Avoiding the Gribov problem by dynamical gauge fixing

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We present a method to quantize SU(N) gauge theories which does not use gauge fixing, and therefore circumvents the problematics around gauge fixing and Gribov copying. The method is illustrated in the Abelian theory where no Gribov ambiguity is present, and it is shown to be equivalent to Landau gauge fixing in this case. It is shown that there are fundamental differences between Abelian and non-Abelian theories. These differences imply that in Abelian theories the gauge particle must be massless, but this is not necessarily true in non-Abelian theories.

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I. INTRODUCTION

In order to quantize a gauge theory and develop a perturbative expansion, i.e., to have well-defined gauge potential propagators, it is necessary to eliminate the redundant gauge degrees of freedom from the functional integral representation of the generating functional. The way in which this is done is by means of gauge fixing. Faddeev and Popov derived the modified form of the functional integral when gauge fixing is imposed [1]. A basic assumption made in the derivation of Faddeev and Popov is that the gauge-fixing condition imposed determines the gauge functions uniquely [1]. Doubt was cast on the correctness of the Faddeev-Popov procedure after the discovery by Gribov that the Coulomb and Landau gauges do not fix the gauge functions uniquely in a non-Abelian theory (Gribov problem) [2]. The same shortcoming was found in various other gauges [3].

A question that immediately arose after Gribov's discovery was whether this problem is a peculiarity of only some specific gauges or whether it was of a more general nature. Singer [4] showed that the problem is of a much more general nature when he proved that in a Euclidean formulation of a compact, semisimple, non-Abelian theory no global, continuous gauge fixing is possible if the boundary conditions imposed on the gauge functions at infinity imply the identification of space-time with S^4. A similar analysis was carried out by Killingback [5] for Euclidean gauge theories on which periodic boundary conditions are imposed, i.e., a gauge theory formulated on the four-torus T^4. Once again the conclusion was that no global continuous gauge fixing is possible for either compact, semisimple, non-Abelian theories, or the Abelian U(1) theory. This has particular implications for lattice theories where periodic boundary conditions are imposed. Whenever gauge-dependent quantities (such as gauge-field propagators) are calculated in these theories, they are affected by the Gribov ambiguity and correctional steps must be taken [6].

The work of Singer and Killingback does not totally exclude the possibility of finding gauges that do determine the gauge functions uniquely. However, such gauges must obviously violate at least one of the conditions required for Singer's argument. Singer pointed out that if no conditions are imposed at infinity such gauges can in fact be found. Alternatively such gauges may exist in Minkowski space. However, for such gauges an analytic continuation to Euclidean space must show that either the conditions imposed by Singer are violated or the pole structure of the gauge-field propagators in these gauges is such that an analytic continuation to Euclidean space is no longer possible. Some gauges do, in fact, exhibit the latter feature [7]. One can, of course, also circumvent Singer's argument by dropping the requirements of a global or continuous gauge-fixing condition.

Keeping the above remarks in mind one notes that a possible resolution of the Gribov ambiguity is to resort to gauges which are not subject to Singer's argument. This possibility was pursued for a considerable time in the literature and several gauges of this type have been invented [7,8]. However, these gauges are usually plagued by technical difficulties such as noncovariance or the propagators of the gauge potentials in these gauges are very complicated and in some cases the pole structure of these propagators does not allow an analytic continuation to Euclidean space.

A different stance taken in the literature towards the Gribov ambiguity is that the Faddeev-Popov procedure is correct even in the presence of Gribov copying [9,10]. In this case it is argued that the integral

$$\eta=\int d[g]\Delta_F({}^gA)\delta(F({}^gA)-C)$$

over all gauge transformations g, with $\Delta_F({}^gA)$ the Faddeev-Popov determinant and $F({}^gA)$ the gauge-fixing condition, does not depend on the gauge potential A_μ or C. It should be noted that the Faddeev-Popov determinant and not its absolute value (as would be required for a change of the volume element under a change of variables) appears in this integral. The independence of η from A_μ and C ensures that when the Faddeev-Popov procedure is applied in the presence of Gribov copying it only leads to a multiplicative constant which cancels when the generating functional is normalized. The independence of η from A and C was, however, only rigorously proved on a lattice [10]. Furthermore, this procedure fails when $\eta=0$.

Yet another approach to the quantization of gauge theories was developed in Ref. [11]. Here the gauge potentials A_μ are eliminated in favor of the field strengths

$F_{\mu\nu}$. Once again this can only be done with an appropriate choice of gauge. This approach was originally not developed with the sole purpose of avoiding the Gribov ambiguity, but rather as an alternative, and more natural, approach to the quantization of gauge theories which uses the physical degrees of freedom, $F_{\mu\nu}$, rather than the gauge potentials A_μ which contain unphysical gauge degrees of freedom. Feynman rules for Abelian theories can be derived in this approach but, unfortunately, the derivation of Feynman rules for non-Abelian theories becomes very difficult.

The approach originally proposed by Gribov to avoid the gauge-fixing ambiguity was to restrict the domain of integration over the gauge potentials to a region in which the Faddeev-Popov determinant remains positive [2]. This approach was elaborated on by Zwanziger [12]. In these works he succeeded in deriving an approximate effective action which is also practically implementable from a perturbative point of view.

Several other investigations into the Gribov ambiguity and its possible implications can be found in Ref. [13]. The common conclusion drawn from all the abovementioned approaches seems to be that the Gribov ambiguity is not crucial for the perturbative aspects of a gauge theory, but it may be essential for the understanding of nonperturbative aspects and the infrared behavior of a gauge theory [2,7,12,13]. In particular the infrared divergencies seem to be much less severe when due attention is paid to the Gribov problem.

There is an alternative approach to the quantization of gauge theories, not mentioned until now, which does not explicitly depend on a gauge fixing condition and therefore circumvents the problem of Gribov copies. This approach is (as far as we can establish) due to Popov [14]. The idea is essentially a very simple generalization of the Faddeev-Popov gauge-fixing procedure. In the normal Faddeev-Popov approach one assumes a gauge-fixing condition $F({}^gA)=0$ which determines the gauge transformations g uniquely. Then one writes the identity in the form

$$1=\Delta_F[A]\int d[g]\delta(F({}^gA))$$

with $\Delta_F[A]$ the Faddeev-Popov determinant which is, by invariance of the group measure, a gauge-invariant quantity. Inserting this identity into the functional integral and using the gauge invariance of the action and Faddeev-Popov determinant one can factorize the functional integral into an integral over the local gauge group and a remaining integral which is no longer gauge invariant because of the gauge-fixing condition. The divergent integral over the gauge group cancels when the generating functional is normalized. One can generalize this procedure by considering any function $F(A)$ with the property that the integral

$$I(A)=\int d[g]F({}^gA)$$

exists. By invariance of the group measure $I(A)$ is gauge invariant. One then inserts the identity

$$1=\int d[g]F({}^gA)/I(A)$$

into the functional integral and as before one uses the gauge invariance of the action and $I(A)$ to factorize the functional integral into an integral over the local gauge group and a remaining integral which is no longer gauge invariant if $F(A)$ is gauge noninvariant. Note that in the conventional Faddeev-Popov approach explicit gauge-fixing is required and that the identity

$$1=\Delta_F(A)\int d[g]\delta(F({}^gA))$$

only holds in the absence of Gribov copies, while in the latter approach explicit gauge fixing is never mentioned. This idea was further pursued by Zwanziger [15] and Parrinello and Jona-Lasinio [16] with specific choices of the function $F(A)$.

The approach we present here follows very much the same line of thought as described in the above paragraph. However, the resolution of the identity we use here as well as the factorization of the functional integral is technically different and resembles more closely the choice of a unitary gauge in spontaneously broken gauge theories.

The paper is organized in the following way. In Sec. (II) the method is illustrated in detail for an Abelian U(1) gauge theory. The generalization to larger unitary groups is fairly straightforward. A detailed discussion of SU(2) is presented in Sec. III and the generalization to SU(N) is outlined in Sec. IV. Section V contains a discussion and conclusions.

II. ABELIAN THEORY

Although the Abelian U(1) gauge theory is free of Gribov ambiguities we prefer to develop, for pedagogical reasons, our method in this simple setting. The generalization to other unitary groups is fairly straightforward and is described in the next two sections.

Consider therefore a U(1) gauge theory:

$$\mathcal{L}_{\rm YM}=-\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}\ ,\qquad F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu\ . \tag{1}$$

Introducing sources J^μ,

$$\partial_\mu J^\mu=0\ , \tag{2}$$

the generating functional

$$Z[J]\propto\int[dA]\exp\left[i\int d^4x(\mathcal{L}_{\rm YM}-J_\mu A^\mu)\right] \tag{3}$$

is undefined since the integration involves an integration over unphysical gauge degrees of freedom which, because of gauge invariance, leads to a divergent factor. Stated differently the propagator of the free gauge potential A_μ does not exist. In the Faddeev-Popov procedure one avoids this difficulty by introducing gauge fixing. The integration over gauge degrees of freedom can then be factorized from (3) and cancels when the generating functional is normalized.

Here we seek to factorize the integration over the gauge degrees of freedom from (3) without resorting to gauge fixing, but rather using a technique similar to the one devised by Popov and described in the introduction. To this end we introduce the identity

$$1 \propto \int [d\eta^*][d\eta][d\phi^*][d\phi] \exp\left[i \int d^4x \left[\frac{1}{2}(D_\mu\phi)^*(D^\mu\phi) + \frac{1}{2}(D_\mu\eta)^*(D^\mu\eta) - \frac{\mu^2}{2}(\phi^*\phi+\eta^*\eta) - \frac{\lambda}{4!}(\phi^*\phi+\eta^*\eta)^2 \right]\right] , \tag{4}$$

where ϕ is a complex scalar field and η is a complex scalar Grassmann (ghost) field. The covariant derivative is defined as usual

$$D_\mu = \partial_\mu + igA_\mu , \quad D_\mu^* = \partial_\mu - igA_\mu , \tag{5}$$

where g is an arbitrary coupling constant, not related to any physical coupling constant.

The trick needed to verify (4) is to introduce the identity

$$\exp\left[i\int d^4x \left[-\frac{\lambda}{4!}(\phi^*\phi+\eta^*\eta)^2 \right]\right] \propto \int [d\chi] \exp\left[i\int d^4x \left[\frac{\lambda}{4!}[\chi^2 + 2\chi(\phi^*\phi+\eta^*\eta)] \right]\right] . \tag{6}$$

Using (6) in (4) gives

$$\begin{aligned} &\int [d\eta^*][d\eta][d\phi^*][d\phi][d\chi] \exp\left[i\int d^4x \left[\frac{1}{2}(D_\mu\phi)^*(D^\mu\phi) + \frac{1}{2}(D_\mu\eta)^*(D^\mu\eta) - \frac{\mu^2}{2}(\phi^*\phi+\eta^*\eta) \right.\right. \\ &\qquad\qquad \left.\left. + \frac{\lambda}{4!}[\chi^2 + 2\chi(\phi^*\phi+\eta^*\eta)] \right]\right] \\ &\quad \propto \int [d\chi] \exp\left[i\int d^4x \left[\frac{\lambda}{4!}\chi^2 \right]\right] |D^\mu D_\mu - \mu^2 + \tfrac{1}{6}\lambda\chi| |D^\mu D_\mu - \mu^2 + \tfrac{1}{6}\lambda\chi|^{-1} \\ &\quad \propto 1 . \end{aligned} \tag{7}$$

Note that the normalization of this identity does not depend on μ or g, but that it does depend on λ. Using the identity (4) we can write for the generating functional:

$$\begin{aligned} Z[J] \propto \int [dA][d\phi^*][d\phi][d\eta^*][d\eta] \exp\left[i\int d^4x \left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - J_\mu A^\mu + \frac{1}{2}(D_\mu\phi)^*(D^\mu\phi) + \frac{1}{2}(D_\mu\eta)^*(D^\mu\eta) \right.\right. \\ \left.\left. - \frac{\mu^2}{2}(\phi^*\phi+\eta^*\eta) - \frac{\lambda}{4!}(\phi^*\phi+\eta^*\eta)^2 \right]\right] . \end{aligned} \tag{8}$$

Next we introduce the following change of variables in (8):

$$\begin{aligned} &\phi(x) = \xi(x)e^{i\theta(x)} , \quad \phi^*(x) = \xi(x)e^{-i\theta(x)} ; \\ &\xi(x) \in (-\infty, \infty) , \quad \theta(x) \in [0, \pi) . \end{aligned} \tag{9}$$

Note that in (9) we allowed the radial coordinate $\xi(x)$ to run from $-\infty$ to ∞, for this reason the angular coordinate $\theta(x)$ is restricted to the interval $[0,\pi)$ and not $[0,2\pi)$ as usual. Under (9) the measure of the path-integral transforms to

$$[d\phi^*][d\phi] = [|\xi| d\xi][d\theta] \tag{10}$$

and

$$(D_\mu\phi)^*(D^\mu\phi) = (\partial_\mu\xi)(\partial^\mu\xi) + g^2\left[A_\mu + \frac{1}{g}\partial_\mu\theta \right]\left[A^\mu + \frac{1}{g}\partial^\mu\theta \right]\xi^2 . \tag{11}$$

By means of a gauge transformation (actually a change of variables)

$$A_\mu \to A_\mu - (1/g)\partial_\mu\theta , \quad \eta \to e^{i\theta}\eta , \quad \eta^* \to e^{-i\theta}\eta^* , \tag{12}$$

we obtain from (11) and (8)

$$\begin{aligned} Z[J] \propto &\int [dA][|\xi| d\xi][d\eta^*][d\eta] \\ &\times \exp\left[i\int d^4x \left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - J_\mu A^\mu + \frac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi) + \frac{g^2}{2}\xi^2 A_\mu A^\mu + \frac{1}{2}(D_\mu\eta)^*(D^\mu\eta) - \frac{\mu^2}{2}(\xi^2+\eta^*\eta) \right.\right. \\ &\qquad \left.\left. - \frac{\lambda}{4!}(\xi^2+\eta^*\eta)^2 \right]\right] \int [d\theta] . \end{aligned} \tag{13}$$

The last factor represents the integration over the gauge group which cancels in the normalized generating functional. Note also that the λ dependency of the normalization of the identity cancels in the normalized generating functional. Hence we have for the normalized generating functional:

$$W[J]=\int[dA][|\xi|d\xi][d\eta^*][d\eta]\times\exp\left[i\int d^4x\left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}-J_\mu A^\mu+\frac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi)+\frac{g^2}{2}\xi^2A_\mu A^\mu+\frac{1}{2}(D_\mu\eta)^*(D^\mu\eta)-\frac{\mu^2}{2}(\xi^2+\eta^*\eta)-\frac{\lambda}{4!}(\xi^2+\eta^*\eta)^2\right]\right]\Big/Z[0]\,. \tag{14}$$

Note that (14) no longer possesses the U(1) symmetry.

As a final step we write the functional measure for ξ in an exponential form. To do this we note

$$[|\xi|d\xi]\alpha\prod_x|\xi_x|d\xi_x\propto\prod_x d\xi_x\exp\left[\sum_x\frac{1}{2}\ln\xi_x^2\right]\propto[d\xi]\exp\left[i\int d^4x\left[-\frac{i}{2}\delta^4(0)\ln\xi^2\right]\right]. \tag{15}$$

Using (15) in (14) gives the final result:

$$W[J]=\int[dA][d\xi][d\eta^*][d\eta]\exp\left[i\int d^4x\left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}-J_\mu A^\mu+\frac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi)+\frac{g^2}{2}\xi^2A_\mu A^\mu+\frac{1}{2}(D_\mu\eta)^*(D^\mu\eta)-\frac{\mu^2}{2}(\xi^2+\eta^*\eta)-\frac{\lambda}{4!}(\xi^2+\eta^*\eta)^2-\frac{i}{2}\delta^4(0)\ln\xi^2\right]\right]\Big/Z[0]\,. \tag{16}$$

Note that the normalized generating functional, $W[J]$, is independent of g, μ, and λ.

To enhance our understanding of this result, let us investigate it from a nonperturbative as well as a perturbative point of view. First we investigate it nonperturbatively. Let us split the gauge field in (16) into its transverse and longitudinal parts

$$A^\mu=A_T^\mu+A_L^\mu\,, \tag{17}$$

where

$$A_L^\mu=\partial^\mu(\Box^{-1}\partial A)\,. \tag{18}$$

Consider the terms

$$\frac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi)+\frac{g^2}{2}\xi^2A_\mu A^\mu=\tfrac{1}{2}[\partial_\mu\xi-ig(A_{T,\mu}+\partial_\mu\Box^{-1}\partial A)\xi][\partial^\mu\xi+ig(A_{T,\mu}+\partial^\mu\Box^{-1}\partial A)\xi] \tag{19}$$

$$=\tfrac{1}{2}[(\partial_\mu-igA_{T,\mu})\phi^*][(\partial^\mu+igA_{T,\mu})\phi]\,, \tag{20}$$

where

$$\phi=\xi\exp(ig\Box^{-1}\partial A)\,. \tag{21}$$

Since all other terms are gauge invariant and the sources are divergenceless, we can replace A^μ by A_T^μ in all the other terms. For the measure of the path integral we note that

$$[dA][|\xi|d\xi]\propto[dA_T][dA_L][|\xi|d\xi]\propto[dA_T][d\phi^*][d\phi]\,. \tag{22}$$

Now we note from the identity (4) and $\xi^2=\phi^*\phi$ that the integration over ϕ, ϕ^*, η, and η^* can be done to yield

$$W[J]=\int[dA_T]\exp\left[i\int d^4x\left[-\frac{1}{4}F_T^{\mu\nu}F_{T,\mu\nu}-J_\mu A_T^\mu\right]\right]\Big/Z[0]\,. \tag{23}$$

We note that the redundant gauge degree of freedom that occurred originally in (3) has been eliminated. The role of the two real Grassmann fields is now clear. They serve to cancel the longitudinal component of the gauge field as well as the real boson field left after transforming to the unitary gauge. Counting the number of degrees of freedom, we note that we started with an integral over four bosonic degrees of freedom in (3). Then we introduced, via the identity (4), two bosonic and two fermionic degrees of freedom which cancel exactly. Going to the unitary gauge we integrated out one bosonic degree of freedom, leaving us with an integral over five bosonic (four gauge field and one real scalar field) and two fermionic degrees of freedom. Finally, we noted above that one gauge degree of freedom and the real scalar field cancel against the fermionic degrees of freedom, leaving us with an integral which goes over only three transversal gauge degrees of freedom.

Considering (23) we note that we have recovered standard QED in the Landau gauge (the inclusion of fermions does not affect any of the above arguments since the fermion and gauge field coupling is gauge invariant). This follows since we can write (23) as an integral over four gauge degrees of freedom with the constraint that the longitudinal component vanishes, i.e.,

$$W[J]=\int [dA]\delta(\partial A)\exp\left[i\int d^4x\left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}-J_\mu A^\mu\right]\right]\Big/Z[0]\ . \tag{24}$$

which is the generating functional in the Landau gauge [17].

It is clear that (16) does not lend itself to a perturbative calculation in the phase where $\langle\xi\rangle=0$ since the $\ln\xi^2$ term cannot be expanded around this point and the unperturbed gauge field propagator is also not defined. On the other hand, when $\langle\xi\rangle:=v\neq0$ the gauge field becomes massive so that the unperturbed gauge field propagator exists and (16) lends itself to a perturbative calculation. On the tree level we can give a nonzero expectation value to ξ by choosing $\mu^2<0$ and $\lambda>0$. To calculate v to higher order we have to minimize the renormalized effective potential for ξ. This is, of course, equivalent to shifting $\xi=v+\rho$ and demanding that the tadpoles sum to zero. We introduce the following renormalized quantities:

$$\begin{aligned} g&=Z_g g_R=g_R+(Z_g-1)g_R\ ,\\ \mu^2&=Z_\mu\mu_R^2=\mu_R^2+(Z_\mu-1)\mu_R^2=\mu_R^2+\delta\mu^2\ ,\\ \lambda&=Z_\lambda\lambda_R=\lambda_R+(Z_\lambda-1)\lambda_R=\lambda_R+\delta\lambda\ . \end{aligned} \tag{25}$$

At the one loop order, to which we perform the present calculations, no wave-function renormalization is required and we set $Z_A=Z_\xi=Z_\eta=1$.

Performing the shift, we note from (16) the contribution of a massive gauge-field loop to the tadpole condition. As is well known, the massive gauge-field loop gives rise to a quartic divergence when a cutoff regularization procedure is used. This is, however, cancelled by the quartic divergence coming from the measure of the path integral for the ξ field, i.e., the term proportional to $\delta^4(0)$ in (16) [18]. Hence one is left with only quadratic and logarithmic divergencies which can be absorbed into the mass and coupling constant renormalization. A dimensional regularization scheme, which we use below, is even simpler since the contribution to the gauge-field loop coming from the part of the propagator which behaves as a constant at high momentum, vanishes. Similarly, the contribution to the tadpole condition coming from the measure of the path integral vanishes. Using a minimal subtraction scheme we have for the tadpole condition

$$v[-\mu_R^2-(\lambda_R/6)v^2+\hbar f_1(v)]=0\ , \tag{26}$$

where

$$f_1(v)=\frac{1}{(4\pi)^2}\left[(1-3\gamma)g_R^2m_A^2+(1-\gamma)\left[\frac{\lambda_R m_\rho^2}{2}-\frac{\lambda_R m_\eta^2}{3}\right]+3g_R^2m_A^2\ln\left[\frac{4\pi M^2}{m_A^2}\right]+\frac{1}{2}\lambda_R m_\rho^2\ln\left[\frac{4\pi M^2}{m_\rho^2}\right]-\frac{1}{3}\lambda_R m_\eta^2\ln\left[\frac{4\pi M^2}{m_\eta^2}\right]\right]\ , \tag{27}$$

with M an arbitrary mass scale and

$$m_A^2=g_R^2v^2\ ,\quad m_\rho^2=\mu_R^2+\frac{\lambda_R v^2}{2}\ ,\quad m_\eta^2=\mu_R^2+\frac{\lambda_R v^2}{6}\ . \tag{28}$$

This gives

$$v=v_0+\hbar v_1\ , \tag{29}$$

with

$$v_0=\sqrt{-6\mu_R^2/\lambda_R}\ ,\quad \mu_R^2<0\ ,\quad \lambda_R>0 \tag{30}$$

and

$$v_1 = 3v_0 f_1(v_0)/\lambda_R v_0^2 \ . \tag{31}$$

Next we calculate the inverse gauge-field propagator to one-loop order:

$$\Gamma(p) = \Gamma^0(p) + i\Pi(p) \ , \tag{32}$$

where $\Gamma^0(p)$ is the zero-order massive inverse propagator

$$\Gamma^0_{\mu\nu}(p) = i[(p^2 - g^2 v^2) g_{\mu\nu} - p_\mu p_\nu] \tag{33}$$

and $\Pi(p)$ is the vacuum polarization to one-loop order. Note the appearance of a mass counterterm for the gauge field. This arises from shifting $\xi = v + \rho$ and renormalizing the coupling constant $g = Z_g g_R$:

$$\tfrac{1}{2} g^2 v^2 A^2 = \tfrac{1}{2} Z_g^2 g_R^2 v^2 A^2 = \tfrac{1}{2} g_R^2 v^2 A^2 + \tfrac{1}{2}(Z_g^2 - 1) g_R^2 v^2 A^2 \ . \tag{34}$$

We also observe that the ρ tadpole and exchange graphs combine to ensure $\Pi(p=0) \propto m_A^2$, the gauge-invariant ghost diagrams vanishing in this limit.

We determine the renormalization constant for the coupling, Z_g, by imposing the physical renormalization condition

$$\Gamma(p=0) = 0 \ , \tag{35}$$

which gives

$$Z_g^2 = 3g_R^2/8\pi^2\varepsilon + \text{finite parts} \tag{36}$$

in a dimensional regularization scheme. The inverse propagator is then given to one-loop order by

$$\Gamma_{\mu\nu}(p) = i\left[[p^2 + g_R^2 F_1(p^2)] g_{\mu\nu} - \left[1 + g_R^2 F_2(p^2) + \frac{g_R^2}{8\pi^2\varepsilon} \right] p_\mu p_\nu \right] , \tag{37}$$

where $F_1(p^2)$ and $F_2(p^2)$ are finite. The only divergence left as the dimensional parameter $\varepsilon \to 0$ is the term $-(ig_R^2/8\pi^2\varepsilon) p_\mu p_\nu$. This term, however, plays a role analogous to a Landau gauge-fixing term where one identifies the gauge parameter $\xi = 8\pi^2\varepsilon/g_R^2$. Indeed the propagator is well defined in the limit $\varepsilon \to 0$ and is given by

$$D_{\mu\nu}(p) = \frac{i}{p^2 + g_R^2 F_1(p^2)} \left[-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right] . \tag{38}$$

Since we calculate the propagator to finite order in perturbation theory it depends on the renormalization scheme that is used. For a renormalization scheme different from the physical one imposed here, such as minimal subtraction, the gauge field obtains a finite mass. We impose the physical renormalization condition here since we know from the nonperturbative argument given above [see (23)] that the gauge particle is massless. Furthermore, we note that the propagator (38), calculated to finite order in perturbation theory, obtains a g_R dependency. On the other hand, we know that the full propagator should be independent of g_R since the generating functional is independent of g_R. It is only when the propagator is calculated to all orders in perturbation theory that the dependency on the renormalization scheme and g_R disappears. We can recover the full propagator from (38) by noting that since g_R is arbitrary we can take the limit $g_R \to 0$. Since perturbation theory is exact in this limit, and the full propagator does not depend on g_R, taking this limit in (38) yields the full propagator. In this limit the propagator (38) coincides with the propagator in the Landau gauge, i.e.,

$$D_{\mu\nu}(p) = \frac{i}{p^2} \left[-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right] . \tag{39}$$

This result was to be expected from (23). Note also that this result does not depend on the renormalization condition. The renormalization constant Z_g^2 can be changed by any finite amount and (39) will remain unchanged.

III. NON-ABELIAN SU(2) THEORY

Consider an SU(2) Yang-Mills theory:

$$\langle \varphi | \varphi \rangle \propto \int [dA] \exp\left[i \int d^4x \left[-\frac{1}{4} F_a^{\mu\nu} F_{a\mu\nu} \right] \right]$$

with (40)

$$F_{a\mu\nu} = \partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} - g f_{abc} A_{b\mu} A_{c\nu} \ .$$

Introduce the identity

$$1 \propto \int [d\eta^*][d\eta][d\varphi^*][d\varphi] \times \exp\left[i \int d^4x \left[\frac{1}{2}(D_\mu\varphi)^\dagger (D^\mu\varphi) + \frac{1}{2}(D_\mu\eta)^\dagger (D^\mu\eta) - \frac{\mu^2}{2}(\varphi^\dagger\varphi + \eta^\dagger\eta) - \frac{\lambda}{4!}(\varphi^\dagger\varphi + \eta^\dagger\eta)^2 \right] \right] , \tag{41}$$

where φ and η are doublets of scalar complex and Grassmann fields, respectively:

$$\varphi := \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} , \quad \eta := \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} , \tag{42}$$

and D_μ is the usual covariant derivative

$$D_\mu = \partial_\mu + i g T_a A_{a\mu} \tag{43}$$

in the fundamental two-dimensional representation of SU(2).

There is an important difference between the Abelian and non-Abelian theory in that the coupling constant g appearing in the identity (41) via the covariant derivative must be the same as the coupling constant determining the cubic and quartic gauge-field couplings of Eq. (40). In the Abelian theory this coupling constant was completely arbitrary and not related to any physical coupling constant. The proof that (41) is indeed an identity is the same as in the Abelian case (see Sec. II) and we do not repeat it here.

We insert the identity (41) into (40) and make the following change of variables:

$$\varphi=U\rho\chi\ ,\qquad \varphi^\dagger=\chi^\dagger\rho U^\dagger\ ,\qquad \chi=\begin{bmatrix}0\\1\end{bmatrix}\ . \tag{44}$$

Here $\rho\in(-\infty,\ \infty)$ and $U\in$ SU(2) where we restrict the angle of rotation around the z axis to lie in the interval $[0,\pi)$, instead of $[0,2\pi)$ to make provision for the fact that we allow $\rho\in(-\infty,\ \infty)$. Under (44) the measure of the path integral transforms to

$$[d\varphi^*][d\varphi]=[|\rho|^3 d\rho][dU]\ . \tag{45}$$

Here $[dU]$ denotes the invariant measure of SU(2). Introducing the gauge transformation,

$$A_\mu\to UA_\mu U^\dagger+(i/g)(\partial_\mu U)U^\dagger\ ,\qquad \eta\to U\eta\ ,\qquad \eta^\dagger\to\eta^\dagger U^\dagger\ , \tag{46}$$

and using the gauge invariance of the Yang-Mills Lagrangian we obtain

$$\langle\varphi|\varphi\rangle\propto\int[dA][|\rho|^3d\rho][d\eta^*][d\eta]\times\exp\left[i\int d^4x\left[-\frac{1}{4}F_a^{\mu\nu}F_{a\mu\nu}+\frac{1}{2}(\partial_\mu\rho)(\partial^\mu\rho)+\frac{1}{2}g^2\rho^2(\chi^\dagger T_aT_b\chi)A_{a\mu}A_b^\mu+\frac{1}{2}(D_\mu\eta)^\dagger(D^\mu\eta)-\frac{\mu^2}{2}(\rho^2+\eta^\dagger\eta)-\frac{\lambda}{4!}(\rho^2+\eta^\dagger\eta)^2\right]\right]\int[dU]\ . \tag{47}$$

The last factor represents the integration over the manifold of the gauge group which cancels in the normalized generating functional. We note that in the remaining integral on the right of (47) the SU(2) gauge symmetry is explicitly broken.

In the final step we write the functional measure for ρ in an exponential form using

$$[|\rho|^3d\rho]\alpha\prod_x|\rho_x|^4d\rho_x\propto\prod_x d\rho_x\exp\left[\sum_x\frac{3}{2}\ln\rho_x^2\right]\propto[d\rho]\exp\left[\int d^4x\left[-\frac{3i}{2}\delta^4(0)\ln\rho^2\right]\right]\ . \tag{48}$$

Using (48) in (47) gives for the normalized generating functional

$$W[J]=\int[dA][d\rho][d\eta^*][d\eta]\times\exp\left[i\int d^4x\left[-\frac{1}{4}F_a^{\mu\nu}F_{a\mu\nu}-J_{a\mu}A_a^\mu+\frac{1}{2}(\partial_\mu\rho)(\partial^\mu\rho)+\frac{1}{2}g^2\rho^2(\chi^\dagger T_aT_b\chi)A_{a\mu}A_b^\mu+\frac{1}{2}(D_\mu\eta)^\dagger(D^\mu\eta)-\frac{\mu^2}{2}(\rho^2+\eta^\dagger\eta)-\frac{\lambda}{4!}(\rho^2+\eta^\dagger\eta)^2-\frac{3i}{2}\delta^4(0)\ln\rho^2\right]\right]\Big/Z[0]\ . \tag{49}$$

We immediately note a couple of fundamental differences between the Abelian and non-Abelian theories. Firstly, the result (49) is independent of μ and λ, but not of g since g features in the Yang-Mills Lagrangian. Secondly, the argument given in the Abelian theory which led to the result (23) of a massless field in the Landau gauge fails in the non-Abelian theory. The reason for this is that it is no longer possible to do the integration over the longitudinal components of the gauge fields due to the self-interactions. Thirdly, when doing perturbation theory, we note that the renormalization condition which determines the renormalization constant Z_g is no longer as arbitrary as in the Abelian case. Instead Z_g has to be determined from the vertex functions. Consequently, in contrast to the Abelian case, there is no freedom left when the inverse gauge-field propagator is calculated.

IV. NON-ABELIAN SU(N) THEORIES

Consider now a SU(N) Yang-Mills theory

$$\langle\varphi|\varphi\rangle \propto \int [dA]\exp\left[i\int d^4x\left[-\frac{1}{4}F_a^{\mu\nu}F_{a\mu\nu}\right]\right],$$
$$F_{a\mu\nu}=\partial_\mu A_{a\nu}-\partial_\nu A_{a\mu}-gf_{abc}A_{b\mu}A_{c\nu}\,. \qquad (50)$$

Introduce the identity

$$1\propto\int[d\eta^\dagger][d\eta][d\varphi^\dagger][d\varphi]\exp\left[i\int d^4x\left[\frac{1}{2}(D_\mu\varphi)^\dagger(D^\mu\varphi)+\frac{1}{2}(D_\mu\eta)^\dagger(D^\mu\eta)-\frac{\mu^2}{2}(\varphi^\dagger\varphi+\eta^\dagger\eta)-\frac{\lambda}{4!}(\varphi^\dagger\varphi+\eta^\dagger\eta)^2\right]\right], \qquad (51)$$

where φ and η are now N-tuplets of scalar complex and Grassmann fields, respectively:

$$\varphi:=\begin{bmatrix}\varphi_1\\ \varphi_2\\ \vdots\\ \varphi_N\end{bmatrix},\qquad \eta:=\begin{bmatrix}\eta_1\\ \eta_2\\ \vdots\\ \eta_N\end{bmatrix}, \qquad (52)$$

and

$$D_\mu=\partial_\mu+igT_aA_{a\mu} \qquad (53)$$

is the covariant derivative in the fundamental N-dimensional representation of SU(N). The proof of (51) is as before and, as for SU(2), the coupling constants in (53) and (50) are the same.

As before the identity (51) is inserted in (49) and the following change in variables is made:

$$\varphi=U\rho\chi\,,\qquad \varphi^\dagger=\chi\rho U^\dagger\,,\qquad \chi=\begin{bmatrix}0\\0\\ \vdots\\1\end{bmatrix}. \qquad (54)$$

Here $\rho\in(-\infty,\infty)$ and

$U\in SU(N)/SU(N-1)$

with suitable restriction on the angles to make provision for the fact that $\rho\in(-\infty,\infty)$. Under (54) the measure of the path integral transforms to

$$[d\varphi^\dagger][d\varphi]=[|\rho|^{2N-1}d\rho][dU]\,, \qquad (55)$$

where $[dU]$ is the invariant measure on the coset space SU(N)/SU($N-1$). Making the gauge transformation

$$A_\mu\to UA_\mu U^\dagger+(i/g)(\partial_\mu U)U^\dagger\,,\qquad \eta\to U\eta\,,\qquad \eta^\dagger\to\eta^\dagger U^\dagger\,, \qquad (56)$$

one arrives at

$$\langle\varphi|\varphi\rangle\propto\int[dA][|\rho|^{2N-1}d\rho][d\eta^*][d\eta]\times\exp\left[i\int d^4x\left[-\frac{1}{4}F_a^{\mu\nu}F_{a\mu\nu}+\frac{1}{2}(\partial_\mu\rho)(\partial^\mu\rho)+\frac{1}{2}g^2\rho^2(\chi^\dagger T_aT_b\chi)A_{a\mu}A_b^\mu+\frac{1}{2}(D_\mu\eta)^\dagger(D^\mu\eta)-\frac{\mu^2}{2}(\rho^2+\eta^\dagger\eta)-\frac{\lambda}{4!}(\rho^2+\eta^\dagger\eta)^2\right]\right]\int_{SU(N)/SU(N-1)}[dU]\,. \qquad (57)$$

The last factor represents the integration over the coset space SU(N)/SU($N-1$) and cancels in the normalized generating functional. In contrast to SU(2) the SU(N) gauge symmetry is not completely broken in the remaining integral in (57), but there is still an SU($N-1$) gauge symmetry left as can immediately be seen from (54). As long as there is a gauge symmetry left, the propagator of the gauge field will not be well defined. Hence we have to break the remaining SU($N-1$) gauge symmetry as well. This can obviously be done in exactly the same way as described above [Eqs. (51)–(57)]. Hence the original SU(N) gauge symmetry can be broken down systematically as follows:

$$SU(N) \supset SU(N-1) \supset SU(N-2) \supset \cdots \supset SU(2) , \qquad (58)$$

where the final step involves the breaking of the SU(2) symmetry as described in the previous section. At each step of the breaking an integration over the coset space $SU(N-m)/SU(N-m-1)$ $(m=0,1,2,\ldots,N-3)$ appears in (57) and the final step involves an integration over the SU(2) manifold. Altogether these terms involve the integration over N^2-1 real angles and represent the integration over the SU(N) manifold which again cancels in the normalized generating functional. Furthermore, every step involves the introduction of one real scalar field and $N-m$ $(m=0,1,2,\ldots,N-2)$ complex Grassmann fields. In total there are therefore $N-1$ real scalar fields and $\frac{1}{2}(N-1)(N+2)$ ghost fields left.

V. DISCUSSION AND CONCLUSIONS

Here we have presented a way of quantizing SU(N) gauge theories which does not involve gauge fixing and therefore circumvents all the problematics of gauge fixing and the Gribov ambiguity. We have discussed this procedure in detail for the Abelian theory and showed that the procedure amounts, in this case, to Landau gauge fixing. There is a fundamental difference between Abelian and non-Abelian theories. In a non-Abelian theory the present procedure does not simply correspond to Landau gauge fixing. Furthermore, the inverse gauge field propagator is fully determined once a renormalization condition has been imposed on the vertex functions to determine the renormalization constant Z_g. The consequences of this is currently being investigated for an SU(2) theory.

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