\lesssim10^{-2}$ GeV^2, and therefore

$$C_N(M^2)\delta W_N^2\ll\delta m_N-A_N(M^2)\mu . \quad (4.7)$$

Thus, clearly we need a nonzero and negative value of γ to satisfy (4.1). Again, since negative values of γ and positive ones for δW^2 contribute with the same sign to $\delta m_N-A_N(M^2)\mu$, we obtain an upper bound for $|\gamma|$ by setting $\delta W_N^2=0$. This turns out to be $|\gamma|=2\times10^{-3}$ for $\mu=3.3$ MeV. More precisely, with $\delta m_N=2.05$ MeV, $\mu=3.3$ MeV, and $\delta W_N^2=8\times10^{-3}$ GeV^2, Eq. (4.1) yields

$$\gamma=-(1.5\text{–}2.0)\times10^{-3} \quad (4.8)$$

for $0.8\le M^2\le1.2$ GeV^2. The nucleon sum rule provides a much stronger upper bound on $|\gamma|$ than the Ξ sum rule for the case of large μ: even for $\mu=5$ MeV we find that $|\gamma|$ is restricted to $|\gamma|<3\times10^{-3}$. However, it is clear that there is a dependence on the Borel parameter M^2 in these sum rules, which indicates that higher-order terms in the OPE are non-negligible and deteriorate the accuracy of the result. Therefore, our conservative conclusion from the consideration of the sum rules (3.34)–(3.36) with respect to γ is

$$\gamma=(-2\pm1)\times10^{-3} . \quad (4.9)$$

Let us now study the sum rules (3.11), (3.12), (3.23), (3.24), (3.29), and (3.30) which contain more information, as it is possible to extract $\delta\tilde{\lambda}^2$ in two ways from each of the pairs of sum rules, and check if they coincide and depend weakly on M^2. We have plotted in Fig. 1(a) the result for $\delta\tilde{\lambda}_N^2$ as calculated from (3.11) [curves labeled (1)] and (3.12) [curves labeled (2)] for $\mu=3.3$ MeV, $\gamma=-2\times10^{-3}$, and $\delta W_1^2=\delta W_2^2=1.0\times10^{-2}$ GeV^2 (solid curves). The agreement is satisfactory and the M^2 dependence is weak. In fact, the agreement tends to be more pronounced for slightly smaller values of $|\gamma|$. On the other hand, we also find that the sum rules cannot be made consistent for larger values of $|\gamma|$, such as the value $|\gamma|\gtrsim6\times10^{-3}$ which was used in [6,7,12]. A closer look at the linear combination of the sum rules (3.11) and (3.12) that eliminates $\delta\tilde{\lambda}_N^2$, with the condition $\delta W_1^2=\delta W_2^2$ removed, reveals that it can only be satisfied for $\delta W_1^2\gg\delta W_2^2$. This being favored in the case of large $|\gamma|$,

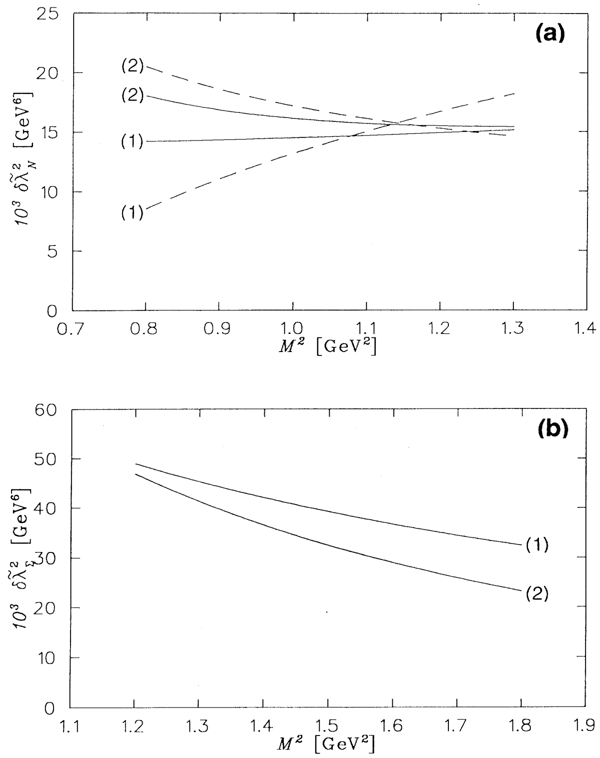

FIG. 1. (a) The values of $\delta\tilde{\lambda}_N^2$ calculated from the nucleon sum rules (3.11) (labeled 1) and (3.12) (labeled 2) for $\mu = 3.3$ MeV. The solid curves correspond to $\gamma = -2\times 10^{-3}$ while the dashed ones were obtained with $\gamma = -6\times 10^{-3}$. (b) $\delta\tilde{\lambda}_\Sigma^2$ calculated from the Σ-hyperon sum rules (3.23) (labeled 1) and (3.24) (labeled 2) for $\mu = 3.3$ MeV. γ as in (a).

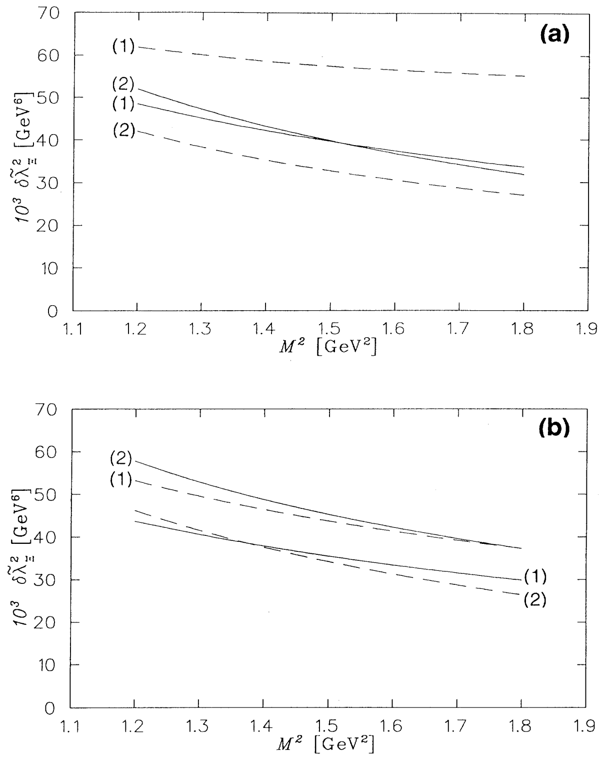

Fig. 2

FIG. 2. (a) The values of $\delta\tilde{\lambda}_\Xi^2$ calculated from the Ξ-hyperon sum rules (3.29) (labeled 1) and (3.30) (labeled 2) for $\mu = 3.3$ MeV. The solid curves correspond to $\gamma = -2\times 10^{-3}$ while the dashed ones were obtained with $\gamma = -6\times 10^{-3}$. (b) Same as (a), only for $\mu = 4.3$ MeV (solid lines) and $\mu = 2.3$ MeV (dashed lines).

we plot in Fig. 1(a) $\delta\tilde{\lambda}_N^2$ as predicted by each of the equations (3.11) and (3.12) for $\gamma = -6\times 10^{-3}$ and $\delta W_1^2 = 25\times 10^{-3}$ GeV^{-2}, $\delta W_2^2 = 0$ (dashed lines). A strong discrepancy between the sum rules is evident. We would like to stress that the unreasonably large value for δW_1^2 used serves to *reduce* the discrepancy between the curves. Indeed, Eqs. (3.11) and (3.12) show that reducing δW_1^2 or increasing δW_2^2 *increases* the discrepancy.

In the same way we can investigate the domain of small $|\gamma|$. We find in a similar manner that a value of $\gamma = 0$ can only be tolerated at the expense of a large difference between δW_1^2 and δW_2^2, for instance, $\delta W_1^2 = 0$ and $\delta W_2^2 = 10\times 10^{-3}$ GeV^2. This being unreasonable, we can safely exclude $\gamma = 0$.

We present in Fig. 2(a) the results for the analogous investigation of the Ξ-hyperon sum rules, Eqs. (3.29) and (3.30). The solid lines correspond to $\gamma = -2\times 10^{-3}$, $\delta W_1^2 = \delta W_2^2 = 3\times 10^{-3}$ GeV^2, whereas the dashed lines were produced with $\gamma = -6\times 10^{-3}$ and $\delta W_1^2 = \delta W_2^2 = 15\times 10^{-3}$ GeV^2. Again, the values of $\delta W_{1,2}^2$ were chosen such as to maximize the agreement between the curves. As before, good agreement is achieved for the first case, disagreement for the second.

Figure 1(b) shows the result of evaluating Eqs. (3.23) and (3.24) for the Σ-hyperon, with $\gamma = -2\times 10^{-3}$, $\mu = 3.3$ MeV, and $\delta W_1^2 = \delta W_2^2 = 0$.

Finally, we would like to investigate the dependence of the sum rules on the quark mass difference. As the Ξ sum rules are most sensitive to this parameter, we shall focus on those. We have plotted in Fig. 2(b) the prediction for $\delta\tilde{\lambda}_\Xi^2$ from Eqs. (3.29) and (3.30) [labeled (1) and (2), respectively] for $\gamma = -2\times 10^{-3}$ and $\delta W_1^2 = \delta W_2^2 = 3\times 10^{-3}$ GeV^2 for two additional values of μ, namely, $\mu = 2.3$ MeV (dashed lines) and $\mu = 4.3$ MeV (solid lines) [see Fig. 2(a) for the intermediate value of μ]. It is clear that both choices lead to a serious disagreement between the curves that can only be resolved assuming a large difference between δW_1^2 and δW_2^2, which appears unreasonable. Comparing Figs. 2(a) and 2(b) we conclude that μ should be close to 3 MeV with an error (conservatively) of about 1 MeV.

Our final results then for the quantities γ and μ as implied by the sum rules (3.11), (3.12), (3.23), (3.24), (3.29), and (3.30) are

$$\gamma = (-2\pm 1)\times 10^{-3} , \tag{4.10}$$

$$\mu = m_d - m_u = (3.0\pm 1.0)\ \text{MeV} . \tag{4.11}$$

In order to put this result into perspective, let us comment on the approximations going into it. It is known [16,18] that higher-order terms in the OPE neglected here are small for the sum rule for the nucleon mass and cannot change this value by more than 10%. In the sum rules studied above, the contributions to the sum rule coming from the highest dimension terms ($d=6$) in the chirality-preserving structure (terms of the order $\sim\mu m_0^2 a$) are always small, smaller than 15% of the main term. We find that contributions of the type γa^3 of dimension $d=9$ change the value of γ only by a few per cent.

Let us now discuss α_s corrections. The α_s corrections to the terms proportional to γ, γa, and γa^2 can easily be calculated using the results of Ref. [21]. They turn out to be small ($\lesssim 10\%$). The α_s corrections to the main terms of the OPE proportional to μ are presently unknown. In the (isospin-symmetric) sum rules for the proton mass the α_s corrections to the main term are relatively large. However, as can be shown using the formulas of Ref. [21], they mainly change the value of the residue $\tilde{\lambda}_N^2$ (increasing it by about 25–30 %) while only slightly changing the pole position (diminishing the proton mass by about 5%).[2] We believe that these conclusions carry over to the sum rules including isospin violation presented here. In any case we expect the corrections to γ to fall within the conservatively chosen error bars included in the result (4.10). (The Ξ sum rules imply that solely increasing $\tilde{\lambda}^2$ is unreasonable as it leads to much smaller values of $|\gamma|$.)

V. DISCUSSION AND COMPARISON WITH PREVIOUS WORK

In most instances of previous work, values of the order $\gamma=-(6-10)\times 10^{-3}$, significantly different from ours, were obtained. Let us therefore examine some of the earlier results.

In the paper by Paver, Riazzudin, and Scadron [7], γ was calculated in a constituent quark model. Hatsuda, Høgaasen, and Prakash [4] and Adami and Brown [5] (the latter in one of their approaches) used the Nambu–Jona-Lasinio model. We believe that the values for γ obtained in these approaches are unreliable for the following reasons. In QCD, as well as PCAC (partial conservation of axial-vector current) type Lagrangians, the value of the current quark masses can be obtained with rather good accuracy. In QCD furthermore, it is obvious that the value of γ is strongly correlated with $\mu=m_d-m_u$. However, in the above-mentioned model approaches, the relation of μ to the other model parameters is obscure. (E.g., in QCD, μ as well as the condensates are renormalization-scale dependent. This concept is absent in quark model and Nambu–Jona-Lasinio-type approaches.)

In Refs. [9–12], γ was obtained by calculating the polarization operators of the *divergence* of vector and axial-vector currents in the framework of QCD sum rules. According to current algebra, γ is related to $\Pi_V(0)$, the vector-polarization operator at zero momentum transfer, leading to $\gamma=-9\times 10^{-3}$ [12]. We are rather skeptical towards this approach as it is well known [24] that the QCD sum-rule method fails in the scalar and pseudoscalar channels. Indeed, it cannot explain the strong violation of the Okubo-Zweig-Iizuka rule in the pseudoscalar channel [25]. Also, there are very serious problems related to subtractions in this approach.

The analysis in Refs. [8,5] is more closely related to the one presented here. In [8], γ was determined from mass splittings in the baryon octet via the QCD sum-rule method, leading to a value $\gamma=-6\times 10^{-3}$. This approach differs from ours in several points: (i) a baryon current different from the one adopted here was used, (ii) the mixed condensate (3.18) as well as anomalous dimensions were ignored, (iii) a different set of parameters was used, namely, $m_s(0.5\text{ GeV})=260$ MeV, $\langle\bar{s}s\rangle=0.5\langle\bar{u}u\rangle$ as opposed to our $m_s(0.5\text{ GeV})=150$ MeV, $\langle\bar{s}s\rangle=0.8\langle\bar{u}u\rangle$. We have serious doubts about the procedure to choose a mixing angle t adopted in Ref. [8]. On the one hand, t is constrained to be the same for all members of the octet, on the other hand it is chosen by requiring that the continuum contributions in the isospin-violating structures vanishes. Also, a vanishing continuum leads to large higher-order terms in the OPE side of the sum rule for baryons, since it is impossible to approximate the functional dependence of the exponential $\exp(-m^2/M^2)$ on the LHS with only a few terms that have a power-law dependence on $1/M^2$ on the RHS.

In Ref. [5], the neutron-proton mass difference was considered in the framework of QCD sum rules, and the polarization operator was calculated, however, without taking into account the continuum. Also, a systematic analysis of the ensuing sum rules was not performed, as its principal aim was to show the mechanism which made the proton-neutron mass difference vary with density. Nevertheless, moving the mass difference used by Adami and Brown ($\mu=4$ MeV) towards the one adopted here improves the agreement between the latter and the present work.

We now turn to chiral perturbation theory, specifically to the results obtained by Gasser and Leutwyler [6] for the parameter γ. Their equation contains an unknown subtraction term, which, however, can be written in terms of the flavor SU(3)-breaking condensate parameter β defined in (2.8). The final result from Ref. [6] is then

$$\gamma=-\frac{\mu}{m_s-(m_u+m_d)/2}\left[-\beta+\frac{1}{16\pi^2F_\pi^2}\left[m_K^2-m_\pi^2-m_\pi^2\ln\frac{m_K^2}{m_\pi^2}\right]\right], \tag{5.1}$$

[2]This is the case choosing the normalization point $\bar{\mu}=0.5$ GeV adopted here (as in Refs. [15–19]) rather than $\bar{\mu}=0.2$ GeV as in Ref. [21].

where $F_\pi = 93$ MeV and m_K and m_π are the kaon and pion masses. Numerically,

$$\gamma = 2.3\times10^{-2}\beta - 3\times10^{-3} = -7.6\times10^{-3} \qquad (5.2)$$

for $\beta = -0.20$. As we argued in Sec. IV, such a large value of $|\gamma|$ is excluded in our QCD sum-rule analysis. We do not see any loopholes in our arguments which could possibly accommodate the result (5.2). We should keep in mind, however, that (5.1) was obtained in first-order chiral perturbation theory. The suspicion persists that higher-order terms in the series could significantly alter (5.2). Indeed, the second term in the square brackets of Eq. (5.1) amounts to about 40% of the total (5.2), and arises as a loop correction in chiral perturbation theory. It is nonanalytic in the quark mass (being proportional to $m_q \ln m_q$) and of the same order of magnitude as the first term in (5.1). Terms nonanalytic in the quark mass are, however, absent in the polarization operator for the baryon current (2.1). Should the result from chiral perturbation theory prove to be stable, it would imply that higher-order corrections to the OPE are unusually large. We surmise that the calculation of higher-order terms in chiral perturbation theory, as well as those resulting from α_s corrections to the isospin-violating QCD sum rules for baryons, will help to resolve this discrepancy.

Our final remark is connected with the proposed [4,5] explanation of the Nolen-Schiffer anomaly. Using (3.34) and our value for γ it is easy to estimate how the neutron-proton mass difference would behave if the value of the quark condensate is reduced by some amount compared to its vacuum value. We find that for $\gamma = -2\times10^{-3}$ a 10% reduction of the quark condensate in the nucleus results in a decrease of the neutron-proton mass difference by 1 MeV—just the value needed for a resolution of the NS anomaly. A 10% decrease of the quark condensate inside the nucleus appears to be quite reasonable.

Note added in proof. In a recent paper [26] the neutron-proton mass difference was calculated using QCD sum rules. The authors claim that good agreement with phenomenological data can be achieved for $\gamma = -6.6\times10^{-3}$, in contradiction with the results presented here. The main difference between the calculation in [6] and ours is that while we subtract the *electromagnetic* mass difference from the experimental value and construct the QCD sum rules for the remaining (strong-interaction) piece, in Ref. [26] the electromagnetic interaction was accounted for in the sum rules by introducing a fitting parameter, thus calculating the entire neutron-proton mass difference. Examination of the sum rules presented in [26] shows that the electromagnetic $n-p$ mass difference as determined from the chirality conserving sum rule (Eqs. (22) and (24) of Ref. [26]) is equal to $(m_n - m_p)_{\rm elec} = -0.11$ MeV while the sum rule for the chirality-violating structure (Eqs. (23) and (25) of Ref. [26]) yields $(m_n - m_p)_{\rm elec} = -4.5$ MeV. These numbers contradict each other as well as the phenomenological value (3.2). The latter contradiction was noted by the authors by observing strongly contradicting values for the coupling constants for the two sum rules. In conclusion, we find that the agreement with the phenomenological value of the $n-p$ mass difference obtained with a value $\gamma = -6.6\times10^{-3}$ in Ref. [26] arises from a spurious compensation of electromagnetic and strong interaction effects. In general, the standard QCD sum-rule method cannot account for electromagnetic effects, as the continuum model of pole+continuum cannot reflect $N+\gamma$ excited states. As a consequence, factorization of the four-quark condensate can no longer be assumed. Note that if electromagnetic interactions are turned off in Eqs. (16) and (17) and (20) and (21) of Ref. [26], they coincide with the equations of this paper and lead to the same conclusions as presented here.

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