### Final-state-interaction simulation of T violation in top-quark semileptonic decay

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The standard electroweak final-state interaction induces a false T-odd correlation in the top-quark semileptonic decay. The correlation parameter is calculated in the standard model and found to be considerably larger than those that could be produced by genuine T-violation effects in a large class of theoretical models.

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## I. INTRODUCTION

Final-state interactions play an important role in the determination of CP and T violation. A test for CP violation is to compare the partial decay rates of a particle and its antiparticle. In this case final-state interactions are necessary since in their absence the partial decay rates are equal from CPT invariance even if CP is violated. A general formalism for calculating such partial rate differences based on CPT invariance and unitarity has recently been developed [1], and its application to B meson decays (Ref. [1]) and to t-quark decays [2] has revealed some interesting relations between final-state interaction and CP-violation observables in weak decays.

A test [3] for T violation is to observe a "T-odd correlation," such as those of the form  $\sigma \cdot (\mathbf{p}_1 \times \mathbf{p}_2)$  where  $\sigma$  is a spin and  $\mathbf{p}_1$  and  $\mathbf{p}_2$  are momenta. In contrast with the partial decay difference, a T-odd correlation can be produced by final-state interactions even if T invariance holds. Thus, to use such correlations as a test of T violation the final-state-interaction effect must be negligible or calculable.

This article will be concerned with the *t*-quark semileptonic decay  $t \rightarrow bW \rightarrow bv_l \overline{l}$  in the standard model. Copious production of t quarks at future high-energy colliders such as the Superconducting Super Collider (SSC) and CERN Large Hadron Collider (LHC) have aroused considerable interest in exploring the origin of CP and Tviolation via t-quark interactions [4]. In particular, a recent study [5] of the possibility of using the T-odd correlation has shown that it has a reasonable sensitivity to some nonstandard sources of T violation. Since such correlations can be produced by standard model physics alone, it is timely to undertake a computation of the final-state-interaction effect due entirely to the standard electroweak interaction, which, up to the one-loop level, respects T and CP invariance in Cabibbo-allowed weak decays such as  $t \rightarrow bW^+ \rightarrow bv_l \overline{l}$ .

# **II. FINAL-STATE-INTERACTION EFFECT**

The computation of final-state-interaction effects on the *T*-odd correlation has long been of interest. Early examples of the calculation involved nuclear  $\beta$  decay [6], hyperon semileptonic decay [7], and  $K_{l_3}^{\pm,0}$  decays [8]. The parameter of interest is the coefficient of the *T*-odd correlation term in the decay spectrum, which in nuclear  $\beta$  decay, for instance, has the following form in the leading approximation:

$$\frac{d\Gamma}{d\Omega_e d\Omega_{\nu_e} dE_e} \sim 1 + a \frac{\mathbf{p}_e \cdot \mathbf{p}_{\nu_e}}{E_e E_{\nu_e}} + \sigma \cdot \left[ A \frac{\mathbf{p}_e}{E_e} + B \frac{\mathbf{p}_{\nu_e}}{E_{\nu_e}} + D \frac{\mathbf{p}_e \times \mathbf{p}_{\nu_e}}{E_e E_{\nu_e}} \right],$$
(1)

where  $\sigma$  is the polarization of the parent nucleus and  $\mathbf{p}_e(E_e)$  and  $\mathbf{p}_{v_e}(E_{v_e})$  are the electron and neutrino momentum (energy), respectively. In this example, the dominant contribution arises from electromagnetic finalstate interaction. The effect depends, among other things, on the recoil of the decaying particle, and thus the size of the T-odd correlation parameter D is of order  $D \sim \alpha E_e / M \ (Z \alpha E_e / M)$  in neutron (nuclear)  $\beta$  decay, where M is a nucleon mass. Since  $E_{e}$  is typically of order 1 MeV, the recoil effect, which is characterized by the ratio  $E_e/M$ , is rather tiny. Hence D is highly suppressed in neutron  $\beta$  decay with D typically of the order of  $10^{-5}-10^{-6}$ . A considerably larger result  $(10^{-3}-10^{-4})$ can be obtained in some nuclear  $\beta$  decays due to the enhancement  $Z \gg 1$  (Ref. [6]). The typical value of the T-odd correlation is between  $10^{-3}$  and  $10^{-4}$  in a neutral  $K_{l3}^0$  decay. The result in a charged  $K_{l3}^{\pm}$  decay is still smaller (10<sup>-5</sup>-10<sup>-6</sup>), because there the final-state pion is neutral and the effect can only arise from two-loop graphs.

In terms of weak-current interactions, the *t*-quark semileptonic decay is analogous in many respects to the nuclear  $\beta$  decay. However, the disparity between  $m_t$  and  $m_b$  implies that the *T*-odd correlation in the decay  $t \rightarrow b v_l \overline{l}$  does not have a recoil suppression. Indeed, compared to nuclear  $\beta$  decay, where the recoil effect is of order 10<sup>-3</sup>, in the *t* semileptonic decay such effects are given by  $E_{\overline{e}}/m_t$ , which is of order unity. As a consequence, we expect that the final-state-interaction contri-

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bution to the T-odd correlation parameter is roughly

$$D(t \to bW \to bv_e \overline{e}) \sim \alpha |Q_d| \frac{E_{\overline{e}}}{m_t} \sim \frac{\alpha}{9} \sim 10^{-3} , \qquad (2)$$

where  $Q_d = -\frac{1}{3}$  is the *b*-quark charge, and we have taken  $E_{\overline{e}}/m_t \sim \frac{1}{3}$ .

In what follows we will concentrate on the decay  $t \rightarrow bW \rightarrow bv_e \overline{e}$ . Insofar as the lepton mass can be ignored, our result holds for the other *t*-quark semileptonic decays as well.

A large  $m_t$  implies that the decay  $t \rightarrow b v_e \overline{e}$  proceeds dominantly through the W resonance. The smallness of the W width ( $\Gamma_W/M_W \approx 0.026$ ) then makes the calculation of the leading final-state-interaction effect very simple. Neglecting the b-quark and lepton masses, the leading contributions are generated by graphs displayed in Fig. 1 with

$$M(t \rightarrow b \nu_e \overline{e}) = \left(\frac{ig}{\sqrt{2}}\right)^2 \frac{[\overline{u}_{\nu_e}(p_{\nu_e})\gamma_{\lambda}Lv_{\overline{e}}(p_{\overline{e}})][\overline{u}_b(p')\Gamma^{\lambda}u_t(p)]}{k^2 - M_W^2 + i\Gamma_W M_W},$$
(3)

where k = p - p' is the momentum transfer carried by the W, L and R are the helicity projection operators, and the effective vertex  $\Gamma^{\lambda}$ , which includes one-loop interaction corrections from Figs. 1(b) and 1(c), can be parametrized as

$$\Gamma^{\lambda} = F_1(k^2) \gamma^{\lambda} L - i F_2(k^2) m_t \sigma^{\lambda \mu} k_{\mu} R , \qquad (4)$$

where  $\sigma^{\lambda\mu} = (i/2)|\gamma^{\lambda}, \gamma^{\mu}|$ . Terms of the form  $\gamma^{\lambda}R$  and  $\sigma^{\lambda\mu}k_{\mu}L$  vanish in the limit  $m_b = 0$ . Also, the  $k^{\lambda}$  term drops out for  $m_e = m_{\nu_e} = 0$ . While the form factor  $F_1 = 1 + O(\alpha/\pi)$  introduces a correction to the weak interaction charge g,  $F_2$  gives an anomalous moment to the btW vertex.

In analogous to nuclear  $\beta$  decay one may define a *T*-odd correlation parameter *D*:

$$\frac{d\Gamma}{d\Omega} = \frac{g^4}{(2\pi)^5} \frac{m_t E_{\nu_e} E_{\overline{e}}}{|k^2 - M_W^2 + i\Gamma_W M_W|^2} \times \left[ \left( 1 - \frac{k^2}{2m_t E_{\nu_e}} \right) + D \left( 1 - \frac{2E_{\overline{e}}}{m_t} \right) \sigma_t \cdot \frac{\mathbf{p}_{\overline{e}} \times \mathbf{p}_{\nu_e}}{E_{\overline{e}} E_{\nu_e}} \right] + \cdots$$
(5)

with

$$D = m_t^2 \mathrm{Im} F_2(M_W^2) \tag{6a}$$



FIG. 1. Feynman graphs generating the dominant contributions to the *T*-odd correlation. The calculation is carried out in the Feynman-'t Hooft gauge.  $\phi$  is the Higgs-Goldstone boson.

evaluated at  $k^2 = M_W^2$ . The ellipses in Eq. (5) refer to the other terms of no interest to us and  $d\Omega = (d^3 \mathbf{p}_{\overline{e}}/2E_{\overline{e}})(d^3 \mathbf{p}_{v_e}/2E_{v_e})(d^3 \mathbf{p}'/2p'_0)$ . In reaching (6a) we have taken  $F_1 = 1$ .

The final-state interaction in nuclear  $\beta$  decay takes place between the daughter nucleus and the electron. By contrast. the dominant effect in the decay  $t \rightarrow bW^+ \rightarrow b\overline{e}v_e$  arises from  $bW \rightarrow bW$  rescattering. By employing the unitarity formula given by Wolfenstein (Ref. [1]) one can show that the relevant interactions are those which scatter a  $bW^+$  state to other  $bW^+$  states with different spin configurations. As a result, the T-odd correlation parameter is directly proportional to the absorptive part of the form factor  $F_2$  which connects hadron states with different helicities. We find (the detail of the calculation is summarized in the Appendix)

$$\operatorname{Im} F_{2}(M_{W}^{2}) = -\frac{\alpha Q_{d}}{2m_{t}^{2}} \left[ 1 - \frac{1}{2} \frac{M_{W}^{2}}{m_{t}^{2}} \right] + \frac{\alpha (1 + 2Q_{d}s^{2})}{8(m_{t}^{2} - M_{W}^{2})} \left[ \left[ -\frac{1}{c^{2}} - \frac{1}{s^{2}} \right] I_{1} + \frac{2}{s^{2}} I_{2} \right],$$
(6b)

where  $s^2 = \sin^2 \theta_W$ ,  $c^2 = \cos^2 \theta_W$ , and

$$\begin{split} I_1 = & 2 + \left[ 1 + \frac{2m_t^2 M_Z^2}{(m_t^2 - M_W^2)^2} \right] \ln \frac{M_Z^2 m_t^2}{M_Z^2 m_t^2 + (m_t^2 - M_W^2)^2} , \\ I_2 = & \left[ 1 - \frac{M_W^2}{m_t^2} \right] \left[ 1 - \frac{1}{2} \frac{M_W^2}{m_t^2} + 2 \frac{M_Z^2}{m_t^2 - M_W^2} + 3 \frac{M_W^2 M_Z^2}{(m_t^2 - M_W^2)^2} \right] \\ & + \frac{M_Z^2}{m_t^2 - M_W^2} \left[ 2 + 2 \frac{M_Z^2}{m_t^2 - M_W^2} + 3 \frac{M_W^2 M_Z^2}{(m_t^2 - M_W^2)^2} \right] \ln \frac{M_Z^2 m_t^2}{M_Z^2 m_t^2 + (m_t^2 - M_W^2)^2} . \end{split}$$

(6c)

In Eq. (6b) the first term comes from the photon graphs and the second from the Z. For a very heavy top the result is dominated by the Z exchange diagram and has a logarithmic dependence on  $m_t$ . Asymptotically it approaches

$$\lim_{m_t \to \infty} D \sim \frac{\alpha}{6} \left\{ 1 - \frac{3}{4} \left[ 1 - \frac{2s^2}{3} \right] \times \left[ \frac{2}{c^2} + \left[ \frac{1}{c^2} + \frac{1}{s^2} \right] \ln \frac{M_Z^2}{m_t^2} \right] \right\}.$$
 (7)

The numerical results for D from Eqs. (6a) to (6c) are summarized [9] in Table I for  $m_t$  between 100 and 200 GeV. One sees that D is between  $1 \times 10^{-3}$  and  $6 \times 10^{-3}$ , as we expected from the simple dimensional argument Eq. (2). The result shows a slow increase with larger values of  $m_t$  in this region.

The *T*-odd correlation may be reparametrized in terms of an asymmetry parameter *A*, which is related to the difference of the decay  $W^+ \rightarrow \overline{e}v_e$  occurring in the opposite sides of the  $\sigma_t \times \mathbf{p}$  plane (Ref. [5]):

$$A = -\frac{3(m_t^2 - M_W^2)}{4(m_t^2 + 2M_W^2)} \frac{m_t M_W \text{Im}(F_1 F_2^*)}{|F_1|^2} \\ \approx -\frac{3(m_t^2 - M_W^2)}{4(m_t^2 + 2M_W^2)} m_t M_W \text{Im}F_2^* , \qquad (8)$$

where  $\text{Im}F_2^*$  is given by Eqs. (6b) and (6c) with an additional overall minus sign. The results for A are summarized in the last column of Table I. They vary from  $1 \times 10^{-4}$  to  $1 \times 10^{-3}$  for  $m_t = 100-200$  GeV. In comparison with the maximal-allowed T violation effect in the models considered in Ref. [5] in which  $A < 5 \times 10^{-5} \sim 5 \times 10^{-4}$ , the standard model final-state interaction produces a much larger false effect.

It is difficult to calculate the *T*-odd parameter to an accuracy of  $\sim 30\%$ . The major theoretical uncertainties of the present calculation come from neglecting QCD corrections, which introduce a sizable interference be-

TABLE I. The result for the *T*-odd correlation in the *t*-quark semileptonic decay. The parameters D and A are defined in Eqs. (5) and (9), respectively.

$m_t$ (GeV)	D	A
100	$1.1 \times 10^{-3}$	$1.0 \times 10^{-4}$
110	$1.4 \times 10^{-3}$	$1.8 \times 10^{-4}$
120	$1.9 \times 10^{-3}$	$2.8 \times 10^{-4}$
130	$2.4 \times 10^{-3}$	$3.9 \times 10^{-4}$
140	$2.9 \times 10^{-3}$	$5.1 \times 10^{-4}$
150	$3.5 \times 10^{-3}$	$6.4 \times 10^{-4}$
160	$4.0 \times 10^{-3}$	$7.6 \times 10^{-4}$
170	$4.6 \times 10^{-3}$	$8.7 \times 10^{-4}$
180	$5.1 \times 10^{-3}$	$9.8 \times 10^{-4}$
190	$5.6 \times 10^{-3}$	$1.1 \times 10^{-3}$
200	$6.1 \times 10^{-3}$	$1.2 \times 10^{-3}$

tween the absorptive part of  $F_1$  from electroweak interactions and the real part of  $F_2$  from QCD. An order of  $\sim (1 \sim 10)\%$  correction due to this effect alone is possible. A still more complicated contribution arises from the interference between Im $F_2$  calculated above and the real part of  $F_1$  due to QCD. Other uncertainties arise from neglecting (1) the WZ threshold effect (relevant if  $m_t > M_W + M_Z + m_b$ ) and (2) all the box diagrams. The contribution of the latter also depends on the angle between  $\mathbf{p}_{\bar{e}}$  and  $\mathbf{p}'$  in a rather complicated way. All of these contributions are suppressed by the ratio  $\Gamma_W/M_W$ , however. The calculation of these next-leading terms would be crucial should future experiments approach the precision of  $D \sim 10^{-3}$ .

*T*-odd correlations of the form  $\sigma_{\bar{e}} \cdot (\mathbf{p}_{v_e} \times \mathbf{p}_{\bar{e}})$ ,  $\sigma_b \cdot (\mathbf{p}_{v_e} \times \mathbf{p}_{\bar{e}})$  and *P*- and *CP*-odd correlations of the form  $\sigma_i \cdot (\sigma_b \times \mathbf{p}')$  are much more difficult to measure experimentally, and thus will not be considered in this paper.

# **III. CONCLUSION**

We have calculated the *T*-odd correlation  $\sigma_t \cdot (\mathbf{p}_{\overline{e}} \times \mathbf{p}_{v_a})$ induced by the standard electroweak final-state interactions in the decay  $t \rightarrow bW^+ \rightarrow b\overline{l}v_l$ , and found that the result has a logarithmic dependence on the t-quark mass and is dominated by the  $bW \rightarrow bW$  rescattering due to a Z exchange in the heavy top limit. For  $m_t$  in the range 100-200 GeV the correlation parameter D defined in Eq. (5) is between  $1 \times 10^{-3}$  and  $6 \times 10^{-3}$ , and the asymmetry parameter A given by Eq. (8) is between  $1 \times 10^{-4}$  and  $1 \times 10^{-3}$ . It is shown that the standard model physics can simulate a false T-odd signal, with its magnitude exceeding genuine T-violation effects of a size that could possibly be produced in a large class of theoretical models. To get rid of this pure final-state interaction effect one may consider comparing the asymmetry parameter for both  $t \rightarrow bW^+$  and  $\overline{t} \rightarrow \overline{b}W^-$ , as in the study of CPviolating parameters  $\alpha + \overline{\alpha}$  and  $\beta + \overline{\beta}$  in the  $\Lambda$  decays [10].

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#### APPENDIX

We give some details of the calculation in this appendix. The technique is standard [11] except that we use the Minkowskian metric  $g^{\lambda\beta} = \text{diag}(1, -1, -1, -1)$ . The one- and two-point functions are defined as FINAL-STATE-INTERACTION SIMULATION OF T...

$$A(m) = -i\mu_0^{(n-4)} \int \frac{d^n K}{(2\pi)^n} \frac{1}{K^2 - m^2 + i\epsilon} ,$$
(A1)  

$$B(m, m, k) = -i\mu_0^{(n-4)} \int \frac{d^n K}{(2\pi)^n} \frac{1}{K}$$

$$B(m_1, m_2; k) = -i\mu_0^{(n-4)} \int \frac{d^n K}{(2\pi)^n} \frac{1}{[K^2 - m_1^2 + i\epsilon][(K+k)^2 - m_2^2 + i\epsilon]},$$

where  $\epsilon \rightarrow 0_+$ , and we use dimensional regularization to isolate the ultraviolet divergences. The only relevant threepoint function is

$$C_0 = -i \int \frac{d^4 K}{(2\pi)^4} \frac{1}{[K^2 - M_Z^2 + i\epsilon][(K - k)^2 - M_W^2 + i\epsilon][(K + p')^2 - m_b^2 + i\epsilon]}$$
(A2)

We find

$$Im A(m) = 0,$$

$$Im B(m_1, m_2; k) = \frac{1}{16\pi k^2} \sqrt{\lambda(k^2, m_1^2, m_2^2)} \theta[k^2 - (m_1 + m_2)^2],$$

$$Im C_0 = \frac{1}{16\pi \sqrt{\lambda(m_t^2, M_W^2, m_b^2)}} ln \frac{M_Z^2 m_t^2}{M_Z^2 m_t^2 + \lambda(m_t^2, M_W^2, m_b^2)} \theta[m_t^2 - (M_W + m_b)^2],$$
(A3)

where  $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ . In evaluating ImC<sub>0</sub> we have put all the external lines on their mass shell.

Neglecting the *b*-quark and lepton masses, the finalstate-interaction effect due to a photon exchange is

$$\Gamma^{\lambda}(\gamma) = -e^2 Q_d 4m_t p'^{\lambda} R(a_1 + b_1) , \qquad (A4)$$

where  $a_1$  and  $b_1$  are the coefficients defined in the integrals

$$-i\int \frac{d^{4}K}{(2\pi)^{4}} \frac{K^{\lambda}}{K^{2}[(K-k)^{2}-M_{W}^{2}][(K+p')^{2}-m_{b}^{2}]} = a_{1}k^{\lambda}+a_{2}p'^{\lambda}, \quad (A5)$$
$$-i\mu_{0}^{(n-4)}\int \frac{d^{n}K}{(2\pi)^{n}} \frac{K^{\lambda}K^{\beta}}{K^{2}[(K-k)^{2}-M_{W}^{2}][(K+p')^{2}-m_{b}^{2}]} = b_{1}(k^{\lambda}p'^{\beta}+k^{\beta}p'^{\lambda})+b_{2}g^{\lambda\beta}+b_{3}k^{\lambda}k^{\beta}+b_{4}p'^{\lambda}p'^{\beta}. \quad (A6)$$

We find

$$a_{1} = \frac{B(M_{W}, 0; k) - B(M_{W}, 0; p)}{m_{t}^{2} - M_{W}^{2}},$$

$$b_{1} = -\frac{1}{2} \frac{A(M_{W}) - M_{W}^{2}B(0, M_{W}; p)}{m_{t}^{2}(m_{t}^{2} - M_{W}^{2})}.$$
(A7)

It then follows from Eqs. (A3), (A4), and (A7) that the absorptive part of  $\Gamma^{\lambda}(\gamma)$  is

$$\Gamma_{\rm abs}^{\lambda}(\gamma) = \frac{\alpha Q_d}{m_t} \left[ 1 - \frac{M_W^2}{2m_t^2} \right] p^{\prime \lambda} R \quad . \tag{A8}$$

It can be written in a more conventional form by applying the Gordon identity

$$[\overline{u}_b(p')p'^{\lambda}Ru_t(p)] = \frac{i}{2}[\overline{u}_b(p')\sigma^{\lambda\mu}k_{\mu}Ru_t(p)] + \cdots .$$
(A9)

The result due to Z exchange is

$$\Gamma^{\lambda}(Z) = -e^{2}(1+2Q_{d}s^{2})m_{t}p'^{\lambda}R$$

$$\times \left[ \left[ -\frac{1}{c^{2}} - \frac{1}{s^{2}} \right](-a_{1}' + a_{2}' + C_{0}) - \frac{2}{s^{2}}(a_{1}' + b_{1}') \right], \quad (A10)$$

where the coefficients  $a'_1$ ,  $a'_2$ , and  $b'_1$  are defined analogously:

$$-i\int \frac{d^{4}K}{(2\pi)^{4}} \frac{K^{\lambda}}{[K^{2}-M_{Z}^{2}][(K-k)^{2}-M_{W}^{2}][(K+p')^{2}-m_{b}^{2}]} = a_{1}'k^{\lambda} + a_{2}'p'^{\lambda},$$

$$-i\mu_{0}^{(n-4)}\int \frac{d^{n}K}{(2\pi)^{n}}$$
(A11)

$$\times \frac{K^{\lambda}K^{\beta}}{[K^2 - M_Z^2][(K-k)^2 - M_W^2][(K+p')^2 - m_b^2]}$$
  
=  $b'_1(k^{\lambda}p'^{\beta} + k^{\beta}p'^{\lambda}) + b'_2g^{\lambda\beta} + b'_3k^{\lambda}k^{\beta} + b'_4p'^{\lambda}p'^{\beta}.$ 

We find

$$a_{1}^{\prime} = \frac{1}{m_{t}^{2} - M_{W}^{2}} [B(M_{Z}, M_{W}; k) - B(M_{W}, 0; p) - M_{Z}^{2}C_{0}], \qquad (A12)$$

$$a_{2}^{\prime} = -\frac{1}{m_{t}^{2} - M_{W}^{2}} [B(M_{Z}, 0; p^{\prime}) - B(M_{W}, 0; p) - M_{Z}^{2}C_{0}] - \frac{2M_{W}^{2}}{(m_{t}^{2} - M_{W}^{2})^{2}} [B(M_{Z}, M_{W}; k) - B(M_{W}, 0; p) - M_{Z}^{2}C_{0}], \qquad (A13)$$

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$$b_{1}' = -\frac{1}{(m_{t}^{2} - M_{W}^{2})^{2}} \left\{ \frac{1}{2} \left[ 1 - \frac{M_{W}^{2}}{m_{t}^{2}} \right] [A(M_{W}) - M_{W}^{2}B(M_{W}, 0; p)] + \frac{1}{2} [A(M_{Z}) - M_{Z}^{2}B(M_{W}, M_{Z}; k)] + 2M_{Z}^{2} [B(M_{W}, 0; p) - B(M_{Z}, 0; p')] - 3\frac{M_{W}^{2}M_{Z}^{2}}{m_{t}^{2} - M_{W}^{2}} [B(M_{W}, M_{Z}; k) - B(M_{W}, 0; p)] + \left[ 2 + 3\frac{M_{W}^{2}}{m_{t}^{2} - M_{W}^{2}} + \frac{m_{t}^{2} - M_{W}^{2}}{M_{Z}^{2}} \right] M_{Z}^{4}C_{0} \right\}.$$
(A14)

One can check that in the limit  $M_Z = 0$ ,  $a_1$  and  $a'_1$  become identical and so do  $b_1$  and  $b'_1$ . The logarithmic dependence on  $m_t$  in the limit  $m_t \to \infty$  arises because  $\Gamma^{\lambda}(Z)$  has a term which is directly proportional to  $C_0$  [see (A10)].

It then follows that the absorptive part of  $\Gamma^{\lambda}(Z)$  is

$$\Gamma_{\rm abs}^{\lambda}(Z) = -\frac{\alpha(1+2Q_d s^2)}{4(m_t^2 - M_W^2)} m_t p'^{\lambda} R\left[ \left[ -\frac{1}{c^2} - \frac{1}{s^2} \right] I_1 + \frac{2}{s^2} I_2 \right], \tag{A15}$$

where  $I_{1,2}$  are given by Eq. (6c). Adding (A8) and (A15) we obtain the results given by Eqs. (6a)–(6c) of the text.

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