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Low energy effects of new interactions in the electroweak boson sector

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Novel strong interactions in the electroweak bosonic sector are expected to induce effective interactions between the Higgs doublet field and the electroweak gauge bosons which lead to anomalous WWZ and $WW\gamma$ vertices once the Higgs field acquires a vacuum expectation value. Using a linear realization of the Goldstone bosons, we consider a complete set of dimension-six operators which are SU(2)×U(1) gauge invariant and conserve C and P. This approach allows us to study effects of new physics which originates above 1 TeV and the Higgs boson mass dependence of the results can be investigated. Four of the dimension-six operators affect low energy and present CERN LEP experiments at the tree level. Another five influence neutral and charged current experiments at the one-loop level and three of these lead to anomalous WWZ and $WW\gamma$ vertices. Their loop contributions are at most logarithmically divergent, and these logarithmic divergences can be understood as renormalizations of the four operators which contribute at the tree level. Constraints on the remaining five operators can be obtained if one assumes the absence of cancellations between the tree level and one-loop contributions. The resulting bounds on anomalous triple gauge boson couplings are modest, which emphasizes the importance of direct measurements of the triple gauge boson vertices, e.g., in W^+W^- production at LEP II.

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I. INTRODUCTION

Many aspects of the standard model (SM) have been beautifully confirmed by the recent experiments at the CERN e^+e^- collider (LEP) and SLAC Linear Collider (SLC), in particular the gauge theory predictions for the couplings of the vector bosons to the fermions. On the other hand the precise dynamics of the spontaneous breaking of the SU(2)×U(1) gauge symmetry remains one of the major open questions. The search for the Higgs boson or the measurement of longitudinal weak boson scattering cross sections at the Superconducting Super Collider (SSC) or the CERN Large Hadron Collider (LHC) will be crucial to shed light on the Higgs sector. More generally we need to determine experimentally whether the SM predictions for the interactions in the bosonic sector are adequate descriptions of nature. This includes the measurement of, e.g., the WWZ and $WW\gamma$ triple vector boson couplings in $e^+e^- \to W^+W^-$ at LEP II [1, 2] and in vector boson pair production at future hadron colliders [3].

While the production of electroweak gauge boson pairs will test the SM predictions for the gauge boson self-interactions at the tree level, one would expect that some new physics which leads to large deviations from the SM in these production experiments would also give indirect effects (virtual corrections) in precision experiments at energies below the pair production threshold. Effects on the anomalous magnetic moment of the electron or the muon [4, 5] and deviations from the SM predictions in four fermion amplitudes as measured, e.g., in deep inelastic scattering, atomic parity violation, or in W and Z production and decay have been analyzed in the past [6–8]. Usually the deviations from the SM were introduced in such a way as to violate SU(2)×U(1) gauge invariance when the scale of new physics, Λ, is taken to be large. As a result the one-loop contributions from anomalous WWV ($V=\gamma, Z$) interactions to observable oblique parameters [9] such as $\delta\rho$ [10], the S, T, U parameters of Peskin and Takeuchi [11], or other related parameters [12, 13] turn out to be quadratically or even quartically divergent [6]. Because of these divergencies the new physics at the high mass scale does not decouple and for sufficiently large values of Λ quantum corrections become much larger than the lowest order effects [14]. These problems simply indicate that the assumed effective Lagrangian becomes inconsistent for a large scale Λ. In order to avoid such an unphysical situation, significant deviations from the gauge theory WWV couplings should imply either a low new physics scale ($\Lambda \sim m_W$) [15] or the existence of an extra contribution to the oblique parameters which exactly cancels the apparent quadratic and quartic sensitivity to Λ [16]. In both cases, we need to know details of the model in order to find a constraint on the WWV couplings from low energy precision experiments.

In a previous Letter [17] we reanalyzed the low energy bounds on the anomalous triple vector boson couplings

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$\Delta\kappa_\gamma$ and $\Delta\kappa_Z$ in a framework which manifestly respects the SU(2)×U(1) gauge invariance of the SM and which uses a linear realization of the symmetry-breaking sector. We found that contributions to observable quantities depend at most logarithmically on the cutoff scale Λ and present low energy bounds are rather weak, in particular for small values of the Higgs boson mass. These mild constraints on the new interactions are universal to all models which possess SU(2)×U(1) gauge invariance and a light Higgs boson originating from a single doublet field. They do not depend on other details of the underlying model.

In this paper we consider more general observable effects at low energy, due to some new interactions which involve the Higgs sector and the electroweak gauge bosons. We assume that the weak vector bosons and the photon are indeed the gauge bosons of an SU(2)×U(1) local symmetry which is broken spontaneously because some order parameter, which transforms as an SU(2) doublet, acquires a vacuum expectation value (VEV). In order to allow for the possible existence of a light Higgs boson (which might be a composite object as, e.g., in top-quark condensate models [18]) we choose a linear realization of the symmetry-breaking sector in the form of the conventional Higgs doublet field Φ. The new interactions lead to the opening of new thresholds at a high energy scale Λ. At low energies their effects are described by an effective Lagrangian which we approximate by considering operators up to dimension six only. The building blocks of this effective Lagrangian are the Higgs doublet field and the gauge fields [14, 19, 20], while any effects on the known quarks and leptons are assumed to be induced by SM gauge boson exchange.

We present a complete analysis of four-fermion amplitudes including all dimension-six operators in the gauge-boson–Higgs-boson sector which are SU(2)×U(1) gauge invariant and which are even under charge conjugation and parity. Of these operators, four (to be called $\mathcal{O}_{BW}$, $\mathcal{O}_{DW}$, $\mathcal{O}_{DB}$, and $\mathcal{O}_{\Phi,1}$) affect the neutral current (NC) and charged current (CC) amplitudes at the tree level [21] and another five induce effects at the one-loop level. Of these five, three operators lead to anomalous triple vector boson couplings, as shown in Sec. II. We explicitly calculate the quadratically and logarithmically divergent contributions of the latter five operators to CC and NC amplitudes and demonstrate that these divergent contributions are equivalent to a renormalization of the four operators $\mathcal{O}_{BW}$, $\mathcal{O}_{DW}$, $\mathcal{O}_{DB}$, and $\mathcal{O}_{\Phi,1}$ which contribute at tree level, and of the SM parameters (which we take to be α, the Z-boson mass m_Z, and the Fermi constant as measured in μ decay, G_F). These one-loop calculations are described in Sec. IV.

Because the leading one-loop contributions can be understood in terms of the tree level effects of $\mathcal{O}_{BW}$, $\mathcal{O}_{DW}$, $\mathcal{O}_{DB}$, and $\mathcal{O}_{\Phi,1}$, we have inserted in Sec. III a discussion of the low energy effects of these operators [21]. We consider in this paper new physics in the gauge-boson—Higgs-boson sector that affects low energy experiments only via a virtual gauge boson exchange. Hence only oblique corrections to the SM appear and all results can be described in the improved Born approximation [9]. In Sec. III we review the results of a recent analysis of the oblique correction parameters [22] which will be used in Sec. V to derive bounds on the various operators. We adopt the formalism of Ref. [22] since it allows for the running of the oblique form factors between zero momentum transfer and the Z boson mass scale, as caused by some of the dimension-six operators that we study.

The tree level bounds on $\mathcal{O}_{BW}$, $\mathcal{O}_{DW}$, $\mathcal{O}_{DB}$, and $\mathcal{O}_{\Phi,1}$ can be translated into constraints on the other five operators (at the one-loop level) and hence on anomalous triple boson vertices if we assume that there are no cancellations between the contributions of different operators. This is done in Sec. V. For the anomalous WWZ and $WW\gamma$ couplings λ and $\Delta\kappa$ one finds that deviations as large as $\lambda = \pm 0.5$ or $\Delta\kappa = 0.5$ are not excluded by the present precision experiments. We also discuss the Higgs boson mass and top-quark mass dependence of these bounds. Because of the assumptions which need to be made to establish any constraints, these bounds must be considered as order of magnitude estimates only, and they are about as stringent as constraints derived from tree level unitarity considerations [23]. A final discussion of our results is given in Sec. VI.

Some of the technical details are relegated to two Appendixes. Appendix A lists the dimension-six operators which we consider and decomposes them into operators with two, three, or four fields, which allows us to immediately read off the Feynman rules for the various non-standard-model vertices. We also show the SM Lagrangian in the same notation in order to fix all sign conventions. In Appendix B we have collected the full formulas for the new physics contributions to the gauge boson two-point functions and the Vff gauge boson fermion vertex functions. In addition, results are given for models in which no light Higgs boson exists.

II. EFFECTIVE LAGRANGIAN DESCRIPTION OF NEW PHYSICS IN THE BOSONIC SECTOR

We are concerned with the low energy effects of new strong interactions in the electroweak symmetry-breaking sector. Denoting by Λ the characteristic scale of the new physics, we are interested in the residual interactions between the light degrees of freedom, i.e., the particles of mass $M \ll \Lambda$. These are taken as the SU(2)×U(1) gauge bosons and an SU(2) doublet field Φ, which acquires a vacuum expectation value $v/\sqrt{2}$, and thus gives rise to the three Goldstone bosons which are absorbed as the longitudinal modes of the W and the Z. The fourth real field contained in Φ is the Higgs boson, which may be light compared to the scale Λ. We use this linear realization of the Goldstone bosons in order to be able to discuss Higgs mass effects and the decoupling of the new physics for $\Lambda \gg v$. Integrating out the heavy degrees of freedom, the residual interactions between the gauge bosons and the Higgs doublet field are described by an effective Lagrangian

$$\mathcal{L}_{\rm eff} = \sum_{n=1}^{\infty} \sum_i \frac{f_i^{(n)}}{\Lambda^n} \mathcal{O}_i^{(n+4)} \quad . \tag{2.1}$$

We will assume that the new physics respects the local SU(2)×U(1) symmetry and that this symmetry is broken spontaneously only, by the vacuum expectation value of Φ. As a result the operators in $\mathcal{L}_{\rm eff}$ must be invariant under the full gauge symmetry. In addition we only consider operators which separately conserve parity and charge conjugation invariance.

A general analysis of the allowed operators with energy dimension $d = n + 4 < 8$ can be found in Refs. [19, 24]. Operators which differ by total derivatives only can be identified with each other and the classical equations of motion provide additional relations. Nevertheless, several dozen independent operators remain when taking into account operators which involve fermions as well as bosons. Here we are interested in low energy effects of the electroweak symmetry-breaking sector only. We thus expect operators involving fermions to be suppressed by powers of m_f/Λ, making them negligible except, perhaps, when involving the top quark. Dropping all terms involving fermions requires that we do not use the equations of motion for the gauge fields since their equations of motion give the fermionic parts of the isospin and hypercharge currents which are not suppressed for small fermion masses.

With these restrictions eleven independent operators remain. Four of them, namely,

$$\mathcal{O}_{DW} = {\rm Tr}([D_\mu, \hat{W}_{\nu\rho}]\,[D^\mu, \hat{W}^{\nu\rho}])\ , \tag{2.2a}$$

$$\mathcal{O}_{DB} = -\frac{g'^2}{2}(\partial_\mu B_{\nu\rho})(\partial^\mu B^{\nu\rho})\ , \tag{2.2b}$$

$$\mathcal{O}_{BW} = \Phi^\dagger \hat{B}_{\mu\nu}\hat{W}^{\mu\nu}\Phi\ , \tag{2.2c}$$

and

$$\mathcal{O}_{\Phi,1} = (D_\mu\Phi)^\dagger\Phi\ \Phi^\dagger(D^\mu\Phi)\ , \tag{2.2d}$$

affect the gauge boson two-point functions at the tree level [21]. Here Φ denotes the Higgs doublet field. The covariant derivative for an isospin doublet with hypercharge $Y = \frac{1}{2}$ is

$$D_\mu = \partial_\mu + \frac{i}{2}\,g'\,B_\mu + i\,g\,\frac{\sigma^a}{2}W^a_\mu\ , \tag{2.3}$$

and $W_{\mu\nu}$ and $B_{\mu\nu}$ denote the full (non-Abelian) field strengths of the W and the B gauge fields:

$$[D_\mu, D_\nu] = \hat{B}_{\mu\nu} + \hat{W}_{\mu\nu} = i\,\frac{g'}{2}\,B_{\mu\nu} + i\,g\,\frac{\sigma^a}{2}W^a_{\mu\nu}\ . \tag{2.4}$$

When replacing Φ by its VEV, $(0, v/\sqrt{2})^T$, and keeping terms bilinear in the gauge fields only, one finds a contribution

$$\mathcal{L}_{VV} = f_{DW}\frac{g^2}{2\Lambda^2}\,W^a_{\mu\nu}\partial^2 W^{a\mu\nu} + f_{DB}\frac{g'^2}{2\Lambda^2}\,B_{\mu\nu}\partial^2 B^{\mu\nu} + f_{BW}sc\frac{m_Z^2}{2\Lambda^2}W^3_{\mu\nu}B^{\mu\nu} + f_{\Phi,1}\frac{v^2}{4\Lambda^2}m_Z^2 Z_\mu Z^\mu \tag{2.5}$$

to the kinetic energy part of the Lagrangian. $\mathcal{O}_{BW}$ introduces B-W^3 mixing and hence gives a contribution to the S parameter. $\mathcal{O}_{\Phi,1}$ contributes to the Z boson mass but not to the W mass and hence leads to deviations of the ρ parameter from 1. Finally, $\mathcal{O}_{DW}$ and $\mathcal{O}_{DB}$ lead to an anomalous running of the QED fine structure constant and of the weak mixing angle [21]. These effects will be considered in more detail in Sec. III.

Of the remaining seven operators two solely affect the Higgs self-interactions at the tree level

$$\mathcal{O}_{\Phi,2} = \frac{1}{2}\partial_\mu(\Phi^\dagger\Phi)\partial^\mu(\Phi^\dagger\Phi), \tag{2.6a}$$

$$\mathcal{O}_{\Phi,3} = \frac{1}{3}(\Phi^\dagger\Phi)^3. \tag{2.6b}$$

They do not enter in our subsequent analysis, since at the one-loop level their effects can be absorbed into a change of the Higgs potential and hence into a renormalization of the SM parameters.

The other five operators are

$$\mathcal{O}_{WWW} = {\rm Tr}[\hat{W}_{\mu\nu}\hat{W}^{\nu\rho}\hat{W}_\rho{}^\mu], \tag{2.7a}$$

$$\mathcal{O}_{WW} = \Phi^\dagger\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\Phi, \tag{2.7b}$$

$$\mathcal{O}_{BB} = \Phi^\dagger\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\Phi, \tag{2.7c}$$

$$\mathcal{O}_{W} = (D_\mu\Phi)^\dagger\hat{W}^{\mu\nu}(D_\nu\Phi), \tag{2.7d}$$

$$\mathcal{O}_{B} = (D_\mu\Phi)^\dagger\hat{B}^{\mu\nu}(D_\nu\Phi)\ . \tag{2.7e}$$

As we shall see they all contribute to four-fermion amplitudes at the one-loop level. In addition $\mathcal{O}_{WWW}$, $\mathcal{O}_W$, and $\mathcal{O}_B$ give rise to nonstandard triple gauge boson couplings. Conventionally the WWV vertices ($V = Z, \gamma$) are parametrized by the effective Lagrangian [2]

$$\mathcal{L}_{\rm eff}^{WWV} = i\,g_{WWV}\left(g_1^V(W^+_{\mu\nu}W^{-\,\mu} - W^{+\,\mu}W^-_{\mu\nu})V^\nu + \kappa_V\,W^+_\mu W^-_\nu V^{\mu\nu} + \frac{\lambda_V}{m_W^2}W^{+\,\nu}_\mu W^{-\,\rho}_\nu V_\rho{}^\mu\right), \tag{2.8}$$

where the overall coupling constants are defined as $g_{WW\gamma} = -e$ and $g_{WWZ} = -e\,\cot\theta_W$. Within the standard model, the couplings are given by $g_1^Z = g_1^\gamma = \kappa_Z = \kappa_\gamma = 1$, and $\lambda_Z = \lambda_\gamma = 0$. While the value of g_1^γ is fixed by electromagnetic gauge invariance, the presence of the operators $\mathcal{O}_{WWW}$, $\mathcal{O}_W$, and $\mathcal{O}_B$ in the effective Lagrangian of Eq. (2.1) will change the other values to

$$g_1^Z = 1 + f_W\,\frac{m_Z^2}{2\Lambda^2}\ , \tag{2.9a}$$

$$\kappa_Z = 1 + [f_W - s^2(f_B + f_W)]\,\frac{m_Z^2}{2\Lambda^2}\ , \tag{2.9b}$$

$$\kappa_\gamma = 1 + (f_B + f_W)\,\frac{m_W^2}{2\Lambda^2}\ , \tag{2.9c}$$

$$\lambda_\gamma = \lambda_Z = \frac{3m_W^2 g^2}{2\Lambda^2}\,f_{WWW} = \lambda\ , \tag{2.9d}$$

with $s = \sin\theta_W$.

At first sight it would appear that the operator $\mathcal{O}_{WW}$ would also give rise to anomalous values of κ_γ or κ_Z when Φ is replaced by its vacuum expectation value, $(0, v/\sqrt{2})^T$, and, hence, $\Phi^\dagger\Phi \to v^2/2$. One immediately finds, however, that the resulting term and the analogous one from the operator $\mathcal{O}_{BB}$ are directly proportional to the SM kinetic energy terms of the SU(2) and U(1) gauge bosons and they can be absorbed into a finite renormalization of the W and B fields, respectively. In addition they give rise to processes such as $H \to \gamma\gamma$ and $Z \to H\gamma$. However, they will not change the SM three boson vertices and they affect the gauge boson propagators at the one-loop level only.

In addition to the three operators $\mathcal{O}_{WWW}$, $\mathcal{O}_W$, and $\mathcal{O}_B$ the operators $\mathcal{O}_{BW}$ and $\mathcal{O}_{DW}$ also give rise to anomalous triple gauge boson couplings [25]. Because they affect the gauge boson propagators in addition, their effects are *not* equivalent to any of the terms in Eq. (2.8) when describing processes such as e.g., $e^+e^- \to W^+W^-$ [14]. Instead of considering modifications of the gauge boson propagators one can use the equations of motion for the gauge fields to rewrite the operators $\mathcal{O}_{BW}$ and $\mathcal{O}_{DW}$ in terms of $\mathcal{O}_W$ or $\mathcal{O}_B$ and $\mathcal{O}_{WWW}$, respectively, *plus* additional anomalous gauge boson-fermion interactions [14]. A parametrization of deviations from the SM, which only considers anomalous values of g_1, κ, or λ, is thus valid only when the coefficients of the four operators $\mathcal{O}_{BW}$, $\mathcal{O}_{DW}$, $\mathcal{O}_{DB}$, and $\mathcal{O}_{\Phi,1}$ are substantially smaller than the coefficients of $\mathcal{O}_{WWW}$, $\mathcal{O}_W$, or $\mathcal{O}_B$. Given the stringent constraints on the former (see Refs. [14, 21] and results in Sec. V) this is exactly the interesting situation for vector boson pair production experiments at LEP II and at hadron colliders. Consequently we have neglected the contributions from $\mathcal{O}_{BW}$ and $\mathcal{O}_{DW}$ to the triple gauge boson couplings.

A remarkable feature of the anomalous triple gauge boson couplings in Eq. (2.9) are the relations

$$g_1^Z = \kappa_Z + \frac{s^2}{c^2}(\kappa_\gamma - 1)\;, \tag{2.10a}$$

$$\lambda_\gamma = \lambda_Z = \lambda\;. \tag{2.10b}$$

These relations result from our restriction to gauge invariant dimension-six operators in the effective Lagrangian. At the dimension-eight level, operators such as

$$\mathcal{L}_\lambda^{(8)} = i\frac{f_\lambda^{(8)}}{\Lambda^4}\,\epsilon^{ijk}\,\hat{W}_\mu^{i\,\nu}\hat{W}_\nu^{j\,\rho}\,\Phi^\dagger\frac{\sigma^k}{2}\,[D_\rho, D^\mu]\,\Phi \tag{2.11}$$

or

$$\mathcal{L}_\kappa^{(8)} = \frac{f_\kappa^{(8)}}{\Lambda^4}\,(D_\mu\Phi)^\dagger(D_\nu\Phi)\,\Phi^\dagger[D^\mu, D^\nu]\Phi \tag{2.12}$$

lift the degeneracies of Eq. (2.10). One finds that λ_Z and λ_γ are no longer equal,

$$\frac{\lambda_Z - \lambda_\gamma}{\lambda_Z + \lambda_\gamma} = \frac{1}{48c^2}\left(\frac{v}{\Lambda}\right)^2 \frac{f_\lambda^{(8)}}{f_{WWW} + \frac{f_\lambda^{(8)}}{12}\left(\frac{v}{\Lambda}\right)^2\left(1+\frac{1}{4c^2}\right)}\;, \tag{2.13}$$

and that κ_Z (but not κ_γ or g_1^Z) gets an extra contribution

$$\Delta\kappa_Z = -\frac{v^2}{4\Lambda^2}\frac{m_Z^2}{\Lambda^2}f_\kappa^{(8)}\;. \tag{2.14}$$

While the relations of Eq. (2.10) are no longer valid at the dimension-eight level, deviations from these relations are naturally suppressed by factors of v^2/Λ^2 and only for very small scales Λ may one expect an appreciable violation of, e.g., the relation $\lambda_\gamma = \lambda_Z$.

In our discussion we have emphasized the use of a linear realization of the Goldstone boson sector. For the case of a very heavy Higgs boson (or in models without a scalar resonance) the use of a chiral Lagrangian is more appropriate. Effectively the chiral Lagrangian is obtained by the replacements $\Phi \to \exp(i\sigma^a\chi^a/v)(0, v/\sqrt{2})^T$ and $\Lambda \to 4\pi v$. As a main effect our counting of the operator dimensions no longer holds, and, e.g., the dimension-six and dimension-eight operators which contribute to κ_V formally appear at the same level. Hence the relations of Eq. (2.10) between the various anomalous couplings are lost, as has been emphasized in Ref. [16]. Clearly these relations are not due to the imposition of SU(2)$\times$U(1) gauge invariance, but rather they follow from neglecting dimension-eight and higher operators in the effective Lagrangian. We believe that the ordering implied by the use of the linear realization should nevertheless be kept in mind. Even in models where new thresholds open at 1 TeV, the suppression factors v^2/Λ^2 may be small enough to lead to the relations of Eq. (2.10) up to corrections which are smaller by more than a factor 10. This possibility combined with the ability to discuss Higgs mass effects strongly motivates us to prefer the formulation with a linear realization of the Goldstone boson sector.

In what follows we shall neglect operators of energy dimension larger than six.

III. PRECISION TESTS OF FOUR-FERMION AMPLITUDES AND CONSTRAINTS ON NEW PHYSICS

To a large extent our knowledge of electroweak interactions stems from precise measurements of four-fermion S-matrix elements. This includes the recent LEP data, neutrino scattering experiments, atomic parity violation, μ decay, and the W-mass measurement at hadron colliders. We are interested in experimental constraints, due to such precision experiments, on the coefficients of the dimension-six operators which were discussed in Sec. II. Contributions of these operators will be considered at the lowest level at which they appear. Thus the operators of Eq. (2.2) will be considered at tree level only, while the one-loop contributions of the operators (2.7) will be discussed in Sec. IV. In this section we describe the general framework for including the dimension-six operators in the four-fermion S-matrix elements and study the tree level contributions of $\mathcal{O}_{DW}$, $\mathcal{O}_{DB}$, $\mathcal{O}_{BW}$, and $\mathcal{O}_{\Phi,1}$ as an example. Constraints on the operators are obtained by subtracting the SM contributions to the amplitudes. Since present experiments are sensitive to one-loop radiative corrections involving SM physics, the derivation of experimental constraints on new physics must include these one-loop effects. Our analysis, which is based on the work of Ref. [22], considers the full one-loop SM radiative corrections, including vertex and box diagrams.

The new physics effects which we consider are universal for all external fermion species, they respect SU(2)$\times$U(1) gauge invariance, albeit in the spontaneously broken phase of the theory, and they do not yield box contributions at the lowest level. Hence they satisfy the conditions of generalized universality [9, 21] and they can be fully described within the improved Born approximation (IBA), in which four-fermion amplitudes for massless external fermions are given by

$$\mathcal{M}(p_1,p_2,p_3,p_4) = I(q^2)\; J_\mu(p_1,p_2)J^\mu(p_3,p_4)\;. \tag{3.1}$$

Here the J_μ only depend on the wave functions of the external fermions and the helicity dependent $I(q^2)$ are given by

$$I_{\rm CC}(q^2) = \frac{\bar{g}_W^2(q^2)/2}{q^2 - m_W^2 + im_W\Gamma_W} \tag{3.2}$$

for CC amplitudes of left-handed fermions. For NC amplitudes we may write

$$I_{\rm NC}(q^2) = \frac{\bar{e}^2(q^2)}{q^2}Q_{f_1}Q_{f_3} + \frac{\bar{g}_Z^2(q^2)}{q^2-m_Z^2+im_Z\Gamma_Z}\left(T_3^{f_1} - \bar{s}^2(q^2)Q_{f_1}\right)\left(T_3^{f_3} - \bar{s}^2(q^2)Q_{f_3}\right)\;, \tag{3.3}$$

where Q_{f_i} denotes the electric charge of fermion f_i in units of $\sqrt{4\pi\alpha} = \bar{e}(0)$ and the helicity dependence is incorporated by setting $T_3^{f_i} = \pm\frac{1}{2}$ for left-handed fermions and $T_3^{f_i} = 0$ for right-handed fermions.

The effects of the four operators $\mathcal{O}_{DW}$, $\mathcal{O}_{DB}$, $\mathcal{O}_{BW}$, and $\mathcal{O}_{\Phi,1}$ on the oblique correction form factors $\bar{e}^2(q^2)$, $\bar{g}_W^2(q^2)$, $\bar{g}_Z^2(q^2)$, and $\bar{s}^2(q^2)$ arise solely from their contributions to the four electroweak gauge-boson propagators. In order to extract information on these new physics contributions we consider the one-loop SM radiative corrections at the same time. For notational convenience, we express the transverse parts of the gauge boson propagators as

$$\Pi_T^{\gamma\gamma}(q^2) = e^2\Pi_T^{QQ}(q^2)\;, \tag{3.4a}$$

$$\Pi_T^{\gamma Z}(q^2) = eg_Z\left[\Pi_T^{3Q}(q^2) - s^2\Pi_T^{QQ}(q^2)\right]\;, \tag{3.4b}$$

$$\Pi_T^{ZZ}(q^2) = g_Z^2\left[\Pi_T^{33}(q^2) - 2s^2\Pi_T^{3Q}(q^2) + s^4\Pi_T^{QQ}(q^2)\right]\;, \tag{3.4c}$$

$$\Pi_T^{WW}(q^2) = g^2\Pi_T^{11}(q^2)\;. \tag{3.4d}$$

The couplings ($e = gs = g_Zsc$) and the propagators are renormalized in the modified minimal subtraction ($\overline{\rm MS}$) scheme and the SM one-loop contributions are calculated in the 't Hooft–Feynman gauge. We separate the SM and new physics contributions to the propagators as

$$\Pi_T^{AB}(q^2) = \Pi_T^{AB}(q^2)_{\rm SM} + \Delta\Pi_T^{AB}(q^2)\;. \tag{3.5}$$

The explicit forms of the SM parts, $\Pi_T^{AB}(q^2)_{\rm SM}$, are found in the Appendix of Ref. [26]. The contributions of the four dimension-six operators are

$$\Delta\Pi_T^{QQ}(q^2) = 2\frac{q^2}{\Lambda^2}\left((f_{DW}+f_{DB})q^2 - f_{BW}\frac{m_W^2}{g^2}\right)\;, \tag{3.6a}$$

$$\Delta\Pi_T^{3Q}(q^2) = 2\frac{q^2}{\Lambda^2}\left(f_{DW}q^2 - f_{BW}\frac{m_W^2}{2g^2}\right)\;, \tag{3.6b}$$

$$\Delta\Pi_T^{33}(q^2) = 2\frac{q^2}{\Lambda^2}f_{DW}q^2 - f_{\Phi,1}\frac{m_W^2}{\Lambda^2}\frac{v^2}{2g^2}\;, \tag{3.6c}$$

$$\Delta\Pi_T^{11}(q^2) = 2\frac{q^2}{\Lambda^2}f_{DW}q^2\;. \tag{3.6d}$$

The form factors $\bar{\alpha}(q^2) = \bar{e}^2(q^2)/4\pi$ and $\bar{s}^2(q^2)$ can be expressed in terms of the above two-point functions as

$$\frac{1}{\bar{\alpha}(q^2)} = \frac{4\pi}{e^2} + 4\pi\,{\rm Re}\,\Pi_{T,\gamma}^{QQ}(q^2) + 8\pi\frac{\Pi_T^{3Q}(0)}{m_W^2}\;, \tag{3.7a}$$

$$\frac{\bar{s}^2(q^2)}{\bar{\alpha}(q^2)} = \frac{4\pi}{g^2} + 4\pi\,{\rm Re}\,\Pi_{T,\gamma}^{3Q}(q^2) + 8\pi\frac{\Pi_T^{3Q}(0)}{m_W^2}\;, \tag{3.7b}$$

which are explicitly renormalization group invariant at the one-loop level. We adopt the shorthand notation

$$\Pi_{T,V}^{AB}(q^2) = \frac{\Pi_T^{AB}(q^2) - \Pi_T^{AB}(m_V^2)}{q^2 - m_V^2} \tag{3.8}$$

for the propagator factors after the gauge boson mass renormalization ($m_\gamma = 0$). Our definition of the running parameters $\bar{\alpha}(q^2)$ and $\bar{s}^2(q^2)$ agrees with the ones adopted by the LEP working group [27]. We note here that the parameter $\bar{\alpha}(q^2)$ is measured accurately only at $q^2 = 0$ [$\bar{\alpha}(0) = \alpha = 1/137.036$], whereas $\bar{s}^2(q^2)$ is measured most accurately at $q^2 = m_Z^2$. We therefore introduce a theoretical quantity $\bar{\alpha}(m_Z^2)_{\rm SM}$,

$$\frac{1}{\bar{\alpha}(m_Z^2)_{\rm SM}} = \frac{1}{\bar{\alpha}(m_Z^2)} - 4\pi\,{\rm Re}\left[\Delta\Pi_{T,\gamma}^{QQ}(m_Z^2) - \Delta\Pi_{T,\gamma}^{QQ}(0)\right]\;, \tag{3.9}$$

whose numerical value is known precisely [28]:

$$\frac{1}{\bar{\alpha}(m_Z^2)_{\rm SM}} = 128.73 \pm 0.12 + \frac{8}{3\pi}\left(1+\frac{\alpha_s}{\pi}\right){\rm Re}\left[B_3(m_Z^2;m_t,m_t) - B_3(0;m_t,m_t)\right]\;. \tag{3.10}$$

The top-quark contribution in the last term is uncertain but its numerical value is found to be smaller than 0.03 for $m_t > 100$ GeV and hence is negligible. Our conventions for the B functions [29] are those of Ref. [26]. The running of the $\bar{s}^2(q^2)$ parameter between $q^2 = 0$ and $q^2 = m_Z^2$ is then obtained from

$$\bar{s}^2(0) = \frac{\alpha}{\bar{\alpha}(m_Z^2)}\bar{s}^2(m_Z^2) - 4\pi\alpha \,\mathrm{Re}\left[\Pi_{T,\gamma}^{3Q}(m_Z^2) - \Pi_{T,\gamma}^{3Q}(0)\right] . \tag{3.11}$$

Within the SM the main uncertainty in the running of $\bar{s}^2(q^2)$ arises from the ambiguities in the hadronic contributions, which we estimate along with Refs. [28] and [30]:

$$\bar{s}^2(0)_{\rm SM} = 0.939\,39\,\bar{s}^2(m_Z^2) + 0.022\,17 \pm 0.000\,24$$
$$-\frac{\alpha}{\pi}\left[1 - \frac{8}{3}\,\bar{s}^2(m_Z^2)\right]\left(1 + \frac{\alpha_s}{\pi}\right) \mathrm{Re}\left[B_3(m_Z^2; m_t, m_t) - B_3(0; m_t, m_t)\right] . \tag{3.12}$$

The top-quark contribution in the last term is at most $-0.000\,03$ for $m_t > 100$ GeV, and hence is negligible.

The remaining effective couplings that are associated with Z and W propagators are most conveniently parametrized as

$$\frac{16\pi}{\bar{g}_Z^2(q^2)} = \frac{4\bar{s}^2(m_Z^2)\bar{c}^2(m_Z^2)}{\bar{\alpha}(m_Z^2)_{\rm SM}} - S_Z(q^2) , \tag{3.13a}$$

$$\frac{16\pi}{\bar{g}_W^2(q^2)} = \frac{4\bar{s}^2(m_Z^2)}{\bar{\alpha}(m_Z^2)_{\rm SM}} - S_W(q^2) , \tag{3.13b}$$

in terms of the S_Z and S_W form factors [22]. At $q^2 = 0$, they are expressed as

$$S_Z(0) = 16\pi \left\{\mathrm{Re}\left[\Pi_{T,\gamma}^{3Q}(m_Z^2) - \Pi_{T,Z}^{33}(0)\right] - \bar{s}^2(m_Z^2)\bar{c}^2(m_Z^2)\,\mathrm{Re}\left[\Delta\Pi_{T,\gamma}^{QQ}(m_Z^2) - \Delta\Pi_{T,\gamma}^{QQ}(0)\right]\right\} , \tag{3.14a}$$

$$S_W(0) = 16\pi \left\{\mathrm{Re}\left[\Pi_{T,\gamma}^{3Q}(m_Z^2) - \Pi_{T,W}^{11}(0)\right] - \bar{s}^2(m_Z^2)\,\mathrm{Re}\left[\Delta\Pi_{T,\gamma}^{QQ}(m_Z^2) - \Delta\Pi_{T,\gamma}^{QQ}(0)\right]\right\} , \tag{3.14b}$$

where the second terms reflect the difference (3.9) between the actual $\bar{\alpha}(m_Z^2)$ and the calculated quantity $\bar{\alpha}(m_Z^2)_{\rm SM}$ that is used in the definitions (3.13). The above form factors can be regarded as possible exact definitions of the S and U parameters of Ref. [11], which may be expressed as

$$S_Z(0) \equiv S , \tag{3.15a}$$

$$S_W(0) \equiv S + U . \tag{3.15b}$$

By dropping the new physics contributions to the running of $\bar{\alpha}(q^2)$ and by replacing the operation (3.8) by differentiation, the expressions for S and U reduce to their original definitions. Within the SM they depend on the as yet unknown masses of the Higgs boson and the top quark. For $m_t > m_Z/2$, i.e., $z_t = \frac{m_t^2}{m_Z^2} > \frac{1}{4}$, this dependence is given by

$$S_{\rm SM} = 0.010 + \frac{1}{4\pi}h\left(\frac{m_H^2}{m_Z^2}\right) + \frac{1}{6\pi}\left(1 + \frac{\alpha_s}{\pi}\right)\left[22z_t - \ln z_t - (1 + 11z_t)[a(z_t) + \pi\sqrt{4z_t - 1}]\right] , \tag{3.16a}$$

$$U_{\rm SM} = -0.010 + \frac{1}{4\pi}\left[h\left(\frac{m_H^2}{m_W^2}\right) - h\left(\frac{m_H^2}{m_Z^2}\right)\right] + \frac{1}{2\pi}\left(1 + \frac{\alpha_s}{\pi}\right)\left[\frac{1}{2}w_t + w_t^2 + (1 - w_t)^2(2 + w_t)\ln\left|1 - \frac{1}{w_t}\right|\right.$$
$$\left. -2z_t + \ln z_t - (1 - z_t)[a(z_t) + \pi\sqrt{4z_t - 1}]\right] . \tag{3.16b}$$

Here $w_t = \frac{m_t^2}{m_W^2}$ and the two functions a and h are given by

$$a(x) = \begin{cases} -2\sqrt{4x - 1}\,\arctan^{-1}\sqrt{4x - 1} & \text{for } x > \frac{1}{4}, \\ 2\sqrt{1 - 4x}\,\ln\dfrac{1 + \sqrt{1 - 4x}}{2\sqrt{x}} & \text{for } 0 < x < \frac{1}{4}, \end{cases} \tag{3.17}$$

and

$$h(x) = -\frac{79}{18} + \frac{3}{2}x - \frac{x^2}{3} + \left(3 - x + \frac{x^2}{6} + \frac{3}{1 - x}\right)x\ln x - \left(2x - \frac{2x^2}{3} + \frac{x^3}{6}\right)a\left(\frac{1}{x}\right) . \tag{3.18}$$

While the functional dependence on m_t and m_H is shown explicitly, a numerical value for the contributions due to the gauge boson loops is given for both S and U. Present measurements of m_Z and m_W leave little uncertainty in these contributions.

The effective W coupling $\bar{g}_W^2(q^2)$ is measured accurately only at $|q^2| \ll m_W^2$, e.g., in μ decay and in low-energy charged current processes. The effective Z coupling $\bar{g}_Z^2(q^2)$ is measured both in low energy neutral current experiments ($|q^2| \ll m_Z^2$) and at $q^2 = m_Z^2$. We therefore need the difference

$$\frac{16\pi}{\bar{g}_Z^2(m_Z^2)} - \frac{16\pi}{\bar{g}_Z^2(0)} = S_Z(0) - S_Z(m_Z^2) = 16\pi \left\{ \mathrm{Re}\left[\Pi_{T,Z}^{33}(m_Z^2) - \Pi_{T,Z}^{33}(0)\right] - 2\bar{s}^2(m_Z^2)\,\mathrm{Re}\left[\Pi_{T,Z}^{3Q}(m_Z^2) - \Pi_{T,Z}^{3Q}(0)\right] + \bar{s}^4(m_Z^2)\,\mathrm{Re}\left[\Pi_{T,Z}^{QQ}(m_Z^2) - \Pi_{T,Z}^{QQ}(0)\right] \right\} . \tag{3.19}$$

Within the SM the running of S_Z is given by

$$S_Z(m_Z^2)_{\mathrm{SM}} = S_Z(0)_{\mathrm{SM}} + 0.988 \pm 0.020 , \tag{3.20}$$

for $m_t > 100$ GeV and $m_H > 50$ GeV.

Finally, in theories with a custodial SU(2) symmetry,

$$\frac{1}{\hat{\rho}} \equiv \frac{g^2}{g_Z^2}\frac{\hat{m}_Z^2}{\hat{m}_W^2} = 1 , \tag{3.21}$$

where $\hat{m}_W$ and $\hat{m}_Z$ are the vector boson masses in the $\overline{\mathrm{MS}}$ scheme. This yields one additional constraint among the effective parameters and one obtains the relation

$$\frac{1}{\bar{\rho}} \equiv \frac{\bar{g}_W^2(0)}{\bar{g}_Z^2(0)}\frac{m_Z^2}{m_W^2} = 1 - \alpha T , \tag{3.22}$$

where

$$T = \frac{4\sqrt{2}G_F}{\alpha(1+\delta_G)}\left[\Pi_T^{33}(0) - \Pi_T^{11}(0)\right] . \tag{3.23}$$

$G_F = 1.166\,39 \times 10^{-5}\ \mathrm{GeV}^{-2}$ is the Fermi coupling and $\delta_G = \frac{g^2}{8\pi^2}\left[3 - \left(\frac{7}{4s^2} - 1\right)\ln\frac{m_Z^2}{m_W^2}\right] \sim 0.0071$ is the extra vertex and box correction to the muon decay lifetime. Within the SM the T parameter shows the well-known quadratic top-mass dependence:

$$T_{\mathrm{SM}} = -0.1115 + \frac{3G_F}{8\sqrt{2}\,\pi^2\alpha(1+\delta_G)}\left[\left(1+\frac{\alpha_s}{\pi}\right)m_t^2 + m_H^2\left(\frac{m_Z^2}{m_Z^2 - m_H^2}\ln\frac{m_H^2}{m_Z^2} - \frac{m_W^2}{m_W^2 - m_H^2}\ln\frac{m_H^2}{m_W^2}\right) + \frac{7}{6}\frac{m_Z^2 - m_W^2}{m_Z^2}\right] , \tag{3.24}$$

where again the gauge boson contribution is given by its numerical value. Finally, the above expressions for S and T together with the definition of the Fermi constant G_F allow us to express $\bar{s}^2(m_Z^2)$ as

$$\bar{s}^2(m_Z^2) = \frac{1}{2} - \sqrt{\frac{1}{4} - \bar{\alpha}(m_Z^2)_{\mathrm{SM}}\left(\frac{S}{4} + \frac{(1-\alpha T)(1+\delta_G)}{\sqrt{2}G_F m_Z^2}\pi\right)} . \tag{3.25}$$

In particular we obtain the SM value $\bar{s}^2(m_Z^2)_{\mathrm{SM}}$ by using this identity and the SM expressions for S and T.

The above parameters suffice to describe the existing precision data on four-fermion amplitudes. An analysis in terms of these parameters has recently been performed in Ref. [22] and here we merely cite their results. The LEP and SLC data can be summarized in terms of

$$\bar{g}_Z^2(m_Z^2) = 0.5520 + 0.000\,224\left(\frac{m_t}{100\ \mathrm{GeV}}\right)^2 \pm 0.0017 , \tag{3.26a}$$

$$\bar{s}^2(m_Z^2) = 0.2318 - 0.000\,04\left(\frac{m_t}{100\ \mathrm{GeV}}\right)^2 \pm 0.0011 , \tag{3.26b}$$

with a correlation of $\rho = 0.315$ and a minimal χ^2 of

$$\chi^2_{\min}(m_t) = 3.430 - 0.180\left(\frac{m_t}{100\ \mathrm{GeV}}\right)^2 + 0.153\left(\frac{m_t}{100\ \mathrm{GeV}}\right)^4 . \tag{3.27}$$

The top-mass dependence of the fit is mainly due to the dependence of the $Zb\bar{b}$ vertex corrections on m_t. Since the data must be corrected for the vertex contributions prior to the extraction of the oblique parameters, a slight dependence on m_t results. The quoted errors already contain the uncertainty in the determination of the strong coupling constant, for which $\alpha_s(m_Z) = 0.12 \pm 0.01$ has been taken.

In a similar fashion the low energy data on neutrino scattering and atomic parity violation can be summarized by

$$\bar{g}_Z^2(0) = 0.5462 \pm 0.0035 , \tag{3.28a}$$

$$\bar{s}^2(0) = 0.2359 \pm 0.0048 \ , \tag{3.28b}$$

with a correlation of $\rho = 0.531$. These results are obtained in Ref. [22] after process dependent vertex and box contributions of the SM have been corrected for in the 't Hooft–Feynman gauge. Finally, the W-mass measurement at hadron colliders together with the input value of G_F as measured in μ decay can be translated into a measurement of $\bar{g}_W^2(0)$:

$$\bar{g}_W^2(0) = 0.4217 \pm 0.0027 \ . \tag{3.29}$$

These data must then be compared to the predicted values which are a sum of the SM contributions given earlier, and the new physics contributions. For the four dimension-six operators which contribute at tree level the latter can be summarized by

$$\Delta\delta\rho = \alpha\Delta T = -\frac{v^2}{2\Lambda^2} f_{\Phi,1} \ , \tag{3.30a}$$

$$\Delta S_Z(q^2) = -32\pi\frac{m_Z^2}{\Lambda^2}\,(c^2 f_{DW} + s^2 f_{DB}) - 4\pi\frac{v^2}{\Lambda^2} f_{BW} - 32\pi\frac{q^2 - m_Z^2}{\Lambda^2}\,(c^4 f_{DW} + s^4 f_{DB}) \ , \tag{3.30b}$$

$$\Delta S_W(q^2) = \Delta S_Z(m_Z^2) - 32\pi\frac{q^2 - m_W^2}{\Lambda^2}\, f_{DW} \ , \tag{3.30c}$$

$$\Delta\bar{s}^2(q^2) = \frac{1}{c^2 - s^2}\left(\frac{\alpha}{4}\Delta S_Z(0) - c^2 s^2 \Delta\delta\rho\right) + 8\pi\alpha\frac{q^2 - m_Z^2}{\Lambda^2}\,(c^2 f_{DW} - s^2 f_{DB}) \ , \tag{3.30d}$$

$$\Delta\frac{\bar{\alpha}(q^2) - \bar{\alpha}(0)}{\bar{\alpha}(0)} = -8\pi\alpha\frac{q^2}{\Lambda^2}\,(f_{DW} + f_{DB}) \ . \tag{3.30e}$$

Numerical results for the constraints on the four coefficients f_{DW}, f_{DB}, f_{BW}, and $f_{\Phi,1}$ will be given in Sec. V together with the constraints on the remaining operators which only contribute at the one-loop level.

IV. NEW PHYSICS CONTRIBUTIONS AT THE ONE-LOOP LEVEL

The five operators $\mathcal{O}_{WWW}$, $\mathcal{O}_{WW}$, $\mathcal{O}_{BB}$, $\mathcal{O}_W$, and $\mathcal{O}_B$ affect the four-fermion amplitudes at the one-loop level only. Contributions can arise either from corrections to the gauge boson two-point functions or from corrections to the fermion–gauge-boson vertices. For the CC amplitudes the relevant Feynman graphs are shown in Figs. 1 and 2. We are neglecting dimension-eight operators, which would enter with a coefficient Λ^{-4}, and are hence suppressed by an additional factor m_Z^2/Λ^2 in the low energy observables. For consistency we must also drop contributions with two anomalous vertices which formally are of the same order. In the vertex corrections only the anomalous triple gauge boson couplings enter. The two-point functions, on the other hand, depend on the new gauge-boson–Higgs-boson and the related Goldstone-boson–Higgs-boson vertices in addition. As we shall see these Higgs contributions are essential.

The calculation has been performed in a general R_ξ gauge which allows us to check the calculation by verifying the gauge invariance of the four-fermion amplitudes. In turn this requires the use of a gauge invariant regularization scheme for which we have chosen dimensional regularization. Working in $d = 4 - 2\epsilon$ dimensions, we identify the poles at $d = 2$ with quadratic divergencies and the poles at $d = 4$ with logarithmic divergencies. These are related to the cutoff scale Λ in a momentum cutoff regularization by the identification

$$4\pi\mu^2\left(\frac{1}{\epsilon - 1} + 1\right) = \Lambda^2 \ , \tag{4.1a}$$

$$\frac{1}{\epsilon} - \gamma_E + \ln(4\pi\mu^2) + 1 = \ln\Lambda^2 \ , \tag{4.1b}$$

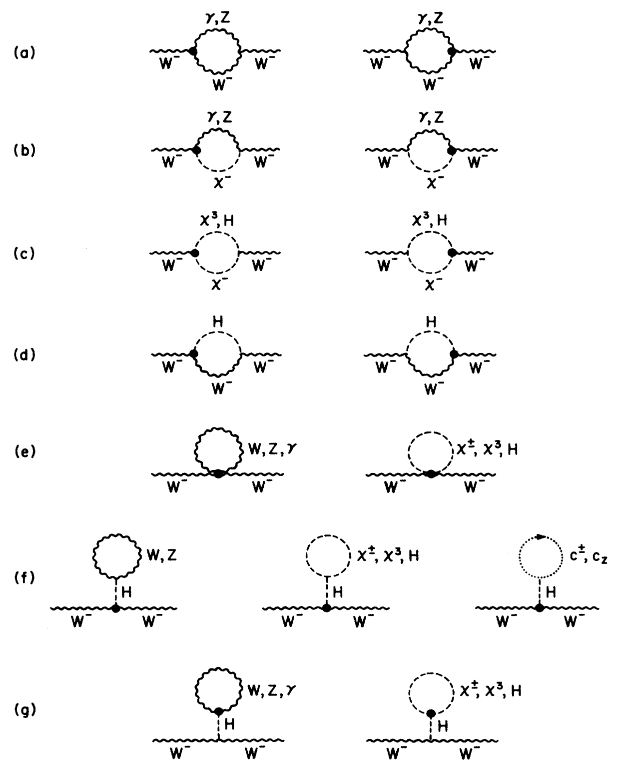

FIG. 1. Feynman graphs for the anomalous contributions to the WW two-point function. The vertices carrying a blob represent the anomalous boson interactions as described by the dimension-six operators of Eq. (2.7). $\chi^\pm$ and χ^3 denote the Goldstone boson propagators while $c^\pm$, c_Z label the Faddeev-Popov ghosts.

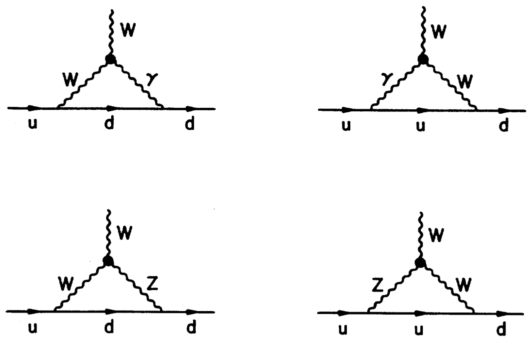

FIG. 2. Feynman graphs for the new physics contributions to the Wud vertex. For massless fermions, only the anomalous triple gauge boson couplings affect the fermion–gauge-boson vertices at the one-loop level.

where μ denotes the unit of mass of the dimensional regularization. The above identification is found by evaluating appropriate tadpole diagrams in the two regularization schemes.

We have calculated all divergent contributions of the dimension-six operators to the four-fermion amplitudes. Vertex corrections appear for left-handed fermions only because in all graphs a virtual W is coupled to the fermion line. Neglecting any fermion mass effects, the vertex functions are given in terms of a scalar form factor:

$$\Delta\Gamma_\mu^{Vf_1f_2}(q) = \gamma_\mu \frac{1}{2}(1-\gamma_5)\Delta\Gamma_L^{Vf_1f_2}(q^2) \, , \tag{4.2}$$

where q denotes the momentum of the virtual gauge boson V. For the couplings of the Z or γ to fermions the antisymmetry of all anomalous triple gauge boson couplings in the W^+ and the W^- fields ensures that up- and down-type fermions receive vertex corrections of opposite sign. In addition we find that the divergent parts of the vertex corrections are independent of the fermion charge. Hence we can write, for both $V=\gamma, Z$,

$$\Delta\Gamma_L^{Vff}(q^2) = g\, T_3^f \, \Delta\Gamma_L^V(q^2) \, , \tag{4.3}$$

where T_3^f denotes the third component of the fermion's isospin and the form factor $\Delta\Gamma_L^V(q^2)$ is independent of the fermion flavor.

For the W coupling to fermions an explicit calculation of the Feynman graphs depicted in Fig. 2 yields the flavor independent result

$$\begin{aligned}\Delta\Gamma_L^{Wf_1f_2}(q^2) &= \frac{g}{\sqrt{2}}\Delta\Gamma_L^W(q^2) \\ &= \frac{g}{\sqrt{2}}\,\frac{\alpha}{8\pi s^2}\,\frac{m_W^2}{\Lambda^2}\ln\frac{\Lambda^2}{\mu^2}\left(3g^2 f_{WWW}\,\frac{q^2}{m_W^2} + \frac{3}{4} f_W\,(\xi_W+\xi_Z+2)\right) .\end{aligned} \tag{4.4}$$

The vertex corrections induced by the operator $\mathcal{O}_W$ depend on the gauge parameters ξ_W and ξ_Z. Vertex corrections induced by the operator $\mathcal{O}_B$ on the other hand are finite and do not appear in Eq. (4.4). The gauge dependence of the vertex functions is exactly canceled by corresponding terms in the gauge boson self-energies. For CC amplitudes we need to consider the transverse part of the W vacuum polarization tensor, $\Delta\Pi_T^{WW}(q^2)$, as shown in Fig. 1. Retaining the f_{WWW} and f_W terms only, the result is

$$\begin{aligned}\Delta\Pi_T^{WW}(q^2) = -\frac{\alpha}{8\pi s^2\Lambda^2}\Bigg\{ & f_W\,(2q^2-3m_W^2)\,\Lambda^2 \\ & +\Bigg[f_W\left(\frac{(q^2)^2}{3}+\frac{q^2}{2}(21m_W^2+m_Z^2-m_H^2)+\frac{m_W^2}{2}(9m_W^2-6m_Z^2+3m_H^2)\right) \\ & -\frac{3}{2}f_W m_W^2(q^2-m_W^2)(\xi_W+\xi_Z+2) - 6g^2 f_{WWW}\, q^2(q^2-6m_W^2)\Bigg]\ln\frac{\Lambda^2}{\mu^2}\Bigg\} .\end{aligned} \tag{4.5}$$

The general result, including all nine operators of Eqs. (2.2) and (2.7), is given in Appendix B.

Any charged current amplitude only depends on the linear combination

$$\Delta\overline{\Pi}_T^{WW}(q^2) = \Delta\Pi_T^{WW}(q^2) - 2(q^2-m_W^2)\Delta\Gamma_L^W(q^2) \, . \tag{4.6}$$

In this combination the gauge dependent terms in $\Delta\Gamma_L^W$ and $\Delta\Pi_T^{WW}$ exactly cancel, as is explicit from Eqs. (4.4) and (4.5). In the same fashion the NC amplitudes only depend on gauge invariant combinations of vertex and vacuum polarization functions:

$$\Delta\overline{\Pi}_T^{\gamma\gamma}(q^2) = \Delta\Pi_T^{\gamma\gamma}(q^2) - 2s\, q^2\, \Delta\Gamma_L^\gamma(q^2) \, , \tag{4.7a}$$

$$\Delta\overline{\Pi}_T^{\gamma Z}(q^2) = \Delta\Pi_T^{\gamma Z}(q^2) - s\, q^2\, \Delta\Gamma_L^Z(q^2) - c\,(q^2-m_Z^2)\,\Delta\Gamma_L^\gamma(q^2) \, , \tag{4.7b}$$

$$\Delta\overline{\Pi}_T^{ZZ}(q^2) = \Delta\Pi_T^{ZZ}(q^2) - 2c\,(q^2-m_Z^2)\,\Delta\Gamma_L^Z(q^2) \, . \tag{4.7c}$$

We have explicitly verified that the four combinations in Eqs. (4.6) and (4.7) are indeed independent of the gauge parameters ξ_W and ξ_Z, which constitutes an important check of our calculation.[1]

Explicit formulas for all vertex functions and the effective vacuum polarization functions $\Delta\overline{\Pi}_T^{V_1V_2}(q^2)$ including the effects of all operators are given in Appendix B. Here we are interested in the observable effects on the CC and NC four-fermion amplitudes only. As shown in Sec. III these effects can be summarized in terms of four form factors and by the W to Z mass ratio. We express these quantities in terms of the ρ parameter as measured by the ratio of NC/CC neutrino scattering, $S_Z(q^2)$, $S_W(q^2)$, the weak mixing angle $\bar{s}(q^2)$ as defined in Sec. III, and by the running of the QED fine-structure constant. When including the contributions of the four operators $\mathcal{O}_{DW}$, $\mathcal{O}_{DB}$, $\mathcal{O}_{BW}$, and $\mathcal{O}_{\Phi,1}$ which already entered at the tree level, the new physics contributions to these five quantities are given by

$$\Delta\delta\rho = \alpha\Delta T = -\frac{v^2}{2\Lambda^2}\, f^r_{\Phi,1}\,, \tag{4.8a}$$

$$\Delta S_Z(q^2) = -32\pi\frac{m_Z^2}{\Lambda^2}\,(c^2 f^r_{DW} + s^2 f^r_{DB}) - 4\pi\frac{v^2}{\Lambda^2} f^r_{BW} -32\pi\frac{q^2-m_Z^2}{\Lambda^2}\left(c^4 f^r_{DW} + s^4 f^r_{DB} + \frac{1}{768\pi^2}(f_B+f_W)\ln\frac{m_H^2}{m_W^2}\right)\,, \tag{4.8b}$$

$$\Delta S_W(q^2) = \Delta S_Z(m_Z^2) + \frac{s^2}{3\pi}\frac{m_Z^2}{\Lambda^2}(f_W - f_B)\ln\frac{m_H^2}{m_W^2} - 32\pi\frac{q^2-m_W^2}{\Lambda^2}\left(f^r_{DW} + \frac{1}{768\pi^2}(f_B+f_W)\ln\frac{m_H^2}{m_W^2}\right)\,, \tag{4.8c}$$

$$\Delta\bar{s}^2(q^2) = \frac{1}{c^2-s^2}\left(\frac{\alpha}{4}\Delta S_Z(0) - c^2 s^2\Delta\delta\rho\right) + 8\pi\alpha\frac{q^2-m_Z^2}{\Lambda^2}\,(c^2 f^r_{DW} - s^2 f^r_{DB})\,, \tag{4.8d}$$

$$\Delta\frac{\bar{\alpha}(q^2)-\bar{\alpha}(0)}{\bar{\alpha}(0)} = -8\pi\alpha\frac{q^2}{\Lambda^2}\,(f^r_{DW} + f^r_{DB})\,. \tag{4.8e}$$

The four parameters f^r_{DW}, f^r_{DB}, f^r_{BW}, and $f^r_{\Phi,1}$ which enter in Eq. (4.8) are independent of q^2. They can be written in terms of the coefficients of the nine operators which contain electroweak gauge fields:

$$f^r_{DW} = f_{DW} - \frac{1}{192\pi^2}\left(f_W\,\ln\frac{\Lambda^2}{m_W^2} + \frac{f_B - f_W}{4}\ln\frac{m_H^2}{m_W^2}\right)\,, \tag{4.9a}$$

$$f^r_{DB} = f_{DB} - \frac{1}{192\pi^2}\left(f_B\,\ln\frac{\Lambda^2}{m_W^2} - \frac{f_B - f_W}{4}\ln\frac{m_H^2}{m_W^2}\right)\,, \tag{4.9b}$$

$$f^r_{BW} = f_{BW} + \frac{\alpha}{96\pi s^2}\Bigg\{3\frac{m_H^2}{m_W^2}(f_B+f_W)\left(\ln\frac{\Lambda^2}{m_H^2}+\frac{1}{2}\right) - 5\left(\frac{1}{c^2}-2\right)(f_B-f_W)\ln\frac{m_H^2}{m_W^2} + \left[\left(20+\frac{7}{c^2}\right)f_B - \left(12+\frac{3}{c^2}\right)f_W + 36g^2 f_{WWW}\right]\ln\frac{\Lambda^2}{m_W^2} - 24\left(f_{WW}+\frac{s^2}{c^2}f_{BB}\right)\ln\frac{\Lambda^2}{m_H^2}\Bigg\}\,, \tag{4.9c}$$

$$f^r_{\Phi,1} = f_{\Phi,1} + \frac{3\alpha}{8\pi c^2}\left[\frac{m_H^2}{v^2}f_B\left(\ln\frac{\Lambda^2}{m_H^2}+\frac{1}{2}\right) + 3\frac{m_W^2}{v^2}(f_B+f_W)\ln\frac{\Lambda^2}{m_W^2} - \left(\frac{m_Z^2}{v^2}f_B + \frac{m_W^2}{v^2}f_W\right)\ln\frac{m_H^2}{m_W^2}\right]\,. \tag{4.9d}$$

[1]Incidentally, the leading $(q^2)^2$ terms arising from the operator $\mathcal{O}_{WWW}$ exactly cancel as well in the gauge independent combinations of Eqs. (4.6) and (4.7). As a result we find no contribution to the running of $\bar{\alpha}(q^2)$ due to $\mathcal{O}_{WWW}$, in apparent conflict to Ref. [14]. The discrepancy is due to our use of the gauge invariant combinations $\Delta\overline{\Pi}_T$ when defining the oblique parameters and we found complete agreement of the full four-fermion amplitudes. We thank M. B. Gavela and E. Massó for their help in establishing agreement of our results.

In addition to the divergent terms (proportional to $\ln \Lambda$) we also have included all terms which depend quadratically or logarithmically on m_H for large values of the Higgs mass ($m_H > m_Z$).

The above equations constitute the main result of our work. Their interpretation is straightforward: the one-loop effects involving the insertion of single dimension-six operators yield logarithmic divergencies which can be absorbed completely into the renormalization of those dimension-six operators which already contribute to the four-fermion amplitudes at tree level. The parameters f^r_{DW}, f^r_{DB}, f^r_{BW}, and $f^r_{\Phi,1}$ are the renormalized coefficients of the four operators $\mathcal{O}_{DW}$, $\mathcal{O}_{DB}$, $\mathcal{O}_{BW}$, and $\mathcal{O}_{\Phi,1}$ at the mass scale of the weak bosons. The logarithmic terms in Eq. (4.9) describe the mixing of these four operators with $\mathcal{O}_{WWW}$, $\mathcal{O}_{WW}$, $\mathcal{O}_{BB}$, $\mathcal{O}_{W}$, and $\mathcal{O}_{B}$ due to the evolution between the boson masses and the new physics scale Λ, as governed by the renormalization group equations.

In intermediate steps of the calculation we actually encounter quadratic divergencies as well. One example is the first term in Eq. (4.5). The q^2 independent part of the quadratic divergence is absorbed into a finite mass shift for the W boson. The cutoff scale Λ^2 and the Λ^{-2} factor from the normalization of the dimension-six operators cancel and yield a mass shift which is independent of the scale of new physics:

$$\delta m_W^2 = -\Delta\Pi_T^{WW}(m_W^2) = -\frac{\alpha}{8\pi s^2}\, f_W\, m_W^2 + \cdots . \tag{4.10}$$

Additional terms due to other operators can be inferred from the complete expressions in Appendix B. Because the Z boson mass receives an analogous shift, these leading effects of the new physics cancel in the mass ratio or the ρ parameter.

The remaining quadratically divergent pieces in the W self-energy vanish for $q^2 = m_W^2$ and give a constant contribution to $\overline{\Pi}_{T,W}^{WW}(q^2)$. They are absorbed into the renormalization of the W coupling $\bar{g}_W^2(0)$ or, equivalently, into the renormalization of the Fermi constant G_F. In the same fashion all quadratic divergencies merely lead to a renormalization of the SM parameters α, m_Z, and G_F; no quadratic divergencies appear in the observable form factors $S_Z(q^2)$, $S_W(q^2)$, $\bar{s}^2(q^2)$, $\bar{\alpha}(q^2)$, and $\delta\rho$, once these are expressed in terms of the renormalized SM parameters. As a result any observable effects of the new interactions vanish for large Λ at least as fast as $\frac{1}{\Lambda^2}\ln\Lambda^2$ and, hence, the new physics decouples in the limit $\Lambda \to \infty$.

These results were to be expected: the one-loop corrections involving dimension-six operators lead to quadratic divergencies multiplying all allowed dimension-four operators, logarithmically divergent terms multiplying the allowed dimension-six operators,[2] and additional finite terms, which we have not calculated completely. Preserving $SU(2)\times U(1)$ gauge invariance at all stages of the calculation, only gauge invariant operators can appear multiplying the divergencies. Since the SM Lagrangian contains all gauge invariant dimension-four operators, the quadratic divergencies necessarily can be absorbed into a renormalization of the SM parameters. Similarly, since only the dimension-six operators $\mathcal{O}_{DW}$, $\mathcal{O}_{DB}$, $\mathcal{O}_{BW}$, and $\mathcal{O}_{\Phi,1}$ can contribute directly to the four-fermion amplitudes at tree level, all logarithmic divergencies are equivalent to a renormalization of the coefficients of these four operators.

In order to achieve the cancellation of quadratic divergencies in the oblique correction form factors, the contributions from Feynman graphs involving the Higgs boson are essential. In these Higgs boson graphs quadratic divergencies as well as m_H^2 terms only arise from the scalar integral[3]

$$\begin{aligned} B_{22}(q^2; m_H, m) &= -\frac{1}{32\pi^2}\Gamma(\epsilon-1)(4\pi\mu^2)^\epsilon \int_0^1 dx[x m_H^2 + (1-x)m^2 - q^2 x(1-x)]^{1-\epsilon} \\ &= \frac{1}{32\pi^2}\left[-\Lambda^2 + \frac{1}{2}\left(m_H^2 + m^2 - \frac{q^2}{3}\right)\left(\ln\frac{\Lambda^2}{M^2} + \frac{1}{2}\right) + \cdots\right] . \end{aligned} \tag{4.11}$$

The scale M^2 appearing in the argument of the logarithm is the larger of m^2 and m_H^2 and the ellipsis represents terms which remain small as m_H becomes large. As we have seen before, the leading quadratic divergencies disappear in all observable quantities when *including* the graphs with Higgs boson propagators. According to Eq. (4.11) dropping these graphs is analogous to the replacement $\frac{1}{2}m_H^2(\ln\frac{\Lambda^2}{m_H^2} + \frac{1}{2}) \to \Lambda^2$, as far as the quadratic divergencies are concerned. With this replace-

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[2] We thank M. Lüscher for making us aware of this point. For a related discussion, see Refs. [5, 31].

[3] Following the identification of the poles at $\epsilon = 1$ and $\epsilon = 0$ with the quadratic and logarithmic dependence on the cutoff Λ as in Eq. (4.1), we define the finite (Λ-independent) terms of a function $f(\epsilon)$ via

$$f_{\text{finite}} = \lim_{\epsilon\to 0}\left[f(\epsilon) - R(1)\left(\frac{1}{\epsilon-1}+1\right) - R(0)\left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi + 1\right)\right] ,$$

where $R(1)$ and $R(0)$ are the residues of the poles at $\epsilon = 1$ and $\epsilon = 0$, respectively.

ment in Eqs. (4.9c) and (4.9d) we qualitatively recover the quadratic divergencies in S and $\delta\rho = \alpha T$ which were obtained in earlier analyses that did not include Higgs exchange contributions [6, 7]. At the same time we find that these quadratic divergencies are indeed physical: they correspond to strong enhancements in the mixing of the dimension-six operators in the absence of a light Higgs boson. In Appendix B we also give the expressions for the renormalized parameters f^r_{DW}, f^r_{DB}, f^r_{BW}, and $f^r_{\Phi,1}$ when dropping the Higgs exchange graphs completely. These results are directly related to the ones one would obtain by using a nonlinear realization for the symmetry-breaking sector.

V. CONSTRAINTS FROM PRECISION EXPERIMENTS

The existing data on four-fermion amplitudes as summarized in Sec. III provide experimental constraints on the new physics contributions to the oblique correction form factors. Apart from the "finite terms" in Eqs. (4.8b) and (4.8c) which are proportional to $\ln\frac{m_H}{m_W}$ and which are small numerically, all the new physics contributions considered in this paper can be described in terms of the renormalized coefficients f^r_{DW}, f^r_{DB}, f^r_{BW}, and $f^r_{\Phi,1}$ of the four operators which already contribute at tree level. This implies that complete cancellations between the operators are possible in principle and no rigorous bounds can be derived from the low energy data alone. One immediate example is given by the operators $\mathcal{O}_{WWW}$, $\mathcal{O}_{WW}$, and $\mathcal{O}_{BB}$ which only contribute via f^r_{BW} and can therefore cancel each other.

Low energy constraints can only be derived if one assumes that cancellations between different new physics contributions do not occur at a serious level. The resulting bounds directly reflect the degree of cancellation which is still considered "natural." In the following we shall first consider the situation where one operator only has a nonzero coefficient, i.e., we do not allow for any cancellations between the various operators. Using the coefficients of the operators at the scale of new physics as the free parameters (i.e., the unrenormalized f_{DW}, f_W, etc.), this procedure provides bounds for all nine operators which contribute up to one loop. The tree level contributions due to the four operators $\mathcal{O}_{DW}$, $\mathcal{O}_{DB}$, $\mathcal{O}_{BW}$, and $\mathcal{O}_{\Phi,1}$ are then analyzed in the more general context, allowing for cancellations. This second analysis is essentially equivalent to a determination of model-independent constraints on the renormalized coefficients f^r_{DW}, f^r_{DB}, f^r_{BW}, and $f^r_{\Phi,1}$ from existing data.

We have performed a χ^2 analysis of the data, which is given by the experimental results on $\bar{g}^2_Z(m^2_Z)$, $\bar{s}^2(m^2_Z)$, $\bar{g}^2_Z(0)$, $\bar{s}^2(0)$, and $\bar{g}^2_W(0)$ as summarized in Eqs. (3.26)–(3.29). The SM and the new physics contributions are added linearly in $\delta\rho = \alpha T$, $S_Z(m^2_Z)$, $S_Z(0)$, $S_W(0)$, and $\bar{s}^2(0)$. The theoretical expectations for the five data points are obtained from these via Eq. (3.25) to determine $\bar{s}^2(m^2_Z)$ and by then using Eq. (3.13) to determine $\bar{g}^2_Z(m^2_Z)$, $\bar{g}^2_Z(0)$, and $\bar{g}^2_W(0)$.

In Table I the values and 1σ errors of the coefficients f_i are given for all nine dimension-six operators assuming that one f_i at a time differs from zero. The columns correspond to four different Higgs mass values. A top mass of 140 GeV and $\Lambda = 1$ TeV are assumed. The coefficient of $\mathcal{O}_{WWW}$ is scaled such that the value given in the table corresponds to the extracted value of the anomalous WWV coupling $\lambda = \frac{3m^2_W g^2}{2\Lambda^2} f_{WWW}$. The other coefficients of the one-loop operators are rescaled with a factor $\frac{m^2_W}{2\Lambda^2}$. The entries for f_B and f_W thus correspond to a determination of κ_γ from the low energy data. One finds that for those operators which contribute at the one-loop level only, values of $|f_i|\frac{m^2_W}{2\Lambda^2}$ larger than ≈ 0.2 require cancellation between the various new physics contributions. For the tree level contributions this level is reached already for values of $|f_i|$ which are smaller by two orders of magnitude.

The bounds given in Table I assume a fixed value for the top mass. Large cancellations are possible, however, between the quadratic m_t dependence of $\delta\rho = \alpha T$ and

TABLE I. Low-energy constraints on the coefficients of the nine operators which contribute to the oblique corrections up to one-loop order. Only one f_i at a time is assumed to be different from zero. 1σ errors are quoted, assuming $m_t = 140$ GeV, $\Lambda = 1$ TeV. The couplings which occur at one loop only are scaled such that the entries correspond to a determination of $\kappa_\gamma - 1$ for f_B and f_W.

	m_H (GeV)			
	60	200	400	800
f_{DW}	0.52±0.51	0.46±0.51	0.39±0.51	0.31±0.51
f_{DB}	−0.54±1.9	0.6±1.9	1.3±1.9	2.0±1.9
f_{BW}	−0.17±0.33	0.06±0.33	0.22±0.33	0.39±0.33
$f_{\Phi,1}$	0.05±0.05	0.01±0.05	−0.01±0.05	−0.04±0.05
$\lambda = f_{WWW}3g^2\frac{m^2_W}{2\Lambda^2}$	−0.08±0.16	0.03±0.16	0.10±0.16	0.19±0.16
$f_B\frac{m^2_W}{2\Lambda^2}$	0.06±0.10	0.042±0.061	0.008±0.031	−0.005±0.020
$f_W\frac{m^2_W}{2\Lambda^2}$	0.043±0.056	0.009±0.087	0.10±0.21	0.087±0.077
$f_{WW}\frac{m^2_W}{2\Lambda^2}$	0.037±0.071	−0.022±0.124	−0.14±0.22	−1.0±0.9
$f_{BB}\frac{m^2_W}{2\Lambda^2}$	0.12±0.24	−0.08±0.41	−0.47±0.72	−3.5±3.0

the operators $\mathcal{O}_B$ and $\mathcal{O}_W$ which mix into the operator $\mathcal{O}_{\Phi,1}$ at the weak scale [Eq. (4.9d)]. This behavior is demonstrated in Fig. 3 where 90% C.L. contours are shown in the $\Delta\kappa_\gamma$-m_t plane for nonzero values of f_W and f_B, respectively, setting all other $f_i = 0$. For small values of the Higgs mass one finds a strong positive correlation between f_W and m_t which is due to the contribution of f_W to $f^r_{\Phi,1}$. For large Higgs masses the contribution of f_W to f^r_{BW} is enhanced by a factor of m_H^2 and the constraint on the S parameter dominates the bound on f_W. As a result the positive correlation is lost. The reverse is true for $f_B \neq 0$: for large Higgs masses the contribution to $f^r_{\Phi,1}$ dominates and a strong correlation with the top-quark mass results. Figure 3 clearly demonstrates the dependence of the f_i bounds on assumptions about the top-quark mass and the Higgs boson mass. While the m_t dependence is due to cancellations between new physics and SM contributions, the m_H dependence can largely be traced to the presence of the Higgs field in the dimension-six operators, namely to Higgs exchange graphs contributing to the gauge boson two-point functions.

Experiments at the Fermilab Tevatron are sensitive to anomalous values of κ_γ and λ_γ via the $W\gamma$ production process. Hence correlations in the κ_γ-λ_γ plane are of particular relevance. In Fig. 4 we show the 90% C.L. contour lines in this plane for three representative top mass values and for $\Lambda = 1$ TeV, $m_H = 100$ GeV. In addition, the choice $f_B = f_W$ has been made. Anomalous $WW\gamma$ couplings of order 0.5 are clearly allowed by the present low energy data, which raises the possibility of observable effects at the Tevatron [32].

Apart from small contributions proportional to f_B and f_W, all other logarithmically enhanced one-loop contributions to the oblique parameters only arise via the running of the four operators which contribute at tree level. As a result complete cancellations are possible in principle. In addition one finds strong correlations between the tree level operators $\mathcal{O}_{DW}$, $\mathcal{O}_{DB}$, $\mathcal{O}_{BW}$, and $\mathcal{O}_{\Phi,1}$. For small enough values of the f_i the χ^2 function is just a quadratic polynomial in these four coefficients $f = (f_1, f_2, f_3, f_4) = (f_{DW}, f_{DB}, f_{BW}, f_{\Phi,1})$:

$$\chi^2 = \chi_0^2 + \sum_{i,j=1}^{4} \left(f_i - \bar{f}_i\right) V_{ij}^{-1} \left(f_j - \bar{f}_j\right) . \tag{5.1}$$

One finds that the minimum of χ^2 is slightly dependent on m_H and m_t (via the corresponding dependence of the SM oblique correction parameters) while the covariance matrix V shows a negligible m_H and m_t dependence. Instead of V^{-1} in Eq. (5.1), we here give the extracted central values $(\bar{f}_i)$ and 1σ errors $(\sqrt{V_{ii}})$ of the f_i and the correlation matrix. The central values depend on the top-quark and Higgs boson masses which we parametrize in terms of

$$x_t = \frac{m_t - 140 \text{ GeV}}{100 \text{ GeV}}, \tag{5.2a}$$

$$x_H = \frac{m_H}{200 \text{ GeV}} . \tag{5.2b}$$

Within better than 5% of the 1σ errors, and in the ranges 90 GeV $< m_t <$ 250 GeV and 60 GeV $< m_H <$ 800 GeV, this dependence is given by

$$f_{DW} = 0.56 - 0.32\, x_t \pm 0.79 , \tag{5.3a}$$

$$f_{DB} = -8.0 \pm 11.9 , \tag{5.3b}$$

$$f_{BW} = 1.9 + 0.132\, x_t^2 + 0.077 \ln x_H \pm 2.9 , \tag{5.3c}$$

$$f_{\Phi,1} = 0.105 + 0.100\, x_t^2 + 0.319\, x_t - 0.029 \ln x_H \pm 0.20 , \tag{5.3d}$$

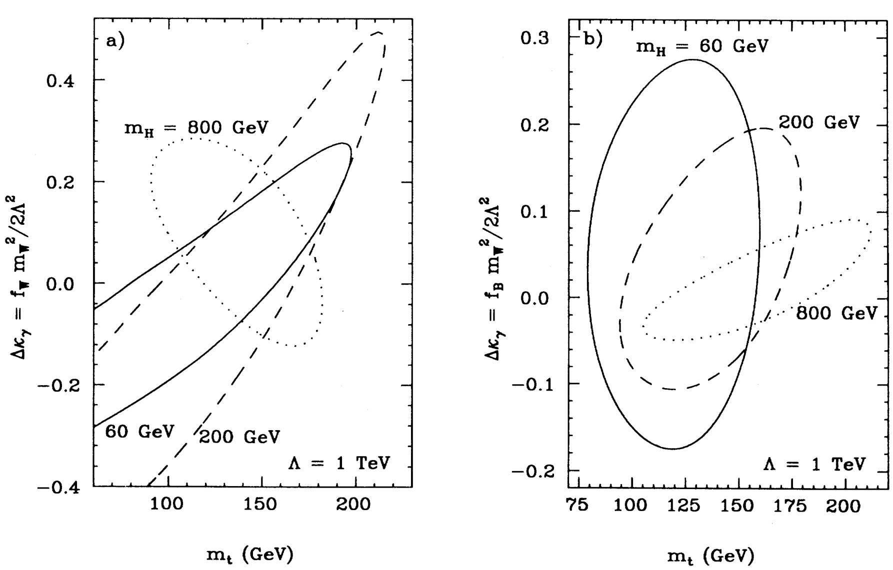

FIG. 3. Low energy constraints on $\Delta\kappa_\gamma$ and the top-quark mass for (a) $f_W \neq 0$ and (b) $f_B \neq 0$. The coefficients of all other dimension-six operators are set to $f_i = 0$. The curves are 90% C.L. contour lines assuming $\Lambda = 1$ TeV and three different values of the Higgs boson mass: $m_H = 60$ GeV, $m_H = 200$ GeV, and $m_H = 800$ GeV.

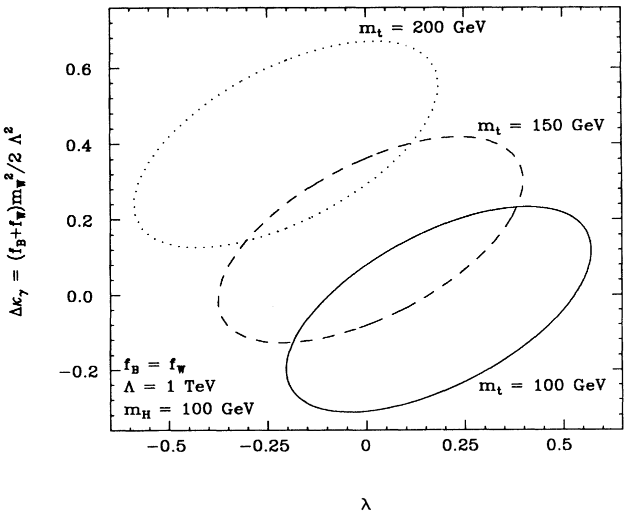

FIG. 4. Correlations in the κ_γ-λ_γ plane for three values of the top-quark mass. Shown are 90% C.L. contours for $f_B = f_W$ and $\Lambda = 1$ TeV, $m_H = 100$ GeV. The anomalous couplings are given by $\kappa_\gamma = 1 + (f_B + f_W)\frac{m_W^2}{2\Lambda^2}$ and $\lambda_\gamma = \lambda = \frac{3m_W^2 g^2}{2\Lambda^2} f_{WWW}$. All other coefficients of the dimension-six operators are set to $f_i = 0$.

assuming $\Lambda = 1$ TeV. The correlation matrix C is found to be

$$C = \frac{V_{ij}}{\sqrt{V_{ii}V_{jj}}} = \begin{pmatrix} 1 & -0.206 & 0.014 & -0.317 \\ & 1 & -0.963 & -0.763 \\ & & 1 & 0.892 \\ & & & 1 \end{pmatrix} . \tag{5.4}$$

Both the 1σ errors and the correlation matrix elements are independent of m_H and m_t to high precision.

Obviously, strong correlations exist in the allowed region. In particular, cancellations between f_{DB}, f_{BW}, and $f_{\Phi,1}$ are possible. This results in substantially increased errors for these three parameters as compared to the entries in Table I, where no correlations were considered. Neglecting the small "finite terms" in Eq. (4.8), the results of Eq. (5.3) can be taken as a measurement of the corresponding renormalized tree level parameters. These may then be used to analyze the bounds on the one-loop operators in the presence of correlations. As one example consider f_{BW}, whose error has increased by a factor 9. Since anomalous values of λ only enter via their contribution to f^r_{BW}, see Eq. (4.9c), the measurement of λ weakens to

$$\lambda = 0.89 \pm 1.35 \,, \tag{5.5}$$

when the correlations with f_{DW}, f_{DB}, and $f_{\Phi,1}$ are taken into account. This may be compared to the error $\Delta\lambda = \pm 0.16$ as given in Table I. If we allow a cancellation in f^r_{BW} between the tree-level term f_{BW} and the one-loop contribution proportional to λ, then no rigorous bound on either of these quantities is possible. Similar results hold for all the other coefficients of the dimension-six operators.

VI. DISCUSSION AND CONCLUSIONS

The appearance of anomalous triple gauge boson couplings constitutes only one possible consequence of new physics in the bosonic sector. As we have seen, a consistent discussion of low energy effects of such new physics, in particular at the loop level, must allow for deviations from the SM in the gauge boson propagators as well as in the interaction terms involving the gauge bosons and the Higgs boson. We have shown that the leading one-loop effect of anomalous triple gauge boson couplings is the renormalization of new physics contributions to the gauge boson propagators. Therefore low energy bounds on the three gauge boson couplings cannot be obtained without making assumptions on how the new physics will alter the propagators.

In more precise terms we have analyzed the effects on four-fermion amplitudes of a complete set of SU(2)×U(1) gauge invariant dimension-six operators which can be constructed from the electroweak gauge bosons and the Higgs doublet field. Of the five operators which first contribute to NC and CC amplitudes at the one-loop level, three induce anomalous WWV couplings at tree level. The divergent (cutoff dependent) contributions of these five operators to four-fermion amplitudes amount to a renormalization of the input parameters of the SM (α, G_F, and m_Z) and of the four operators ($\mathcal{O}_{DW}$, $\mathcal{O}_{DB}$, $\mathcal{O}_{BW}$, $\mathcal{O}_{\Phi,1}$) which affect the gauge boson propagators already at tree level. Our calculation explicitly demonstrates the well-known fact that a consistent calculation within a nonrenormalizable theory leads to finite and cutoff independent results, albeit at the price of having to introduce additional free parameters to describe the predictions of the theory [33, 31, 5]. Because we limited ourselves to a single insertion of dimension-six operators, only a finite set of such additional parameters was needed: f^r_{DW}, f^r_{DB}, f^r_{BW}, $f^r_{\Phi,1}$. Going beyond this approximation (as would be needed when extrapolating our results to higher energies where E^2/Λ^2 is no longer small) additional parameters will appear.

Previous analyses of these one-loop effects (including our own) have generally been incomplete as only small subsets of the possible higher dimensional (nonrenormalizable) interactions were considered [4, 6–8, 14, 17, 34]. This partial analysis of the problem led to logarithmically divergent results. In addition, quadratic divergencies were observed in some previous calculations, which can be identified [17] with a quadratic Higgs mass dependence for large values of m_H (see also Ref. [34]). We have discussed how the neglect of Higgs exchange diagrams, which naturally accompany anomalous WWV vertices in our gauge invariant formulation of the problem, leads to the appearance of these quadratic divergencies.

Low energy bounds on nonstandard WWV couplings which exploit the enhancement arising from a quadratic cutoff dependence (Λ^2/m_W^2 terms) can consequently only be valid for models without a light Higgs boson of mass $m_H < \Lambda$. In addition, the appearance of the quadratic cutoff dependences implies that the quantum corrections are sensitive to details of the physics at and above the cutoff scale, and hence no reliable constraints are obtained without specifying the model [16]. The logarithmic cutoff dependence of previous results [$\ln(\Lambda^2/m_W^2)$ terms] are now understood as operator mixing between the scale of new physics, Λ, and the weak boson mass scale. To very good approximation, only the coefficients of the four operators that modify the gauge boson propagators at tree level are measurable at low energy. Contributions of the other operators are observable through their mixing with these four operators. As a result it is practically impossible to derive rigorous low energy bounds on the coefficients of those operators which first contribute to four-fermion amplitudes at the one-loop level. The anomalous WWV couplings which arise from these operators are hence only very weakly constrained by the present precision experiments at and below the Z mass scale.

One may argue that it is "unnatural" that large cancellations appear between the tree level and the various one-loop contributions in the running coefficients f_{DW}^r, f_{DB}^r, f_{BW}^r, $f_{\Phi,1}^r$, as measured at the weak boson mass scale. Negating such cancellations we have derived 90% C.L. bounds, which roughly translate into

$$0.7 < \kappa_\gamma < 1.7, \qquad (6.1a)$$

$$|\lambda_\gamma| < 0.6\,, \qquad (6.1b)$$

for $m_H = 100$ GeV and 100 GeV$< m_t < 200$ GeV. A substantial dependence on the values of the top-quark mass and the Higgs boson mass remains. These bounds may be compared with present experimental bounds as determined by the UA2 Collaboration [35],

$$\kappa_\gamma = 1^{+2.6}_{-2.2}\,, \quad -3.5 < \kappa_\gamma < 5.9 \text{ at } 95\% \text{ C.L.}\,, \quad (6.2a)$$

$$\lambda_\gamma = 0^{+1.7}_{-1.8}\,, \quad -3.6 < \lambda_\gamma < 3.5 \text{ at } 95\% \text{ C.L.}\,, \quad (6.2b)$$

and are comparable to the sensitivity expected in the Tevatron experiments [32].

A stronger "naturalness" assumption has recently been advocated by De Rújula *et al.* [14]. The four operators which contribute to the four-fermion amplitudes at tree level are very strongly constrained, even when considering full correlations between them. Since there is no apparent symmetry distinguishing between these four operators and the other five, which we found to contribute only at the one-loop level, one should expect that all coefficients f_i are of the same order of magnitude. Given the constraints $|f_{DW}|$, $|f_{BW}| < 1$, as derived from Table I for a new physics scale $\Lambda = 1$ TeV (and hence, by the strong naturalness assumption, $|f_W| < 1$, $|f_B| < 1$), one must expect, e.g.,

$$|\kappa_\gamma - 1| = |f_B + f_W|\frac{m_W^2}{2\Lambda^2} < 0.007\,, \qquad (6.3)$$

a value too small to lead to any observable effects, either in W^+W^- production at LEP [1] or in $W\gamma$ production at future hadron colliders [36].

While this estimate may well be correct, it is in no ways rigorous. Even the less stringent low energy bounds of Eq. (6.1) or of Table I are not free of naturalness assumptions. Naturalness arguments can never substitute for the direct measurements, which are possible in vector boson pair production experiments. Clearly these measurements must be carried out in order to exclude anomalous triple gauge boson interactions. The naturalness arguments highlight the fact, however, that a positive signal in these experiments would point to an intricate dynamics in the bosonic sector.

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APPENDIX A: ELECTROWEAK LAGRANGIAN WITH DIMENSION-SIX OPERATORS

In this appendix, we present details of the standard model Lagrangian augmented by the eleven gauge invariant dimension-six operators which involve the gauge bosons and the Higgs doublet field. The electroweak part of the complete Lagrangian is expressed in the general covariant renormalizable gauge as

$$\mathcal{L}_{EW} = \mathcal{L}_V + \mathcal{L}_\Phi + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm FP} + \mathcal{L}_F + \Sigma_i \frac{f_i}{\Lambda^2}\mathcal{O}_i, \qquad (A1)$$

where $\mathcal{L}_V$ denotes the SU(2) and U(1) gauge boson kinetic and self-interaction terms, $\mathcal{L}_\Phi$ denotes the Higgs boson kinetic term and the Higgs potential, $\mathcal{L}_{\rm GF}$ is the covariant gauge fixing term and $\mathcal{L}_{\rm FP}$ the associated Faddeev-Popov term. $\mathcal{L}_F$ contains the fermion kinetic terms and their couplings to the gauge bosons and the Higgs bosons, which do not play an essential role in our loop calculation and hence are omitted below for brevity. The additional dimension-six terms $\mathcal{O}_i$ are scaled by the common dimensionful parameter Λ with dimensionless coefficients f_i, which may be different for each operator.

We first list the eleven gauge invariant dimension-six operators [19] which contain the gauge boson fields and the Higgs doublet field. Three of them are independent of the Higgs doublet field:

$$\mathcal{O}_{WWW} = {\rm Tr}[\hat{W}_{\mu\nu}\hat{W}^{\nu\rho}\hat{W}_\rho{}^\mu], \qquad (A2a)$$

$$\mathcal{O}_{DW} = {\rm Tr}([D_\mu, \hat{W}_{\nu\rho}]\,[D^\mu, \hat{W}^{\nu\rho}]), \qquad (A2b)$$

$$\mathcal{O}_{DB} = -\frac{g'^2}{2}(\partial_\mu B_{\nu\rho})(\partial^\mu B^{\nu\rho}). \qquad (A2c)$$

Five of them are bilinear in the Higgs field Φ:

$$\mathcal{O}_{WW} = \Phi^\dagger \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\Phi, \tag{A3a}$$
$$\mathcal{O}_{BB} = \Phi^\dagger \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\Phi, \tag{A3b}$$
$$\mathcal{O}_{BW} = \Phi^\dagger \hat{B}_{\mu\nu}\hat{W}^{\mu\nu}\Phi, \tag{A3c}$$
$$\mathcal{O}_{W} = (D_\mu\Phi)^\dagger \hat{W}^{\mu\nu}(D_\nu\Phi), \tag{A3d}$$
$$\mathcal{O}_{B} = (D_\mu\Phi)^\dagger \hat{B}^{\mu\nu}(D_\nu\Phi). \tag{A3e}$$

Finally, two operators contain four Higgs fields and the last one has six:[4]

$$\mathcal{O}_{\Phi,1} = (D_\mu\Phi)^\dagger\Phi\Phi^\dagger(D^\mu\Phi), \tag{A4a}$$
$$\mathcal{O}_{\Phi,2} = \frac{1}{2}\partial_\mu(\Phi^\dagger\Phi)\partial^\mu(\Phi^\dagger\Phi), \tag{A4b}$$
$$\mathcal{O}_{\Phi,3} = \frac{1}{3}(\Phi^\dagger\Phi)^3. \tag{A4c}$$

Among the above eleven operators, four ($\mathcal{O}_{DW}$, $\mathcal{O}_{DB}$, $\mathcal{O}_{BW}$, $\mathcal{O}_{\Phi,1}$) contribute at the tree level to the electroweak precision experiments at low energies ($\sqrt{s} \lesssim m_Z$) [21], and five ($\mathcal{O}_{WWW}$, $\mathcal{O}_{DW}$, $\mathcal{O}_{BW}$, $\mathcal{O}_{W}$, $\mathcal{O}_{B}$) give rise to nonstandard weak boson self-interactions. All of them contribute to the low energy processes at the one-loop level.

The covariant derivative of the standard $SU(2)\times U(1)$ electroweak theory is expressed as

$$\begin{aligned} D_\mu &= \partial_\mu + igT^aW^a_\mu + ig'YB_\mu \\ &= \partial_\mu + i\frac{g}{\sqrt{2}}(W^+_\mu T^+ + W^-_\mu T^-) \\ &\quad + ig_Z(T^3 - s^2Q)Z_\mu + ieQA_\mu \end{aligned} \tag{A5}$$

where g and g' are the SU(2) and U(1) gauge coupling, respectively, $c = \cos\theta_W$ and $s = \sin\theta_W$ are the weak mixing factors

$$\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}. \tag{A6}$$

$Q = T^3 + Y$ is the electric charge, $g_Z = g/\cos\theta_W$ and $e = g\sin\theta_W = g'\cos\theta_W$ are, respectively, the Z and the photon (A) couplings. The SU(2) generators are normalized as $\mathrm{Tr}[T^aT^b] = \frac{1}{2}\delta^{ab}$, $T^\pm = T^1 \pm iT^2$, and the charged weak boson fields are defined as

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu), \tag{A7}$$

accordingly. The operators with carets are obtained as

$$[D_\mu, D_\nu] = \hat{W}_{\mu\nu} + \hat{B}_{\mu\nu}, \tag{A8}$$

with

$$\begin{aligned} \hat{W}_{\mu\nu} &= igT^aW^a_{\mu\nu}, \\ \hat{B}_{\mu\nu} &= ig'YB_{\mu\nu}. \end{aligned} \tag{A9}$$

The use of the above operators with carets automatically takes account of the associated gauge coupling factor and the Hermiticity of each operator. In terms of the standard tensors, the first eight operators (A2) and (A3) are expressed as

$$\mathcal{O}_{WWW} = -i\frac{3}{2}g^3W^+_{\mu\nu}W^{-\nu\rho}W^3{}_\rho{}^\mu, \tag{A10a}$$
$$\mathcal{O}_{DW} = -g^2\left[(D_\mu W_{\nu\rho})^+(D^\mu W^{\nu\rho})^- + \frac{1}{2}(D_\mu W_{\nu\rho})^3(D^\mu W^{\nu\rho})^3\right], \tag{A10b}$$
$$\mathcal{O}_{DB} = -\frac{g'^2}{2}(\partial_\mu B_{\nu\rho})(\partial^\mu B^{\nu\rho}), \tag{A10c}$$
$$\mathcal{O}_{WW} = -\frac{g^2}{2}(\Phi^\dagger\Phi)\left[W^+_{\mu\nu}W^{-\mu\nu} + \frac{1}{2}W^3_{\mu\nu}W^{3\mu\nu}\right], \tag{A10d}$$
$$\mathcal{O}_{BB} = -\frac{g'^2}{4}(\Phi^\dagger\Phi)B_{\mu\nu}B^{\mu\nu}, \tag{A10e}$$
$$\mathcal{O}_{BW} = -\frac{gg'}{4}[\sqrt{2}(\Phi^\dagger T^+\Phi)W^+_{\mu\nu} + \sqrt{2}(\Phi^\dagger T^-\Phi)W^-_{\mu\nu} + (\Phi^\dagger\sigma^3\Phi)W^3_{\mu\nu}]B^{\mu\nu}, \tag{A10f}$$
$$\mathcal{O}_{W} = \frac{ig}{2}[\sqrt{2}(D^\mu\Phi)^\dagger T^+(D^\nu\Phi)W^+_{\mu\nu} + \sqrt{2}(D^\mu\Phi)^\dagger T^-(D^\nu\Phi)W^-_{\mu\nu} + (D^\mu\Phi)^\dagger\sigma^3(D^\nu\Phi)W^3_{\mu\nu}], \tag{A10g}$$
$$\mathcal{O}_{B} = \frac{ig'}{2}(D^\mu\Phi)^\dagger(D^\nu\Phi)B_{\mu\nu}. \tag{A10h}$$

Here the field operators without carets are

[4]A fourth operator, $\mathcal{O}_{\Phi,4} = \Phi^\dagger\Phi(D_\mu\Phi)^\dagger(D^\mu\Phi)$, which was listed in Ref. [19], is equivalent to $-\mathcal{O}_{\Phi,2}$ and contributions to the Higgs potential $V(\Phi^\dagger\Phi)$ as can be shown by a partial integration of $\partial_\mu(\Phi^\dagger\Phi)\partial^\mu(\Phi^\dagger\Phi)$ and the use of the Higgs equation of motion. Here the Higgs potential $V(\Phi^\dagger\Phi)$ also contains cubic terms which are proportional to the operator $\mathcal{O}_{\Phi,3}$.

$$W^{\pm}_{\mu\nu} = \partial_\mu W^{\pm}_\nu - \partial_\nu W^{\pm}_\mu \pm ig(W^3_\mu W^{\pm}_\nu - W^3_\nu W^{\pm}_\mu),$$
$$W^3_{\mu\nu} = \partial_\mu W^3_\nu - \partial_\nu W^3_\mu + ig(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu), \tag{A11a}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \tag{A11b}$$

and

$$(D_\mu W_{\nu\rho})^{\pm} = \partial_\mu W^{\pm}_{\nu\rho} \pm ig(W^3_\mu W^{\pm}_{\nu\rho} - W^{\pm}_\mu W^3_{\nu\rho}),$$
$$(D_\mu W_{\nu\rho})^3 = \partial_\mu W^3_{\nu\rho} + ig(W^+_\mu W^-_{\nu\rho} - W^-_\mu W^+_{\nu\rho}). \tag{A11c}$$

The standard Higgs field Φ is a doublet with hypercharge $Y = \frac{1}{2}$, which has the form

$$\Phi = \left(1 + \frac{H + i\sigma^a\chi^a}{v}\right)\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} i\chi^1 + \chi^2 \\ v + H - i\chi^3 \end{pmatrix} = \begin{pmatrix} i\chi^+ \\ \frac{v+H-i\chi^3}{\sqrt{2}} \end{pmatrix}, \tag{A12}$$

in the renormalizable gauge. Here σ^a are the Pauli matrices ($T^a = \sigma^a/2$ for doublets) and χ^a are the Goldstone bosons. The charged Goldstone boson fields are defined as $\chi^{\pm} = \frac{1}{\sqrt{2}}(\chi^1 \mp i\chi^2)$. We also note

$$D_\mu\Phi = \begin{pmatrix} [i\partial_\mu - (\frac{1}{2} - s^2)g_Z Z_\mu - eA_\mu]\chi^+ + i\frac{g}{2}W^+_\mu(v + H - i\chi^3) \\ \frac{1}{\sqrt{2}}(\partial_\mu - i\frac{g_Z}{2}Z_\mu)(v + H - i\chi^3) - \frac{g}{\sqrt{2}}W^-_\mu\chi^+ \end{pmatrix}. \tag{A13}$$

It is useful to express all the operators in terms of the component fields χ^a and the physical Higgs boson H. The following expressions are sufficient to obtain all the Feynman rules. The scalar terms

$$\Phi^\dagger\Phi = \frac{v^2}{2} + vH + \frac{H^2 + (\chi^3)^2}{2} + \chi^+\chi^-, \tag{A14a}$$
$$\Phi^\dagger\sigma^3\Phi = -\frac{v^2}{2} - vH - \frac{H^2 + (\chi^3)^2}{2} + \chi^+\chi^-, \tag{A14b}$$
$$\sqrt{2}\Phi^\dagger T^{\pm}\Phi = \mp i(v + H)\chi^{\mp} - \chi^3\chi^{\mp}, \tag{A14c}$$

appear in the operators $\mathcal{O}_{WW}$, $\mathcal{O}_{BB}$, $\mathcal{O}_{BW}$, and $\mathcal{O}_{\Phi,3}$. Four-vectors

$$\begin{aligned}\Phi^\dagger D_\mu\Phi = {}& \frac{v}{2}(\partial_\mu H - i\partial_\mu\chi^3) + \frac{1}{2}[H\partial_\mu H + \chi^3\partial_\mu\chi^3 - iH\overleftrightarrow{\partial}_\mu\chi^3] + \chi^-\partial_\mu\chi^+ \\ & + i\frac{g_Z}{2}Z_\mu\left[-\frac{v^2}{2} - vH - \frac{H^2 + (\chi^3)^2}{2} + \chi^+\chi^-\right] + i[-s^2 g_Z Z_\mu + eA_\mu]\chi^+\chi^- \\ & + \frac{g}{2}W^+_\mu\chi^-[v + H - i\chi^3] - \frac{g}{2}W^-_\mu\chi^+[v + H + i\chi^3],\end{aligned} \tag{A15a}$$
$$\partial_\mu(\Phi^\dagger\Phi) = v(\partial_\mu H) + H\partial_\mu H + \chi^3\partial_\mu\chi^3 + (\partial_\mu\chi^+)\chi^- + \chi^+(\partial_\mu\chi^-), \tag{A15b}$$

appear in $\mathcal{O}_{\Phi,1}$ and $\mathcal{O}_{\Phi,2}$. Here $A\overleftrightarrow{\partial}_\mu B = A(\partial_\mu B) - (\partial_\mu A)B$. The Feynman rules are then obtained by inserting (A11)–(A15) into (A10) and (A4).

Although the complete expressions for the operators $\mathcal{O}_W$ and $\mathcal{O}_B$ are rather lengthy, those terms which contribute to the vector boson propagators in the one-loop order are relatively simple. We find

$$\mathcal{O}_W = \frac{g}{2}[W^{+\mu\nu}C_{\mu\nu} + W^{-\mu\nu}C^*_{\mu\nu} + W^{3\mu\nu}(D_{\mu\nu} - E_{\mu\nu})], \tag{A16a}$$
$$\mathcal{O}_B = \frac{g'}{2}B^{\mu\nu}(D_{\mu\nu} + E_{\mu\nu}), \tag{A16b}$$

where

$$C_{\mu\nu} = -i(\hat{m}_W W^-_\mu + \partial_\mu\chi^-)(\hat{m}_Z Z_\nu + \partial_\nu\chi^3 + i\partial_\nu H) + \frac{g}{2}W^-_\mu(H\partial_\nu H + \chi^3\partial_\nu\chi^3 + 2\chi^+\partial_\nu\chi^-) + \cdots, \tag{A17a}$$
$$\begin{aligned}D_{\mu\nu} = {}& i(\hat{m}_W W^-_\mu + \partial_\mu\chi^-)(\hat{m}_W W^+_\nu + \partial_\nu\chi^+) \\ & + \left[\left(\frac{1}{2} - s^2\right)g_Z Z_\mu + eA_\mu\right][\hat{m}_W(\chi^+W^-_\nu + \chi^-W^+_\nu) + \chi^+\partial_\nu\chi^- + \chi^-\partial_\nu\chi^+] + \cdots,\end{aligned} \tag{A17b}$$

$$E_{\mu\nu} = (\partial_\mu H)(\hat{m}_Z Z_\nu + \partial_\nu \chi^3) - \frac{g_Z}{2} Z_\mu (H\partial_\nu H + \chi^3 \partial_\nu \chi^3) + \cdots, \tag{A17c}$$

with

$$\begin{aligned} \hat{m}_W &= \tfrac{1}{2} g v, \\ \hat{m}_Z &= \tfrac{1}{2} g_Z v. \end{aligned} \tag{A18}$$

The operators $\mathcal{O}_{WW}$ and $\mathcal{O}_{BB}$ are found simply by inserting (A14a) into (A10d) and (A10e), respectively. The relevant part of $\mathcal{O}_{BW}$ is found to be

$$\mathcal{O}_{BW} = \frac{gg'}{4} B^{\mu\nu} [iv(W^+_{\mu\nu}\chi^- - W^-_{\mu\nu}\chi^+) + W^3_{\mu\nu}(\Phi^\dagger \sigma^3 \Phi)]. \tag{A19}$$

Finally, we present the standard model Lagrangian in our notation since the signs of all the terms are relevant for cancellation of divergencies and for gauge independence. The sum of the gauge boson term, the Higgs term, and the gauge fixing term is

$$\begin{aligned}
\mathcal{L}_V + \mathcal{L}_\Phi + \mathcal{L}_{\rm GF} = {} & W^+_\alpha \left[(\partial^2 + \hat{m}_W^2) g^{\alpha\beta} + \left(\frac{1}{\xi_W} - 1\right)\partial^\alpha \partial^\beta\right] W^-_\beta + \frac{1}{2} Z_\alpha \left[(\partial^2 + \hat{m}_Z^2) g^{\alpha\beta} + \left(\frac{1}{\xi_Z} - 1\right)\partial^\alpha \partial^\beta\right] Z_\beta \\
& + \frac{1}{2} A_\alpha \left[\partial^2 g^{\alpha\beta} + \left(\frac{1}{\xi_A} - 1\right)\partial^\alpha \partial^\beta\right] A_\beta - \chi^+(\partial^2 + \xi_W \hat{m}_W^2)\chi^- - \frac{1}{2}\chi^3(\partial^2 + \xi_Z \hat{m}_Z^2)\chi^3 \\
& - \frac{1}{2} H(\partial^2 + \hat{m}_H^2) H + \frac{\lambda}{4} v^4 - (\lambda v^2 + \mu^2)\left[\chi^+\chi^- + \frac{(\chi^3)^2}{2} + \frac{(v+H)^2}{2}\right] \\
& - 6\lambda \left[v \frac{H^3}{3!} + \frac{H^4}{4!} + \frac{(\chi^3)^4}{4!}\right] - 2\lambda v H \left[\chi^+\chi^- + \frac{(\chi^3)^2}{2}\right] \\
& - 2\lambda \left[\left(\frac{H^2}{2} + \frac{(\chi^3)^2}{2}\right)\chi^+\chi^- + \frac{H^2}{2}\frac{(\chi^3)^2}{2}\right] - 4\lambda \frac{(\chi^+\chi^-)^2}{4} \\
& + \frac{g}{2} W^{-\mu} [H \overleftrightarrow{\partial}_\mu \chi^+ + i\chi^3 \overleftrightarrow{\partial}_\mu \chi^+] + \frac{g}{2} W^{+\mu} [H \overleftrightarrow{\partial}_\mu \chi^- - i\chi^3 \overleftrightarrow{\partial}_\mu \chi^-] \\
& + \frac{g_Z}{2} Z^\mu H \overleftrightarrow{\partial}_\mu \chi^3 + i\left[\left(\frac{1}{2} - s^2\right) g_Z Z^\mu + e A^\mu\right] \chi^+ \overleftrightarrow{\partial}_\mu \chi^- \\
& + \left[\frac{s^2 g g_Z}{2} Z^\mu - \frac{eg}{2} A^\mu\right] [(W^+_\mu \chi^- + W^-_\mu \chi^+)\chi^3 + i(W^+_\mu \chi^- - W^-_\mu \chi^+)(v+H)] \\
& + \left[\frac{g^2}{2} W^+_\mu W^{-\mu} + 2\left(\frac{1}{2} - s^2\right)^2 g_Z^2 \frac{Z_\mu Z^\mu}{2} + (1 - 2s^2) g_Z e Z_\mu A^\mu + 2e^2 \frac{A_\mu A^\mu}{2}\right] \chi^+\chi^- \\
& + \left[\frac{g^2}{2} W^+_\mu W^{-\mu} + \frac{g_Z^2}{2} \frac{Z_\mu Z^\mu}{2}\right] \left[vH + \frac{H^2}{2} + \frac{(\chi^3)^2}{2}\right] \\
& - ig[g^{\alpha\beta}(W^+_\alpha \overleftrightarrow{\partial}^\gamma W^-_\beta) W^3_\gamma + g^{\beta\gamma}(W^-_\beta \overleftrightarrow{\partial}^\alpha W^3_\gamma) W^+_\alpha + g^{\gamma\alpha}(W^3_\gamma \overleftrightarrow{\partial}^\beta W^+_\alpha) W^-_\beta] \\
& + g^2(g^{\alpha\beta} g^{\gamma\delta} + g^{\alpha\delta} g^{\beta\gamma} - 2 g^{\alpha\gamma} g^{\beta\delta}) \left[\frac{1}{2} W^+_\alpha W^3_\beta W^-_\gamma W^3_\delta - \frac{1}{4} W^+_\alpha W^-_\beta W^+_\gamma W^-_\delta\right].
\end{aligned} \tag{A20}$$

Note the tree level constraints

$$\lambda v^2 + \mu^2 = 0, \tag{A21}$$

and

$$\hat{m}_H^2 = 2\lambda v^2. \tag{A22}$$

It is easy to read off the Feynman rules directly from the above expression. The Lagrangian including all eleven dimension-six operators is invariant under the Becchi-Rovet-Stora (BRS) transformation

$$\begin{aligned}
\delta W_\mu^\pm &= (\partial_\mu \pm igW_\mu^3)c^\pm \mp igW_\mu^\pm c^3, \\
\delta Z_\mu &= \partial_\mu c_Z + ig\cos\theta_W (W_\mu^+ c^- - W_\mu^- c^+), \\
\delta A_\mu &= \partial_\mu c_A + ie(W_\mu^+ c^- - W_\mu^- c^+), \\
\delta H &= \frac{g}{2}(\chi^+ c^- + \chi^- c^+) + \frac{g_Z}{2}\chi^3 c_Z, \\
\delta\chi^3 &= i\frac{g}{2}(\chi^+ c^- - \chi^- c^+) - \frac{g_Z}{2}(v+H)c_Z, \\
\delta\chi^\pm &= -g\frac{v+H\mp i\chi^3}{2}c^\pm \mp i\chi^\pm\left[\left(\frac{1}{2}-s^2\right)g_Z c_Z + ec_A\right].
\end{aligned} \tag{A23}$$

The associated Faddeev-Popov term is

$$\begin{aligned}
\mathcal{L}_{\rm FP} = &-\bar{c}^-(\partial^2+\xi_W\hat{m}_W^2)c^+ - \bar{c}^+(\partial^2+\xi_W\hat{m}_W^2)c^- - \bar{c}_Z(\partial^2+\xi_Z\hat{m}_Z^2)c_Z - \bar{c}_A\partial^2 c_A \\
&+ig(\partial^\mu\bar{c}^-)[W_\mu^3 c^+ - W_\mu^+ c^3] + ig(\partial^\mu\bar{c}^+)[W_\mu^- c^3 - W_\mu^3 c^-] + ig(\partial^\mu\bar{c}^3)[W_\mu^+ c^- - W_\mu^- c^+] \\
&-\frac{g}{2}\xi_W\hat{m}_W[\bar{c}^-(H-i\chi^3)c^+ + \bar{c}^+(H+i\chi^3)c^-] + i\left(\frac{1}{2}-s^2\right)g_Z\xi_W\hat{m}_W[\bar{c}^+\chi^- c_Z - \bar{c}^-\chi^+ c_Z] \\
&+ie\xi_W\hat{m}_W[\bar{c}^+\chi^- c_A - \bar{c}^-\chi^+ c_A] - \frac{g_Z}{2}\xi_Z\hat{m}_Z\bar{c}_Z H c_Z + i\frac{g}{2}\xi_Z\hat{m}_Z[\bar{c}_Z\chi^+ c^- - \bar{c}_Z\chi^- c^+],
\end{aligned} \tag{A24}$$

with

$$\begin{aligned}
c^3 &= \cos\theta_W c_Z + \sin\theta_W c_A, \\
\bar{c}^3 &= \cos\theta_W \bar{c}_Z + \sin\theta_W \bar{c}_A.
\end{aligned} \tag{A25}$$

APPENDIX B: SELF-ENERGIES, VERTEX FUNCTIONS, AND OBLIQUE PARAMETERS

In this appendix we give full details on our analytic results. We first list the one-loop contributions to the vertex functions in a general R_ξ gauge. As described in Sec. IV the gauge dependent terms cancel in the combinations $\Delta\overline{\Pi}_T(q^2)$ of vertex and two-point functions which appear in the four-fermion amplitudes [see Eqs. (4.6) and (4.7)]. Full expressions for the $\Delta\overline{\Pi}_T(q^2)$ will be given below, including all divergent terms. When using a nonlinear realization of the Goldstone bosons, the Higgs exchange graphs would not appear in our calculation. We therefore also give in this appendix the expressions for the oblique correction parameters when keeping anomalous gauge boson interactions only [25].

The divergent contributions to the gauge boson fermion vertices only depend on the operators $\mathcal{O}_W$ and $\mathcal{O}_{WWW}$ which induce anomalous triple gauge boson couplings. The W-fermion vertex is flavor independent and given by the form factor

$$\begin{aligned}
\Delta\Gamma_L^{Wf_1f_2}(q^2) &= \frac{g}{\sqrt{2}}\Delta\Gamma_L^W(q^2) \\
&= \frac{g}{\sqrt{2}}\,\frac{3\alpha}{8\pi s^2}\,\left(\frac{q^2}{\Lambda^2}g^2 f_{WWW} + \frac{m_W^2}{\Lambda^2}f_W\,\frac{1}{4}(\xi_W+\xi_Z+2)\right)\ln\frac{\Lambda^2}{\mu^2}\,.
\end{aligned} \tag{B1}$$

Similarly the corrections to the Zff and γff vertex functions only depend on the third component of the fermion's isospin, T_3^f, and are given by

$$\Delta\Gamma_L^{Vff}(q^2) = g\,T_3^f\,\Delta\Gamma_L^V(q^2)\,, \tag{B2}$$

with

$$\Delta\Gamma_L^\gamma(q^2) = s\,\frac{3\alpha}{8\pi s^2}\,\frac{q^2}{\Lambda^2}\,g^2 f_{WWW}\,\ln\frac{\Lambda^2}{\mu^2}\,, \tag{B3}$$

and

$$\Delta\Gamma_L^Z(q^2) = c\,\frac{3\alpha}{8\pi s^2}\,\left(\frac{q^2}{\Lambda^2}\,g^2 f_{WWW} + \frac{m_Z^2}{\Lambda^2}\,f_W\,\frac{1}{2}(\xi_W+1)\right)\ln\frac{\Lambda^2}{\mu^2}\,. \tag{B4}$$

The gauge dependent terms in the vertex functions are exactly canceled by corresponding terms in the two-point functions: only the gauge invariant combinations

$$\Delta\overline{\Pi}_T^{WW}(q^2) = \Delta\Pi_T^{WW}(q^2) - 2(q^2 - m_W^2)\Delta\Gamma_L^W(q^2)\ , \tag{B5a}$$

$$\Delta\overline{\Pi}_T^{\gamma\gamma}(q^2) = \Delta\Pi_T^{\gamma\gamma}(q^2) - 2s\, q^2\, \Delta\Gamma_L^{\gamma}(q^2)\ , \tag{B5b}$$

$$\Delta\overline{\Pi}_T^{\gamma Z}(q^2) = \Delta\Pi_T^{\gamma Z}(q^2) - s\, q^2\, \Delta\Gamma_L^{Z}(q^2) - c\,(q^2 - m_Z^2)\, \Delta\Gamma_L^{\gamma}(q^2)\ , \tag{B5c}$$

$$\Delta\overline{\Pi}_T^{ZZ}(q^2) = \Delta\Pi_T^{ZZ}(q^2) - 2c\,(q^2 - m_Z^2)\, \Delta\Gamma_L^{Z}(q^2)\ , \tag{B5d}$$

enter in the four-fermion amplitudes. Instead of these we give below the linear combinations $\Delta\overline{\Pi}_T^{QQ}$, etc., as defined in Eq. (3.4):

$$\begin{aligned}
\Delta\overline{\Pi}_T^{QQ}(q^2) = {}& 2\frac{q^2}{\Lambda^2}\left[(f_{DW}+f_{DB})q^2 - f_{BW}\frac{m_W^2}{g^2}\right] - \frac{q^2}{16\pi^2\Lambda^2}18g^2 f_{WWW} m_W^2 \ln\frac{\Lambda^2}{\mu^2} \\
& -\frac{q^2}{16\pi^2\Lambda^2}(f_{WW}+f_{BB})\left[\left(1+\frac{m_Z^2+2m_W^2}{m_H^2}\right)\Lambda^2 - m_H^2\left(1+3\frac{m_Z^4+2m_W^4}{m_H^4}\right)\ln\frac{\Lambda^2}{\mu^2}\right] \\
& -\frac{q^2}{16\pi^2\Lambda^2}(f_W+f_B)\left[\Lambda^2 + \left(3m_W^2+\frac{q^2}{6}\right)\ln\frac{\Lambda^2}{\mu^2}\right]\ ,
\end{aligned} \tag{B6a}$$

$$\begin{aligned}
\Delta\overline{\Pi}_T^{3Q}(q^2) = {}& \frac{q^2}{\Lambda^2}\left(2f_{DW}q^2 - f_{BW}\frac{m_W^2}{g^2}\right) - \frac{q^2}{16\pi^2\Lambda^2}\frac{33}{2}g^2 f_{WWW} m_W^2 \ln\frac{\Lambda^2}{\mu^2} \\
& -\frac{q^2}{16\pi^2\Lambda^2}f_{WW}\left[\left(1+\frac{m_Z^2+2m_W^2}{m_H^2}\right)\Lambda^2 - m_H^2\left(1+3\frac{m_Z^4+2m_W^4}{m_H^4}\right)\ln\frac{\Lambda^2}{\mu^2}\right] \\
& +\frac{q^2}{16\pi^2\Lambda^2}(s^2 f_{BB} - c^2 f_{WW})m_Z^2 \ln\frac{\Lambda^2}{\mu^2} - \frac{q^2}{16\pi^2\Lambda^2}f_W\left[\Lambda^2 + \left(6m_W^2+\frac{q^2}{6}\right)\ln\frac{\Lambda^2}{\mu^2}\right] \\
& +\frac{q^2}{16\pi^2\Lambda^2}\frac{f_W - f_B}{8}(m_H^2 - m_Z^2 + 10m_W^2)\ln\frac{\Lambda^2}{\mu^2}\ ,
\end{aligned} \tag{B6b}$$

$$\begin{aligned}
\Delta\overline{\Pi}_T^{33}(q^2) = {}& +2\frac{q^2}{\Lambda^2}f_{DW}q^2 - f_{\Phi,1}\frac{m_W^2}{\Lambda^2}\frac{v^2}{2g^2} - \frac{q^2}{16\pi^2\Lambda^2}15g^2 f_{WWW} m_W^2 \ln\frac{\Lambda^2}{\mu^2} \\
& -\frac{q^2}{16\pi^2\Lambda^2}f_{WW}\left[\left(1+\frac{m_Z^2+2m_W^2}{m_H^2}\right)\Lambda^2 - m_H^2\left(1-2\frac{m_W^2}{m_H^2}+3\frac{m_Z^4+2m_W^4}{m_H^4}\right)\ln\frac{\Lambda^2}{\mu^2}\right] \\
& -\frac{m_W^2}{16\pi^2\Lambda^2}3f_{WW}\frac{m_W^2}{m_H^2}\left[\Lambda^2 - (m_Z^2+2m_W^2)\ln\frac{\Lambda^2}{\mu^2}\right] - \frac{m_Z^2}{16\pi^2\Lambda^2}s^4 f_{BB}\frac{m_Z^2}{m_H^2}\left(\Lambda^2 - 3m_Z^2\ln\frac{\Lambda^2}{\mu^2}\right) \\
& +\frac{m_W^2}{16\pi^2\Lambda^2}\frac{3}{2}f_W\left[\Lambda^2 - \frac{1}{2}(m_H^2+m_Z^2)\ln\frac{\Lambda^2}{\mu^2}\right] - \frac{q^2}{16\pi^2\Lambda^2}f_W\left[\Lambda^2 - \frac{1}{4}\left(m_H^2 - m_Z^2 - 21m_W^2 - \frac{2}{3}q^2\right)\ln\frac{\Lambda^2}{\mu^2}\right] \\
& +\frac{m_Z^2}{16\pi^2\Lambda^2}s^2\frac{1}{2}f_B\left[\Lambda^2 - \frac{3}{2}\left(m_H^2+m_Z^2-\frac{5}{9}q^2\right)\ln\frac{\Lambda^2}{\mu^2}\right]\ ,
\end{aligned} \tag{B6c}$$

$$\Delta\overline{\Pi}_T^{11}(q^2) = \Delta\overline{\Pi}_T^{33}(q^2) + f_{\Phi,1}\frac{m_W^2}{\Lambda^2}\frac{v^2}{2g^2} + \frac{m_Z^2}{16\pi^2\Lambda^2}\frac{3s^2}{4}\left[f_B m_H^2 + 3(f_B+f_W)m_W^2\right]\ln\frac{\Lambda^2}{\mu^2}\ . \tag{B6d}$$

In the above equations the $1/m_H^2$ terms correspond to Higgs tadpole graphs such as the ones in Figs. 1(f) and 1(g). They merely contribute to a renormalization of the Higgs vacuum expectation value, or, equivalently, to the renormalization of G_F.

Feynman graphs involving Higgs boson loops are absent in models with a nonlinear realization of the Goldstone boson sector. Hence, we also give the results of Sec. IV when eliminating all graphs involving the exchange of the physical Higgs boson. The expressions for the oblique correction form factors of Eq. (4.8) remain valid except for the anticipated replacement of $\ln\frac{m_H^2}{m_W^2}$ by $\ln\frac{\Lambda^2}{m_W^2}$. The replacement rules for the quadratic divergencies are not as simple because of a subtlety of using dimensional regularization: the quadratic divergence is defined as the pole at $d = 2$ dimensions [see Eq. (4.1a)] and hence one gets $d\Lambda^2 = 2\Lambda^2$ whereas $d\ \ln\Lambda = 4\ \ln\Lambda$. Because of this nontrivial d dependence the quadratic divergencies for the theory without Higgs graphs are only qualitatively reproduced by the replacement $\frac{1}{2}m_H^2(\ln\frac{\Lambda^2}{m_H^2}+\frac{1}{2}) = \Lambda^2$ as suggested by Eq. (4.11).

More precisely we need to subtract the Higgs contributions from the results of Sec. IV. These Higgs contributions simultaneously give rise to the m_H^2 terms in Eqs. (4.9c) and (4.9d) and to quadratic divergencies via the scalar integral

$$\begin{aligned}
B_{22}(q^2; m_H, m) &= -\frac{1}{32\pi^2}\Gamma(\epsilon-1)(4\pi\mu^2)^\epsilon \int_0^1 dx[xm_H^2 + (1-x)m^2 - q^2x(1-x)]^{1-\epsilon} \\
&= -\frac{\mu^2}{8\pi}\left(\frac{1}{\epsilon-1}+1\right) + \frac{1}{64\pi^2}\left(m_H^2+m^2-\frac{q^2}{3}\right)\left(\frac{1}{\epsilon}-\gamma_E+\ln(4\pi)+1\right) + \cdots\ ,
\end{aligned} \tag{B7}$$

which is multiplied by some function of $f(d = 4 - 2\epsilon)$ in the full two-point functions. According to Eq. (4.1) the quadratic and logarithmic singularities of the two-point functions are defined via the residues of the poles at $\epsilon = 1$ and $\epsilon = 0$, respectively, and these residues get different contributions from the factor $f(d)$ as long as $f(d = 2) \neq f(d = 4)$. Since m_H^2 terms only appear in the logarithmic singularity, dropping the Higgs exchange graphs leads to the replacement rule

$$f(4)\frac{1}{2}m_H^2\left(\ln\frac{\Lambda^2}{m_H^2}+\frac{1}{2}\right) \to f(2)\Lambda^2\,, \tag{B8}$$

and the function $f(d)$ must be known.

Keeping track of the powers of the space-time dimension d, we have calculated the renormalized coefficients of the four dimension-six operators which contribute at tree level, in the absence of the Higgs exchange graphs:

$$f_{DW}^r = f_{DW} - \frac{3f_W + f_B}{768\pi^2}\,\ln\frac{\Lambda^2}{m_W^2}\,, \tag{B9a}$$

$$f_{DB}^r = f_{DB} - \frac{3f_B + f_W}{768\pi^2}\,\ln\frac{\Lambda^2}{m_W^2}\,, \tag{B9b}$$

$$f_{BW}^r = f_{BW} + \frac{\alpha}{32\pi s^2}\left\{-\frac{\Lambda^2}{m_W^2}(f_B+f_W) + \left[\left(10+\frac{2}{3c^2}\right)f_B - \left(\frac{22}{3}-\frac{2}{3c^2}\right)f_W + 12g^2 f_{WWW}\right]\,\ln\frac{\Lambda^2}{m_W^2}\right\}\,, \tag{B9c}$$

$$f_{\Phi,1}^r = f_{\Phi,1} - \frac{\alpha}{8\pi c^2}\left[f_B\left(\frac{\Lambda^2}{v^2}+\frac{3m_Z^2 s^2}{v^2}\ln\frac{\Lambda^2}{m_W^2}\right) - \frac{6m_W^2}{v^2}(f_B+f_W)\ln\frac{\Lambda^2}{m_W^2}\right]\,. \tag{B9d}$$

One finds that even the sign of the quadratic divergence in f_{BW}^r and $f_{\Phi,1}^r$ has changed compared to the naive replacement $m_H \to \Lambda$. This sign flip may be taken as indicative of the strong model dependence of the quadratically enhanced terms in the operator mixing.

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