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Nucleosynthesis bounds on the Schmidt-Greiner-Heinz-Müller theory of gravitation

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We present a detailed calculation of light-element production in the Schmidt-Greiner-Heinz-Müller theory of gravitation. The comparison of our results with current observations implies very strong bounds on the allowed deviation from general relativity. These bounds lead to cosmological models which do not significantly differ from the standard Friedmann-Robertson-Walker (FRW) predictions. This result, together with those previously obtained in the literature for other scalar-tensor theories, allows us to conclude that there still does not exist any alternative metric theory of gravitation with a scalar field leading to a viable cosmological evolution distinguishable from the standard FRW one.

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I. INTRODUCTION

The Einstein equivalence principle generates a whole class of theories (called metric theories) describing gravitational interaction. Among the metric theories alternative to general relativity (GR), particular attention has been paid to scalar-tensor theories which contain, in addition to the metric tensor $g_{\mu\nu}$, a dynamical scalar field ϕ and a coupling function $\omega(\phi)$, which determines the relative importance of the scalar field.

Several scalar-tensor theories have been proposed: (1) the *Brans-Dicke* theory [1], where $\omega=\text{const}\neq-\frac{3}{2}$; (2) the *Dirac* theory [2], with $\omega=-\frac{3}{2}$; (3) the *Barker* theory [3], where the gravitational coupling constant is effectively constant; (4) the *Bekenstein* theory [4], with a variable rest mass; and (5) the *Schmidt-Greiner-Heinz-Müller* theory [5], which also includes a possible mass term for the scalar field. They are all particular cases of the general scalar-tensor of Bergmann, Wagoner, and Nordtvedt [6]. In addition, the *scale-covariant* theory of Canuto [7] has a mathematical representation similar in many aspects to Dirac's gravitation but with a nondynamical scalar function β.

The viability of a given alternative gravity theory can be analyzed by means of two kinds of tests [8]: those which examine its weak field limit and those which prove its full exact formulation. The first mainly consists of comparing the theory predictions in that limit with post-Newtonian experiments and binary pulsar observations. The only metric theory which is discarded by these experiments is Dirac's gravitation.

On the other hand, the most important type of strong field tests consists of comparing the cosmological predictions of a theory with present observations of cosmological interest. The astronomical data leading to the strongest bounds on the alternative theories are the observed light-element abundances, which have to be explained as an outcome of the primordial nucleosynthesis (PN) process. Cosmological models and light-element production has been previously analyzed in the Brans-Dicke [9], Barker [10], Bekenstein [11], and Canuto [12] theories. The bounds obtained on the free parameters which characterize each one of these theories are always very stringent and imply that the only viable models are those whose predictions do not significantly differ from the standard GR cosmological predictions up to at least temperatures of 10^{12} K.

The Schmidt-Greiner-Heinz-Müller (SGHM) theory has been proposed in order to prevent the collapse of massive dense objects [5,13]. In this theory, the gravitational constant depends on a scalar field ϕ which couples to the surrounding masses via the curvature scalar $\mathcal{R}$. This coupling is such that the gravitational interaction decreases with the strength of the scalar field. The original purpose of this theory is inviable because the effective gravitational constant can only vary within a narrow interval and the collapse of a massive object cannot be stopped. However, we have shown in a previous paper [14] (hereafter paper I) that it has a cosmological interest because, although its predictions at the present time are very close to those of GR, the cosmic evolution during earlier epochs can be different from the standard one, also giving rise to different conditions in the early Universe. In that paper, we had also shown that cosmological tests related to the matter-dominated evolution of the Universe do not constrain considerably its free parameters. The SGHM gravitation is then the only presently proposed metric theory with a scalar field which could give viable alternative cosmological models. The aim of this paper is to compute the light-element abundances predicted by this theory in order to elucidate if it can actually provide a viable alternative description of gravity.

The paper is arranged as follows. We begin outlining in Sec. II the SGHM theory and the basic equations to analyze light-element production. Predicted abundances for a sweep of initial conditions and the constraints obtained from comparison with observations are shown in Sec. III. Finally, conclusions and a summary of our results are given in Sec. IV.

II. THE SGHM THEORY

The starting point of the SGHM theory is the conformally invariant equation for a massless scalar field [15], which is generalized by adding a mass term and allowing for an arbitrary coupling constant β between ϕ and the scalar curvature $\mathcal{R}$:

$$\left[\Box+\frac{\beta}{6}\mathcal{R}+\mu^2\right]\phi=0 , \tag{1}$$

where μ has dimensions of $(\mathrm{sec})^{-1}$ because it includes a factor c^2/h, h being the Planck constant.

Equation (1) can be obtained from the action integral $I=I_G+I_M$, with I_M, as in general relativity, but with

$$I_G=\int\left[\frac{1}{2}\phi_{,\mu}\phi^{,\mu}-\frac{1}{2}\mu^2\phi^2-\frac{\beta}{12}\phi^2\mathcal{R}+\gamma\mathcal{R}-2\gamma\Lambda\right]\sqrt{-g}\,d^4x , \tag{2}$$

where

$$\gamma=\frac{c^2}{16\pi G} \tag{3}$$

is half of the inverse gravitational constant, Λ is the cosmological constant, and $g\equiv\det(g_{\mu\nu}),g_{\mu\nu}$ being the metric tensor.

From the action integral (2), one deduces that the effective inverse gravitational coupling constant is

$$\gamma_{\mathrm{eff}}=\gamma-\frac{\beta}{12}\phi^2 . \tag{4}$$

That is, the gravitational constant, as measured, e.g., by a Cavendish scale, depends on ϕ and is then a function of space-time coordinates. Note that β has to be negative in order that γ_{eff} decreases when ϕ increases.

The variation of Eq. (2) with respect to ϕ and $g_{\mu\nu}$ leads to the SGHM field equations

$$\left[\gamma-\frac{\beta}{12}\phi^2\right](\mathcal{R}_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}\mathcal{R})=-\tfrac{1}{2}T_{\mu\nu}-\phi_{,\mu}\phi_{,\nu}-\tfrac{1}{2}g_{\mu\nu}(\phi_{,\alpha}\phi^{,\alpha}-\mu^2\phi^2)+\frac{\beta}{12}[(\phi^2)_{;\mu;\nu}-g_{\mu\nu}(\phi^2)^{;\alpha}_{;\alpha}] , \tag{5}$$

$$\Box\phi+\frac{\beta}{6}\mathcal{R}\phi+\mu^2\phi=0 , \tag{6}$$

which satisfy the usual conservation law $T^{\mu\nu}_{;\nu}=0$, where $T^{\mu\nu}$ is the energy-momentum tensor.

In order to build up cosmological models, we consider a homogeneous and isotropic Universe. The energy-momentum tensor then corresponds to a perfect fluid and the line element has a Robertson-Walker form:

$$ds^2=-dt^2+R^2(t)\left[\frac{dr^2}{1-Kr^2}+r^2d\Omega^2\right] , \tag{7}$$

where $K=0,\pm1$ and $R(t)$ is the scale factor. The field equations (5) and (6) can then be written, in terms of $H\equiv\dot{R}/R$ and $D\equiv\dot{\phi}$, as

$$\dot{H}=\frac{-3}{\beta^2\phi^2+12\gamma_{\mathrm{eff}}}\left[P+\frac{1}{6}(3-2\beta)D^2+\frac{1}{6}(3-2\beta)\mu^2\phi^2+\frac{1}{3}\beta\phi DH+\left[\frac{1}{3}\beta^2\phi^2+2\gamma_{\mathrm{eff}}\right]\frac{c^2K}{R^2}+\left[\frac{2}{3}\beta^2\phi^2+6\gamma_{\mathrm{eff}}\right]H^2\right] , \tag{8}$$

$$\dot{D}=\frac{3\beta\phi}{\beta^2\phi^2+12\gamma_{\mathrm{eff}}}\left[P+\frac{1}{6}(3-2\beta)D^2-2\gamma_{\mathrm{eff}}\frac{c^2K}{R^2}-2\gamma_{\mathrm{eff}}H^2\right]-\frac{3}{\beta^2\phi^2+12\gamma_{\mathrm{eff}}}[(\tfrac{2}{3}\beta^2\phi^2+12\gamma_{\mathrm{eff}})DH-(\tfrac{1}{2}\beta\phi^2+4\gamma_{\mathrm{eff}})\phi\mu^2] \tag{9}$$

together with the algebraic equation

$$6\gamma_{\mathrm{eff}}\frac{c^2K}{R^2}=\rho c^2-6\gamma_{\mathrm{eff}}H^2+\tfrac{1}{2}D^2-\frac{\mu^2}{2}\phi^2+\beta\phi DH . \tag{10}$$

The dynamical evolution of the temperature T can be obtained from the energy-momentum conservation law and the standard state equation

$$\frac{dt}{dT}=-\frac{d\rho_1/dT}{3H(\rho_1+P_1/c^2)} , \tag{11}$$

where $\rho_1=\rho_b+\rho_e+\rho_\gamma$, $P_1=P_\epsilon+P_\gamma$, and subscripts b, e, and γ refer to baryon electron-positron and photon, respectively.

Equations (8), (9), and (11) constitute the basic set of equations to build up cosmological models in the SGHM theory. In these equations, we have considered as independent functions the scalar field ϕ, the Hubble parameter H, $D \equiv \dot{\phi}$, and the photon temperature T. The algebraic equation (10) gives, at any time, c^2K/R^2 as a function of ϕ, H, D, and T.

In order to build up cosmological models compatible with astronomical observations, we take as initial conditions the present values of the dynamical functions (ϕ_0, H_0, D_0, c^2K/R_0^2, T_0) and we integrate the field equations backwards in time (for fixed values of the β and μ parameters). By present values we mean their current observational values or their limits from observational data. Some of these initial data can be expressed in terms of the others, or can be directly known from observations. In fact, since the gravitational constant is well known from experiments such as, e.g., the Cavendish scale, the present value of $\gamma_{\rm eff}$ must be equal to the general relativistic γ value. Therefore, if we impose $\gamma_{\rm eff}=\gamma$ at the present time, Eq. (4) implies

$$\phi_0 = 0 \ . \tag{12}$$

Furthermore, if $\Omega \equiv \rho/\rho_c$, where ρ_c is the critical density needed to close the Universe, Eqs. (10) and (12) yield to

$$D_0 = -\sqrt{12\gamma H_0^2(1-\Omega^{\rm FRW}/\Omega)} \ . \tag{13}$$

As we have shown in paper I, if $\Omega_0 = \Omega_0^{\rm FRW} \equiv \rho_0 c^2/[6\gamma H_0^2]$ the SGHM theory reduces to GR at any time. However, if we consider Ω_0 as a free parameter different from $\Omega_0^{\rm FRW}$, the SGHM models never reduce to GR even for vanishing β and μ. The difference between both types of models can be parametrized by $\Delta_0 \equiv 1-\Omega_0^{\rm FRW}/\Omega_0$. Only models with $\Delta_0 \neq 0$ will be considered in this paper.

Equations (4) and (12) also imply post-Newtonian parameters equal to the standard ones and a vanishing present value of $\gamma_{\rm eff}$ (even for $D_0 \neq 0$). Consequently, there does not exist any observational limit on the SGHM parameters β and μ.

III. RESULTS

Light-element production in the SGHM theory has been computed by taking $T_0 = 2.735 \pm 0.017$ [16], $N_\nu = 3$ [17], and using η_{10}, Δ_0, β, and μ as free parameters, η_{10} being the baryon-to-photon ratio in units of 10^{-10}. The results do not depend too much on H_0 and the neutron half-life τ_n, which have been taken to be 50 km/(sec Mpc) [18] and 889.9 sec [19], respectively, in order to avoid an excessive number of free parameters.

Figure 1 shows the ratio $\xi = H/H^{\rm FRW}$ at $T = 10^{10}$ K as a function of $\log_{10}(\Delta_0)$ for different values of β and μ. Clearly, when $\Delta_0 \to 0$, any model reduces to GR independently on β and μ parameters. This is coherent with our previous result that the difference between FRW and SGHM models can be uniquely parametrized by Δ_0. From Fig. 1 we can also see that the expansion rate of the

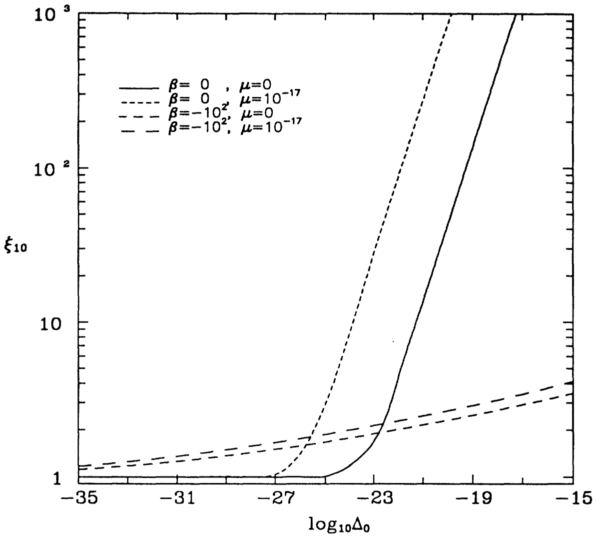

FIG. 1. Expansion rate $\xi = H/H^{\rm FRW}$ at $T = 10^{10}$ K as a function of $\log_{10}(\Delta_0)$ for several values of β and μ.

Universe at nucleosynthesis, ξ_{10}, is always faster than in GR.

In order to analyze the qualitative behavior of light-element production in the framework of this theory, we present (Figs. 2–5) primordial abundances of ^{4}He, D/H, (D+^{3}He)/H, and ^{7}Li for $\eta_{10} = 3$ and $\beta = \mu = 0$. This qualitative behavior with respect to Δ_0 is similar to that of other scalar-tensor theories [20]. When Δ_0 increases, the expansion rate an nucleosynthesis ξ_{10} is also faster and the temperature at which the n/p ratio freezes out is then higher. Consequently, the abundance of ^{4}He, Y_p, grows with Δ_0 until a maximum is reached. After this maximum, Y_p decreases because, for too high values of

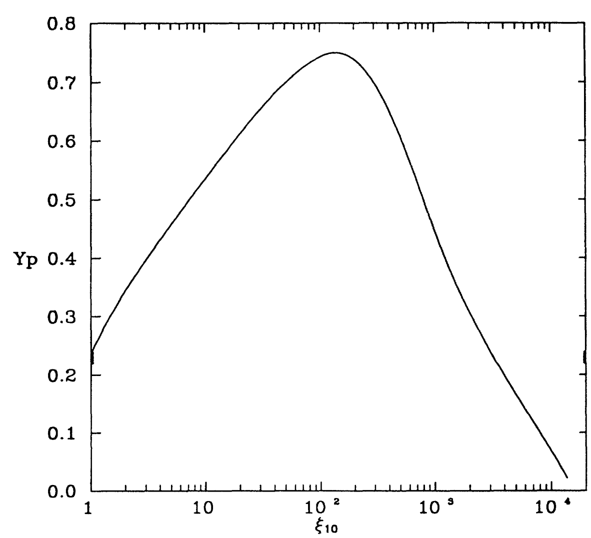

FIG. 2. ^{2}He abundance as a function of the expansion rate for $\eta_{10} = 3$ and $\beta = \mu = 0$.

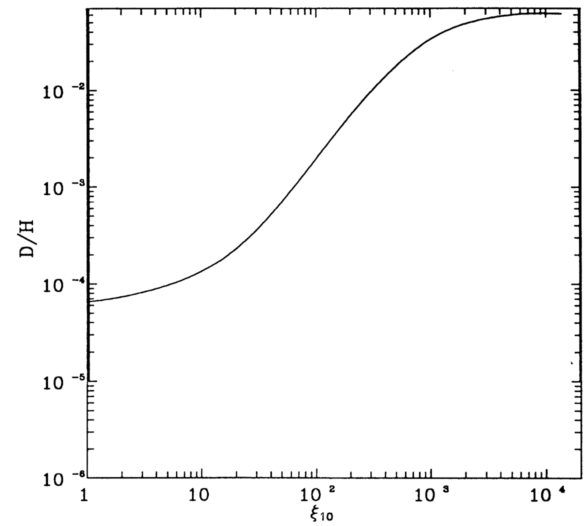

FIG. 3. Same as Fig. 2 for D/H.

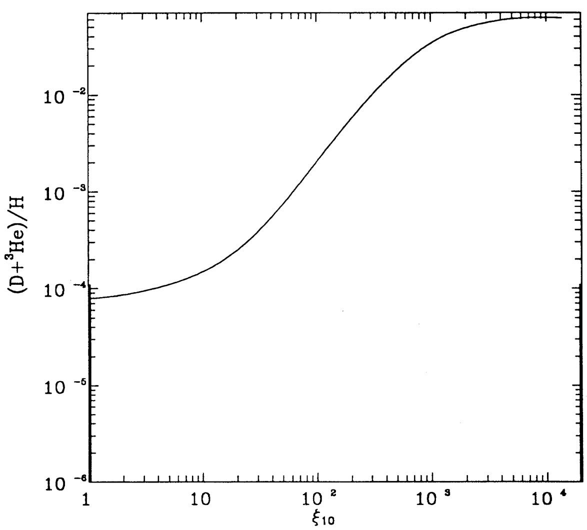

FIG. 4. Same as Fig. 2 for $(D+{}^3He)/H$.

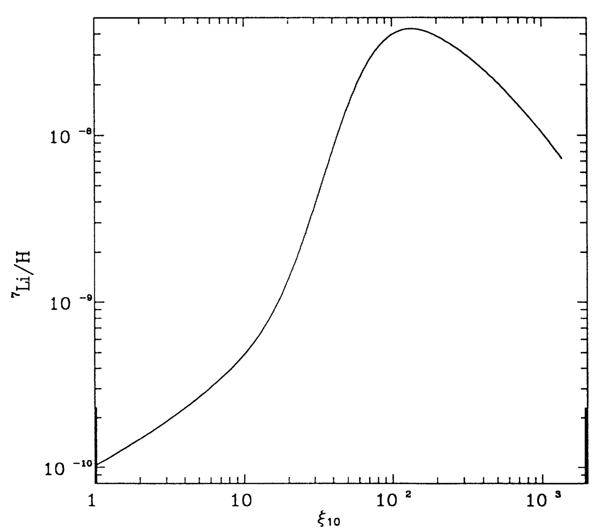

FIG. 5. Same as Fig. 2 for $^7Li/H$.

ξ_{10}, nuclear reactions do not have enough time to produce ^{4}He.

The D and ^{3}He productions also grow with Δ_0 (or ξ_{10}) until a maximum is reached (Figs. 3 and 4). This maximum is located at high values of ξ_{10} when burning nuclear reactions have no time to occur and most $D+{}^3He$ produced at the onset of the primordial nucleosynthesis survives the cooling of the Universe. The comparison of Figs. 2 and 4 shows the correlation between the maximum in the $D+{}^3He$ abundance and the decreasing zone in the ^{4}He production. However, for very high values of ξ_{10}, nuclear reactions giving rise to D and ^{3}He have no time to occur and their abundances decrease with Δ_0. A similar behavior is found in the variation of the ^{7}Li production with Δ_0 (Fig. 5).

In order to analyze the compatibility among observed and predicted abundances, we have depicted the observational bounds [21] by a thick line on the abundance axes of Figs. 2–5. As can be seen from these figures, only the compatibility regions placed at low ξ_{10} values are simultaneously consistent for all light elements.

Primordial nucleosynthesis bounds on the allowed ξ_{10} and Δ_0 values in the less restrictive case $\beta=\mu=0$ can be obtained by performing a similar computation as before for several values of η_{10}. Figures 6–9 show light-element abundances for a range of Δ_0 values which correspond to expansion rates not very different from unity. As can be noticed from these figures, a right D, $D+{}^3He$, and ^{7}Li production is obtained in some η_{10} interval even for high values of Δ_0. However, right ^{4}He abundances can only be found for $\Delta_0 \leq 5\times10^{-24}$. The compatibility regions in the plane (η_{10},Δ_0) for ^{4}He, D, $D+{}^3He$, and ^{7}Li are depicted in Fig. 10. The intersection of these regions gives the η_{10} and Δ_0 values for which light-element abundances are simultaneously consistent with observations. We find

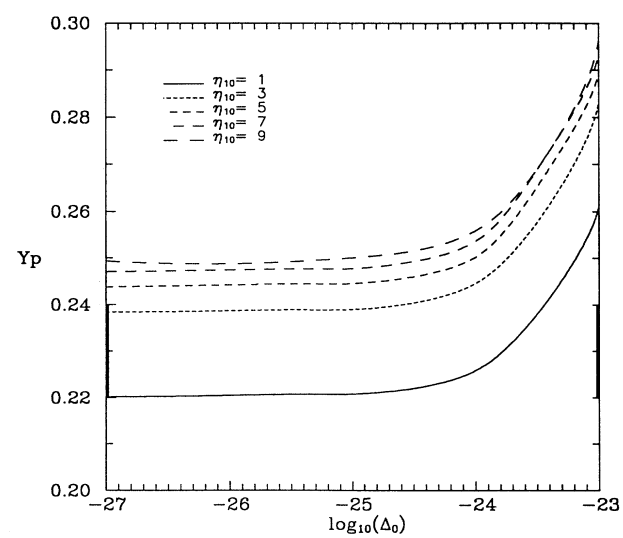

FIG. 6. ^{4}He abundance as a function of $\log_{10}(\Delta_0)$ for a range of values which correspond to expansion rates not very different from unity and for $\beta=\mu=0$ and several values of η_{10}.

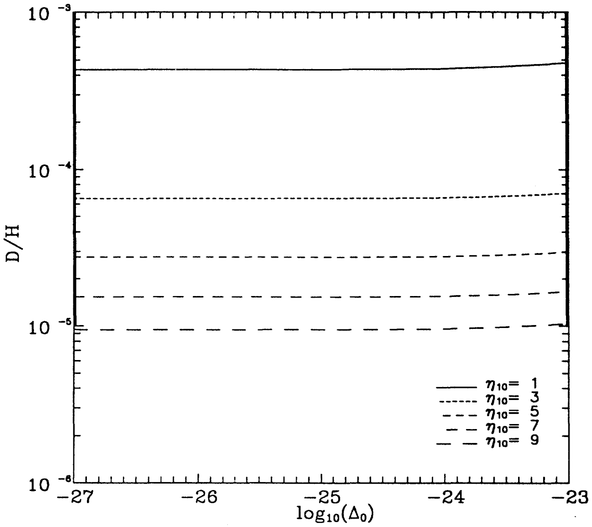

FIG. 7. Same as Fig. 6 for D/H.

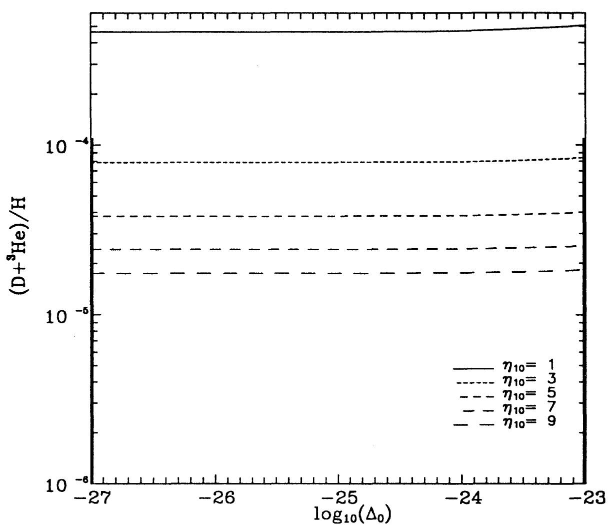

FIG. 8. Same as Fig. 6 for $(D+^3He)/H$.

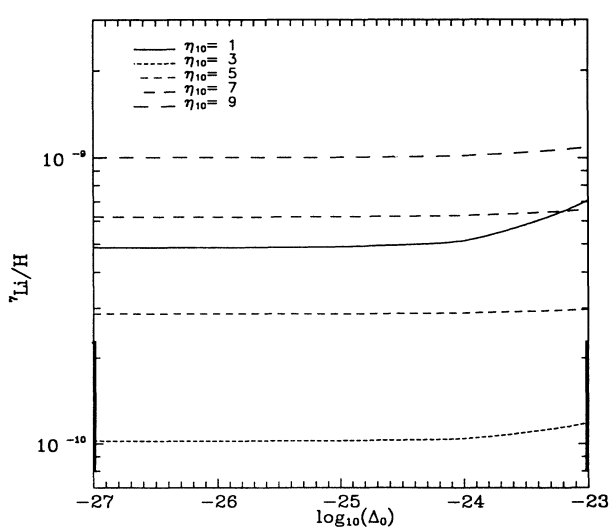

FIG. 9. Same as Fig. 6 for $^7Li/H$.

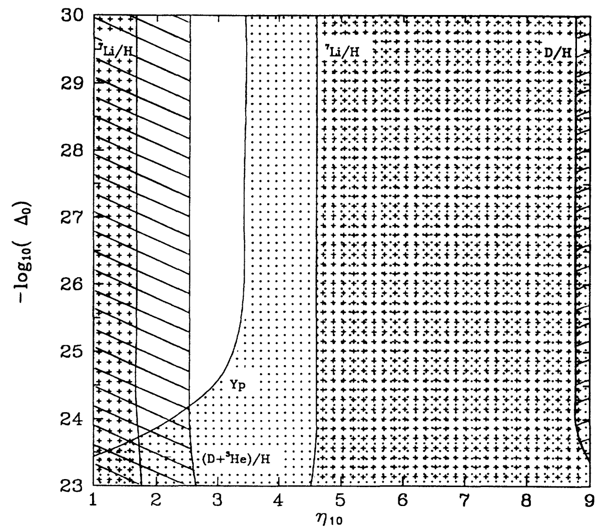

FIG. 10. Compatibility regions in the plane $[\eta_{10}, -\log_{10}(\Delta_0)]$ for $\beta=\mu=0$. Shaded regions are forbidden for ^4He (dots), D/H (hatched zone on the right-hand side of figure), $(D+^3He)/H$ (hatched zone on the left-hand side of figure), and $^7Li/H$ (crosses).

from this figure that, when $\beta=\mu=0$, right abundances are obtained for $\Delta_0 \leq 5\times10^{-25}$ (or $\xi_{10} \leq 1.02$).

Much more stringent bounds on Δ_0 are found if we consider nonvanishing β or μ values (for example, $\Delta_0 \leq 10^{-28}$ if $\beta=0$ and $\mu=10^{-17}$; $\Delta_0 \leq 10^{-45}$ if $\beta=10^{-2}$ and $\mu=0$ or 10^{-17}). However, these bounds imply a similar constraint on ξ_{10} (≤ 1.02).

IV. CONCLUSIONS

In order to test the viability of SGHM cosmological models, primordial light-element abundances have been calculated for a sweep of present values of the dynamical functions and parameters which cover the space of initial conditions compatible with astronomical data.

Since the SGHM theory reduces to GR when $\Delta_0=1-\Omega_0^{\rm FRW}/\Omega_0=0$ (for any β and μ), the difference between both theories can be parametrized by Δ_0 and no bounds on β or μ can be obtained. Our results imply that the expansion rate at primordial nucleosynthesis can be faster than in FRW models by at most 2%. Consequently, in the less restrictive case ($\beta=\mu=0$), the Δ_0 parameter must be smaller than 5×10^{-24}. This bound on Δ_0, together with the cosmological evolution of SGHM shown in paper I, imply that the only viable models are those whose predictions do not significantly differ from the standard FRW predictions. That is, SGHM cosmological models are observationally indistinguishable from the FRW ones up to at least primordial nucleosynthesis epoch.

This result, together with that previously obtained in the literature for other alternative theories, implies that there do not exist, to date, any alternative metric theories of gravitation with a scalar field which can predict a viable cosmological evolution distinguishable from the

FRW one. Primordial nucleosynthesis is then a very strong test for the viability of gravitational theories. Very small deviations from GR during the early stages of the evolution of the Universe imply a light-element production inconsistent with present observations. However, we cannot assure that these theories are also indistinguishable from GR at very early epochs before nucleosynthesis or with a nonstandard composition of the Universe [22].

ACKNOWLEDGMENT

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