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Cosmological consequences of spontaneous lepton number violation in SO(10) grand unification

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Cosmological constraints on grand-unified theories with spontaneous lepton number violation are analyzed. We concentrate on SO(10), the simplest of the models possessing this property. It has been noted previously that the consistency of these models with the observed baryon asymmetry generically implies strict upper bounds on the light neutrino masses. In this paper, we analyze the situation in detail. We find that minimal models of fermion masses face difficulties, but that it is possible for these models to generate an adequate baryon asymmetry via nonequilibrium lepton-number-violating processes when the right-handed neutrino masses are near their maximum possible values. This condition uniquely picks out the minimal gauge symmetry-breaking scheme. A nonminimal model is also analyzed, with somewhat different conclusions due to the nature of the imposed symmetries.

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I. INTRODUCTION

The ability of grand-unified theories (GUT's) to produce the observed baryon asymmetry of the Universe is arguably one of their most attractive features [1]. However, for some time it has been realized [2,3] that grand-unified theories which also exhibit spontaneous lepton number violation are in danger of eliminating this baryon asymmetry at later times, when one considers anomalous $(B+L)$-violating processes [4] to be in thermal equilibrium. Generally this danger is averted by prescribing a bound on the neutrino masses such that lepton-number-violating processes are not in equilibrium at temperatures near or below the scale of the spontaneous lepton number violation. The actual situation is somewhat more complicated because such out-of-equilibrium processes can create an asymmetry, approximately proportional to *CP*-violating parameters. A full calculation must be considered.

The purpose of this paper is to analyze leptogenesis in these models and the consequent constraints on grand unification, specifically SO(10) models. SO(10) is the smallest candidate possessing the required spontaneous lepton number violation, and many larger groups can have symmetry breaking chains which contain an SO(10) stage. We consider the phenomenologically viable breaking pattern $\mathrm{SO}(10)\rightarrow\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{SU}(4)$ [5]. SU(4) breaks to $\mathrm{SU}(3)_c\times\mathrm{U}(1)_{B-L}$, $\mathrm{SU}(2)_R$ breaks to $\mathrm{U}(1)_{I_{3R}}$, and the subsequent breaking of $\mathrm{U}(1)_{B-L}\times\mathrm{U}(1)_{I_{3R}}$ to $\mathrm{U}(1)_Y$ provides the spontaneous violation of lepton number. Constraints on the intermediate scales arising from gauge coupling unification and Yukawa coupling unification are analyzed in Ref. [6].

Of course, one must realize that it is hard to make definitive statements about the grand-unification program based on fermion mass phenomenology. Many constraints can be avoided by introducing arbitrary complications in the Higgs sector. However, if it is found that many (or all) of the minimal prescriptions are not viable, then the model can lose its attractiveness. Therefore, our philosophy is to analyze certain specific prescriptions for fermion masses, bringing to bear all the known constraints. After such an analysis we can attempt to abstract those features which will remain true generally and those which seem specifically model dependent. It is not too difficult to construct attractive models which are consistent with all known low-energy parameters, but it is more difficult to construct models which survive the various cosmological constraints.

In this paper we consider certain minimal models which can give rise to realistic fermion masses and mixings. As discussed in Ref. [6], a detailed analysis of the situation, with emphasis on neutrino masses and lepton-number-violating processes in the early Universe, has been required for some time. For any such model, there are two issues to address. First, there is the question of whether or not the lepton sector mixing implied by a given fermion mass model is sufficient to render the neutrino mass bounds applicable to all generations. Second, there is the question of radiative corrections. If one estimates the lepton-number-violating cross sections naively, assuming that the appropriate Dirac neutrino mass is of order the top quark mass, then one finds that the right-handed neutrino Majorana mass scale must be quite large, greater than 10^{16} GeV at least [3]. This is so large that it is not possible to reconcile it with determinations of the intermediate scales via gauge-coupling unification. In Ref. [6] it was pointed out that, when the radiative corrections are taken into account, this constraint is reduced to a level that is consistent with gauge-coupling unification. These are the issues which we set out to understand with the present work. We find that the constraints are quite severe for the models that we examine;

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nevertheless these models are just adequate to the task of producing the observed baryon asymmetry of the universe via out-of-equilibrium $(B-L)$-violating processes. One can speculate on the importance of the fact that both gauge-coupling unification and baryogenesis concerns point to a unique intermediate unification scale.

We also examine a specific nonminimal prescription for fermion masses. The behavior of this model is quite different from the minimal model; as we discuss below, in some sense these two models encapsulate the various possibilities.

II. A MINIMAL REALISTIC MODEL

The minimal Yukawa interactions necessary to generate realistic fermion masses and mixings at low energies are

$$h_{ij}\psi^i\psi^j H(10)+f_{ij}\psi^i\psi^j\overline{\Delta(126)} \ . \tag{2.1}$$

The $H(10)$ is actually two real **10**-dimensional representations of SO(10) combined into a complex field. The coupling of the $H(10)$ gives the well-known tree-level mass relations $m_b=m_\tau$, $m_t=m_{\nu_\tau}^{\text{Dirac}}$, and similarly for the other generations. Radiative corrections upset these relations, making the τ-bottom mass ratio acceptable. The coupling of the $\Delta(126)$ gives rise to a Majorana mass matrix for the right-handed neutrinos upon spontaneous breaking of $\mathrm{U}(1)_{B-L}$. A $\mathrm{U}(1)_{\mathrm{PQ}}$ Peccei-Quinn symmetry [7,8] forbids the appearance of a Yukawa coupling to the $H(10)$.

The minimal gauge symmetry breaking scheme has SO(10) breaking through one intermediate stage, $\mathrm{SO}(10)\rightarrow\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{SU}(4)\rightarrow\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y\times\mathrm{SU}(3)_c$. The required scalar representations are a **210** or **54** to break SO(10), a **126** to break the intermediate group and generate right-handed neutrino Majorana masses, and a **10** to break the electroweak group. As pointed out in Ref [9], an extra representation is required in order to prevent the $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry from surviving to the electroweak scale and thereby creating an unacceptable weak-scale axion. A **16** can be used for this purpose.

At first sight, the simplicity of the Yukawa couplings seems to preclude realistic fermion mixing. However, it has been demonstrated that mixing in the scalar sector can induce small corrections which will generate nontrivial, viable mixing and masses for fermions [8]. Thus the fermion mass model given by Eq. (2.1) is very attractive. There is essentially one free parameter in this model, the $\Delta(126)$ vacuum expectation value (VEV) v_R. Given this parameter, the charged fermion masses and the Cabibbo-Kobayashi-Maskawa (CKM) matrix determine the neutrino masses and the lepton mixing matrix [8]. The diagram which induces the small admixture of **126**-like Yukawa couplings for the $H(10)$ is shown in Fig. 1(a). The VEV's which appear are of the $(1,3,\overline{10})$ content of the $\Delta(126)$.

Integrating out the scalars with masses of order M_{GUT}, we write the Yukawa couplings for the effective theory below that scale in the form

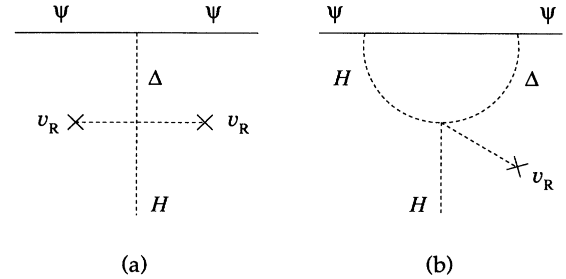

FIG. 1. Scalar mixing induced terms giving rise to the effective coupling of the $H(10)$.

$$\begin{gathered}
\overline{U_L}(h+f\delta_u)U_R\phi_u \ , \\
\overline{D_L}(h+f\delta_d)D_R\phi_d \ , \\
\overline{N_L}(h-3f\delta_u)N_R\phi_u \ , \\
\overline{L_L}(h-3f\delta_d)L_R\phi_d \ , \\
\overline{N_R^c}fN_R\Phi \ ,
\end{gathered} \tag{2.2}$$

where δ_u and δ_d are (complex) parameters derived from the scalar mixing diagram illustrated in Fig. 1(a). The dimension-four Peccei-Quinn invariant operator which induces the mixing shown in Fig. 1(a) is

$$\lambda\Delta(126)\overline{\Delta(126)}\Delta(126)H(10) \ . \tag{2.3}$$

Therefore the sizes of δ_u and δ_d are determined by the ratio v_R^2/M_Σ^2, where M_Σ is the mass of the scalar field $\Sigma(2,2,15)\in\Delta(126)$. Notice that if $\delta_u=\delta_d$ then quark mixing would vanish. The difference in δ_u and δ_d is generated by $\mathrm{SU}(2)_R$ breaking, which splits the degeneracy of the $\mathrm{SU}(2)_L$ doublets contained in Σ. Phases in δ_u and δ_d can be generated by effective soft Peccei-Quinn-breaking terms.

The diagram in Fig. 1(b) is a one-loop contribution to the CP-violating effective H coupling resulting from the operator

$$\lambda'\Delta(126)\Delta(126)H(10)H(10) \ , \tag{2.4}$$

which played a role in the inflaton scenario introduced in Ref. [10]. This diagram generates an effective coupling

$$\frac{\lambda'}{16\pi^2}\frac{v_R^2}{M_\Sigma^2}\overline{N_L}fhN_R\phi_u \ . \tag{2.5}$$

This contribution is only relevant for the right-handed neutrinos due to an insertion of the Majorana mass matrix. Also, because of the family structure of the coupling, the contribution is only important for the heavy generation. Thus, in principle this operator will give a contribution to the asymmetry in decays of the third generation right-handed neutrino. However, there are two justifications for neglecting this operator in our analysis. First, we expect that $\lambda\simeq\lambda'$, so that the numerical importance of the loop-induced operator is decreased by a relative factor of $1/(16\pi^2)$. Second, it is a general feature of the out-of-equilibrium lepton asymmetry production that the lightest of the right-handed neutrinos is most important, since an asymmetry produced by the decays of the heavier right-handed neutrinos will generally be restored by equilibrium lepton-number-violating scattering involving exchange of the lighter right-handed neutrinos. Therefore, the effect of the operator (2.5) is numerically less than the contributions already considered, and we do not consider it further.

In the following, we will consider a low-energy theory containing two Higgs doublets. One could consider a low-energy theory with one Higgs doublet by choosing a linear combination of ϕ_u and $\tilde{\phi}_d$ to become ultramassive. Note that the renormalization group analysis of the Yukawa couplings becomes complicated in the two-doublet case due to nonlinearities which cannot be ignored. Thus the solutions found in Ref. [8] cannot be taken over wholly. We are in the process of a full renormalization group analysis for the Yukawa couplings in this model, and details regarding the evolution of these couplings are postponed to a future paper [11].

There is some inconvenience in the determination of the model parameters from our knowledge of charged fermions. Clearly, the low-energy parameters can be used only to fix the product $\delta_u f_{ij}$ and the ratio δ_u/δ_d. Therefore the coupling matrix f_{ij} is actually determined only to within an overall scale connected to our inability to determine the scale of δ_u and δ_d. This scale ambiguity translates into an overall scale ambiguity for the masses of the right-handed neutrinos, and we often write the Majorana mass matrix for the right-handed neutrinos in the form

$$M_{ij}^{\rm Maj}=v_R f_{ij}=\frac{1}{R}\delta_u f_{ij} v_u \ , \tag{2.6}$$

where v_u is the VEV of ϕ_u and $R=v_u\delta_u/v_R$. A further difficulty is connected with the implementation of CP violation. CP violation is spontaneous, manifested as phases in the propagators of the Σ fields, and the two independent CP phases can be rotated into the induced VEV parameters δ_u and δ_d. Strictly speaking, because of our lack of knowledge of the phases in the mixing matrix for right-handed currents, it is not actually possible to completely determine the coupling matrices h_{ij} and f_{ij} from low-energy data. One could work from the GUT scale down, but this shoot and miss approach is cumbersome. However, for small values of the phases a perturbative determination is possible, as discussed in Ref. [8]; we determine the couplings assuming that CP violation is absent, and then calculate the CP-violating corrections to the fermion masses and mixings. This leaves only sign ambiguities in various fermion masses, which can be chosen so that solutions to the constraints are generated. As commented in Ref. [8], large CP phase solutions to the constraints are not found.

Once the coupling matrices h_{ij} and f_{ij} and the parameters δ_u and δ_d are prescribed, the history of the lepton number asymmetry in the early Universe is fixed. The dominant lepton-number-violating effect is the decay of the heavy right-handed neutrino. This out-of-equilibrium decay, together with C and CP violation, will create a lepton asymmetry. Subsequently, this lepton asymmetry will be distributed into a baryon asymmetry by the action of anomalous $B+L$ violation [2,3]. Counter to this simple mechanism is the effect of the dimension-five lepton-number-violating operators which are associated with the light neutrino masses:

$$\mathcal{O}_5=\frac{m_{\nu i}}{v^2}\phi^0\phi^0\nu_{Li}\nu_{Li} \ , \tag{2.7}$$

where v is the weak scale VEV, and ϕ^0 is the neutral member of a low-energy Higgs doublet.

The competition between the CP-violating parts of the right-handed neutrino decays, which produce a lepton asymmetry, and the processes $\nu_{Li}\nu_{Li}\rightleftharpoons\phi^0\phi^0$, which subsequently destroy it, is governed by the Boltzmann equations for right-handed neutrino density and the $B-L$ asymmetry. The framework for this calculation was established in Ref. [12], where the necessary rates were calculated and the Boltzmann equations were derived and integrated for several generic cases. We choose to write the Boltzmann equations in a slightly different form. Let Y_i indicate the number density of species i, normalized to entropy, and let $\Delta_i=Y_{N_{Ri}}-Y_{N_{Ri}}^{\rm eq}$, where $Y^{\rm eq}$ is the equilibrium number density. Let $x=M_{N_{R1}}/T$, where N_{R1} is the lightest of the heavy neutrinos, and let $x_i=m_i/T$ for a generic species i. The basic decay and scattering processes of interest to us are

$$\begin{aligned}
&D_{ij}:\ N_{Ri}\to\nu_{Lj}\phi^0 \ ,\\
&N_{ij}:\ \nu_{Li}\nu_{Lj}\to\phi^0\phi^0 \ ,\\
&Hs_{ij}:\ N_{Ri}L_{Lj}\to Q_L\bar{t}_R \ ,\\
&Ht_{ij}:\ N_{Ri}\bar{t}_R\to L_{Lj}Q_L \ .
\end{aligned} \tag{2.8}$$

Then we have the Boltzmann equations

$$\frac{d}{dx}\Delta_i=-\frac{d}{dx}Y_{N_{Ri}}^{\rm eq}-\frac{x}{H(M_{N_{R1}})}\Delta_i\sum_j\left[\tilde{\gamma}_{D_{ij}}+\frac{\tilde{\gamma}_{Ht_{ij}}+\tilde{\gamma}_{Hs_{ij}}}{Y_{N_{Ri}}^{\rm eq}}\right] , \tag{2.9}$$

$$\frac{d}{dx}Y_{(B-L)_i}=\frac{x}{H(M_{N_{R1}})}\times\sum_j\left[\epsilon_{ij}\Delta_j\tilde{\gamma}_{D_{ji}}-\frac{Y_{(B-L)_i}}{Y_{\rm leptons}^{\rm eq}}S_{ji}^{BL}\right] , \tag{2.10}$$

where

$$S_{ji}^{BL}=\left[\tilde{\gamma}_{N_{ji}}+\left[1+\frac{\Delta_j}{Y_{N_{Rj}}^{\rm eq}}\right]\tilde{\gamma}_{Hs_{ji}}+\tilde{\gamma}_{Ht_{ji}}\right] , \tag{2.11}$$

and where $\tilde{\gamma}_{D_{ij}}$ is the thermally averaged decay rate for $N_{Ri}\to\nu_{Lj}\phi^0$:

$$\tilde{\gamma}_{D_{ij}}=\frac{K_1(x_i)}{K_2(x_i)}\Gamma(N_{Ri}\to\nu_{Lj}\phi^0)=\frac{K_1(x_i)}{K_2(x_i)}\frac{h_{ji}h_{ji}^*}{16\pi}M_{N_{Ri}} \ . \tag{2.12}$$

The Hubble rate at temperature T has been denoted $H(T)$ above. The various thermally averaged reduced cross sections, dimensionalized to rates by a factor of the entropy density, are given by

$$\tilde{\gamma}_k = \frac{45T}{64\pi^6 g_*}\int_{\sqrt{s_{\min}}/T}^{\infty} dy\, y^2 K_1(y)\hat{\sigma}_k(T^2y^2) \tag{2.13}$$

with

$$\hat{\sigma}_{Ht_{ij}}(T^2y^2) = \frac{m_t^2 h_{ji}h_{ji}^*}{\pi v^2}\left[1-\frac{x_i^2}{y^2} + \frac{x_i^2}{y^2}\ln\left[\frac{y^2-x^2+x_{\phi^0}^2}{x_{\phi^0}^2}\right]\right],$$

$$\hat{\sigma}_{Hs_{ij}}(T^2y^2) = \frac{m_t^2 h_{ji}h_{ji}^*}{2\pi v^2}\left[1-\frac{x_i^2}{y^2}\right]^2, \tag{2.14}$$

$$\hat{\sigma}_{N_{ij}} = \frac{1}{2\pi}\sum_d h_{id}h_{dj}h_{id}^*h_{dj}^*\frac{x_d^2}{y^2}g_d(y^2/x_d^2),$$

and g_* is the effective number of degrees of freedom at $T\simeq 10^{12}$ GeV, $g_*\simeq 120$. The function $g_i(z)$ is calculated in Ref. [12], but appears with a typographical error in that reference. The correct equation is

$$g_i(z) = z + \frac{z}{D_i(z)} + \frac{z^2}{D_i(z)^2} - \left[1+\frac{1+z}{D_i(z)}\right]\ln(1+z), \tag{2.15}$$

where $D_i(z)$ is defined in Ref. [12]. As in that reference, we have neglected the interference terms in $\hat{\sigma}_{N_{ij}}$, assuming that, for a given i and j, one heavy neutrino exchange dominates the others. The most important of the exhibited processes, of course, are the heavy neutrino decays and the lepton-number-violating scatterings induced by the dimension-five operators. The CP-violating parameters ϵ_{ij} in Eq. (2.10), associated with the decay rates $\tilde{\gamma}_{D_{ji}}$, are given by

$$\epsilon_{ij} = \frac{1}{\pi h_{ij}h_{ij}^*}\sum_{k,l}\text{Im}(h_{kj}h_{kl}^*h_{il}^*h_{ij})f(M_{N_{Rl}}^2/M_{N_{Rj}}^2). \tag{2.16}$$

Here, $f(z)=\sqrt{z}\,[1-(1+z)\ln((1+z)/z)]$. Notice that the quantity $\epsilon_{ij}\Gamma(N_{Rj}\to\phi^0\nu_{Li})$, which appears in the Boltzmann equation for the lepton asymmetries, is not a rephasing invariant. However, when summed over i, as it appears in the equation for the total lepton asymmetry, it becomes rephasing invariant, as it must since the total lepton asymmetry is independent of the phase convention.

It is also useful to know how $B+L$ is depleted. Given the rate per unit volume for $(B+L)$-violating processes as [13]

$$\gamma_{B+L} = C(\alpha_2 T)^4, \tag{2.17}$$

it is easy to derive the exponential depletion equation [14]

$$Y_{B+L}(x) = Y_{B+L}(0)e^{-\kappa x}, \tag{2.18}$$

where $\kappa\simeq C(10^{12}\text{ GeV}/M_{N_{R1}})$, C being of order unity in the high temperature limit.

Before proceeding to the calculation which is relevant for the model at hand, it is interesting to calculate the baryon number depletion caused by the $(B+L)$- and $(B-L)$-violating processes. We imagine (unrealistically) that the CP violation in the lepton sector vanishes. By calculating the depletion of an initial baryon asymmetry as a function of the right-handed scale, we can calculate the lower bound on that scale, assuming that the initial baryon asymmetry must survive to low temperatures. This is the standard argument for bounding neutrino masses, transferred to our specific model and utilizing a full numerical computation. We integrate the Boltzmann equations numerically, using an implicit differencing scheme to insure computational stability [15]. In Fig. 2 we show the depletion of an initial asymmetry $Y_B\simeq 3\times 10^{-9}$ when the lightest right-handed neutrino has mass $M_{N_1}=2\times 10^{12}$ GeV. The linearity of the equations implies that the initial asymmetry would be required to be greater than 10^{-5} in order for an acceptable baryon number to survive to low temperatures. Alternatively, the mass of the lightest right-handed neutrino would be required to be greater than 2×10^{13} GeV. This is difficult to accommodate within the context of this model since the right-handed scale is bounded by gauge-coupling unification criteria. The example shown, with $m_t=150$ GeV, in the absence of leptonic CP violation, would be ruled out. This is a generic feature for SO(10) grand unification; the $\text{U}(1)_{I_{3R}}\times\text{U}(1)_{B-L}$-breaking scale is bounded above by the minimum of the $\text{SU}(2)_R$- and

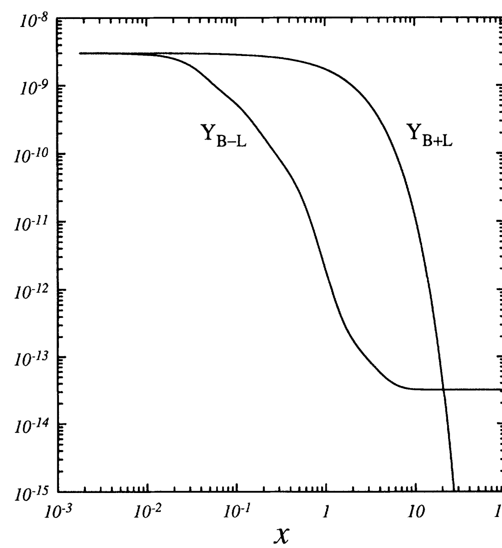

FIG. 2. Baryon number depletion in the absence of leptonic CP violation. The lightest right-handed neutrino has mass $M_{N_1}=2\times 10^{12}$ GeV, and $x=M_{N_1}/T$.

TABLE I. Some results on baryon asymmetry production and neutrino masses for the minimal model of fermion masses.

	$1/R$	N_R Masses (GeV)	ν_i Masses (eV)	Y_B
Set 1	3×10^{14}	$2\times10^{12},6\times10^{12},3\times10^{13}$	$4\times10^{-6},2\times10^{-4},0.03$	0.9×10^{-10}
Set 1	2×10^{14}	$10^{12},4\times10^{12},2\times10^{13}$	$7\times10^{-6},3\times10^{-4},0.05$	0.5×10^{-10}
Set 1	5×10^{13}	$3\times10^{11},10^{12},6\times10^{12}$	$3\times10^{-5},10^{-3},0.2$	6.0×10^{-12}
Set 2	2×10^{14}	$10^{12},4\times10^{12},2\times10^{13}$	$7\times10^{-6},3\times10^{-4},0.05$	2.5×10^{-12}
Set 2	5×10^{13}	$3\times10^{11},10^{12},6\times10^{12}$	$3\times10^{-5},10^{-3},0.2$	0.8×10^{-12}

SU(4)-breaking scales, and therefore must be less than $M_{R_0}^{\max}\simeq3\times10^{13}$ GeV. This implies, by a triviality bound discussed in Ref. [6], that the heaviest right-handed neutrino must be less than approximately $1.5M_{R_0}^{\max}/g_R\simeq2\times10^{14}$ GeV. There is some freedom in the conclusions, since lowering m_t will decrease the neutrino Dirac masses and thus diminish the effect of the lepton-number-violating scattering processes. Again, we defer a catalogue of parameter space until later [11]. Thus we now turn to leptonic *CP* violation in order to explain the observed baryon asymmetry in the context of this model, without recourse to strict bounds on the top quark mass.

Using the approach of Ref. [8], with the appropriate renormalization group evolution of the Yukawa couplings, we find many solutions for the Yukawa couplings which give rise to acceptable fermion masses and mixings. Using these solutions we calculate the *CP*-violating parameters ϵ_{ij}, and the neutrino masses and mixings. Recall that the right-handed neutrino mass scale is an extra input parameter. There is no special correlation between the coupling matrices f_{ij} and h_{ij}; therefore, the leptons are well mixed, as indicated by the absence of a large hierarchy in the neutrino Dirac-Yukawa couplings (see the Appendix). Given these parameters, we calculate the lepton asymmetry. Figure 3 displays the results of one such calculation, with $1/R=3\times10^{14}$ so that the right-handed neutrino masses are $M_{N_i}=(2\times10^{12}$ GeV, 6×10^{12} GeV, 3×10^{13} GeV). We have chosen a set of Yukawa couplings giving as large an asymmetry as we could find. The largest values of ϵ_{ij} are of order 10^{-5}. This is sensible since the *CP*-violation in the neutrino sector is related to that in the quark sector via the unification *Ansatz*. The baryon asymmetry produced by sphaleron conversion is related to the $B-L$ symmetry by [3]

$$\Delta B\simeq\tfrac{1}{3}\Delta(B-L)\ . \tag{2.19}$$

The observed baryon asymmetry is $Y_B^{\text{obs}}\simeq(0.6-1.0)\times10^{-10}$ [16]. The $B-L$ asymmetry illustrated in Fig. 3 is sufficient to explain the observed baryon asymmetry.

Furthermore, we comment that the produced asymmetry is approximately proportional to the scale of the right-handed neutrino masses. This is easy to understand, since the decay rates which produce the Δ_i are proportional to the masses, and the scattering processes which destroy $Y_{(B-L)_i}$ are approximately inversely proportional to the squares of the masses. (This is exactly true, of course, at low energies, but the behavior at energies near the right-handed neutrino masses is more complicated.) Therefore, we must take the right-handed neutrino masses as large as possible [17]. This is the manifestation of the neutrino mass bounds [2,3]. Note that we can derive an upper bound on the right-handed neutrino masses by a "triviality" argument, as discussed in Ref. [6]; $|f_{ij}|\lesssim1.5$, so that the heaviest right-handed neutrino should satisfy $M_N\lesssim1.5v_R$, implying $1/R\lesssim3\times10^{14}$.

In the Appendix, we have collected the specific parameter values for those sets of Yukawa couplings for which we give results. The results on ΔB production are presented in Table I.

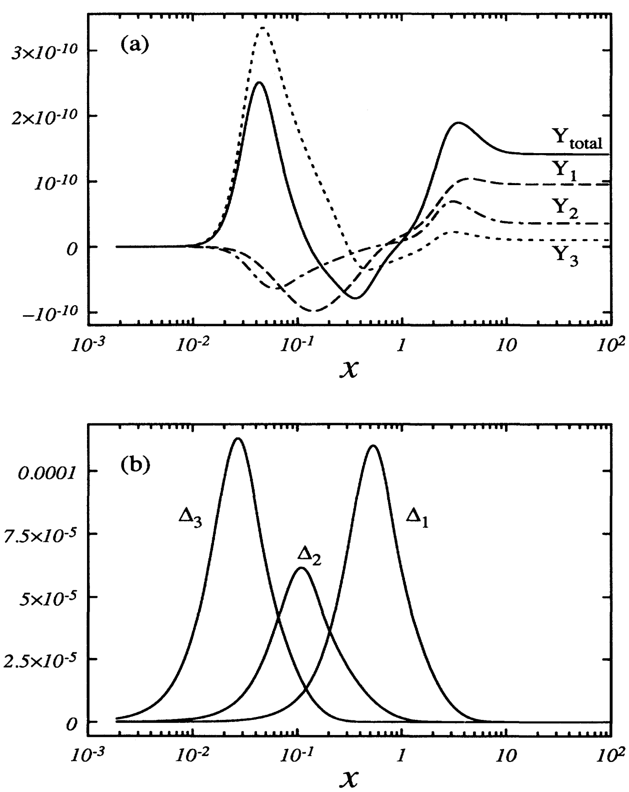

FIG. 3. Lepton asymmetry production for set 1, as described in the text, with $1/R=3\times10^{14}$. See also the Appendix. (a) shows the $B-L$ asymmetry for each generation, as well as the total asymmetry, normalized to the entropy density. (b) shows the N_R deviations from equilibrium Δ_i as defined in the text.

III. NONMINIMAL MODELS

The above class of models illustrates that the out-of-equilibrium processes implied by the necessary suppression of $B-L$ violation contain the hidden potential for production of a baryon asymmetry of the correct order of magnitude via leptogenesis. However, the pattern of neutrino masses in these minimal models is very rigid. We can ask how much of this rigidity is due to the details of the model. As an attempt to understand this, we investigate certain nonminimal prescriptions for fermion masses.

The first possible extension is the introduction of extra heavy **126** fields. In this way a nontrivial phase structure in the Majorana mass matrix can be spontaneously generated. These fields will affect the Dirac-Yukawa couplings of the **10** exactly as in Fig. 1 so that mixing and *CP* violation in the quark sector will again be suppressed naturally by ratios of order $v_R^2/M_{\Sigma_i}^2$. However, *CP* violation from the neutrino Majorana mass matrix can be full strength. Of course, any predictive power is lost with such an addition, unless we are willing to consider specific *Ansätze.* We content ourselves with the observation that the net effect of such an addition will be the introduction of full-strength *CP*-violating phases which can be rotated into the neutrino Dirac mass matrix upon diagonalization of the right-handed neutrino mass matrix. Since the neutrino Dirac-Yukawa couplings are still dominated by the direct couplings to the **10**, they will not change appreciably in magnitude. Therefore the enhancement in *CP* violation for right-handed neutrino decays will be given at most by the ratio of the angles which appear. Comparing to the analysis above, we see that this will provide at most an enhancement of a factor of 5 in the produced asymmetry, since the *CP* phase angles which appeared above were $\simeq\pi/10$. Thus we do not expect that a qualitatively different light neutrino mass spectrum is possible with a generic introduction of extra heavy **126** fields.

Next, let us examine a particular family of nonminimal *Ansätze.* Consider the popular Fritzsch-type *Ansätze* [18], with the incorporation of the Georgi-Jarlskog asymptotic relation $m_s \simeq m_\mu/3$ [19]. A specific model which is phenomenologically interesting is presented in Ref. [20]. We do not address how such a mass matrix *Ansatz* could arise from the underlying theory. There are several ways in which nonminimal scalar representations and mixing in the scalar sector could be introduced in order to generate such mass matrices. The neutrino Dirac-Yukawa couplings take the form

$$h_\nu^{\rm Dirac} = \frac{1}{v_u}\begin{bmatrix} 0 & \sqrt{m_u m_c} & 0 \\ \sqrt{m_u m_c} & 0 & -3\sqrt{m_c m_t} \\ 0 & -3\sqrt{m_c m_t} & m_t \end{bmatrix}, \tag{3.1}$$

and the Majorana mass matrix for the right-handed neutrinos takes the form

$$M_\nu^{\rm Maj} = \begin{bmatrix} M_1 e^{i\gamma_1} & 0 & 0 \\ 0 & M_2 e^{i\gamma_2} & M_3 e^{i\gamma_3} \\ 0 & M_3 e^{i\gamma_3} & 0 \end{bmatrix}. \tag{3.2}$$

The masses which appear in $h_\nu^{\rm Dirac}$ are, of course, the asymptotic values obtained by a renormalization group evolution from low energies. Triviality bounds still require the eigenvalues of $M_\nu^{\rm Maj}$ to be bounded by $\simeq 1.5 v_R$, but otherwise we assume the right-handed neutrino masses are arbitrary. $M_\nu^{\rm Maj}$ is diagonalized by the transformation

$$U^T M_\nu^{\rm Maj} U, \tag{3.3}$$

where $U=KV$ is unitary, with V orthogonal and K a diagonal matrix of phases. From the form of $M_\nu^{\rm Maj}$ we have

$$V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}, \tag{3.4}$$

and we let σ_i be the phases in K:

$$K = {\rm diag}(e^{i\sigma_1}, e^{i\sigma_2}, e^{i\sigma_3}). \tag{3.5}$$

Because of the imposed symmetries, which have determined the form of the mass matrices, neutrino masses in this model are very different from the minimal model considered above. The neutrino states are not "well mixed" at all, and the diagonalization of the right-handed neutrino masses will not upset the large hierarchy in the Dirac-Yukawa couplings. Thus the right-handed neutrino which has mass M_1 will have very weak Dirac-Yukawa couplings, and will decay very slowly. Furthermore, the lepton-number-violating cross sections for the light family will be highly suppressed.

The first issue we must examine is whether or not an initial baryon asymmetry is able to survive the period of lepton-number-violating interactions. As done above for the minimal models, we set the phases in the Majorana mass matrix to zero and consider the evolution of an initially nonzero baryon asymmetry from the GUT scale through the intermediate temperature regime. The depletion of $B+L$ is complete and is exactly as shown in Fig. 2. In Fig. 4 we show the depletion of $B-L$, and thus B, for right-handed neutrino masses all equal 10^{11} GeV. As we expect from the poor mixing of the light generation, approximately one-third of the initial asymmetry will survive the lepton-number-violating temperature regime, and thus this model avoids the neutrino

TABLE II. Depletion of $B-L$ asymmetry as a function of the lightest right-handed neutrino mass for the Fritzsch *Ansatz* model.

M_{N1} (GeV)	$Y_{B-L}^{\rm final}/Y_{B-L}^{\rm initial}$
10^{10}	1.0
10^{9}	0.8
10^{8}	0.1

TABLE III. Production of a baryon asymmetry via leptogenesis in the Fritzsch *Ansatz* model.

μ_1,μ_2,μ_3 (GeV)	$\sigma_1,\sigma_2,\sigma_3$	ν_i Masses (eV)	Y_B
$10^{12},10^{12},10^{12}$	20°,45°,60°	$10^{-11},6\times10^{-3},5$	2×10^{-7}
$10^{11},10^{11},10^{11}$	20°,45°,60°	$10^{-10},0.06,50$	10^{-8}
$3\times10^{10},3\times10^{11},3\times10^{11}$	20°,45°,60°	$3\times10^{-10},0.02,16$	3×10^{-9}
$3\times10^{10},3\times10^{11},3\times10^{11}$	6°,5°,4°	$3\times10^{-10},0.02,16$	2×10^{-10}
$5\times10^{10},5\times10^{10},5\times10^{10}$	20°,45°,60°	$2\times10^{-10},0.1,97$	3×10^{-10}
$10^{10},10^{10},10^{10}$	20°,45°,60°	$10^{-9},0.6,490$	2×10^{-14}

mass bounds. In fact, we find that the right-handed neutrino masses can be taken as small as 10^9 GeV before lepton-number-violating effects begin to deplete the asymmetry associated with the light generation. The depletion factor $Y_{B-L}^{\text{final}}/Y_{B-L}^{\text{initial}}$ as a function of the lightest right-handed neutrino mass is given in Table II.

Given the above, we might expect that little more could be said about this model. However, the introduction of nonzero phases in the Majorana mass matrix creates a constraint of a different type. Because the decay rate of the first generation right-handed neutrino is highly suppressed, it remains out of equilibrium for much longer than it otherwise would. As the temperature decreases, the lepton-number-violating scattering processes quietly become unimportant, but the decays of this neutrino continue to produce a $B-L$ asymmetry. This state of affairs can continue over two decades of temperature. The resulting asymmetry can be very large, and this must be avoided. The results of some calculations (far from exhaustive) for this model are shown in Table III. We have chosen to parametrize the right-handed neutrino Majorana matrix in terms of its positive eigenvalues μ_i and the phases σ_i. In terms of the mass matrix parameters, $\mu_1=M_1$, $\mu_{2,3}=\frac{1}{2}|M_2\pm\sqrt{M_2^2+4M_3^2}\,|$. The mixing angle θ is determined from the μ_i, $\tan\theta=\sqrt{\mu_3/\mu_2}$. The phases

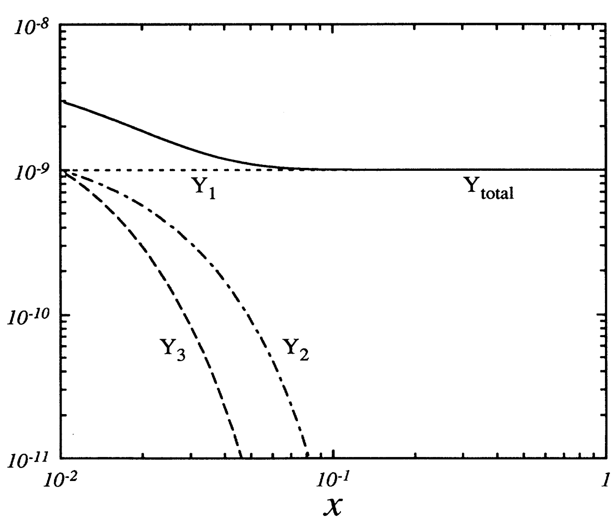

FIG. 4. $B-L$ depletion for the Fritzsch *Ansatz* model with right-handed neutrino masses all equal 10^{11} GeV. Notice that the asymmetry associated with the light family survives unmolested.

σ_i were chosen arbitrarily. Note that the entropy production associated with the decay of the light N_R is no longer negligible, due to its great deviation from equilibrium; using the estimate $\Delta s\simeq M_N/g_* T_{\text{decay}}$ [21,12], the decrease of the asymmetry is approximately a factor of 2, and this is incorporated in the results.

IV. DISCUSSION

The two models examined above have contrasting cosmological consequences, and there is a sense in which they display the two natural possibilities. The minimal model possesses no special symmetries in the fermion couplings to scalars, and therefore displays "democratic" behavior in the language of Ref. [12]. On the other hand, the symmetries which determine the form of the right-handed Majorana mass matrix in the nonminimal model will not allow a relaxation of the Dirac hierarchy via mixing, and this model is "correlated" in the language of Ref. [12]. To display a behavior intermediate to this would almost certainly require the introduction of small parameters which allow the Majorana mass matrix to interpolate between the symmetric form of the nonminimal model and a more generic form. It is not clear how such small parameters could be introduced in a natural way.

We have seen that the minimal models probably do not allow an initial baryon asymmetry to survive the intermediate temperature period at the required level. However, the intermediate scales are such that *CP* violation in the lepton sector can produce a viable $B-L$ asymmetry; sphaleron reprocessing will create a baryon asymmetry of the observed size. The survival question for an initial baryon asymmetry thus becomes irrelevant in this model. The consequences for neutrino mass phenomenology are, of course, somewhat grim. Certainly the light neutrino masses are well below the level relevant to Mikheyev-Smirnov-Wolfenstein (MSW) solar neutrino oscillations [22]. They are also an order of magnitude too large for a vacuum oscillation solution [22]. It is interesting that our bounds on the $U(1)_{B-L}\times U(1)_{I_{3R}}$ breaking scale drive us to a unique gauge symmetry breaking chain, the minimal one $SO(10)\rightarrow SU(2)_L\times SU(2)_R\times SU(4)$.

The results for the nonminimal model which we examined were in marked contrast to those of the minimal model. Because of the imposed symmetric mass matrix structure, an initial baryon asymmetry is able to survive the intermediate temperature period for all interesting values of the right-handed neutrino masses. However, the introduction of leptonic *CP* violation is problematic. Because of the large suppression of the decay rate for one species of right-handed neutrino, the production of a $B-L$ asymmetry can escalate out of control. This effect can be countered by choosing a slight hierarchy in the right-handed neutrino masses and by choosing the angles σ_i of order $\pi/10$. We could find no solution with larger phases, and this seems a little uncomfortable, since there is no mechanism which could ensure this situation. These choices give rise to light neutrino masses which can be phenomenologically interesting, as indicated by the one viable entry in Table III, with light neutrino masses $m_{\nu i} \simeq 3\times10^{-10}$ eV, 0.02 eV, 16 eV. Notice that the third generation neutrino naturally comes out quite heavy, approaching the upper bound from cosmological overclosure arguments [23], making it a hot dark matter candidate. This is another interesting coincidence of the model.

We comment that the identification of the measured baryon asymmetry with a GUT-scale generated asymmetry, as is possible in the nonminimal model, may be plagued with other difficulties. For example, in initial baryon asymmetry will be destroyed by an inflationary period which is probably required to dilute the population of magnetic monopoles which are generated at the GUT scale [24]. There may also be a problem with monopoles generated at the intermediate scale, though there exists a mechanism for the annihilation of these monopoles [25]. Obviously such issues depend on details other than the fermion mass spectrum, and it would be beyond the scope of this paper to address them here.

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APPENDIX

In this appendix we list the two sets of parameter values in the minimal model for which we present results. Most of this does not require comment. We note only that we can accommodate a phenomenologically acceptable value for $|V_{cb}|$, and that the ratio m_d/m_u is typically about 1.5, which is slightly smaller than the value which appears in Ref. [26], but this does not worry us excessively. The neutrino Dirac-Yukawa coupling matrix which we give is expressed in the basis where the right-handed neutrino Majorana mass matrix has been diagonalized. J is the rephasing invariant measure of CP violation in the quark sector [27]. Notice that the CP violation in the quark sector is controlled by the difference in $\arg\delta_u$ and $\arg\delta_d$; in set 2 we have lowered $\arg\delta_u$, but the value of J has increased.

Set 1:

$$m_t=150\text{ GeV}\ ,\quad m_c=1.22\text{ GeV}\quad m_b=-4.35\text{ GeV}\ ,$$

$$\tan\beta=-47.1\ ,\quad |V_{cb}|=0.045\ ,\quad |V_{ub}|=0.006\ ,$$

$$m_u=4\text{ MeV}\ ,\quad m_d=6.5\text{ MeV}\quad m_s=135\text{ MeV}\ ,$$

$$J\simeq2\times10^{-6}\ ,\quad \arg\delta_u=9^\circ\ ,\quad \arg\delta_d=13^\circ\ ,$$

$$M_{N_R}=\frac{1}{R}(0.006\text{ GeV},0.022\text{ GeV},0.12\text{ GeV})\ ,$$

$$h_\nu^{\text{Dirac}}=\begin{bmatrix}0.016 & 0.045 & -0.066\\ 0.045 & 0.13 & -0.19\\ -0.066 & -0.19 & 0.29\end{bmatrix},$$

$$\epsilon_{ij}=\begin{bmatrix}2.7\times10^{-6} & -7.6\times10^{-6} & -2.2\times10^{-6}\\ 1.4\times10^{-6} & -2.3\times10^{-7} & -6.8\times10^{-7}\\ 4.7\times10^{-7} & -1.5\times10^{-6} & 2.9\times10^{-6}\end{bmatrix}.$$

Set 2: As above except $\arg\delta_u=4^\circ$, $J\simeq10^{-5}$, $m_u=4.6$ MeV, and

$$\epsilon_{ij}=\begin{bmatrix}9.7\times10^{-7} & -7.5\times10^{-6} & 1.6\times10^{-5}\\ -5.3\times10^{-7} & -6.9\times10^{-6} & 1.6\times10^{-5}\\ -2.3\times10^{-7} & -6.3\times10^{-6} & 1.4\times10^{-5}\end{bmatrix}.$$

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