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Heavy-quark spin symmetry and D mesons

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The implications of heavy-quark spin symmetry for P-state mesons with one heavy quark are discussed. In particular, we have derived the mass relation $m_{D_1^*}-m_{D_0}=\frac{5}{2}(m_{D_2^*}-m_{D_1})$ for the four P-state mesons. We have shown that the $E1$ transition decay $D_1(2420)\to D+\gamma$ is a prediction of the above symmetry and as such should be tested experimentally. We predict $m_{D_1^*}=2.29$ GeV and $m_{D_0}=2.19$ GeV in the model in which the spin-dependent potential is taken to be a one-gluon-exchange potential. In the second model, with the Cornell potential in the Dirac Hamiltonian, we obtain $m_{D_1^*}=2.01$ GeV and $m_{D_0}=1.91$ GeV.

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For hadrons containing a single heavy quark (Q), the dynamics is simplified by the heavy-quark spin symmetry of QCD [1,2] (for a review see, e.g., Ref. [3]). For a $q\bar{Q}$ or $Q\bar{q}$ system, the spins $\mathbf{S}_q$ and $\mathbf{S}_Q$ decouple. The implications of heavy-quark spin symmetry for the mass spectrum and strong decay widths of such hadrons have been discussed recently [4]. However, the heavy-quark limit for the mass splittings of P-wave D mesons was first considered in Ref. [5].

The purpose of this paper is to discuss the implications of heavy-quark spin symmetry for the masses of P-state mesons with one heavy quark. We also discuss an important consequence of their decay by $E1$ transition.

For a $q\bar{q}$ or $Q\bar{Q}$ bound system, we first combine spins of q and $\bar{q}$ (Q and $\bar{Q}$) and then combine it with orbital angular momentum; i.e., we combine $S=0$ and 1 states with angular momentum $L=1$. In this way one obtains four P states which are designated as ${}^1P_1$, ${}^3P_{J=0,1,2}$. The states ${}^3P_J$ decay to ${}^3S_1$ with the emission of γ rays by $E1$ transition, whereas ${}^1P_1$ decays to ${}^1S_0$. If we follow the same procedure in combining the spin and orbital angular momentum for a $q\bar{Q}$ or $Q\bar{q}$ system, we again get the four states discussed above.

However, if we impose heavy-quark spin symmetry, then the spin of the heavy quark is treated separately and it is natural to combine $\mathbf{j}=\mathbf{L}+\mathbf{S}_q$ with S_Q, i.e., $\mathbf{J}=\mathbf{j}+\mathbf{S}_Q$. For P states, $\mathbf{j}^2$ has eigenvalues $j(j+1)$ with $j=\frac{3}{2}$ or $\frac{1}{2}$. In this case, we get two multiplets [4]: one with $J=(\frac{3}{2}+\frac{1}{2})=2$ and $J=(\frac{3}{2}-\frac{1}{2})=1$ and the other with $J=(\frac{1}{2}+\frac{1}{2})=1$ and $J=(\frac{1}{2}-\frac{1}{2})=0$. For $c\bar{d}$, we designate these two multiplets as (D_2^*,D_1) and (D_1^*,D_0), respectively.

In order to discuss the mass spectrum for a $q\bar{Q}$ or $Q\bar{q}$ bound system, we note that, in the limit $m_Q\to\infty$, the heavy quark or antiquark can be taken as a static source of field in which the light quark or antiquark moves. The situation is like the hydrogen atom. Therefore, we can use the Dirac Hamiltonian for a hydrogen atom as a guide. To order v^2/c^2, this Dirac Hamiltonian is given by [6]

$$H=\frac{\hat{p}^2}{2m}+V(r)-\frac{\hat{p}^{(4)}}{8m^2}-\frac{1}{4m^2}\sigma\cdot(\mathbf{E}\times\hat{\mathbf{p}})-\frac{1}{8m^2}\nabla\cdot\mathbf{E}\ . \tag{1}$$

Thus for a $q\bar{Q}$ or $Q\bar{q}$ system, we may take the Hamiltonian [7] as

$$H=H_0-\frac{\hat{p}^4}{8m^2}+\frac{1}{2m^2}\mathbf{S}_q\cdot\mathbf{L}\left[\frac{1}{r}\frac{dV_1}{dr}\right]+\frac{1}{8m^2}\nabla^2V_1\ , \tag{2}$$

where

$$H_0=\hat{p}^2/2\mu+V(r)\ , \tag{3}$$

$$\mathbf{E}=-\frac{dV_1}{dr}\frac{\mathbf{r}}{r}\ . \tag{4}$$

Note that $\hat{\mathbf{p}}=-i\nabla$, $\mathbf{L}=\mathbf{r}\times\hat{\mathbf{p}}$, $1/\mu=1/m+1/M$; m is the mass of the light quark and M is the mass of the heavy quark. The spin-dependent potential V_1 may or may not be the same as V.

In the heavy-quark symmetry limit $m_{D_2^*}=m_{D_1}$, $m_{D_1^*}=m_{D_0}$ and $m_{D^*}=m_D$. The mass difference between the multiplets (D_2^*,D_1) and (D_1^*,D_0) arises due to the spin-orbit term in Eq. (2). Noting that $\langle\mathbf{S}_q\cdot\mathbf{L}\rangle_{j=3/2}=\frac{1}{2}$ and $\langle\mathbf{S}_q\cdot\mathbf{L}\rangle_{j=1/2}=-1$, we obtain, from Eq. (2),

$$\begin{aligned} m_{D_2^*}&=m_{D_1}=\bar{M}+\tfrac{1}{2}g\ ,\\ m_{D_1^*}&=m_{D_0}=\bar{M}-g\ , \end{aligned} \tag{5}$$

where

$$g=\frac{1}{2m^2}\left\langle\frac{1}{r}\frac{dV_1}{dr}\right\rangle . \qquad (6)$$

The mass difference between members of a multiplet arises from terms of order $1/mM$. Such terms for the S wave and P wave can be added to the Hamiltonian (2) [8,9,5]. Thus we can write the basic Hamiltonian as

$$H=H_0-\frac{\hat{p}^4}{8m^2}+\frac{1}{2m^2}\mathbf{S}_q\cdot\mathbf{L}\left[\frac{1}{r}\frac{dV_1}{dr}\right]+\frac{1}{8m^2}\nabla^2V_1+\frac{2}{3Mm}\mathbf{S}_q\cdot\mathbf{S}_Q\nabla^2V_2+\frac{1}{Mm}\mathbf{S}\cdot\mathbf{L}\left[\frac{1}{r}\frac{dV_2}{dr}\right]$$
$$+\frac{1}{12Mm}[12(\mathbf{S}_q\cdot\hat{\mathbf{r}})(\mathbf{S}_Q\cdot\hat{\mathbf{r}})-4\mathbf{S}_q\cdot\mathbf{S}_Q]\left[\frac{1}{r}\frac{dV_2}{dr}-\frac{d^2V_2}{dr^2}\right] , \qquad (7)$$

where $\mathbf{S}=\mathbf{S}_q+\mathbf{S}_Q$.

In order to discuss the mass difference, we first write the four P states, showing their spin and angular momentum content. According to the scheme outlined above, a P state can be labeled as $|JMjs_Q\rangle$. Thus we can write

$$|D_2^*\rangle=|2M\tfrac{3}{2}\tfrac{1}{2}\rangle=\left[\frac{(2+M)(1+M)}{12}\right]^{1/2}Y_{1M-1}\chi_+^{+1}+\left[\frac{4-M^2}{6}\right]^{1/2}Y_{1M}\chi_+^0+\left[\frac{(2-M)(1-M)}{12}\right]^{1/2}Y_{1M+1}\chi_+^{-1} , \qquad (8)$$

$$|D_1\rangle=|1M\tfrac{3}{2}\tfrac{1}{2}\rangle=-\left[\frac{(2-M)(1+M)}{12}\right]^{1/2}Y_{1M-1}\chi_+^{+1}+\frac{1}{\sqrt{6}}Y_{1M}(M\chi_+^0+2\chi_-^0)+\left[\frac{(2+M)(1-M)}{12}\right]^{1/2}Y_{1M+1}\chi_+^{-1} , \qquad (9)$$

$$|D_1^*\rangle=|1M\tfrac{1}{2}\tfrac{1}{2}\rangle=-\left[\frac{(2-M)(1+M)}{6}\right]^{1/2}Y_{1M-1}\chi_+^{+1}+\frac{1}{\sqrt{3}}Y_{1M}(M\chi_+^0-\chi_-^0)+\left[\frac{(2+M)(1-M)}{6}\right]^{1/2}Y_{1M+1}\chi_+^{-1} , \qquad (10)$$

$$|D_0\rangle=|00\tfrac{1}{2}\tfrac{1}{2}\rangle=(1/\sqrt{3})(Y_{1-1}\chi_+^{+1}-Y_{10}\chi_+^0+Y_{11}\chi_+^{-1}) , \qquad (11)$$

where χ_+^{+1}, χ_+^0, χ_+^{-1} are spin triplet states and χ_-^0 is spin singlet state. These states are eigenstates of $\mathbf{S}^2$ and S_z with $S=1$ and 0, respectively. Using Eqs. (8)–(11) and the Hamiltonian (7), we obtain the masses of four P states:

$$\begin{aligned}
m_{D_2^*}&=\overline{M}+\tfrac{1}{2}g+g_2-\tfrac{2}{5}h ,\\
m_{D_1}&=\overline{M}+\tfrac{1}{2}g-\tfrac{1}{3}g_2+\tfrac{2}{3}h ,\\
m_{D_1^*}&=\overline{M}-g-\tfrac{2}{3}g_2+\tfrac{4}{3}h ,\\
m_{D_0}&=\overline{M}-g-2g_2-4h ,\\
m_{D_1-D_1^*}&=-\frac{\sqrt{2}}{3}g_2+\frac{2\sqrt{2}}{3}h ,
\end{aligned} \qquad (12)$$

where

$$g_2=\frac{1}{Mm}\left\langle\frac{1}{r}\frac{dV_2}{dr}\right\rangle , \qquad (13)$$

$$h=\frac{1}{12Mm}\left\langle\frac{1}{r}\frac{dV_2}{dr}-\frac{d^2V_2}{dr^2}\right\rangle . \qquad (14)$$

Taking V_2 as a short-range Coulomb-like potential [5], i.e., $V_2=-K'/r$, we obtain $g_2=d/Mm$, $h=d/4Mm$, where $d=K'\langle 1/r^3\rangle$. From Eqs. (12), we obtain

$$\begin{aligned}
m_{D_2^*}&=\overline{M}+\frac{1}{2}g+\frac{9d}{10Mm} ,\\
m_{D_1}&=\overline{M}+\frac{1}{2}g-\frac{d}{6Mm} ,\\
m_{D_1^*}&=\overline{M}-g-\frac{d}{3Mm} ,\\
m_{D_0}&=\overline{M}-g-3\frac{d}{Mm} ,\\
m_{D_1-D_1^*}&=-\frac{d}{3\sqrt{2}Mm} .
\end{aligned} \qquad (15)$$

Since the mixing angle is very small, the mixing gives a small correction to the masses m_{D_1} and $m_{D_1^*}$. We will neglect the mixing. In fact, it is a very good approximation (see below). Hence from Eqs. (15), we obtain

$$\frac{m_{D_1^*}-m_{D_0}}{m_{D_2^*}-m_{D_1}}=\frac{5}{2} . \qquad (16)$$

This is a general result and holds for any meson with one heavy quark. Thus the result (16) also holds for B mesons.

In deriving Eq. (16), we have assumed that the potential V_2 is a short-range Coulomb-like potential. The assumption is supported by the fine structure of P states of quarkonium ($c\bar{c}$ and $b\bar{b}$) systems, where V_2 is taken to be a short-range Coulomb-like potential; it is the potential V_1 which has a long-range confining part in addition to a short-range part [7,10]. Since our result (16) does not depend on the structure of V_1, it is on a sound footing. Thus Eq. (16) can be taken as a definite prediction of heavy-quark spin symmetry.

Another thing to be noted from the fine structure of quarkonium systems is that the spin-dependent potentials are not flavor independent [7,10]. Thus V_1 cannot be determined from the quarkonium systems.

Using the experimental values [11], $m_{D_2^*}=2.460$ GeV, $m_{D_1}=2.420$ GeV, so that $m_{D_2^*}-m_{D_1}\approx 40$ MeV, we obtain from Eq. (15) $m_{D_1^*}-m_{D_0}\approx 100$ MeV. In future experiments, by measuring this mass difference, the heavy-quark spin symmetry of QCD can be tested. Using $m=m_d=0.34$ GeV, $M=m_c=1.52$ GeV, we obtain, from [cf. Eqs. (15)]

$$m_{D_2^*}-m_{D_1}=\frac{16}{15}\frac{d}{Mm}\ , \tag{17}$$

$d=0.019$ GeV^3.

To proceed further we have to specify the potential V_1. It is reasonable to use the following two models for the spin-dependent potential V_1. In the first model (to be called model 1) we take $V_1=V_2=-K'/r$. In this case the spin-dependent potential is that given by the one-gluon-exchange potential [5]. In the second model (to be called model 2) we take $V_1=V=(1/b^2)r-K/r$. Note that V is the Cornell potential [12] which has been successfully used to explain the mass spectra of quarkonia.

We first consider model 1. In this model, $g=d/2m^2$. Thus in this model, the masses are given by

$$\begin{aligned} m_{D_2^*}&=\overline{M}+\frac{d}{4m^2}+9\frac{d}{10Mm}\ ,\\ m_{D_1}&=\overline{M}+\frac{d}{4m^2}-\frac{d}{6Mm}\ ,\\ m_{D_1^*}&=\overline{M}-\frac{d}{2m^2}-\frac{d}{3Mm}\ ,\\ m_{D_0}&=\overline{M}-\frac{d}{2m^2}-3\frac{d}{Mm}\ . \end{aligned} \tag{18}$$

Now using $d=0.019$ GeV^3, $m_{D_1}=2.420$ GeV, $m_{D_2}=2.460$ GeV, we get $\overline{M}=2.385$ GeV. Thus we predict $m_{D_1^*}=2.29$ GeV, $m_{D_0}=2.19$ GeV. The mixing angle is given by

$$\tan 2\phi=-\frac{2\sqrt{2m}}{9M+2m}\ . \tag{19}$$

Using $m=0.34$ GeV and $M=1.52$ GeV, we find $\phi\approx-2°$. Thus the mixing between D_1 and D_1^* is indeed negligible. In order to get other predictions of the model, especially for the $1S$ state, we use the following procedure: we treat H_0 as an unperturbed Hamiltonian. The expectation value $\langle H_0\rangle$ is determined by minimizing it. For this purpose we use the Gaussian wave functions for $1S$ and $1P$ states. Thus we write the reduced radial wave function

$$u(y)\left[y=\left[\frac{2\mu}{b^2}\right]^{1/3} r\right]$$

for these states:

$$u_{1s}=\left[\frac{4\beta_{1s}^3}{\sqrt{\pi}}\right]^{1/2} y e^{-1/2\beta_{1s}^2 y^2}\ , \tag{20}$$

$$u_{1p}=\left[\frac{8\beta_{1p}^5}{3\sqrt{\pi}}\right]^{1/2} y^2 e^{-1/2\beta_{1p}^2 y^2}\ . \tag{21}$$

Using $b=2.34$ GeV^{-1}, $K=0.52$ for the Cornell potential and minimizing $\langle H_0\rangle$, we find, for the $c\bar{d}$ system, $\beta_{1s}=0.809$, $\overline{E}_{1s}=\langle H_0\rangle_{1s}$, $\beta_{1p}=0.703$, $\overline{E}_{1p}=\langle H_0\rangle_{1p}=1.192$ GeV.

Using the relations [13]

$$\begin{aligned} |\psi_{1s}(0)|^2&=\frac{2\mu}{4\pi}\left\langle\frac{dV}{dr}\right\rangle_{1s}\\ &=\frac{2\mu}{4\pi}\left[\frac{1}{b^2}+K\left\langle\frac{1}{r^2}\right\rangle_{1s}\right] \end{aligned} \tag{22}$$

and

$$\begin{aligned} 2\mu\left\langle\frac{dV}{dr}\right\rangle_{1p}&=2\mu\left[\frac{1}{b^2}+K\left\langle\frac{1}{r^2}\right\rangle_{1p}\right]\\ &=4\left\langle\frac{1}{r^3}\right\rangle_{1p} \end{aligned} \tag{23}$$

and the wave functions given in Eqs. (20) and (21), we can easily calculate the matrix elements $\langle 1/r^2\rangle_{1/s}$ and $\langle 1/r^2\rangle_{1p}$. Thus from Eqs. (22) and (23), we obtain $|\psi_{1s}(0)|^2\approx 0.015$ GeV^3, $\langle 1/r^3\rangle_{1p}=0.031$ GeV^3. Now, using the relations

$$f_D^2=\frac{6|\psi_{1s}(0)|^2}{m_D}\ , \tag{24}$$

$$d=K'\left\langle\frac{1}{r^3}\right\rangle_{1p}=0.019\ \text{GeV}^3\ , \tag{25}$$

we obtain $f_D=217$ MeV (in the normalization $f_\pi=93$ MeV) and $K'=0.62$. K' comes out to be 20% higher than $K=0.52$ in the Cornell potential.

The mass splitting between $^3S_1$ and $^1S_0$ states is given by the fifth term in the Hamiltonian (7), namely, the Fermi term

$$\begin{aligned} \langle V_F\rangle_{1s}&=\frac{2S(S+1)-3}{6Mm}\langle\nabla^2 V_2\rangle_{1s}\\ &=\frac{2S(S+1)-3}{6Mm}4\pi K'|\psi_{1s}(0)|^2\ . \end{aligned} \tag{26}$$

From Eq. (26), we obtain $m_{D^*}-m_D=148$ MeV, remarkably close to its experimental value [11] of 145 MeV. The Darwin term [the fourth term in Eq. (7)] is given by

$$\begin{aligned} \langle V_D\rangle_{1s}&=\frac{1}{8m^2}\langle\nabla^2 V_1\rangle_{1s}\\ &=\frac{1}{8m^2}4\pi K'|\psi_{1s}(0)|^2\ . \end{aligned} \tag{27}$$

We find $\langle V_D \rangle_{1s} = 124$ MeV.

Hence we write

$$m_{D^*} = m_c + m_d + \bar{E}_{1s} + \langle V_D \rangle_{1s} + \langle V_F \rangle_{\text{triplet}} - A \ , \tag{28}$$

where we have absorbed the term $-\langle \hat{p}^4/8m^2 \rangle_{1s}$ in the constant A. Using $m_{D^*} = 2.010$ GeV as input, we find $A = 0.719$ GeV. Assuming $-\langle \hat{p}^4/8m^2 \rangle_{1p}$ is about the same as for the $1S$ state, we find

$$\bar{M} = m_d + m_c + \bar{E}_{1p} - A \tag{29}$$

to be 2.332 GeV which is about 50 MeV less than the value $\bar{M} = 2.385$ GeV obtained from Eq. (18).

We now discuss the model 2. In this model, we take $V_1 = V = (1/b^2)r - K/r$. For the $1S$ state, the only difference is in the Darwin term. Here we get

$$\langle V_D \rangle_{1s} = \frac{1}{8m^2} \langle \nabla^2 V_1 \rangle_{1s} = \frac{1}{8m^2} \left[\frac{2}{b^2} \left\langle \frac{1}{r} \right\rangle_{1s} + 4\pi K |\psi_{1s}(0)|^2 \right] . \tag{30}$$

Equation (30) gives $\langle V_D \rangle_{1s} = 271$ MeV. Hence we obtain $A = 0.866$ GeV for this model. Here the Darwin term also contributes to the $1P$ state. We obtain

$$\langle V_D \rangle_{1p} = \frac{1}{8m^2} \left[\frac{2}{b^2} \left\langle \frac{1}{r} \right\rangle_{1p} \right] . \tag{31}$$

Equation (31) gives $\langle V_D \rangle_{1p} = 97$ MeV. Hence we get

$$\bar{M} = m_d + m_c + \bar{E}_{1p} + \langle V_D \rangle_{1p} - A = 2.282 \text{ GeV} . \tag{32}$$

In this model

$$g = \frac{1}{2m^2} \left\langle \frac{1}{r} \frac{dV_1}{dr} \right\rangle_{1p} = \frac{1}{2m^2} \left[\frac{1}{b^2} \left\langle \frac{1}{r} \right\rangle_{1p} + K \left\langle \frac{1}{r^3} \right\rangle_{1p} \right] . \tag{33}$$

From Eq. (33), we obtain $g = 263$ MeV. Hence we obtain $m_{D_2^*} = 2.45$ GeV, $m_{D_1} = 2.41$ GeV, $m_{D_1^*} = 2.01$ GeV, and $m_{D_0} = 1.91$ GeV.

It is clear that in this model, we get the masses of $m_{D_2^*}$ and m_{D_1} in agreement with their experimental values. But the predictions for the masses $m_{D_1^*}$ and m_{D_0} are quite different in the two models we have considered. The existing experimental data do not distinguish between the two models. It is suggested that experimentalists should search the D_1^* and D_0 mesons in the regions between 2.15–2.35 GeV and 1.85–2.06 GeV. In the second model the D_1^* and D_0 are almost degenerate in mass with the D^* and D mesons. Although this prediction looks very much like the old parity-doubling realization of chiral symmetry, here it is accidental.

There is another way in which the heavy-quark spin symmetry can be tested in a clean way. From Eqs. (9) and (10), we see that $J = 1$ mesons D_1 and D_1^* both contain the spin singlet state χ_-^0. Thus D_1 and D_1^* can also decay to D with $E1$ transition. Let us concentrate on the $D_1(2420)$ meson. In the conventional theory, D_1 is identified with $^3P_1$ state and it can decay to D^* by $E1$ transition, the other state $^1P_1$ decays only to D. The mixing between them can give only a small amplitude for $D_1 \to D\gamma$. But the heavy-quark spin symmetry predicts the same order of amplitude for $D_1 \to D^*\gamma$ and $D_1 \to D\gamma$ decays.

We now discuss the predictions of heavy-quark spin symmetry for $E1$ transition decay widths. The $E1$ transition decay width can be written in the form

$$\Gamma = (4\alpha/3)|M^{0,+}|^2 k^3 , \tag{34}$$

where superscripts zero and plus correspond to the charge of the decaying particle. From Eq. (9), it is clear that the radiative decays $D_1 \to D\gamma$ and $D_1 \to D^*\gamma$ are possible and that for these decays, the right-hand side of Eq. (34) is to be multiplied by a factor of $\frac{2}{3}$ and $\frac{1}{3}$, respectively. Similarly, from Eq. (10), it follows that for $D_1^* \to D\gamma$ and $D_1^* \to D^*\gamma$ decays, the right-hand side of Eq. (34) is to be multiplied by a factor of $\frac{1}{3}$ and $\frac{2}{3}$, respectively. For D_0 and D_2^* only radiative transitions to D^* are possible as in the conventional theory and the decay width is given by Eq. (34).

In order to make quantitative estimates of $E1$ transition decay widths, we use the nonrelativistic quark model (NQM). However, in this model, we take into account the recoil correction [14]. In the NQM, the amplitudes $M^{0,+}$ are given by

$$M^0 = \mu \left[\frac{2}{3m_c} I_c + \frac{2}{3m_u} I_u \right] , \quad M^+ = \mu \left[\frac{2}{3m_c} I_c - \frac{1}{3m_d} I_d \right] , \tag{35}$$

where I_c or $I_{u,d}$ is the overlap integral [14]

$$I = \frac{1}{\sqrt{3}} \int_0^\infty [j_0(qr) - 2j_2(qr)] u_{1p}(r) u_{1s}(r) r \, dr . \tag{36}$$

Here j_0 and j_2 are spherical Bessel functions and $q = m_{\text{spect}}/m_{D_J} k$. Thus for u and d quarks ($m_u = m_d$), $q = m_c/m_{D_J} k$ and for the c quark $q = m_d/m_{D_J} k \approx 0$. Thus $I_u = I_d$ and using the wave function given in Eqs. (20) and (21), we get [14]

$$I = F e^{-q^2/4\bar{\beta}^2} (1 - q^2/2\bar{\beta}^2) , \tag{37}$$

where

$$\bar{\beta} = (2\mu/b^2)^{1/3}\beta, \quad \bar{\beta}^2 = \tfrac{1}{2}(\bar{\beta}_{1s}^2 + \bar{\beta}_{1p}^2)$$

and

$$F = \frac{1}{\sqrt{2}} (2\mu/b^2)^{-1/3} (\beta_{1s}^3 \beta_{1p}^5)/\beta^5 .$$

Note that $I_c \approx F$. Using the values of β_{1s} and β_{1p} obtained from the analysis of D mesons, I_c and I_d can be easily calculated from Eq. (37). Equations (34) and (35) then give the $E1$ transition decay widths. These decay widths are tabulated in Table I. In Table I, we also give their estimates in the NQM without the recoil correction

TABLE I. $E1$ decay widths for P-wave D mesons in the nonrelativistic quark model (NQM).

Mode	k (MeV)	Decay widths Γ (KeV)	Decay widths with recoil correction Γ (KeV)
$D_2^*(2460)\to D^{*0}\gamma$	409	990	510
$D_2^{*+}\to D^{*+}\gamma$	409	50	7
$D_1^0(2420)\to D^0\gamma$	492	1150	410
$D_1^0\to D^{*0}\gamma$	375	255	143
$D_1^+\to D^+\gamma$	492	59	0.69
$D_1^+\to D^{*+}\gamma$	375	13	2.6
$D_1^{*0}(2290)\to D^0\gamma$	386	280	140
$D_1^{*0}\to D^{*0}\gamma$	263	170	130
$D_1^{*+}\to D^+\gamma$	386	14	1.8
$D_1^{*+}\to D^{*+}\gamma$	263	9	4.1
$D_0^0(2190)\to D^{*0}\gamma$	165	65	57
$D_0^+\to D^{*+}\gamma$	165	3.3	2.4

($I_c=I_u=I_d=F$). It is clear from Table I that recoil correction is important for those decays for which the photon energy k is large.

From Table I, we get

$$\frac{\Gamma(D_1^{0,+}\to D^{0,+}\gamma)}{\Gamma(D_1^{0,+}\to D^{*0,+}\gamma)}\approx 2.9(0.26)\ , \qquad (38)$$

whereas in the NQM without recoil correction this ratio is 4.3. For D_1^* decays, using $m_{D_1^*}=2.29$ as predicted by our model, we get

$$\frac{\Gamma(D_1^{*0,+}\to D^{0,+}\gamma)}{\Gamma(D_1^{*0,+}\to D^{*0,+}\gamma)}\approx 1.1(0.49)\ , \qquad (39)$$

whereas in NQM without recoil correction this ratio is 1.6.

The decay $D_1(2420)\to D\gamma$ may be easy to detect experimentally due to the following reason. The decay $D_1\to D\pi$ is forbidden due to parity conservation and only $D_1\to D^*\pi$ is an allowed strong decay. Thus there is no competing strong decay for the radiative decay $D_1\to D\gamma$. Hence detecting a D meson in the primary decay product of the D_1 meson would give a clear signal for this.

Similarly, $D_1^*\to D\pi$ is forbidden due to the parity selection rule. Thus a D meson in the primary decay product accompanied by a monoenergetic γ-ray with energy $k<490$ MeV but greater than 140 MeV would give a clear signal for the existence of D_1^*.

In our second model the mesons D_1^* and D_0 are almost degenerate in mass with D^* and D mesons, respectively. This has some interesting consequences.

We note that only the radiative decay $D_1^*\to D\gamma$ is energetically possible. The strong decay $D_1^*\to D\pi$ is forbidden by parity conservation and $D_1^*\to D^*\pi$ is not energetically possible. We find in the NQM

$$\Gamma(D_1^{*0,+}\to D^{0,+}\gamma=13(0.67)\ \text{and}\ 12(0.53)\ \text{keV}$$

without and with the recoil correction, respectively. Note that for this case this is also the total decay width of D_1^*. Note also that D_0 is stable for strong and electromagnetic interactions. Thus for this case, D^* has another decay channel available, viz., $D^*\to D_0\gamma$. This is an $E1$ transition and for this case we find

$$\Gamma(D^*\to D_0\gamma)=\Gamma(D_1^*\to D\gamma)\ .$$

The strong channel $D^*\to D_0\pi$ is not allowed by parity conservation.

We now discuss some experimental implications of these predictions. First we note that due to an additional decay channel available to D^*, its radiative decay width is given by

$$\Gamma_{D^*}^{(\gamma)}=\Gamma_{E1}(D^*\to D_0\gamma)+\Gamma_{M1}(D^*\to D\gamma)\ . \qquad (40)$$

It is reasonable [14–16] to take

$$\Gamma_{M1}(D^{*0,+}\to D^{0,+}\gamma)\approx 18(0.61)\ \text{keV}\ .$$

If experimentalists do not take into account the spin parity of the decaying particle of a definite mass and do not determine the spin parity of the final particle of a definite mass in their measurement of the branching ratio, the measured branching ratio $D^*\to D\gamma$ would correspond to the combination

$$\Gamma^{(\gamma)}=\Gamma_{E1}(D^*\to D_0\gamma)+\Gamma_{M1}(D^*\to D\gamma)+\Gamma_{E1}(D_1^*\to D\gamma) \qquad (41)$$

if D_1^* and D_0 are almost degenerate in mass with D^* and D, respectively. Now our estimate gives

$$\Gamma_0^{(\gamma)}\approx 12+18+12\approx 42\ \text{keV}$$

and

$$\Gamma_+^{(\gamma)}\approx 0.53+0.61+0.53\approx 1.7\ \text{keV}\ .$$

Recent CLEO data [17] give $B_\gamma^0=36.4\pm2.3\pm3.3\%$. Taking $33\%\le B_\gamma^0\le 40\%$, and $\Gamma_0^{(\gamma)}\approx 42$ keV, we find

$$63\ \text{keV}\le\Gamma(D^{*0}\to D^0\pi^0)\le 85\ \text{keV}\ .$$

Using isospin symmetry for pionic decays, we get

$$189\ \text{keV}\le\Gamma(D^{*+}\to D^0\pi^+ + D^+\pi^0)\le 255\ \text{keV}\ .$$

To conclude we note that the mass relations

$$(m_{D_1^*}-m_{D_0})/(m_{D_2^*}-m_{D_1})=\tfrac{5}{2}$$

and the decay $D_1(2420)\rightarrow D+\gamma$ are two definite predictions of heavy-quark spin symmetry. Other interesting consequences of this symmetry regarding the radiative decays of P-wave mesons are discussed above. In the end we suggest that experimentalists should look into the above predictions.

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