

Drell-Yan dimuon production with transversely polarized protons

W. Vogelsang and A. Weber

Institut für Physik, Universität Dortmund, D-44221 Dortmund, Germany

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We carefully study dimuon production by transversely polarized protons. It is shown that this process should provide a good tool to uncover the parton distributions of the transversely polarized proton. In particular we calculate the $O(\alpha_s)$ corrections to the total Drell-Yan cross section and the rapidity differential cross section and find that the respective cross section asymmetries are rather stable.

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I. INTRODUCTION

The spin structure of transversely polarized protons has recently attracted growing theoretical interest [1–10]. Up to now nothing is known experimentally about the twist-2 distributions of transversely polarized quarks in a transversely polarized proton, which are usually referred to as “transversity distributions” [11,1,2]. They may, however, be accessible in future experiments at polarized hadron-hadron colliders [12]. It is therefore important to perform detailed theoretical studies of what is to be expected for the various conceivable processes such as (Drell-Yan) dimuon production [1,2,4,5,13], jet [2,6,9,10], prompt photon [2,8], or heavy flavor [2] production. Among these, dimuon production in collisions of two transversely polarized hadrons plays a special role. The reason for this is very simple: As is well known, there are no contributions from incoming gluons to the transverse two-spin cross sections due to angular momentum conservation [3,5,13]. Nevertheless, gluons of course contribute to the unpolarized (spin-averaged) cross sections. The quantities of interest in spin physics are the cross-section asymmetries which are defined in this case as the ratio of the transverse cross section over the unpolarized one. Thus, if incoming gluons contribute strongly to the unpolarized cross section they will tend to drastically reduce the asymmetry. In all processes mentioned above, with the exception of dimuon production, gluons enter the unpolarized cross section already at the leading order (LO), which means that the asymmetries for those processes are expected to be small [2]. This is not the case for the Drell-Yan process since here the LO subprocess is $q\bar{q} \rightarrow \mu^+\mu^-$ annihilation which (in the unpolarized case) receives only $O(\alpha_s)$ corrections from incoming gluons. This feature makes the Drell-Yan process as important for the transversely polarized case as deep-inelastic scattering is for the unpolarized and the longitudinally polarized case [14–16] to define parton densities beyond the leading order.

Furthermore, if studied in *proton-proton* collisions (which will solely be discussed in the following), the Drell-Yan process is very sensitive to the sea quark component of the proton. Among the transversity distributions, the transverse sea is the most unknown (and therefore probably interesting) one since there are arguments

[1] that the transverse valence quark distributions are closely related to their longitudinal counterparts and thus are not completely unconstrained.

In view of the importance of the transversely polarized Drell-Yan process we want to give a detailed study of it in this paper. Since it is well known from the unpolarized case that the higher-order [$O(\alpha_s)$] corrections to the various differential cross sections are sizable [15,17] and crucial for bringing theoretical predictions and experimental results into agreement (“ K factors”), it is necessary to study the corresponding corrections for the transversely polarized case and to examine their influence on the asymmetries. This is the major issue of this work. In Sec. II we present our notation and point out the general framework for our calculations. In Sec. III we briefly deal with the q_T differential Drell-Yan cross section where q_T is the transverse momentum of the virtual photon with respect to the beam axis. It should be noted that in the q_T differential case gluons again enter at LO in the unpolarized cross section since $q\bar{q}$ annihilation in $O(\alpha_s^0)$ only contributes at $q_T=0$ so that our above discussion applies and small asymmetries are to be anticipated. Therefore we turn to the q_T integrated (total) cross section in Sec. IV. We present the $O(\alpha_s)$ corrections to this cross section. Section V is devoted to the cross-section differential in the rapidity y , which is also presented including next-to-leading-order corrections. Finally we draw our conclusions in Sec. VI. The appendixes contain calculational details and the results for the unpolarized cross sections.

II. GENERAL FRAMEWORK

In this section we give all ingredients needed for the calculation of the various Drell-Yan cross sections for transverse polarization to order α_s . In the unpolarized case [15,17] the corresponding results were obtained by simply considering the subprocesses $q\bar{q} \rightarrow \gamma^*$ (including virtual corrections), $q\bar{q} \rightarrow \gamma^*g$, and $qg \rightarrow \gamma^*q$ and taking into account the decay $\gamma^* \rightarrow \mu^+\mu^-$ by multiplying with the factor $\alpha/3\pi Q^2$, which represents the integration over the lepton angular variables. This is unfortunately not possible in the case of transverse polarization for the incoming particles where in order to obtain nonvanishing cross sections it is crucial to keep the azimuthal angle ϕ_1

of one outgoing lepton unintegrated [5]. We therefore have to resort to the full processes $q\bar{q} \rightarrow \mu^+\mu^-g$, $q\bar{q} \rightarrow \mu^+\mu^-g$ if transverse polarization is involved. Note that the gluon-initiated process $qg \rightarrow \mu^+\mu^-q$ is not present in the case of transverse polarization as stated in the Introduction [3,5].

To begin with, let us discuss the LO process $q\bar{q} \rightarrow \mu^+\mu^-$. Our notation of the momenta is fixed in Fig. 1(a). The transverse spin vectors of the incoming quark and antiquark are denoted by s_1 and s_2 , respectively. The color-averaged matrix element is then given by [2,11]

$$|M|_{q\bar{q} \rightarrow \mu^+\mu^-}^2(s_1, s_2) = \frac{2}{3} e^4 e_q^2 \left[\frac{\hat{u}_1^2 + \hat{t}_1^2}{\hat{s}^2} + 4 \frac{(k_1 \cdot s_1)(k_1 \cdot s_2)}{\hat{s}} + \frac{2\hat{t}_1 \hat{u}_1}{\hat{s}^2} (s_1 \cdot s_2) \right], \quad (1)$$

where $\hat{s} \equiv (p_1 + p_2)^2$, $\hat{t}_1 \equiv (p_1 - k_1)^2$, and $\hat{u}_1 \equiv (p_2 - k_1)^2$. In Eq. (1) the first term in square brackets corresponds to the usual unpolarized matrix element, whereas the spin-dependent terms which are the quantities of interest are readily projected out from Eq. (1) by taking the difference

$$\Delta_T |M|_{q\bar{q} \rightarrow \mu^+\mu^-}^2 \equiv \frac{1}{2} [|M|_{q\bar{q} \rightarrow \mu^+\mu^-}^2(s_1, s_2) - |M|_{q\bar{q} \rightarrow \mu^+\mu^-}^2(s_1, -s_2)]. \quad (2)$$

Working in the c.m. system (c.m.s.) of the incoming particles we define the z axis by the direction of the three-momentum of the incoming quark [18]. Furthermore parametrizing the transverse spin vectors and the muon's momentum by

$$\Delta_T |M|^2 = \frac{8C_F g^2 e^4 e_q^2}{3Q^2} \left[\frac{2\hat{s}}{\hat{t}\hat{u}} [(k_1 \cdot s_1)(k_1 \cdot s_2) + (k_2 \cdot s_1)(k_2 \cdot s_2)] + (s_1 \cdot s_2) \left[1 + \frac{Q^2}{\hat{t}\hat{u}} (\hat{s} - k^2) - \frac{\hat{t}_1}{\hat{t}} - \frac{\hat{u}_1}{\hat{u}} + \frac{Q^2}{\hat{t}\hat{u}} (\hat{t}_1 + \hat{u}_1) + 2 \frac{\hat{t}_1 \hat{u}_1}{\hat{t}\hat{u}} \right] \right], \quad (5)$$

where $C_F = \frac{4}{3}$ and we have defined $\hat{t} = (p_1 - q)^2$ and $\hat{u} = (p_2 - q)^2$. Parametrizing the momenta and spin vectors as in Eq. (3) and introducing the angles θ and ϕ via

$$q \equiv (q_0; |\mathbf{q}| \sin\theta \cos\phi, |\mathbf{q}| \sin\theta \sin\phi, |\mathbf{q}| \cos\theta), \quad (6)$$

we find

$$\Delta_T |M|^2 = \frac{8C_F g^2 e^4 e_q^2}{3Q^2} \left[\frac{2\hat{s}}{\hat{t}\hat{u}} \{ |\mathbf{q}|^2 \sin^2\theta \cos(\phi - \phi_a) \cos(\phi - \phi_b) + 2|\mathbf{k}_1|^2 \sin^2\theta_1 \cos(\phi_1 - \phi_a) \cos(\phi_1 - \phi_b) - |\mathbf{q}| |\mathbf{k}_1| \sin\theta \sin\theta_1 [\cos(\phi - \phi_a) \cos(\phi_1 - \phi_b) + \cos(\phi - \phi_b) \cos(\phi_1 - \phi_a)] \} - \cos(\phi_a - \phi_b) \left[1 + \frac{Q^2}{\hat{t}\hat{u}} (\hat{s} - k^2) + \sqrt{\hat{s}} |\mathbf{k}_1| \left[\frac{1 - \cos\theta_1}{\hat{t}} + \frac{1 + \cos\theta_1}{\hat{u}} \right] - 2\sqrt{\hat{s}} |\mathbf{k}_1| \frac{Q^2}{\hat{t}\hat{u}} + 2\hat{s} \frac{|\mathbf{k}_1|^2 \sin^2\theta_1}{\hat{t}\hat{u}} \right] \right], \quad (7)$$

where

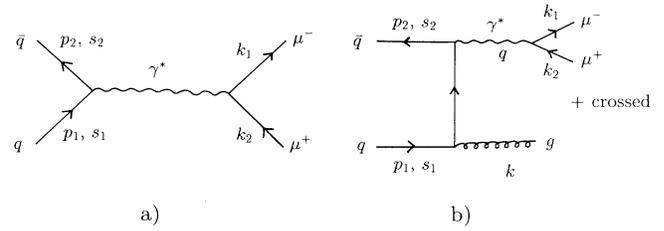


FIG. 1. Feynman diagrams contributing to the transversely polarized Drell-Yan process: (a) $q\bar{q} \rightarrow \mu^+\mu^-$ in $O(\alpha_s^0)$, (b) $q\bar{q} \rightarrow \mu^+\mu^-g$ in $O(\alpha_s)$.

$$\begin{aligned} s_1 &\equiv (0; \cos\phi_a, \sin\phi_a, 0), \\ s_2 &\equiv (0; \cos\phi_b, \sin\phi_b, 0), \\ k_1 &\equiv |\mathbf{k}_1| (1; \sin\theta_1 \cos\phi_1, \sin\theta_1 \sin\phi_1, \cos\theta_1), \end{aligned} \quad (3)$$

$\Delta_T |M|_{q\bar{q} \rightarrow \mu^+\mu^-}^2$ reads

$$\Delta_T |M|_{q\bar{q} \rightarrow \mu^+\mu^-}^2 = \frac{1}{3} e^4 e_q^2 \sin^2\theta_1 \cos(2\phi_1 - \phi_a - \phi_b), \quad (4)$$

in agreement with the result of Refs. [2,4,5,11]. Equation (4) clearly demonstrates that it is crucial to keep the lepton's azimuthal angle ϕ_1 unintegrated. When considering the total Drell-Yan cross section in Sec. IV and the y differential cross section in Sec. V, Eq. (4) will be the starting point for our LO calculations.

Let us now turn to the $2 \rightarrow 3$ process $q\bar{q} \rightarrow \mu^+\mu^-g$ [Fig. 1(b)]. Quite in general, upon phase-space integration this process will show up collinear and infrared singularities. We therefore have to introduce a regulator in order to regularize these singularities. A suitable choice is to introduce a mass for the outgoing gluon ($k^2 \neq 0$) [17]. The polarized matrix element for $k^2 \neq 0$ is then given by

$$|\mathbf{k}_1| = Q^2/2 \{q_0 - |\mathbf{q}| [\sin\theta_1 \sin\theta \cos(\phi - \phi_1) + \cos\theta \cos\theta_1]\} .$$

The $2 \rightarrow 3$ phase space for massive outgoing gluons is given in terms of our angular variables by

$$R_3 = \frac{1}{32\sqrt{\hat{s}}(2\pi)^5} \int dQ^2 \int \frac{|\mathbf{q}| Q^2 d \cos\theta_1 d\phi_1 d \cos\theta d\phi}{\{q_0 - |\mathbf{q}| [\sin\theta_1 \sin\theta \cos(\phi - \phi_1) + \cos\theta \cos\theta_1]\}^2} , \quad (8)$$

where $|\mathbf{q}| = \sqrt{q_0^2 - Q^2}$ and $q_0 = (\hat{s} + Q^2 - k^2)/2\sqrt{\hat{s}}$. Of course it is again crucial to keep ϕ_1 unintegrated. As we want to study transverse momentum distributions of the virtual photon in the next section, we will also keep the variable θ which is connected with q_T via

$$q_T^2 = |\mathbf{q}|^2 \sin^2\theta . \quad (9)$$

The remaining two angles θ_1 and ϕ can be integrated over using the integrals given in Appendix A [19]. We obtain

$$\begin{aligned} & \frac{|\mathbf{q}| Q^2}{32\sqrt{\hat{s}}(2\pi)^5} \int d\phi d \cos\theta_1 \frac{\Delta_T |M|^2}{\{q_0 - |\mathbf{q}| [\sin\theta_1 \sin\theta \cos(\phi - \phi_1) + \cos\theta \cos\theta_1]\}^2} \\ &= \frac{2\alpha_s \alpha^2 e_q^2 C_F}{9\pi} \frac{|\mathbf{q}|}{\sqrt{\hat{s}}} \frac{1}{q_T^2 + k^2 Q^2 / \hat{s}} \cos(2\phi_1 - \phi_a - \phi_b) \left[1 + \frac{2q_T^2}{Q^2} - 3 \ln \left[1 + \frac{q_T^2}{Q^2} \right] \right] . \end{aligned} \quad (10)$$

From Eq. (10) it becomes obvious already how the regulator $k^2 \neq 0$ cures the singularities arising for $q_T^2 \rightarrow 0$, although it is not needed for studying q_T differential cross sections to which we will turn in the next section.

III. THE q_T DIFFERENTIAL CROSS SECTION

In this section we study the transverse momentum (q_T) distribution of the virtual photon which decays into the lepton pair. In order for the photon to acquire a nonvanishing q_T , gluon emission must take place, which means that the process $q\bar{q} \rightarrow \mu^+ \mu^- g$ is the leading-order process for the transversely polarized case. Collecting all factors, setting $k^2=0$ in Eq. (10) and changing from $\sin\theta$ to q_T via Eq. (9), we find, for the transversely polarized subprocess cross section (omitting the fractional quark charge for the moment),

$$\begin{aligned} \frac{d\Delta_T \hat{\sigma}}{dQ^2 d\phi_1 dq_T^2} &= \frac{\alpha^2}{9\hat{s}} C_F \frac{\alpha_s}{2\pi} \frac{4 \cos(2\phi_1 - \phi_a - \phi_b)}{(\hat{s} - Q^2) \left[1 - \frac{4\hat{s}q_T^2}{(\hat{s} - Q^2)^2} \right]^{1/2}} \\ &\times \frac{1}{q_T^2} \left[1 + \frac{2q_T^2}{Q^2} - 3 \ln \left[1 + \frac{q_T^2}{Q^2} \right] \right] , \end{aligned} \quad (11)$$

where $\Delta_T \hat{\sigma}$ is defined in complete analogy with $\Delta_T |M|^2$ in Eq. (2). In order to get the hadronic cross section we have to convolute $\Delta_T \hat{\sigma}$ with the appropriate transverse quark densities $\Delta_T q$, which are defined by

$$\Delta_T q(x) \equiv q^\uparrow(x) - q^\downarrow(x) , \quad (12)$$

q^\uparrow (q^\downarrow) denoting the distribution of transversely polarized quarks with their spin parallel (antiparallel) to the spin of their transversely polarized parent proton [21]. This yields the hadronic cross section

$$\begin{aligned} \frac{d\Delta_T \sigma}{dQ^2 d\phi_1 dq_T^2} &= \int_{\tau_+}^1 dx_1 \int_{\tau_+/x_1}^1 dx_2 \frac{d\Delta_T \hat{\sigma}}{dQ^2 d\phi_1 dq_T^2} \\ &\times \Delta_T H(x_1, x_2, \mu^2) , \end{aligned} \quad (13)$$

where

$$\begin{aligned} \Delta_T H(x_1, x_2, \mu^2) &\equiv \sum_q e_q^2 [\Delta_T q(x_1, \mu^2) \Delta_T \bar{q}(x_2, \mu^2) \\ &+ \Delta_T q(x_2, \mu^2) \Delta_T \bar{q}(x_1, \mu^2)] \end{aligned} \quad (14)$$

and $\tau_\pm \equiv (\sqrt{\tau + \rho} + \sqrt{\rho})^2$ with $\tau \equiv Q^2/S$, $\rho \equiv q_T^2/S$ and the hadronic c.m.s. energy squared S . μ is a typical mass scale of the process not specified in this order. Apart from the cross sections we are interested in the cross-section asymmetry which is defined by

$$A_T(q_T) \equiv \frac{d\Delta_T \sigma}{dQ^2 d\phi_1 dq_T^2} \bigg/ \frac{d\sigma}{dQ^2 d\phi_1 dq_T^2} . \quad (15)$$

Here $d\sigma/dQ^2 d\phi_1 dq_T^2$ is the unpolarized cross section which also receives contributions from incoming gluons and is given in Appendix B. As stated in the Introduction, the transverse quark distributions in the proton are completely unknown up to now. The only constraint on them is that they satisfy for each flavor the relation [1]

$$|\Delta_T \bar{q}^{(-)}(x, Q^2)| \leq \bar{q}^{(-)}(x, Q^2) , \quad (16)$$

where the $\bar{q}^{(-)}$ are the corresponding unpolarized parton distributions for which we shall take the LO ones suggested by Glück, Reya, and Vogt (GRV) [22] for all our calculations.

In order to present numerical results we have to resort to some simple model for the transverse parton distributions. We assume that at some low-resolution scale, $Q^2 = Q_0^2 = 1 \text{ GeV}^2$, the transverse valence quark distributions equal their longitudinal counterparts, i.e.,

$$\begin{aligned} \Delta_T u_V(x, Q_0^2) &= \Delta_L u_V(x, Q_0^2) , \\ \Delta_T d_V(x, Q_0^2) &= \Delta_L d_V(x, Q_0^2) . \end{aligned} \quad (17)$$

Note that such an ansatz is supported by bag-model con-

siderations where $\Delta_T q_V$ only slightly differs from $\Delta_L q_V$ due to different p -wave contributions [1]. We fix the valence input by choosing the longitudinally polarized valence quark densities at $Q_0^2=1 \text{ GeV}^2$ from Ref. [23] which satisfy the constraint (16) for the unpolarized GRV [22] distributions. For the transversely polarized sea $\Delta_T \bar{q}(x, Q_0^2)$ the situation is less clear. In order to study the sensitivity of the asymmetries to the sea we shall assume two different parametrizations: one with a rather small sea [23],

$$x \Delta_T \bar{q}(x, Q_0^2) = -0.446x^{1.14}(1-x)^9 \quad (\text{model a}), \quad (18)$$

and another one with a large transverse sea quark polarization [24],

$$x \Delta_T \bar{q}(x, Q_0^2) = -0.57x^{0.28}(1-x)^{10.8} \quad (\text{model b}), \quad (19)$$

which nevertheless satisfies the constraint (16). In both cases we take the sea to be SU(2) symmetric but assume $\Delta_{TS}(x, Q_0^2)=0$ for the strange quarks. Figure 2 shows the two asymmetries for $\Delta_T \bar{q}/\bar{q}$ using the unpolarized \bar{q} from GRV [22] at $Q_0^2=1 \text{ GeV}^2$ and $Q^2=49 \text{ GeV}^2$. Having fixed our input for the transverse quark densities, we evolve them to higher scales with the help of the well-known evolution kernel for transverse quark densities given in Ref. [5] (see also Sec. IV). Note that the leading-order evolution equations for the transverse case do not involve gluons [5] and therefore only provide a $q \rightarrow q$ transition. This means that all quark distributions evolve in Q^2 separately like nonsinglet combinations. Thus the assumption $\Delta_{TS}(x, Q_0^2)=0$ persists for all Q^2 . For the same reason there would be no transverse sea at all at higher Q^2 if we had assumed $\Delta_T \bar{q}(x, Q_0^2)=0$ instead of Eqs. (18) or (19) as is also conceivable [10]. In this case the transverse Drell-Yan cross sections for pp scattering would be identically zero, and we need not pursue this possibility.

Figure 3 shows our result for the asymmetry $A_T(q_T)$ vs q_T for $Q^2=49 \text{ GeV}^2$ [27] and $\sqrt{S}=100 \text{ GeV}$. We have made, as for all the following numerical evaluations, the optimal choice $\cos(2\phi_1 - \phi_a - \phi_b)=1$ in Eq. (11).

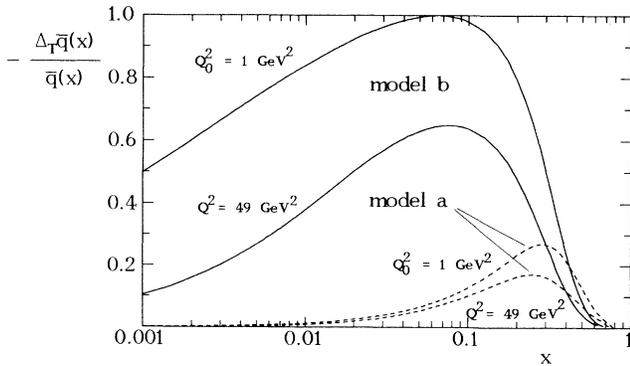


FIG. 2. The asymmetry $-\Delta_T \bar{q}(x)/\bar{q}(x)$ for the two transversely polarized sea quark distributions in Eq. (18) (dashed lines, model a) and Eq. (19) (solid lines, model b) for $Q_0^2=1 \text{ GeV}^2$ and $Q^2=49 \text{ GeV}^2$. The unpolarized sea $\bar{q}(x)$ was taken from [22].

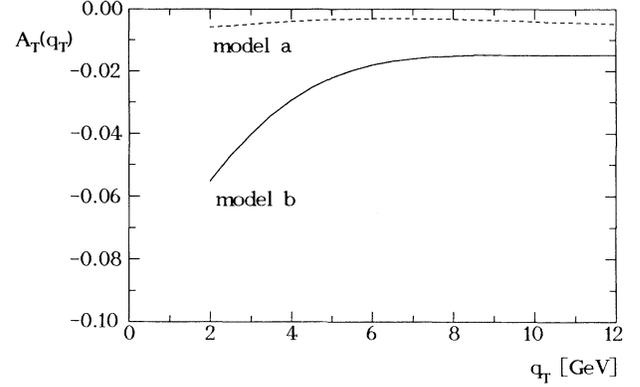


FIG. 3. $A_T(q_T)$ according to Eq. (15) for the two models (18) (dashed line) and (19) (solid line) at $Q^2=49 \text{ GeV}^2$ and $\sqrt{S}=100 \text{ GeV}$. The unpolarized parton distributions are taken from [22].

Furthermore we have chosen the scale in the parton distributions and α_s to be Q^2 . It becomes obvious from Fig. 3 that the asymmetries in the perturbatively safe region $q_T > 4 \text{ GeV}$ are very small for both models for $\Delta_T \bar{q}$. The reason for this was already given in the Introduction. The asymmetry is strongly suppressed by the gluonic contribution to the unpolarized cross section, which is significant especially at small q_T .

The influence of the unpolarized gluon's contribution on the asymmetry is drastically reduced if the LO [$O(\alpha_s^0)$] process [Fig. 1(a)] has to be taken into account as is the case for the asymmetries for the total and the y differential Drell-Yan cross sections. For these the gluonic contribution acts only as an order α_s correction to the unpolarized cross section and is known to be rather small compared to the quark piece even for the pp case where valence-sea annihilation dominates. For this reason we shall now turn to the total and the rapidity differential cross sections to see whether they can give a clue to the transversity distributions. Note that the LO asymmetry for the rapidity differential cross section was already discussed in Refs. [2,4]. As stated in the Introduction it has, however, been shown [15,17] that $O(\alpha_s)$ corrections to the $q\bar{q}$ annihilation process are quite sizable for the total and the y differential unpolarized cross sections. In our opinion it is therefore interesting to study the importance of $O(\alpha_s)$ corrections to the Drell-Yan cross sections also for the transversely polarized case as these might be important for quantitative analyses.

IV. THE TOTAL DRELL-YAN CROSS SECTION

$$d\Delta_T \sigma / dQ^2 d\phi_1$$

The LO result for $d\Delta_T \sigma / dQ^2 d\phi_1$ can be immediately obtained from Eq. (4) by integrating over $\cos\theta_1$. This gives for the subprocess cross section

$$\frac{d\Delta_T \hat{\sigma}}{dQ^2 d\phi_1} = \frac{\alpha^2}{9\hat{s}Q^2} \cos(2\phi_1 - \phi_a - \phi_b) \delta(1-z), \quad (20)$$

where $z \equiv Q^2/\hat{s}$. The $O(\alpha_s)$ corrections stemming from the process $q\bar{q} \rightarrow \mu^+ \mu^- g$ are derived from Eq. (10) by integrating over θ (for calculational details see Appendix A) and are given in the limit $k^2 \rightarrow 0$ by

$$\frac{d\Delta_T\hat{\sigma}^R}{dQ^2d\phi_1} = \frac{\alpha^2}{9\hat{s}Q^2} C_F \frac{\alpha_s}{2\pi} 4 \cos(2\phi_1 - \phi_a - \phi_b) z \left[\delta(1-z) \left[\frac{1}{4} \ln^2 \frac{Q^2}{k^2} - \frac{\pi^2}{12} \right] + \ln \frac{Q^2}{k^2} \frac{1}{(1-z)_+} \right. \\ \left. + 2 \left[\frac{\ln(1-z)}{1-z} \right]_+ - \frac{2 \ln z}{1-z} + \frac{1-z}{z} - \frac{3}{2} \frac{\ln^2 z}{1-z} \right], \quad (21)$$

where the “+” prescription is defined in the usual way [15]:

$$\int_0^1 \frac{f(z)}{(1-z)_+} dz \equiv \int_0^1 \frac{f(z) - f(1)}{1-z} dz. \quad (22)$$

Note the appearance of infrared divergencies in Eq. (21) which show up, e.g., in the term $\sim \ln^2 Q^2/k^2$. They are removed by adding the $O(\alpha_s)$ part of the virtual corrections to the LO process, which can be found in Ref. [17] for the unpolarized case and are trivially transformed to the case of transverse polarization:

$$\frac{d\Delta_T\hat{\sigma}^V}{dQ^2d\phi_1} = \frac{\alpha^2}{9\hat{s}Q^2} \cos(2\phi_1 - \phi_a - \phi_b) \delta(1-z) C_F \frac{\alpha_s}{2\pi} \left[-\ln^2 \frac{Q^2}{k^2} + 3 \ln \frac{Q^2}{k^2} - \frac{7}{2} + \frac{\pi^2}{3} \right]. \quad (23)$$

Upon adding Eqs. (21) and (23) we find

$$\frac{d\Delta_T\hat{\sigma}^{V+R}}{dQ^2d\phi_1} = \frac{\alpha^2}{9\hat{s}Q^2} \frac{\alpha_s}{2\pi} \cos(2\phi_1 - \phi_a - \phi_b) \left\{ 2\Delta_T P_{qq}(z) \ln \frac{Q^2}{k^2} + C_F \left[8z \left[\frac{\ln(1-z)}{1-z} \right]_+ - \frac{8z \ln z}{1-z} - \frac{6z \ln^2 z}{1-z} \right. \right. \\ \left. \left. + 4(1-z) - \frac{7}{2} \delta(1-z) \right] \right\}, \quad (24)$$

where

$$\Delta_T P_{qq}(z) \equiv C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(1-z) - (1-z) \right] \quad (25)$$

is the transverse splitting function for the transition $q \rightarrow q$, which agrees with the one calculated in Ref. [5] and which was already used for the evolution of our transverse (twist-2) quark densities in Sec. III. We shall factorize precisely this logarithmically singular term $\Delta_T P_{qq}(z) \ln Q^2/k^2$ into the polarized quark densities and take the remaining terms in Eq. (24) as the finite $O(\alpha_s)$ corrections to the cross section. Of course there is a well-known arbitrariness in performing the factorization of mass singularities. For instance, we could have alternatively decided to absorb *all* α_s corrections into the quark densities as was suggested for unpolarized deep-

inelastic scattering in Ref. [15] (“DIS scheme”). Any change in the definition of quark densities beyond the leading order is compensated by a corresponding change in the respective two-loop splitting function [28]. However, the latter is not known up to now for the transversely polarized case. This of course limits the validity of our numerical results presented below for which we have to stick to the one-loop evolution kernel $\Delta_T P_{qq}(z)$ in Eq. (25). The present situation is very similar to that in the unpolarized case where also Drell-Yan K factors have been studied before the two-loop anomalous dimensions were calculated [29]. In any case our results will show up their full importance when they will be combined with future calculations of the transverse two-loop splitting functions.

Adding finally all contributions including the LO one we find, for the hadronic cross section,

$$\frac{d\Delta_T\sigma}{dQ^2d\phi_1} = \frac{\alpha^2}{9SQ^2} \cos(2\phi_1 - \phi_a - \phi_b) \int_{\tau/x_1}^1 \frac{dx_1}{x_1} \int_{\tau/x_1}^1 \frac{dx_2}{x_2} \Delta_T H(x_1, x_2, Q^2) \left[\delta(1-z) + C_F \frac{\alpha_s}{2\pi} \Delta_T f(z) \right] \quad (26)$$

with $z = \tau/x_1 x_2$ and

$$\Delta_T f(z) \equiv 8z \left[\frac{\ln(1-z)}{1-z} \right]_+ - \frac{8z \ln z}{1-z} - \frac{6z \ln^2 z}{1-z} \\ + 4(1-z) - \frac{7}{2} \delta(1-z). \quad (27)$$

For the numerical evaluation of Eq. (26) it is convenient to have the Mellin- n moments of $\Delta_T f(z)$ which are defined by

$$\Delta_T f^n \equiv \int_0^1 z^{n-1} \Delta_T f(z) dz, \quad (28)$$

since the corresponding moments of the cross section take the simple form

$$\int_0^1 \tau^{n-1} S Q^2 \frac{d\Delta_T \sigma}{dQ^2 d\phi_1} d\tau$$

$$= \frac{\alpha^2}{9} \cos(2\phi_1 - \phi_a - \phi_b) \Delta_T H^n(Q^2) \left[1 + C_F \frac{\alpha_s}{2\pi} \Delta_T f^n \right], \quad (29)$$

where $\Delta_T H^n(Q^2) \equiv 2 \sum_q e_q^2 \Delta_T q^n(Q^2) \Delta_T \bar{q}^n(Q^2)$. We find

$$\Delta_T f^n = 8 \sum_{k=1}^n \frac{1}{k} \sum_{j=1}^k \frac{1}{j} - 8 \left[\sum_{k=1}^n \frac{1}{k^2} - \frac{\pi^2}{6} \right]$$

$$+ 12 \left[\sum_{k=1}^n \frac{1}{k^3} - \zeta(3) \right] + \frac{4}{n(n+1)} - \frac{7}{2} \quad (30)$$

with $\zeta(3) \approx 1.20206$. The corresponding results for the unpolarized case can be derived from Ref. [17] and are collected in Appendix B. Let us now study the numerical importance of the $O(\alpha_s)$ corrections to the transverse asymmetry of the total Drell-Yan cross section, which is defined in analogy with $A_T(q_T)$ in Sec. III as

$$A_T \equiv \frac{d\Delta_T \sigma / dQ^2 d\phi_1}{d\sigma / dQ^2 d\phi_1}. \quad (31)$$

Obviously A_T is a scaling function in the variable $\tau = Q^2/S$ if the scale dependence of the parton distributions and α_s is neglected. We have checked that A_T is indeed rather independent of the scale chosen, and therefore we present it as a function of τ in Fig. 4. We have used the same sets of parton distributions as introduced in Sec. III and, for definiteness, we have taken the scale $Q^2 = 49 \text{ GeV}^2$ [27]. The almost exact scaling behavior of A_T makes it possible to obtain the value of the asymmetry for arbitrary values of Q^2 and S from Fig. 4. It becomes obvious that the asymmetry is strongly dependent on the size of $\Delta_T \bar{q}$ [30] and that it is much larger now (at least for the large $\Delta_T \bar{q}$) than for the q_T differential case. Comparing the predictions for the leading-order case ($q\bar{q} \rightarrow \mu^+ \mu^-$ only) and for the $O(\alpha_s)$ results we see that the asymmetry is stable, which is mainly due to the fact that the most important corrections [terms $\sim \delta(1-z)$ in

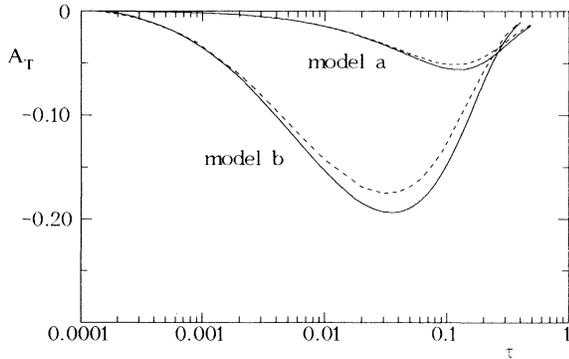


FIG. 4. A_T according to Eq. (31) as a function of $\tau = Q^2/S$ for the two models (18) and (19): $O(\alpha_s^0)$ (dashed lines), $O(\alpha_s)$ (solid lines). The scale of the parton distributions and α_s was chosen to be $Q^2 = 49 \text{ GeV}^2$.

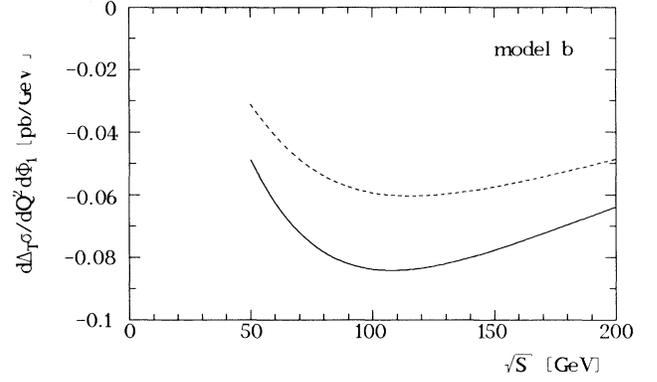


FIG. 5. $d\Delta_T \sigma / dQ^2 d\phi_1$ according to Eq. (26) at $Q^2 = 49 \text{ GeV}^2$ for model b, Eq. (19), in $O(\alpha_s^0)$ (dashed line) and $O(\alpha_s)$ (solid line).

Eqs. (27) and (B5)] are the same for the polarized and the unpolarized cross sections. Note that in contrast with the asymmetry the individual cross sections $d\Delta_T \sigma / dQ^2 d\phi_1$ and $d\sigma / dQ^2 d\phi_1$ receive corrections of order 30%. This can be seen in Fig. 5, where we show the transversely polarized cross section $d\Delta_T \sigma / dQ^2 d\phi_1$ vs \sqrt{S} in $O(\alpha_s^0)$ and $O(\alpha_s)$ for model b, Eq. (19), using again $Q^2 = 49 \text{ GeV}^2$. As discussed above, the expressiveness of the comparison between LO and higher-order results is still limited; nevertheless, it seems that one can be rather confident that at least the asymmetry A will remain stable even when the correct two-loop evolution is implemented. Therefore one may tentatively conclude from Fig. 4 that the asymmetry A should give a good clue to the transverse parton distributions, especially the sea quark distribution, of the proton.

V. THE RAPIDITY DIFFERENTIAL CROSS SECTION

Finally we want to briefly discuss the cross-section differential in the rapidity y of the outgoing virtual photon, which is defined by

$$y \equiv \frac{1}{2} \ln \frac{q_0^* + q_3^*}{q_0^* - q_3^*}, \quad (32)$$

q_0^* and q_3^* being the zeroth and third component of the photon's momentum in the c.m.s. of the incoming protons. Again we start with the result for the LO subprocess cross section, which is

$$\frac{d\hat{\sigma}}{dQ^2 dy d\phi_1} = \frac{\alpha^2}{9SQ^2} \cos(2\phi_1 - \phi_a - \phi_b)$$

$$\times \delta(x_1 - x_1^0) \delta(x_2 - x_2^0) \quad (33)$$

with $x_1^0 \equiv \sqrt{\tau} e^y$ and $x_2^0 \equiv \sqrt{\tau} e^{-y}$. The hadronic cross section is given by

$$\frac{d\Delta_T \sigma}{dQ^2 dy d\phi_1} = \int_{x_1^0}^1 dx_1 \int_{x_2^0}^1 dx_2 \frac{d\Delta_T \hat{\sigma}}{dQ^2 dy d\phi_1}$$

$$\times \Delta_T H(x_1, x_2, Q^2). \quad (34)$$

We now turn to the $O(\alpha_s)$ corrections to this cross sec-

tion. y and $\cos\theta$ (which was defined in Sec. II in the c.m.s. of the incoming *partons*) are related via

$$\cos\theta = \frac{q_0}{|\mathbf{q}|} \frac{x_1^0 x_2 - x_2^0 x_1}{x_1^0 x_2 + x_2^0 x_1}. \quad (35)$$

By changing from $\cos\theta$ to y in Eq. (10) we can obtain the $O(\alpha_s)$ cross section $d\Delta_T \hat{\sigma} / dQ^2 dy d\phi_1$. To this end we use the techniques developed in Ref. [17] which allow for an explicit extraction of the collinear and infrared singularities. After some algebra we find, for the subprocess cross section,

$$\begin{aligned} \frac{d\Delta_T \hat{\sigma}}{dQ^2 dy d\phi_1} &= \frac{\alpha^2}{9S Q^2} C_F \frac{\alpha_s}{2\pi} \frac{4\tau(x_1 x_2 + \tau)}{(x_1 + x_1^0)(x_2 + x_2^0)} \cos(2\phi_1 - \phi_a - \phi_b) \\ &\times \left\{ \delta(x_1 - x_1^0) \delta(x_2 - x_2^0) \left[\frac{1}{2} \ln^2 \frac{(1-x_1^0)(1-x_2^0)}{x_1^0 x_2^0} - \frac{7}{4} + \frac{\pi^2}{6} \right] \right. \\ &+ \delta(x_2 - x_2^0) \left[\frac{1}{(x_1 - x_1^0)_{++}} \ln \frac{2x_1(1-x_2^0)}{x_1^0 x_2^0 (x_1 + x_1^0)} + \left[\frac{\ln(x_1 - x_1^0)}{x_1 - x_1^0} \right]_{++} \right] \\ &+ \delta(x_1 - x_1^0) \left[\frac{1}{(x_2 - x_2^0)_{++}} \ln \frac{2x_2(1-x_1^0)}{x_1^0 x_2^0 (x_2 + x_2^0)} + \left[\frac{\ln(x_2 - x_2^0)}{x_2 - x_2^0} \right]_{++} \right] \\ &+ \frac{1}{[(x_1 - x_1^0)(x_2 - x_2^0)]_{++}} + 2 \frac{(x_1 + x_1^0)(x_2 + x_2^0)}{(x_1 x_2^0 + x_2 x_1^0)^2} - \frac{6 \ln \left[\frac{x_1 x_2 + x_1^0 x_2^0}{x_1 x_2^0 + x_2 x_1^0} \right]}{(x_1 - x_1^0)(x_2 - x_2^0)} \\ &+ \ln \frac{Q^2}{k^2} \left[\delta(x_1 - x_1^0) \delta(x_2 - x_2^0) \left[\frac{3}{2} + \ln \frac{(1-x_1^0)(1-x_2^0)}{x_1^0 x_2^0} \right] \right. \\ &\left. \left. + \delta(x_1 - x_1^0) \frac{1}{(x_2 - x_2^0)_{++}} + \delta(x_2 - x_2^0) \frac{1}{(x_1 - x_1^0)_{++}} \right] \right\}, \quad (36) \end{aligned}$$

where we have defined a new prescription:

$$\begin{aligned} \int_{x_1^0}^1 \frac{f(x_1)}{(x_1 - x_1^0)_{++}} dx_1 &\equiv \int_{x_1^0}^1 \frac{f(x_1) - f(x_1^0)}{x_1 - x_1^0} dx_1, \\ \int_{x_1^0}^1 dx_1 \int_{x_2^0}^1 dx_2 \frac{f(x_1, x_2)}{[(x_1 - x_1^0)(x_2 - x_2^0)]_{++}} &\equiv \int_{x_1^0}^1 dx_1 \int_{x_2^0}^1 dx_2 \frac{f(x_1, x_2) - f(x_1, x_2^0) - f(x_1^0, x_2) + f(x_1^0, x_2^0)}{(x_1 - x_1^0)(x_2 - x_2^0)}. \end{aligned} \quad (37)$$

In Eq. (36) the virtual corrections are already included such that only collinear divergencies are remaining, which are removed by factorization. Analogously to Sec. IV we want to subtract only the singularity proportional to $\Delta_T P_{qq}(z) \ln Q^2 / k^2$. For the y differential cross section this simply means to subtract the $\ln Q^2 / k^2$ terms in Eq. (36), which equal

$$\begin{aligned} \frac{\alpha^2}{9S Q^2} \frac{\alpha_s}{2\pi} C_F \ln \frac{Q^2}{k^2} \cos(2\phi_1 - \phi_a - \phi_b) &\left[\delta(x_1 - x_1^0) \int_{x_2^0}^1 \frac{dz}{z} \Delta_T P_{qq} \left[\frac{x_2^0}{z} \right] \delta(x_2 - z) \right. \\ &\left. + \delta(x_2 - x_2^0) \int_{x_1^0}^1 \frac{dz}{z} \Delta_T P_{qq} \left[\frac{x_1^0}{z} \right] \delta(x_1 - z) \right]. \end{aligned}$$

The remaining terms are regarded as the $O(\alpha_s)$ corrections to the cross section.

For a numerical evaluation of our results we define the asymmetry $A_T(y)$ as before:

$$A_T(y) \equiv \frac{d\Delta_T \sigma / dQ^2 dy d\phi_1}{d\sigma / dQ^2 dy d\phi_1}. \quad (38)$$

The expression for the unpolarized cross section which again also receives $O(\alpha_s)$ contributions from incoming

gluons can be derived with some modifications concerning the factorization scheme from Ref. [17] and is presented in Appendix B. Figure 6 shows our results for the asymmetry $A_T(y)$ for our two different choices for the spin-dependent sea quark distributions for both LO (dashed lines) and $O(\alpha_s)$ (solid lines). We have again chosen $Q^2 = 49 \text{ GeV}^2$ [27] and $\sqrt{S} = 100 \text{ GeV}$. Obviously the asymmetry is again very sensitive to the transverse sea quark distribution. Furthermore, the asymmetry is rather stable to $O(\alpha_s)$ corrections if one does not go to

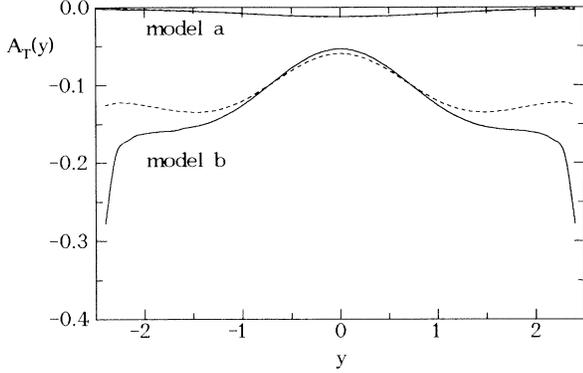


FIG. 6. $A_T(y)$ according to Eq. (38) for the two models (18) and (19) at $Q^2=49 \text{ GeV}^2$ and $\sqrt{S}=100 \text{ GeV}$: $O(\alpha_s^0)$ (dashed lines), $O(\alpha_s)$ (solid lines).

large $|y|$ at the edge of the phase space. We may therefore conclude that also from studying the rapidity differential cross sections one might be able to discriminate between different models for the transverse sea quark distributions in the proton.

VI. CONCLUSIONS

We have studied dimuon production in collisions of two transversely polarized protons. We have presented

two different parametrizations for the transverse sea quark distribution $\Delta_T \bar{q}$ in the proton, which is the quantity that mainly determines the asymmetries. As we have shown, studying the q_T differential cross section does not provide a good tool to measure $\Delta_T \bar{q}$ since the asymmetry is too strongly suppressed by the contribution of gluons to the unpolarized cross section. This does not happen for the total and the rapidity differential cross sections since here the dominant contribution comes from $q\bar{q} \rightarrow \mu^+ \mu^-$ annihilation. Since $O(\alpha_s)$ corrections to these cross sections turned out to be rather large in the unpolarized case, we have presented these corrections also for the case of transverse polarization. Our main result is that although the corrections to the cross sections are of order 30%, the cross-section asymmetries are rather stable against higher-order effects. This is in particular true for the total Drell-Yan cross section and for the rapidity differential cross section at not too large $|y|$. As these are strongly dependent on the transverse sea quark distribution, we can be confident that their measurement at future polarized hadron colliders would allow a determination of the up to now completely unknown transversity distributions in the proton.

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APPENDIX A

In this appendix we want to give some integrals needed for the derivation of Eqs. (10) and (21). Let us define

$$I[f, n] \equiv \int_0^{2\pi} d\phi \int_{-1}^1 d \cos\theta_1 \frac{f(\theta_1, \phi)}{\{q_0 - |\mathbf{q}|[\sin\theta_1 \sin\theta \cos(\phi - \phi_1) + \cos\theta \cos\theta_1]\}^n}. \quad (\text{A1})$$

Equation (10) is then obtained using

$$I[1, 2] = \frac{4\pi}{Q^2}, \quad I[1, 3] = \frac{4\pi q_0}{Q^4}, \quad I[\cos\theta_1, 3] = \frac{4\pi |\mathbf{q}| \cos\theta}{Q^4}, \quad I[\sin^2\theta_1, 4] = \frac{8\pi}{3Q^6} (Q^2 + 2|\mathbf{q}|^2 \sin^2\theta),$$

$$I[\cos^2(\phi - \phi_1), 2] = \frac{4\pi}{Q^2} - \frac{2\pi \ln \left[1 + |\mathbf{q}|^2 \sin^2\theta / Q^2 \right]}{|\mathbf{q}|^2 \sin^2\theta},$$

$$I[\sin^2(\phi - \phi_1), 2] = \frac{2\pi \ln \left[1 + |\mathbf{q}|^2 \sin^2\theta / Q^2 \right]}{|\mathbf{q}|^2 \sin^2\theta},$$

$$I[\sin(\phi - \phi_1) \cos(\phi - \phi_1), 2] = 0$$

$$I[\sin\theta_1 \sin(\phi - \phi_1), 3] = 0, \quad I[\sin\theta_1 \cos(\phi - \phi_1), 3] = \frac{4\pi |\mathbf{q}| \sin\theta}{Q^4}.$$

The ‘‘+’’ prescriptions in Eq. (21) enter via the relations

$$\frac{1}{\hat{s} - Q^2 - k^2} \approx \frac{z}{Q^2} \left[\frac{1}{(1-z)_+} + \frac{1}{2} \ln \left[\frac{4Q^2}{k^2} \right] \delta(1-z) \right], \quad (\text{A2})$$

$$\frac{1}{\hat{s} - Q^2 - k^2} \ln \frac{\sqrt{\hat{s}} |\mathbf{q}|^2 + Q^2 k^2 + \sqrt{\hat{s}} |\mathbf{q}|}{\sqrt{\hat{s}} |\mathbf{q}|^2 + Q^2 k^2 - \sqrt{\hat{s}} |\mathbf{q}|} \approx \frac{z}{Q^2} \left[-\frac{2 \ln z}{1-z} + \frac{1}{(1-z)_+} \ln \frac{Q^2}{k^2} + 2 \left[\frac{\ln(1-z)}{1-z} \right]_+ + \delta(1-z) \left[\frac{1}{4} \ln^2 \frac{Q^2}{k^2} - \frac{\pi^2}{12} \right] \right]. \quad (\text{A3})$$

APPENDIX B

In this appendix we collect the corresponding results for the unpolarized Drell-Yan cross sections. All of them can be derived from Ref. [17]. For the total and the y differential cross sections the results of Ref. [17] have to be transformed from the DIS scheme used in [17] to our scheme in which we simply subtract the $\ln Q^2/k^2$ singularity. For all three cross sections the hadronic cross section is obtained by convoluting the subprocess cross sections with the appropriate combinations of unpolarized parton distributions:

$$D\sigma = \int dx_1 \int dx_2 [D\hat{\sigma}^{q\bar{q}}H(x_1, x_2, Q^2) + D\hat{\sigma}^{qg}K(x_1, x_2, Q^2) + D\hat{\sigma}^{gq}K(x_2, x_1, Q^2)], \quad (\text{B1})$$

where the integration limits for the cases $D = d/dQ^2 d\phi_1 dq_T^2$, $D = d/dQ^2 d\phi_1$, and $D = d/dQ^2 dy d\phi_1$ can be read off from Eqs. (13), (26), and (34), respectively. In Eq. (B1) $H(x_1, x_2, Q^2)$ is the unpolarized analogue of $\Delta_T H(x_1, x_2, Q^2)$ in Eq. (14) and

$$K(x_1, x_2, Q^2) \equiv \sum_q e_q^2 [q(x_1, Q^2) + \bar{q}(x_1, Q^2)] G(x_2, Q^2) \quad (\text{B2})$$

with the unpolarized gluon distribution $G(x, Q^2)$. The subprocess cross sections for the q_T differential case are given by

$$\frac{d\hat{\sigma}^{q\bar{q}}}{dQ^2 d\phi_1 dq_T^2} = \frac{\alpha^2}{9\hat{s}} C_F \frac{\alpha_s}{2\pi} \frac{4}{q_T^2} \frac{1}{(\hat{s} - Q^2) \left[1 - \frac{4\hat{s}q_T^2}{(\hat{s} - Q^2)^2} \right]^{1/2}} \frac{\hat{s}}{Q^2} \left[1 + \frac{Q^4}{\hat{s}^2} - \frac{2q_T^2}{\hat{s}} \right], \quad (\text{B3})$$

$$\begin{aligned} \frac{d\hat{\sigma}^{qg}}{dQ^2 d\phi_1 dq_T^2} &= \frac{\alpha^2}{9\hat{s}} T_R \frac{\alpha_s}{2\pi} \frac{4}{q_T^2} \frac{1}{(\hat{s} - Q^2) \left[1 - \frac{4\hat{s}q_T^2}{(\hat{s} - Q^2)^2} \right]^{1/2}} \frac{\hat{s}}{2Q^2} \left[1 - \frac{Q^2}{\hat{s}} + \frac{q_T^2}{\hat{s}} + \frac{3Q^2 q_T^2}{\hat{s}^2} \right. \\ &\quad \left. - \frac{2Q^2(\hat{s} - Q^2)}{\hat{s}^2} + \frac{2Q^4(\hat{s} - Q^2)}{\hat{s}^3} \right] \\ &= \frac{d\hat{\sigma}^{gq}}{dQ^2 d\phi_1 dq_T^2}, \end{aligned} \quad (\text{B4})$$

with the color factor $T_R = \frac{1}{2}$. For the total Drell-Yan cross section we find

$$\frac{d\hat{\sigma}^{q\bar{q}}}{dQ^2 d\phi_1} = \frac{\alpha^2}{9\hat{s}Q^2} 2 \left\{ \delta(1-z) + \frac{\alpha_s}{2\pi} C_F \left[4(1+z^2) \left[\frac{\ln(1-z)}{1-z} \right]_+ - 4(1+z^2) \frac{\ln z}{1-z} - 4(1-z) - \frac{7}{2} \delta(1-z) \right] \right\}, \quad (\text{B5})$$

$$\begin{aligned} \frac{d\hat{\sigma}^{qg}}{dQ^2 d\phi_1} &= \frac{\alpha^2}{9\hat{s}Q^2} 2 \frac{\alpha_s}{2\pi} T_R \left[\left[z^2 + (1-z)^2 \right] \ln \frac{1-z}{z^2} - \frac{3}{2} z^2 + z - \frac{1}{2} \right] \\ &= \frac{d\hat{\sigma}^{gq}}{dQ^2 d\phi_1}. \end{aligned} \quad (\text{B6})$$

Finally the rapidity differential subprocess cross sections are given by

$$\begin{aligned} \frac{d\hat{\sigma}^{q\bar{q}}}{dQ^2 dy d\phi_1} &= \frac{2\alpha^2}{9SQ^2} \delta(x_1 - x_1^0) \delta(x_2 - x_2^0) \\ &\quad + \frac{4\alpha^2}{9SQ^2} C_F \frac{\alpha_s}{2\pi} \frac{(x_1 x_2 + \tau) [\tau^2 + (x_1 x_2)^2]}{(x_1 x_2)^2 (x_1 + x_1^0)(x_2 + x_2^0)} \\ &\quad \times \left\{ \delta(x_1 - x_1^0) \delta(x_2 - x_2^0) \left[\frac{1}{2} \ln^2 \frac{(1-x_1^0)(1-x_2^0)}{x_1^0 x_2^0} - \frac{7}{4} + \frac{\pi^2}{6} \right] \right. \\ &\quad + \delta(x_2 - x_2^0) \left[\frac{1}{(x_1 - x_1^0)_{++}} \ln \frac{2x_1(1-x_2^0)}{x_1^0 x_2^0 (x_1 + x_1^0)} + \left[\frac{\ln(x_1 - x_1^0)}{x_1 - x_1^0} \right]_{++} - \frac{x_1 - x_1^0}{x_1^2 + (x_1^0)^2} \right] \\ &\quad + \delta(x_1 - x_1^0) \left[\frac{1}{(x_2 - x_2^0)_{++}} \ln \frac{2x_2(1-x_1^0)}{x_1^0 x_2^0 (x_2 + x_2^0)} + \left[\frac{\ln(x_2 - x_2^0)}{x_2 - x_2^0} \right]_{++} - \frac{x_2 - x_2^0}{x_2^2 + (x_2^0)^2} \right] \\ &\quad \left. + \frac{1}{[(x_1 - x_1^0)(x_2 - x_2^0)]_{++}} - 2 \frac{x_1 x_2 (x_1 + x_1^0)(x_2 + x_2^0) \tau}{[\tau^2 + (x_1 x_2)^2] (x_1 x_2^0 + x_2 x_1^0)^2} \right\}, \end{aligned} \quad (\text{B7})$$

$$\frac{d\hat{\sigma}^{gq}}{dQ^2 dy d\phi_1} = \frac{4\alpha^2}{9SQ^2} T_R \frac{\alpha_s}{2\pi} \left[\delta(x_2 - x_2^0) \left[\frac{(x_1^0)^2 + (x_1 - x_1^0)^2}{2x_1^3} \ln \frac{2x_1^2(1-x_2^0)}{x_1^0 x_2^0 (x_1 + x_1^0)} - \frac{1}{2x_1} \right] \right. \\ \left. + \frac{1}{(x_2 - x_2^0)_{++}} \frac{x_2^0(\tau + x_1 x_2)[\tau^2 + (\tau - x_1 x_2)^2]}{x_1^3 x_2^2 (x_1 x_2^0 + x_2 x_1^0)(x_2 + x_2^0)} \right. \\ \left. + \frac{\tau(\tau + x_1 x_2)[x_1 x_2^2 x_1^0 + \tau(x_1 x_2^0 + 2x_2 x_1^0)]}{(x_1 x_2)^2 (x_1 x_2^0 + x_2 x_1^0)^3} \right], \quad (\text{B8})$$

$$\frac{d\hat{\sigma}^{qg}}{dQ^2 dy d\phi_1} = \frac{d\hat{\sigma}^{gq}}{dQ^2 dy d\phi_1} (1 \leftrightarrow 2). \quad (\text{B9})$$

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