Ten-dimensional SO(10) GUT models with dynamical symmetry breaking

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We discuss the derivation of SO(10) GUT models from higher-dimensional theories with intermediate breaking scales. We then present models based on the coset space dimensional reduction scheme with intermediate symmetry breaking induced by four fermion condensates.

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I. INTRODUCTION

The phenomenological advantages of grand unified theory (GUT) models with quark-lepton unification are well known. For example, the proton decay rate may be suppressed. Also, the existence of a right-handed neutrino may account for the missing mass problem in cosmology and provide a mechanism for a light mass left-handed neutrino to exist consistently in nature. GUT models of this kind, such as SO(10) or SU(16), imply the existence of an intermediate mass scale providing greater freedom to incorporate other phenomenological features, such as a reasonable value for $\sin^2\theta_W$ [1].

The concept of grand unification can be extended into the broader framework of superstring theory [2] and superstring-inspired models. These models incorporate the notion of an extended space-time and dimensional reduction to the four dimensions observed in nature. Much work has been done in this field, in particular on models which realize E_6 GUT models in four dimensions [2, 3]. These examples arise most naturally due to the properties of the underlying space-time manifold and the relative ease with which a symmetry-breaking pattern to low-energy physics can be incorporated, such as with symmetry breaking by Wilson lines [4, 5]. This is not the case with SO(10) models [we do not consider SU(16) so that we do not have to assume the existence of mirror families to cancel anomalies]. The existence of an intermediate symmetry [in particular incorporating SU(4) of color] is not easily accommodated within such a framework [5,6]. Interestingly, exploiting these extended symmetry-breaking patterns by the assumed existence of nonrenormalizable higher-order operators arising from spontaneous compactifications can allow significant phenomenology to appear at low energies [7]. For instance, rare kaon decays could be accessible at current machine energies and so be a signature of left-right-symmetric models. These conjectures are supported by only one particular example of reduction from a higher-dimensional theory, formulated by Wetterich [8]. His approach generates chiral fermions by the dimensional reduction onto a noncompact internal manifold of finite volume. Starting from an 18-dimensional theory, a six-dimensional model emerges with an SO(12) gauge symmetry. Considered as a gauge theory on $M^4 \otimes S^2$, the model is reduced to a four-dimensional SO(10) GUT. Higgs-boson fields required for symmetry breaking are introduced into the model from six-dimensional SO(12) representations which exhibit nonzero coupling to the fermions.

While important in its own right, it would be useful to have an alternate description of a higher-dimensional model with intermediate symmetry breaking. Within string-inspired models this would be particularly important due to the naturalness with which SO(10) gauge models emerge with the appropriate fermionic representations [2]. SO(10) models inspired from E_6 gauge theories, arising from manifolds with SU(3) holonomy, have been considered but contain a number of fermions within the **27** of E_6 which are not realized in nature and represent a compromise solution to finding realistic SO(10) models on appropriate manifolds [9]. It has also been demonstrated that the existence of a low-energy supersymmetry breaking could solve many phenomenological problems but a mechanism to implement this remains speculative [10].

A particular string-inspired approach to model building, which has been applied with some success to SU(5) GUT models, is to impose space-time invariance conditions on all the fields, known as coset space dimensional reduction (CSDR) [11, 12]. This has the advantage of producing a finite number of states in four dimensions, as opposed to an infinite tower of states in the harmonic expansion approach usually employed [13], as well as providing a possible origin for the Higgs-boson mechanism. To date, considerations on SO(10) models within CSDR have been "diagonalized" to the standard model or rely upon imaginative applications of Wilson lines so as to avoid the problem of the nonexistence of an intermediate Higgs-boson mechanism [12, 14]. However, there is an alternative approach involving four fermion condensates, breaking symmetries by a dynamical mechanism [15]. Indeed, dynamical symmetry breaking has been the direction taken in some SU(5) models within this framework in order to avoid the problems of electroweak symmetry breaking at the compactification scale [14, 16]. In this paper we will present realistic models which uti-

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lize this mechanism. We will show that the appropriate fermionic representations can emerge from CSDR and we will discuss the construction of such condensates within the constraints of this scheme. By introducing discrete symmetries onto the internal manifold we can produce strong breaking of the SO(10) GUT and, more importantly, eliminate Higgs-boson fields of geometrical origin.

II. COSET SPACE DIMENSIONAL REDUCTION AND SO(10)

In models constructed on extended space-times, gauge fields are introduced as a possible origin for the Higgs-boson mechanism and also because they provide for the existence of low mass flavor chiral fermions in four dimensions by having nontrivial field configurations on the internal manifold [13, 17]. While not in the spirit of a purely gravitational model, this approach allows for an interesting new approach to GUT's by beginning with a larger, more generalized symmetry in higher dimensions. Clearly this is motivated by the emergence of gauge symmetries as a natural part of superstring theory [2].

The space-time manifold in such a scheme is presumed to have the form $M^4 \otimes S/R$, where M^4 is four-dimensional Minkowski space and S/R is a compact coset space. Rather than set to zero the field dependence on the internal coordinates on S/R, CSDR provides a means by which a field dependence can be maintained. The number of space-time dimensions can be consistently reduced by imposing S invariance on all the fields, so producing a finite number of fields in four dimensions. That is, transformations under symmetries of S/R are compensated by gauge transformations. Starting from a principal fiber bundle with bundle group G defined over S/R, S invariant connections are characterized by linear maps from the Lie algebra of S to the Lie algebra of G such that $\Phi_i : R \to G$ is a faithful homomorphism. Corresponding to such an embedding, the gauge fields carrying vector indices corresponding to the additional internal dimensions, which behave as scalar fields under four-dimensional space-time transformations, transform under R as a vector $\mathbf{v}$ specified by the embedding

$$\mathrm{adj}(S) = \mathrm{adj}(R) + \mathbf{v} \ . \tag{2.1}$$

Φ_i, where i corresponds to a generator of $\mathbf{v}$, then satisfy the linear constraint condition

$$[\Phi_i, \Phi_j] = f_{ijk}\Phi_k, \ \ \forall j \in \Im[\mathrm{adj}(R)] \ , \tag{2.2}$$

where f_{ijk} are the structure constants of S, and have arbitrary values. When i corresponds to a generator of R Φ_i are not arbitrary and define a nontrivial R bundle over S/R. The gauge symmetry which survives this procedure, H, is the centralizer of the image of R in G.

It is found, by exploiting Schur's lemma, that an unconstrained scalar field is obtained whenever the tensor product of an induced representation of R over S/R and a representation of R in the adjoint of G contains a singlet. Similarly, the surviving fermionic fields in four dimensions are found by applying Schur's lemma, this time with consideration to the branching rule of the spinor representation of the coset space tangent group under R. Starting from a vectorlike representation, flavor chirality in four dimensions demands Rank$S$ =Rank$R$ [18]. This places severe restrictions on the allowed coset spaces. Imposing the Weyl and Majorana conditions futher requires that the total space-time dimensionality be $D = 2 + 8n$ when the fermionic representations are real. Thus the smallest dimension from which we can construct a model within this scheme is ten.

The scalar fields which emerge, identified with Higgs-boson fields, form a potential in the effective four-dimensional theory from the relevant terms in the ten-dimensional gauge kinetic action. Importantly, it is found that if S has an isomorphic image in G then the four-dimensional symmetry group H breaks to K, the centralizer of S in G. This result is independent of whether the coset space is nonsymmetric, in which case it is otherwise possible to manipulate the radial parameters so as to have a Higgs-boson potential with vanishing order parameter [19]. It turns out that in such cases all fermionic fields become massive, at the order of the compactification scale, after symmetry breaking. Thus, such models are not phenomenologically viable unless these fields can be otherwise eliminated.

For convenience, we list the six-dimensional coset spaces with Rank$S$ =Rank$R$ [20]:

$$\mathrm{SO}(7)/\mathrm{SO}(6),\ \mathrm{SU}(4)/\mathrm{SU}(3)\otimes \mathrm{U}(1),\ \mathrm{SP}(4)/[\mathrm{SU}(2)\otimes\mathrm{U}(1)]_{\mathrm{max}},$$

$$\mathrm{SP}(4)/[\mathrm{SU}(2)\otimes\mathrm{U}(1)]_{\mathrm{nonmax}},$$

$$\mathrm{G}_2/\mathrm{SU}(3),\ \mathrm{SP}(4)\otimes\mathrm{SU}(2)/\mathrm{SU}(2)\otimes\mathrm{SU}(2)\otimes\mathrm{U}(1),$$

$$\mathrm{SU}(2)\otimes\mathrm{SU}(2)\otimes\mathrm{SU}(2)/\mathrm{U}(1)\otimes\mathrm{U}(1)\otimes\mathrm{U}(1),\ \mathrm{SU}(3)\otimes\mathrm{SU}(2)/\mathrm{SU}(2)\otimes\mathrm{U}(1)\otimes\mathrm{U}(1),$$

and

$$\mathrm{SU}(3)/\mathrm{U}(1)\otimes\mathrm{U}(1).$$

As we have mentioned, CSDR has been applied with some success to generating SU(5) GUT models in four dimensions [21–23]. From the set of allowed coset spaces, it is clear that if we wish to have an SO(10) model after dimensional reduction then we must consider gauge fields in ten dimensions with rank at least seven. Furthermore, we choose not to allow horizontal flavor symmetries to emerge. This greatly restricts the range of groups that we can consider. For instance, unitary groups have not played a large role in unified model building within CSDR. In fact, only a $G = \mathrm{SU}(8)$ model dimensionally reduced on the manifold $G_2/\mathrm{SU}(3)$, yielding $H = \mathrm{SU}(5) \otimes \mathrm{U}(1)$, has been considered [16]. Beside the symplectic groups, this leaves E_8 and E_7. By considering E_8 we are implicitly including SO(16) and SO(17), the orthogonal groups of the correct rank which also have real spinorial representations, via the maximal embeddings [24]

$$E_8 \supset \mathrm{SO}(16), \quad \mathrm{SO}(17) \supset \mathrm{SO}(16). \tag{2.3}$$

Indeed, it is sometimes more convenient to consider, for example, SO(16) rather than E_8 since the next smallest irreducible representation to the fundamental **248** of E_8 is the **3875**. By considering SO(16) instead, there is a greater range of choice of starting representations in ten dimensions. Furthermore, in the particular case of SO(16), this allows the scalar and fermionic representations to be separated, i.e., breaking supersymmetry by the initial boundary conditions. This can be useful when discrete symmetries are introduced and the transformation properties of the vector and spinor under such a symmetry are considered [23].

III. INTERMEDIATE SYMMETRY AND THE WILSON FLUX-BREAKING MECHANISM

The nontrivial Higgs scalars which arise from CSDR have $|\Delta I_W| = 1/2$ breaking components but are not sufficient to break the four-dimensional GUT group. Strong breaking of H can be induced, however, by the Wilson flux-breaking mechanism [4, 5]. In this scheme, rather than consider $M^4 \otimes B_0$ where $B_0 = S/R$ is a simply connected manifold, we consider a gauge theory on $M^4 \otimes B$ with $B = B_0/K^{S/R}$ where $K^{S/R}$ is a freely acting symmetry on B_0. [A group K acts freely on B_0 if for any element $k \in K$ other than the identity, the equation $k(y) = y$ has no solution for $y \in B_0$.] The space B is not simply connected with its fundamental group π_1 isomorphic to $K^{S/R}$. This means that there will be contours not contractable to a point in the manifold. The resulting unbroken gauge group turns out to be the centralizer of the homomorphic image, K^H, of $K^{S/R}$ in H. Furthermore it is found that the matter fields which survive have to be invariant under the diagonal sum

$$K^{S/R} \oplus K^H \ .$$

This mechanism is related to the Aharonov-Bohm effect in electrodynamics. The freely acting discrete groups on all possible six-dimensional coset spaces satisfying Rank$R$ =Rank$S$ have already been derived. These fall into two classes corresponding to the center of S and $W = W_S/W_R$ where W_S and W_R are the Weyl groups of S and R, respectively [14]. Under W, the S/R vector and spinor have nontrivial transformation properties. By appropriately embedding in H it becomes possible that eigenstates of fields will not be invariant under the gauge group and so are eliminated from the model. In particular, this provides a way to eliminate Higgs-boson fields of geometrical origin. On symmetric coset spaces, for which symmetry breaking is guaranteed to occur at the compactification scale, this becomes crucial [18]. Since we wish to produce symmetry breaking by fermionic condensates we will always require that the Higgs-boson fields vanish. In this way we avoid the problem that S may have an isomorphic image in G.

Rather than breaking symmetries dynamically, there is an alternate way in which intermediate scale symmetry breaking may be introduced. This approach is more closely related to that of Wetterich [8]. We could introduce fundamental Higgs-boson fields transforming in particular representations of the original gauge group G. Those components which survive in four dimensions, transforming under H, must be R singlets. Note, however, that most of the allowed coset spaces contain U(1) factors in R. Being Abelian, these cannot be "centralized away." Although these factors can be essentially omitted by setting the coupling strengths to zero since they receive separate renormalization from the others [25], they will affect the allowed couplings. This is just a statement of gauge invariance under G. Fermionic fields derived from such manifolds will carry nontrivial U(1) quantum numbers. The relevant tensor product of two such fields with a Higgs-boson field with zero U(1) charge will not produce a gauge singlet.

If we instead consider manifolds without U(1) factors in R, only $S^6 \simeq \mathrm{SO}(7)/\mathrm{SO}(6)$ emerges as a candidate [$G_2/\mathrm{SU}(3)$ would yield an SO(10) model with an additional, unwanted, U(1) gauge symmetry which cannot be omitted]. Indeed, this was the only example where appropriate SO(10) Higgs-boson fields arose as R singlets, particularly the **126** of SO(10). It is, also, on this simplest example of an internal manifold that the naturalness with which SO(10) models emerge can be demonstrated. Starting from an E_8 theory, the $R = \mathrm{SO}(6)$ group is identified with the subgroup appearing in the decomposition [12, 14]

$$\begin{aligned} &E_8 \supset \underbrace{\mathrm{SO}(6)}_{R} \otimes \mathrm{SO}(10) \ , \\ &\mathbf{248} = (\mathbf{15},\mathbf{1}) + (\mathbf{1},\mathbf{45}) + (\mathbf{6},\mathbf{10}) + (\mathbf{4},\mathbf{16}) + (\bar{\mathbf{4}},\overline{\mathbf{16}}) \ . \end{aligned} \tag{3.1}$$

The surviving four-dimensional gauge group will then be $H = C_{\mathrm{E}_8}[\mathrm{SO}(6)] = \mathrm{SO}(10)$. The SO(6) content of the S^6 vector and spinor are **6** and **4**, respectively [18]. Comparing this with (3.1), the surviving Higgs-boson fields transform as a **10** of SO(10) and the left-handed fermions as a **16**, if we choose the ten-dimensional theory to be supersymmetric. Being geometrical in origin, with an order parameter associated with the compactification scale, the **10** will produce electroweak symmetry breaking at a phenomenologically unacceptable large energy. Turning to the Wilson flux-breaking mechanism, we note that this manifold has a Z_2 discrete symmetry in W [14]. Under this discrete symmetry the SO(6) vector and spinor transform as

$$\text{vector}\left\{\mathbf{6} \leftrightarrow \mathbf{6}\right., \qquad \text{spinor}\left\{\mathbf{4} \leftrightarrow \overline{\mathbf{4}}\right. . \tag{3.2}$$

Thus, the Higgs-boson field is unaffected and so survives. So while it is "very satisfying" that SO(10) with the correct fermion representation emerges in a natural way we cannot rely upon this model.

Diagonalization to the standard model for this example has been performed by embedding $Z_2^{S/R}$ into a discrete subgroup of the U(1) appearing in the decomposition [14]

$$\mathrm{SO}(10) \supset \mathrm{SU}(2) \otimes \mathrm{SU}(2) \otimes \mathrm{SU}(4), \quad \mathrm{SU}(4) \supset \mathrm{SU}(3) \otimes \mathrm{U}(1) \tag{3.3}$$

The Wilson flux mechanism breaks the four-dimensional gauge group $H = C_{\mathrm{E}_8}[\mathrm{SO}(6)] = \mathrm{SO}(10)$ down to $H' = C_{\mathrm{E}_8}[\mathrm{SO}(6) \otimes Z_2^H] = \mathrm{SU}(3) \otimes \mathrm{SU}(2) \otimes \mathrm{SU}(2) \otimes \mathrm{U}(1)$. Since $S = \mathrm{SO}(7)$ has an isomorphic image in G the symmetry-breaking of H by the geometrical Higgs boson is known: $K' = C_{\mathrm{E}_8}[\mathrm{SO}(7)] = \mathrm{SO}(9)$. Both symmetry-breaking mechanisms acting together give a final unbroken gauge group $K = K' \cap H' = \mathrm{SU}(2) \otimes \mathrm{U}(1) \otimes \mathrm{SU}(3)$, where the SU(2) in K is the diagonal sum of the previous SU(2)'s.

Alternatively a more ambitious embedding of discrete symmetries may be pursued [12]. For example, suppose S^6 is divided out by $(Z_2 \times Z_2)^{S/R}$ where one Z_2 is the center of SO(7) and the other is in W. One $Z_2^{S/R}$ is identified with a Z_2 subgroup of the U(1) appearing in the decomposition SO(10)⊃SU(5)⊗U(1), for which the Higgs bosons and fermions have the branching rule

$$\mathbf{10} = \mathbf{5}(2) + \overline{\mathbf{5}}(-2), \qquad \mathbf{16} = \mathbf{1}(-5) + \overline{\mathbf{5}}(3) + \mathbf{10}(-1) \; . \tag{3.4}$$

The Z_2 subgroup is chosen to be $Z_2 = \exp[i(n+1)\pi]$, n being the U(1) quantum number. The second $Z_2^{S/R}$ is embedded in a Z_2 subgroup of the hypercharge under SU(5)⊃SU(2)⊗SU(3)⊗U(1), such that all the components of the fundamental representation are invariant. The result is a model yielding the standard model in four dimensions with a family of fermions from the **16** of SO(10) and no surviving scalars. Electroweak symmetry breaking must now rely upon dynamical means in both cases.

Both these approaches attempt to give realistic models in the absence of an intermediate Higgs-boson mechanism. Clearly many other similar examples could be constructed depending on the choice of embedding for the discrete symmetries.

As with this last example we could try embedding $Z_2^{S/R}$ into the U(1) subgroup appearing in (3.3) in such a way as to yield an SU(2)⊗SU(2)⊗SU(3)⊗U(1) model in four dimensions where the **10** is odd under Z_2^H. However, we note that the bidoublet component of the **10** is a singlet under this U(1) and so cannot be odd. Consequently we are still unable to eliminate this field. Any considerations on eliminating this Higgs-boson field therefore rest with $Z_2^{S/R}$.

It should be pointed out that, even if we start with a supersymmetric model in ten dimensions, supersymmetry will never survive to low energies within CSDR. On symmetric coset spaces this is because the constraints explicitly break $N = 1$ supersymmetry [18]. While the constraints on nonsymmetric coset spaces preserve $N = 1$ supersymmetry, there exists a purely geometric term which emerges from the ten-dimensional fermionic kinetic action, written as V [26]. The nonvanishing matrix elements of V correspond to R singlets, so that gaugino fields in four dimensions acquire superheavy masses. It may be possible to overcome this by introducing torsion onto nonsymmetric cosets but this has yet to be demonstrated [12]. This, then, rules out implementing the generalized notion of the seesaw mechanism where the neutrino mass problem can be tackled by assuming a nonzero vacuum expectation value for the scalar superpartner of the right-handed neutrino [10].

IV. MODELS WITH DYNAMICAL SYMMETRY BREAKING

It is known from lattice calculations that it is possible to generalize the Higgs-boson phenomenon to a dynamical symmetry-breaking scheme, described in a gauge invariant way [27]. The existence of a gauge symmetry-breaking potential is associated with four-fermion condensates such that

$$\langle \mathcal{C} \rangle \neq 0 \; , \tag{4.1}$$

where $\mathcal{C}$ is a four-fermion gauge singlet operator. Four-fermion condensates are considered since for chiral gauge theories a quadratic mass condensate does not exist. It can be shown, under a set of general assumptions, that an anomaly free set of fermions, including exotics, which are capable of forming such condensates, has the form [15]

$$\mathbf{R_L} = n\mathbf{16} + \mathbf{144}, \quad \text{where} \quad n = 2, 3, 4 \; , \tag{4.2}$$

for an SO(10) model. Clearly, two forms of gauge singlet condensates can be constructed:

$$\langle \mathcal{C} \rangle = \langle LLLL \rangle \;\text{ or }\; \langle \mathcal{C} \rangle = \langle LL\overline{L}\overline{L} \rangle \; . \tag{4.3}$$

It was argued [15] that only operators of the form $\langle \mathcal{C} \rangle = \langle LLLL \rangle$ should contribute as such condensates will have nontrivial flavor quantum numbers. No rigorous justification was given but this did allow a systematic study to be undertaken. We note, however, that this argument breaks down when condensates with the **144** alone are considered. Necessarily, such condensates will be flavor singlets from the fact that the **144** itself is a flavor singlet.

We have seen that higher-dimensional models in CSDR constructed on internal manifolds without U(1) factors are not phenomenologically acceptable. This means that any gauge-invariant structures arising in acceptable models must be constrained by these factors. Clearly, this will be important for constructing four-fermion operators. Ideally, fermionic states could be derived from ten-dimensional representations with appropriate U(1) factors such that operators such as $\langle \mathcal{C} \rangle = \langle LLLL \rangle$ could be formed. However, we note that intermediate symmetry breaking is associated with four-fermion condensates of the type

$$\langle \mathcal{C} \rangle = \langle (\mathbf{144} \times \mathbf{144}) \times (\mathbf{144} \times \mathbf{144}) \rangle \ . \tag{4.4}$$

Not all the **144** factors can carry the same additional U(1) charge if this is to be a gauge singlet. But if we have **144** representations with differing U(1) quantum numbers then strictly they belong to inequivalent families; i.e., we would have more than one such state. So, while it may be possible, although it seems unlikely, to construct condensates such as

$$\langle \mathcal{C} \rangle = \langle (\mathbf{16} \times \mathbf{16}) \times (\mathbf{144} \times \mathbf{144}) \rangle \ , \tag{4.5}$$

which could include family mixing among **16**'s with different U(1) factors, states of the form $\langle \mathcal{C} \rangle = \langle LLLL \rangle$ will not in general be gauge singlets. On the other hand, condensates which can be written as $\langle \mathcal{C} \rangle = \langle LL\overline{L}\overline{L} \rangle$ can be made to be gauge singlets without resorting to involved family mixing prescriptions. Note that this U(1) factor has arisen before with respect to SU(5) models in CSDR where high color condensates were considered for electroweak symmetry breaking only [16]. In the light of previous discussion we will take these U(1) factors as a model building *constraint* which binds us to condensates of a particular type. Such a constraint could clearly not arise in the original four-dimensional approach.

As we have mentioned, higher-dimensional models yielding SO(10) in four dimensions have already been demonstrated to exist. As with the S^6 example, many of these models fail to eliminate unwanted geometrical Higgs-boson fields in the presence of Wilson lines. For example, an E_8 model on the manifold $CP^3 \simeq SU(4)/SU(3) \otimes U(1)$ will have both **16**'s and **144** fermions arising from the **248** and **3875** as well as **10**'s of Higgs bosons. However, this manifold has no discrete symmetry in W [14]. Consequently, this example is not viable. The manifold $CP^2 \otimes S^2 \simeq SU(3) \otimes SU(2)/SU(2) \otimes U(1) \otimes U(1)$ is also not suitable since strictly CP^2 cannot support a spinor structure [28]. SO(10) models have, however, been considered on this manifold [12]. An example constructed on the manifold $SP(4) \otimes SU(2)/SU(2) \otimes SU(2) \otimes U(1)$ appears promising [12, 29]. Here an SO(10) model with **16**'s of fermions emerges with the Higgs-boson fields transforming as **10**'s. Unfortunately, the **10**(0) Higgs-boson state [the number in brackets corresponding to the U(1) charge] corresponding to the vector component $(\mathbf{2},\mathbf{2})(0)$ of the **6** of SO(6) transforms to itself under $Z_2^{S/R}$ so the **10**(0) Higgs-boson state survives. The manifold $SP(4)/SU(2) \otimes U(1)$ has repeatedly been applied successfully to SU(5) GUT models [12]. In particular, a realistic model on the nonsymmetric version of this manifold has been previously considered [21]. However, SO(10) GUT models with this internal space invariably contain exotic fermions in the **10**, and sometimes other representations, of SO(10). We could attempt to eliminate these by an appropriate embedding of $Z_2^{S/R}$, such as with (3.3), so that all the fields do not transform evenly under Z_2^H. Eigenstates of such fermions under $Z_2^{S/R} \oplus Z_2^H$ may vanish when the Majorana condition is imposed in ten dimensions [14]. However, as mentioned earlier, the bidoublet component of the **10** of SO(10) is a singlet under this U(1) so such definite eigenstates do not arise. While exotic fermions have interesting properties within left-right-symmetric models [30], we wish to remain as close as possible to the minimal anomaly free set (4.2).

We are thus left with two examples, the symmetric manifold $[SU(2)/U(1)]^3$ and the nonsymmetric manifold $SU(3)/U(1) \otimes U(1)$. The interesting advantage of manifolds where R contains more than one U(1) factor lies in the added freedom to manipulate the embedding $\Phi : R \to G$, corresponding to taking new linear combinations of the U(1) generators. Thus we will present candidate models on these manifolds and demonstrate that the required fermionic content arises with Higgs-boson fields of geometrical origin being eliminated.

V. CANDIDATE MODELS

A. Example on a symmetric coset space

We will consider a $G = E_8$ theory on the manifold $M^4 \otimes B_0$ where $B_0 = [SU(2)/U(1)]^3$. A similar model has been previously investigated but not in the context of the dynamical symmetry-breaking scheme we are considering [29]. The $R = [U(1)]^3$ group is chosen to be identified with the $[U(1)]^3$ subgroup of E_8 appearing in the decomposition

$$\begin{aligned} E_8 &\supset E_7 \otimes SU(2) \\ &\supset E_7 \otimes U(1)_{I} \\ &\supset E_6 \otimes U(1)_{I} \otimes U(1)_{II} \\ &\supset SO(10) \otimes \underbrace{U(1)_{I} \otimes U(1)_{II} \otimes U(1)_{III}}_{R} \ . \end{aligned} \tag{5.1}$$

We consider fermions to be transforming in the **3875**-dimensional representation of E_8. Note that now the model is explicitly not supersymmetric. We decompose the representations under (5.1) by employing the branching rules [24]

$$
\begin{aligned}
&\mathrm{E}_8 \supset \mathrm{E}_7 \otimes \mathrm{SU}(2),\\
&\mathbf{248} = (\mathbf{1},\mathbf{3}) + (\mathbf{133},\mathbf{1}) + (\mathbf{56},\mathbf{2}),\\
&\mathbf{3875} = (\mathbf{1},\mathbf{1}) + (\mathbf{56},\mathbf{2}) + (\mathbf{133},\mathbf{3}) + (\mathbf{1539},\mathbf{1}) + (\mathbf{912},\mathbf{2}) \ ,\\
&\mathrm{SU}(2) \supset \mathrm{U}(1),\\
&\mathbf{2} = (1) + (-1),\\
&\mathbf{3} = (2) + (0) + (-2) \ ,\\
&\mathrm{E}_7 \supset \mathrm{E}_6 \otimes \mathrm{U}(1),\\
&\mathbf{56} = \mathbf{1}(3) + \mathbf{27}(1) + \overline{\mathbf{27}}(-1) + \mathbf{1}(-3),\\
&\mathbf{133} = \mathbf{78}(0) + \mathbf{1}(0) + \mathbf{27}(-2) + \overline{\mathbf{27}}(2),\\
&\mathbf{912} = \mathbf{78}(3) + \mathbf{78}(-3) + \mathbf{351}(1) + \overline{\mathbf{351}}(-1) + \overline{\mathbf{27}}(-1) + \mathbf{27}(1),\\
&\mathbf{1539} = \mathbf{1}(0) + \mathbf{27}(4) + \overline{\mathbf{27}}(-4) + \mathbf{27}(-2) + \overline{\mathbf{27}}(2) + \mathbf{78}(0) + \mathbf{351}(-2) + \overline{\mathbf{351}}(2) + \mathbf{650}(0) \ ,\\
&\mathrm{E}_6 \supset \mathrm{SO}(10) \otimes \mathrm{U}(1),\\
&\mathbf{27} = \mathbf{1}(4) + \mathbf{10}(-2) + \mathbf{16}(1),\\
&\mathbf{78} = \mathbf{1}(0) + \mathbf{45}(0) + \mathbf{16}(-3) + \overline{\mathbf{16}}(3),\\
&\mathbf{351} = \mathbf{10}(-2) + \overline{\mathbf{16}}(-5) + \mathbf{16}(1) + \mathbf{45}(4) + \mathbf{120}(-2) + \mathbf{144}(1),\\
&\mathbf{650} = \mathbf{1}(0) + \mathbf{10}(6) + \mathbf{10}(-6) + \mathbf{16}(-3) + \overline{\mathbf{16}}(3) + \mathbf{45}(0) + \mathbf{54}(0) + \mathbf{144}(-3) + \overline{\mathbf{144}}(3) + \mathbf{210}(0) \ .
\end{aligned}
\tag{5.2}
$$

Thus the four-dimensional gauge group will be

$$H = C_{\mathrm{E}_8}[\mathrm{U}(1)^3] = \mathrm{SO}(10)[\otimes \mathrm{U}(1)^3] \ . \tag{5.3}$$

The $R = \mathrm{U}(1)^3$ contents of $[\mathrm{SU}(2)/\mathrm{U}(1)]^3$ vector and spinor are [18]

$$
\begin{aligned}
\underline{6} &= (2a,0,0) + (0,2b,0) + (0,0,2c) + (-2a,0,0) + (0,-2b,0) + (0,0,-2c),\\
\underline{4} &= (a,b,c) + (-a,-b,c) + (-a,b,-c) + (a,-b,-c).
\end{aligned}
\tag{5.4}
$$

Applying the CSDR rules with $a = b = c = 1$ we get (a) scalar fields transforming as $\mathbf{1}(2,0,0)+\mathbf{1}(-2,0,0)$ and (b) fermions transforming as $3 \times \mathbf{16}(1,1,1) + \mathbf{144}(1,1,1) + 3 \times \mathbf{16}(-1,1,1) + \mathbf{144}(-1,1,1)$.

This manifold has a $(Z_2)^3$ symmetry in W, where each Z_2 changes the sign of a, b, and c [14]. We take $Z_2^{S/R} \subset (Z_2)^3$ and embed this into the $\mathrm{U}(1)_{\mathrm{I}} \otimes \mathrm{U}(1)_{\mathrm{II}}$ subgroup of SO(10) appearing in the decomposition

$$
\begin{aligned}
\mathrm{SO}(10) &\supset \mathrm{SU}(5) \otimes \mathrm{U}(1)_{\mathrm{I}}\\
&\supset \mathrm{SU}(2)_L \otimes \mathrm{SU}(3)_C \otimes \mathrm{U}(1)_{\mathrm{I}} \otimes \mathrm{U}(1)_{\mathrm{II}} \ ,
\end{aligned}
\tag{5.5}
$$

and in such a way that all the fields transform evenly under Z_2^H. We choose a solution for the symmetry-breaking matrices U, arising from the homomorphism $Z_2^{S/R} \to G$, such that a maximal number of unbroken generators survive, resulting in the four-dimensional gauge group [5]

$$
\begin{aligned}
K &= C_{\mathrm{E}_8}\{[\mathrm{U}(1)]^3 \otimes Z_2^H\}\\
&= \mathrm{SU}(2) \otimes \mathrm{SU}(2) \otimes \mathrm{SU}(4)\{\otimes[\mathrm{U}(1)]^3\} \ .
\end{aligned}
\tag{5.6}
$$

We could alternatively have chosen the embedding given in (3.3). Under the action of $Z_2^{S/R} \oplus Z_2^H$ eigenstates of the scalar fields do not have definite transformation properties under the four-dimensional gauge group and so are eliminated. It is worthwhile pointing out, however, that gauge singlet scalar fields can be very useful in phenomenological model building. Recalling the Majorana condition, the fermionic fields survive.

We see that we have two sets of fermions transforming as $3\times\mathbf{16}+\mathbf{144}$ under SO(10). Since we do not wish to introduce any mixing between these sets we will choose to identify them by an appropriate choice of discrete symmetry on the complex structure of the internal space such that

$$Z_2 : (1,1,1) \leftrightarrow (-1,1,1) \ . \tag{5.7}$$

Thus we have realized a model in four dimensions with the appropriate set of fermions. Note that we need not have necessarily embedded $Z_2^{S/R}$ in H since we have chosen the fields to transform evenly under Z_2^H. In this case we would have an SO(10) model in four dimensions with the appropriate fermionic content.

B. Example on a nonsymmetric coset space

We will consider now a $G = \mathrm{E}_7$ theory on the manifold $M^4 \otimes B_0$ where $B_0 = \mathrm{SU}(3)/\mathrm{U}(1)\otimes\mathrm{U}(1)$. The $R = \mathrm{U}(1)\otimes \mathrm{U}(1)$ group is chosen to be identified with the $\mathrm{U}(1)\otimes\mathrm{U}(1)$ subgroup of E_7 appearing in the decomposition

$$
\begin{aligned}
\mathrm{E}_7 &\supset \mathrm{SO}(12) \otimes \mathrm{SU}(2)\\
&\supset \mathrm{SO}(12) \otimes \mathrm{U}(1)_{\mathrm{I}}\\
&\supset \mathrm{SO}(10) \otimes \underbrace{\mathrm{U}(1)_{\mathrm{I}} \otimes \mathrm{U}(1)_{\mathrm{II}}}_{R} \ .
\end{aligned}
\tag{5.8}
$$

We take fermions to be transforming in two adjoint **133**'s and one **1463**-dimensional representation of E_7. These representations are decomposed under (5.8) by employing the branching rules [24]

$$
\begin{aligned}
&E_7 \supset SO(12)\otimes SU(2),\\
&\mathbf{133} = (\mathbf{1},\mathbf{3}) + (\mathbf{32}',\mathbf{2}) + (\mathbf{66},\mathbf{1}),\\
&\mathbf{1463} = (\mathbf{66},\mathbf{1}) + (\mathbf{77},\mathbf{3}) + (\mathbf{462},\mathbf{1}) + (\mathbf{352}',\mathbf{2})\ ,\\
&SU(2)\supset U(1),\\
&\mathbf{2} = (1) + (-1),\\
&\mathbf{3} = (2) + (0) + (-2)\ ,\\
&SO(12)\supset SO(10)\otimes U(1),\\
&\mathbf{32}' = \mathbf{16}(-1) + \overline{\mathbf{16}}(1),\\
&\mathbf{66} = \mathbf{1}(0) + \mathbf{45}(0) + \mathbf{10}(2) + \mathbf{10}(-2),\\
&\mathbf{77} = \mathbf{54}(0) + \mathbf{10}(2) + \mathbf{10}(-2) + \mathbf{1}(4) + \mathbf{1}(0) + \mathbf{1}(-4),\\
&\mathbf{352}' = \mathbf{144}(-1) + \overline{\mathbf{144}}(1) + \overline{\mathbf{16}}(-3) + \overline{\mathbf{16}}(1) + \mathbf{16}(-1) + \mathbf{16}(3),\\
&\mathbf{462} = \mathbf{126}(2) + \overline{\mathbf{126}}(-2) + \mathbf{210}(0)\ .
\end{aligned}
\tag{5.9}
$$

Thus the four-dimensional gauge group will be

$$H = C_{E_7}[U(1)\otimes U(1)] = SO(10)[\otimes U(1)\otimes U(1)]\ . \tag{5.10}$$

The $R = U(1)\otimes U(1)$ content of $SU(3)/U(1)\otimes U(1)$ vector and spinor are [18]

$$
\begin{aligned}
\mathbf{6} &= (a,c) + (b,d) + (a+b,c+d)\\
&\quad +(-a,-c) + (-b,-d) + (-a-b,-c-d),\\
\mathbf{4} &= (0,0) + (a,c) + (b,d) + (-a-b,-c-d)\ .
\end{aligned}
\tag{5.11}
$$

We make a particular choice of embedding $\Phi : R \to G$ by setting $a = 1, c = -1, b = 1, d = 0$. Applying the CSDR rules we get (a) scalar fields transforming as $\mathbf{16}(1,-1) + \overline{\mathbf{16}}(-1,1)$ and (b) fermions transforming as $3\times \mathbf{16}(1,-1) + \mathbf{144}(1,-1)$. We have neglected gaugino fermionic fields as they all obtain masses on the order of the compactification scale from the purely geometrical term, V [26], which appears in the fermionic mass matrix.

This manifold has a Z_2 discrete symmetry in W [14]. Under this the R decompositions of the vector and spinor transform as

$$Z_2^{S/R} = \begin{cases} (1,0)\leftrightarrow(2,-1),\\ (1,-1)\leftrightarrow(-1,1),\\ (-1,0)\leftrightarrow(-2,1). \end{cases} \tag{5.12}$$

We can again embed this into the same U(1) subgroup of SO(10) as before such that all the fields transform evenly. Eigenstates of the scalar fields under $Z_2^{S/R}\oplus Z_2^H$ do not have definite transformation properties under the four-dimensional gauge group so they do not survive. Recalling the Majorana condition, the fermionic fields do survive.

Thus again we have realized a model in four dimensions with the appropriate fermionic content. Note that in both models the discrete symmetries can substitute the Majorana condition which we consequently relax.

VI. THE DYNAMICAL SYMMETRY-BREAKING SCHEME

As with Napoly [15], we will take all symmetry breaking from the four-dimensional gauge group down to $SU(3)_C\otimes U(1)_Q$ to originate from the existence of G-symmetric four-fermion condensates. Decomposing $\mathbf{R_L}\times\mathbf{R_L}$ we have [24]

$$
\begin{aligned}
\mathbf{16}\times\mathbf{16} &= \mathbf{10}_S + \mathbf{120}_A + \mathbf{126}_S,\\
(\mathbf{144}\times\mathbf{144})_S &= \mathbf{10} + \mathbf{126} + \overline{\mathbf{126}} + \mathbf{210}' + \mathbf{320} + \mathbf{1728}\\
&\quad +\mathbf{2970} + \mathbf{4950}\\
\mathbf{16}\times\mathbf{144} &= \mathbf{10} + \mathbf{120} + \overline{\mathbf{126}} + \mathbf{320} + \mathbf{1728}\ .
\end{aligned}
\tag{6.1}
$$

Since we are considering condensates of the form $\langle\mathcal{C}\rangle = \langle LL\overline{L}\overline{L}\rangle$, the **4950**-dimensional representation will contribute even though it is complex. The symmetry-breaking pattern can proceed along two possible directions depending on whether we (i) embed $Z_2^{S/R}$ into the four-dimensional gauge group or (ii) choose not to embed $Z_2^{S/R}$. The resulting breaking schemes then have the form

$$\text{(i)}\quad G \xrightarrow{M_{\rm Pl}} \mathrm{SO}(10) \xrightarrow{M_{\rm Pl}} \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{SU}(4) \xrightarrow{M_{\rm I}} \mathrm{SU}(2)_L \otimes \mathrm{SU}(3)_C \otimes \mathrm{U}(1)_Y \xrightarrow{M_{\rm II}} \mathrm{SU}(3)_C \otimes \mathrm{U}(1)_Q ,$$

$$\text{(ii)}\quad G \xrightarrow{M_{\rm Pl}} \mathrm{SO}(10) \xrightarrow{M_{\rm I}} \mathrm{SU}(2)_L \otimes \mathrm{SU}(3)_C \otimes \mathrm{U}(1)_Y \xrightarrow{M_{\rm II}} \mathrm{SU}(3)_C \otimes \mathrm{U}(1)_Q ,$$

where $M_{\rm Pl}$ is the Planck scale and $M_{\rm I}$ and $M_{\rm II}$ are mass scales characterizing each level of symmetry breaking. It is worthwhile emphasizing that all the symmetry breaking is induced either by condensates or topologically. This is true even in case (i) where we employ Wilson lines. It has been pointed out that Wilson lines are similar to ordinary Higgs-boson fields transforming in the adjoint representation of H [2]. However, it is not difficult to show that it is possible, by a more exotic embedding of R into G, to achieve the breaking pattern $G \to \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{SU}(4)$ directly by the CSDR mechanism alone. It is not clear that a similar result holds for SU(5) models. Thus, at the expense of a more involved procedure, we could have arrived at similar conclusions without Wilson lines. We can, therefore, maintain the model building prescription without introducing Higgs-boson fields outside those formed by condensates.

We will assume that only the **144**-dimensional representation is involved in forming Higgs-boson condensates at the scales $M_{\rm I}$ and $M_{\rm II}$. In this way we will be able to give these exotic fermions large masses relative to the **16**'s. At the scale $M_{\rm I}$, then, we need only consider the **126**, $\overline{\mathbf{126}}$, **1728**, **2970**, and **4950** representations in forming effective Higgs-boson fields as these contain a $\mathrm{SU}(2)_L \otimes \mathrm{SU}(3)_C \otimes \mathrm{U}(1)_Y$ singlet [24]. We can therefore consider the condensates, in the notation of Napoly [15]:

$$\begin{aligned}
\langle \mathcal{C}_{\rm I}' \rangle &= \langle [(\mathbf{144}\times\mathbf{144})_{\mathbf{126}} \times (\overline{\mathbf{144}}\times\overline{\mathbf{144}})_{\overline{\mathbf{126}}}]_{\mathbf{1}} \rangle \sim (M_{\rm I}')^6 ,\\
\langle \mathcal{C}_{\rm I}'' \rangle &= \langle [(\mathbf{144}\times\mathbf{144})_{\mathbf{1728}} \times (\overline{\mathbf{144}}\times\overline{\mathbf{144}})_{\mathbf{1728}}]_{\mathbf{1}} \rangle \sim (M_{\rm I}'')^6 ,\\
\langle \mathcal{C}_{\rm I}''' \rangle &= \langle [(\mathbf{144}\times\mathbf{144})_{\mathbf{2970}} \times (\overline{\mathbf{144}}\times\overline{\mathbf{144}})_{\mathbf{2970}}]_{\mathbf{1}} \rangle \sim (M_{\rm I}''')^6 ,\\
\langle \mathcal{C}_{\rm I}'''' \rangle &= \langle [(\mathbf{144}\times\mathbf{144})_{\mathbf{4950}} \times (\overline{\mathbf{144}}\times\overline{\mathbf{144}})_{\overline{\mathbf{4950}}}]_{\mathbf{1}} \rangle \sim (M_{\rm I}'''')^6
\end{aligned} \tag{6.2}$$

The energy scales corresponding to each condensate are assumed to be approximately equal. Unfortunately, a satisfactory topological mechanism to inhibit proton decay has yet to be found within this higher-dimensional scenario [22]. We are therefore moved to set the scale $M_{\rm I}$ greater than about 10^{14} GeV so that

$$M_{\rm I}' \simeq M_{\rm I}'' \simeq M_{\rm I}''' \simeq M_{\rm I}'''' \geq 10^{14}\ \text{GeV} . \tag{6.3}$$

If we choose to realize breaking scheme (i), however, all the baryon-number-violating gauge fields will have been made superheavy at the compactification scale. In this case then we could set the scale $M_{\rm I}$ to be significantly lower ($10^6 - 10^7$ GeV for a light W_R model). Note that we could also have included the condensate $\langle [(\mathbf{144}\times\mathbf{144})_{\overline{\mathbf{126}}} \times (\overline{\mathbf{144}}\times\overline{\mathbf{144}})_{\mathbf{126}}]_{\mathbf{1}} \rangle$. With no compelling reason to the contrary, we will simply take this to be also characterized by the energy scale $M_{\rm I}'$.

An appropriate choice of four-fermion condensates at the scale $M_{\rm II}$ corresponds to effective Higgs-boson fields transforming as **10**, **210′**, and **320** which all contain an $\mathrm{SU}(2)_L$ doublet. We therefore have the condensates

$$\begin{aligned}
\langle \mathcal{C}_{\rm II}' \rangle &= \langle [(\mathbf{144}\times\mathbf{144})_{\mathbf{10}} \times (\overline{\mathbf{144}}\times\overline{\mathbf{144}})_{\mathbf{10}}]_{\mathbf{1}} \rangle \sim (M_{\rm II}')^6 ,\\
\langle \mathcal{C}_{\rm II}'' \rangle &= \langle [(\mathbf{144}\times\mathbf{144})_{\mathbf{210'}} \times (\overline{\mathbf{144}}\times\overline{\mathbf{144}})_{\mathbf{210'}}]_{\mathbf{1}} \rangle \sim (M_{\rm II}'')^6 ,\\
\langle \mathcal{C}_{\rm II}''' \rangle &= \langle [(\mathbf{144}\times\mathbf{144})_{\mathbf{320}} \times (\overline{\mathbf{144}}\times\overline{\mathbf{144}})_{\mathbf{320}}]_{\mathbf{1}} \rangle \sim (M_{\rm II}''')^6 .
\end{aligned} \tag{6.4}$$

It is known that this symmetry breaking occurs at around 10^2 GeV so that

$$M'_{\rm II} \simeq M''_{\rm II} \simeq M'''_{\rm II} \simeq 10^2 \text{ GeV} \quad . \tag{6.5}$$

As has been pointed out [15], the **210**$'$ and **320** also contain Higgs representations, transforming as $(\mathbf{4},\mathbf{1})(1) + (\mathbf{4},\mathbf{1})(-1)$ under $SU(2)_L \otimes SU(3)_C \otimes U(1)_Y$, that can induce breaking to $SU(3)_C \otimes U(1)_Q$. However, these representations always appear with doublets, while doublets can appear alone, so that it seems reasonable to postulate that symmetry breaking occurs by composite Higgs-boson doublets only.

VII. THE FERMION MASS SPECTRUM AND CHIRAL SYMMETRY BREAKING

We propose that the quark and lepton bare masses originate from the condensates

$$\begin{aligned} \langle \mathcal{C}_f \rangle &= \langle [(\mathbf{16}\times\mathbf{16})_{\mathbf{10}} \times (\overline{\mathbf{144}}\times\overline{\mathbf{144}})_{\mathbf{10}}]_{\mathbf{1}}\rangle \neq 0, \\ \langle \mathcal{C}'_f \rangle &= \langle [(\mathbf{16}\times\mathbf{16})_{\mathbf{126}} \times (\overline{\mathbf{144}}\times\overline{\mathbf{144}})_{\overline{\mathbf{126}}}]_{\mathbf{1}}\rangle \neq 0 \quad , \end{aligned} \tag{7.1}$$

while chiral symmetry breaking in the quark sector is associated with

$$\langle \mathcal{C}_c \rangle = \langle [(\mathbf{16}\times\mathbf{16})_{\mathbf{10}} \times (\overline{\mathbf{16}}\times\overline{\mathbf{16}})_{\mathbf{10}}]_{\mathbf{1}}\rangle \sim (M_c)^6 \quad , \tag{7.2}$$

where M_c is the symmetry-breaking scale such that $M_c \ll M_{\rm II}$. Note that this approach divorces the chiral symmetry breaking from the electroweak breaking at $M_{\rm II}$ which the additional U(1) factors in H obstruct in SU(5) models previously considered under CSDR which utilize dynamical symmetry breaking [14]. The condensate $\mathcal{C}_f$ couples fermions to the **10** composite Higgs boson as in a Yukawa coupling. This mass scale is suggested to occur at $M_c \simeq 10^{-1}$ GeV at which the **144** fermions are decoupled. A large Majorana mass for the antineutrino comes from coupling to the $(\overline{\mathbf{144}}\times\overline{\mathbf{144}})_{\overline{\mathbf{126}}}$ composite Higgs boson at the $M_{\rm I}$ scale.

The **144** fermions attain masses at the scales $M_{\rm I}$ and $M_{\rm II}$. The effective $SU(2)_L$ symmetry above $M_{\rm II}$ protects the **144** components associated with symmetry breaking at this scale so that their masses are not larger than $M_{\rm II}$. This gives rise to a TeV scale hadronic spectroscopy as well as charged heavy leptons and neutrinos. Most interestingly, a charge-two heavy lepton emerges.

It is important to note that, unlike the purely phenomenological model, we do not have an exact U(3) family symmetry. Originating from different representations of G, a reduced family symmetry at best can exist. Indeed, it may be possible to find a model in which all the **16**'s originate in different representations in ten dimensions. This would completely eliminate the existence of exactly massless Goldstone bosons arising from breaking family symmetry. Interestingly, we can still eliminate these fields by demanding that they be odd under the $Z_2^{S/R}$ discrete symmetry.

VIII. CONCLUSION

We have presented models which realize an anomaly free set of fermions necessary to yield realistic low-energy theories by the formation of Higgs-boson fields from fermionic condensates. As well as providing an origin for this approach within phenomenological models, we now have a means by which the desert region may be filled in higher-dimensional theories. This is a major problem with SU(5) type models derived from CSDR. The large gap separating the compactification scale, usually taken as the Planck scale, and the electroweak scale is unnatural, yielding apparently no new phenomenology in this region. While, as with the Wetterich model [8], we have set the symmetry-breaking scales by hand we have demonstrated that these symmetry-breaking structures can be associated with fields *derived* from higher-dimensional models. Furthermore, the CSDR scheme has provided an explicit model building constraint for the form of condensates for which only heuristic arguments could be previously used. It is compelling also to speculate that a nontrivial topological signature may arise such that these condensates could have well-defined expectation values.

In the absence of manifolds with the appropriate holonomy or compelling low-energy supersymmetric symmetry-breaking schemes, the introduction of dynamical symmetry breaking provides a consistent approach to higher-dimensional SO(10) unified models. Indeed it has been noted in left-right-symmetric models that conclusions are unaltered if Higgs-boson fields are replaced by fermionic bilinears [31]. Outstanding questions, such as the hierarchy of scales may yet yield to a more exotic geometrical approach, while interesting exotic heavy fermions may provide the experimental signature for these symmetry-breaking mechanisims.

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