Estimates of $m_d - m_u$ and $\langle\bar{d}d\rangle - \langle\bar{u}u\rangle$ from QCD sum rules for D and D^* isospin mass differences

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The recent experimental data on D^+-D^0 and D^{*+}-D^{*0} mass differences are used as inputs in the QCD sum rules to obtain new estimates on the mass difference of light quarks and on the difference of their condensates: $m_d - m_u = 3 \pm 1$ MeV, $\langle\bar{d}d\rangle - \langle\bar{u}u\rangle = -(2.5 \pm 1) \times 10^{-3}\langle\bar{u}u\rangle$ (at a standard normalization point, $\mu = 0.5$ GeV).

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The QCD sum rules invented more than a decade ago are now well known to be a very useful tool to study properties of hadrons at intermediate energies and to get information on the basic parameters of QCD, such as quark masses and nonperturbative condensates. One of the problems addressed already in the pioneering paper [1] by Gasser and Leutwyler before the advent of QCD sum rules was the relation between the isotopic symmetry violation on the level of hadrons and the difference between u- and d-quark masses. Using PCAC (partial conservation of axial-vector current), current algebra, and experimental data on isospin splitting in pseudoscalar nonet, Weinberg [2] found a reliable estimate of the quark mass difference $m_d - m_u = 3$ MeV, as well as for the sum $m_d + m_u = 11$ MeV. The physical effects arising from the nonzero $m_d - m_u$ were then considered in Refs. [3, 4]. The first attempt to determine the isospin violation in quark condensates,

$$\gamma = \langle\bar{d}d\rangle/\langle\bar{u}u\rangle - 1, \tag{1}$$

was performed by Shifman, Vainshtein, and Zakharov [5]. Later the parameter γ was determined in a number of papers, using various approaches. In the chiral perturbation theory Gasser and Leutwyler [6] found $\gamma \approx -8 \times 10^{-2}$, provided $m_d - m_u = 3$ MeV, $\langle\bar{s}s\rangle/\langle\bar{u}u\rangle - 1 = -0.2$ [7]. The same parameter was also determined from hadronic mass splittings in a quark model [8], the Nambu–Jona-Lasinio model [9], as well as in the framework of the QCD sum rule method [10]. The results are spread in the region $\gamma = -(3\text{–}10)\times10^{-3}$. Recently isospin violation in QCD sum rules for the nucleon, Σ, and Ξ was considered [11] and the following results were obtained: $m_d - m_u = 3 \pm 1$ MeV, $\gamma = -(2 \pm 1) \times 10^{-3}$. Thus, while most predictions for $m_d - m_u$ agree and are grouped around 3 MeV, predictions for $\langle\bar{d}d\rangle - \langle\bar{u}u\rangle$ are more diverse and range within an order of magnitude. Moreover, arguments were given in Ref. [12] that m_u may be equal to zero due to instanton contributions to the renormalization of the quark mass. Thus, additional independent estimates of differences between u- and d-quark masses and condensates are certainly welcome.

In this paper we will consider QCD sum rules for isospin mass splittings of D^* and D mesons and make use of the recently reported [13] results on these splittings,

$$\begin{aligned} m_{D^{*+}} - m_{D^{*0}} &= 3.32 \pm 0.08 \pm 0.05 \text{ MeV}, \\ m_{D^+} - m_{D^0} &= 4.80 \pm 0.10 \pm 0.06 \text{ MeV}, \end{aligned} \tag{2}$$

to obtain such estimates. The sum rules are similar to those which were used in Ref. [14] to successfully predict the mass splittings $m_{D_s^*} - m_{D^*} = 110 \pm 20$ MeV and $m_{D_s} - m_D = 120 \pm 20$ MeV.

Let us start with the correlator of two pseudoscalar currents with quantum numbers of D, $j_5 = \bar{c}\gamma_5 q$, where q is either u, or d, at the Euclidean momentum $-q^2 > 1$ GeV^2,

$$C^q = i \int d^4x\, e^{iqx} \langle 0|T\{j_5(x), j_5^\dagger(0)|0\rangle \tag{3}$$

and consider its variation δC^q as the light quark mass rises from zero to its actual value m_q. To estimate C^q it is possible to take into account only the unit operator and the quark condensate (Fig. 1) in the operator product

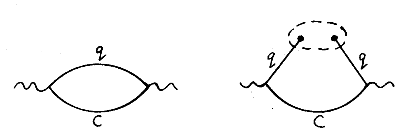

FIG. 1. The diagrams taken into account in the sum rules.

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expansion, since the contribution of operators of higher dimension to the heavy-light correlators is negligible [15]. Using the expansion of the quark condensate in the quark mass,

$$\langle q^a_\alpha(x)\bar{q}^b_\beta(0)\rangle = \left(-\frac{1}{12}\delta_{\alpha\beta}\langle\bar{q}q\rangle + \frac{i}{48}m_q\hat{x}_{\alpha\beta}\langle\bar{q}q\rangle\right)\delta^{ab}, \tag{4}$$

it is easy to see that δC^q is a function of m_q and $\langle\bar{q}q\rangle - \langle\bar{q}q\rangle_0$ where the subsript 0 denotes the chiral limit. On the other hand, saturating the correlators in the standard manner by the corresponding lowest mass resonances D^+ and D^0, subtracting the continuum from the contribution of the unit operator, and applying the Borel transformation [5], $(s+Q^2)^{-1} \to M^{-2}\exp(-s/M^2)$, we arrive at the following sum rule for $\delta C^d - \delta C^u$:

$$-\frac{\beta_{D^+}-\beta_{D^0}}{\beta_D m_D}M^2 + (m_{D^+}-m_{D^0})_{\rm hadr} = \frac{M^4 e^{m_D^2/M^2}}{2\beta_D^2 m_D}\left\{(m_d-m_u)\left[\frac{3m_c}{4\pi^2}\{e^{-x}-e^{-y}-x[E_1(x)-E_1(y)]\}L^{4/9} - \frac{\langle\bar{q}q\rangle}{2M^2}e^{-x}\left(1+\frac{m_c^2}{M^2}\right)\right] + \frac{\langle\bar{d}d\rangle-\langle\bar{u}u\rangle}{M^2}m_c e^{-x}L^{4/9} - (s_{D^+}-s_{D^0})\frac{3}{8\pi^2M^2s_D}(s_D-m_c^2)^2e^{-y}\right\}. \tag{5}$$

Here $E_1(x) = \int_x^\infty dt\, t^{-1}e^{-t}$, m_c is the charmed quark mass, s_D is the continuum threshold, $x = m_c^2/M^2$, $y = s_D/M^2$, and the residue of the D meson into the current j_5 is $\beta_D^2 = f_D^2 m_D^4/m_c^2$, where f_D is the semileptonic decay constant defined by $\langle D|\bar{c}\gamma_\mu\gamma_5 q|0\rangle = -if_D p_\mu$. The dependence of light quark masses and condensates on the normalization point μ in the operator product expansion is given by powers of $L = \ln(M/\Lambda)/\ln(\mu/\Lambda)$ where $\Lambda = 150$ MeV and we take $\mu = 0.5$ GeV which corresponds to $\langle\bar{q}q\rangle = -(0.24\text{ GeV})^3$. Here $\beta_{D^+} = \beta_D + \delta\beta_D^d$, $\beta_{D^0} = \beta_D + \delta\beta_D^u$, and $s_{D^+} = s_D + \delta s_D^d$, $s_{D^0} = s_D + \delta s_D^u$ where β_D and s_D are the residue and the continuum thresholds in the chiral limit and $\delta\beta_D^{u,d}$ and $\delta s_D^{u,d}$ are the deviations of residues and of continuum thresholds from their values in the chiral limit. The anomalous dimension of the current j_5 is not taken into account, since the momentum $|q^2| = 1 - 2$ GeV^2 is comparable with the heavy quark mass.

Similar sum rules can be written for the correlator of two vector currents, $j_\mu = \bar{c}\gamma_\mu q$,

$$C^q_{\mu\nu} = i\int d^4x\, e^{iqx}\langle 0|T\{j_\mu(x), j^\dagger_\nu(0)|0\rangle. \tag{6}$$

In the case of the tensor structure $q_\mu q_\nu$ for which the sum rule is known to work better [16], we obtain

$$-\left(\frac{\beta_{D^{*+}}-\beta_{D^{*0}}}{\beta_{D^*}m_{D^*}} - \frac{(m_{D^{*+}}-m_{D^{*0}})_{\rm hadr}}{m_{D^*}^2}\right)M^2 + (m_{D^{*+}}-m_{D^{*0}})_{\rm hadr} = -\frac{m_{D^*}e^{m_{D^*}^2/M^2}}{2\beta_{D^*}^2}\left[(m_d-m_u)\langle\bar{q}q\rangle e^{-x} + (s_{D^{*+}}-s_{D^{*0}})\frac{e^{-y}M^2}{4\pi^2}\left(1-\frac{3m_c^4}{s_{D^*}^2}+\frac{2m_c^6}{s_{D^*}^3}\right)\right]. \tag{7}$$

It is important to emphasize that since we do not take into account perturbative two-loop diagrams with one-photon exchange in the "theoretical" part of the sum rules, only the hadronic parts of the isospin splittings, $(\Delta m_D)_{\rm hadr}$ and $(\Delta m_{D^*})_{\rm hadr}$, enter Eqs. (5) and (7). To obtain them we use a quark model estimate [17] for the photon cloud part of the mass difference $m_{D^+} - m_{D^0} = 1.7\pm 0.5$ MeV and also take into account the electromagnetic hyperfine splitting

$$\delta m = -\frac{2\pi Q_c Q_q|\Psi(0)|^2}{3m_c m_q}[2S(S+1)-3], \tag{8}$$

where S is the spin of the meson. The quark masses here are the constituent ones, $m_c \sim 1.7$ GeV, $m_q \sim 0.3$ GeV. The quark-antiquark wave function at the origin may be estimated using the relation $f_D^2 = 12|\Psi(0)|^2/m_D$. Using the estimate [15] $f_D = 170$ MeV, we get that the hyperfine electromagnetic interaction contributes ~ 0.5 MeV to $D^+ - D^0$ and ~ -0.17 MeV to $D^{*+} - D^{*0}$ mass splittings. Using the full experimental mass differences from Eq. (2) we thus have the mass splittings to be used in the sum rules in Eqs. (5) and (7):

$$(m_{D^{*+}} - m_{D^{*0}})_{\rm hadr} = 1.8 \pm 0.5\text{ MeV},$$
$$(m_{D^+} - m_{D^0})_{\rm hadr} = 2.6 \pm 0.5\text{ MeV}. \tag{9}$$

From the sum rules in Eqs. (5) and (7) it follows that in the working region for the Borel parameter M^2, the left-hand sides (LHS's) should be close to linear functions in M^2 whose extrapolations to $M^2 = 0$ give the corresponding hadronic mass splittings while the slopes give information on the residue differences, $\beta_{D^+} - \beta_{D^0}$ and $\beta_{D^{*+}} - \beta_{D^{*0}}$. To numerically analyze the sum rules we take $m_D = 1.87$ GeV, $m_D^* = 2.01$ GeV, the standard value of the quark condensate, $\langle\bar{q}q\rangle = -(0.24)^3$ GeV^3, and $m_c = 1.35$ GeV. For the residues and continuum thresholds we take the estimates obtained with sum rules

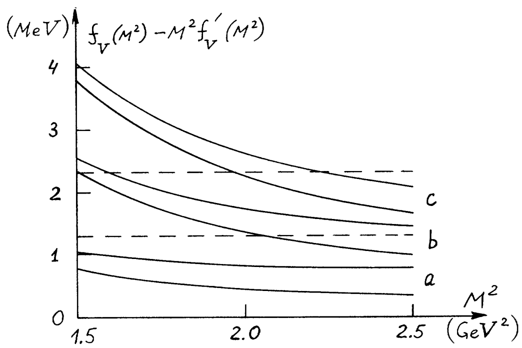

FIG. 2. Sum rule for D^* mass splitting: $f_V(M^2) - M^2 df_V(M^2)/dM^2$ vs M^2, where $f_V(M^2)$ is the RHS of Eq. (7) for $m_d - m_u = 1$ MeV (a), 3 MeV (b), and 5 MeV (c). In each of the three cases the lower and the upper curves correspond to $s_{D^{*+}} - s_{D^{*0}} = 0$ and 0.005 GeV^2, respectively. The dashed horizontal lines are the boundaries set by Eq. (9).

in Refs. [15, 16], $\beta_D^2 \approx \beta_{D^*}^2 \approx 0.2$ GeV^4 and $s_D \approx s_{D^*} \approx 6$ GeV^4.

Thus, $m_d - m_u$, $\langle \bar{d}d \rangle - \langle \bar{u}u \rangle$, the Borel parameter M^2, and the differences of continuum thresholds $s_{D^+} - s_{D^0}$ and $s_{D^{*+}} - s_{D^{*0}}$ are the fitting parameters of the sum rules in Eqs. (5) and (7). The working regions in M^2 determined by the requirement of controllable contributions from nonperturbative power corrections and continuum in the chiral limit for u and d quarks are [15, 16] 1 GeV^2 $< M^2 <$ 2 GeV^2 for the pseudoscalar channel and 1.5 GeV^2 $< M^2 <$ 2.5 GeV^2 for the vector channel. We will let the difference of thresholds vary around an estimate $s_{D^+} - s_{D^0} \sim s_{D^{*+}} - s_{D^{*0}} \sim (m_d - m_u)\sqrt{s_D}$ which implies that the threshold and the quark mass differences are of the same sign.

In Fig. 2 we show the result of the sum rule calculation of the D^* mass difference by plotting $f_V(M^2) - M^2 df_V(M^2)/dM^2$, where $f_V(M^2)$ is the RHS of Eq. (7). From Eq. (7) one sees that only the quark mass difference enters this sum rule and one might hope to fix $m_d - m_u$ from it. The term proportional to $s_{D^{*+}} - s_{D^{*0}}$ has the same sign as the term proportional to $m_d - m_u$ and in the whole working interval in M^2 contributes less than 50% to $f(M^2) - M^2 df_V(M^2)/dM^2$ at $m_d - m_u = 3$ MeV. The values $m_d - m_u > 4$ MeV cannot be made compatible with $(m_{D^{*+}} - m_{D^{*0}})_{\rm hadr}$ from Eq. (9) at any $s_{D^{*+}} - s_{D^{*0}} > 0$. At $m_d - m_u < 4$ MeV the compatibility can be achieved by varying $s_{D^{*+}} - s_{D^{*0}}$. However, at $m_d - m_u < 2$ MeV the continuum term is dominant in the whole working region in M^2 and the sum rule is not reliable. So, from the sum rules for D^* it follows that $m_d - m_u$ is limited from above by a value about 4 MeV and, consequently, the value $m_u = 0$ is excluded, if $m_u + m_d$ =11 MeV.

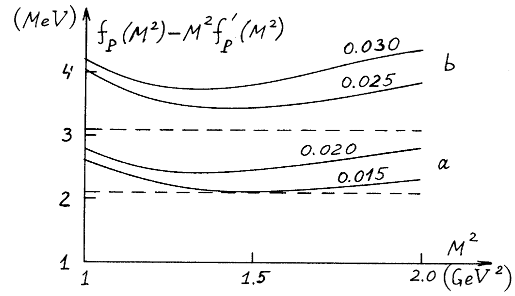

FIG. 3. Sum rule for D mass splitting: $f_P(M^2) - M^2 df_P(M^2)/dM^2$ vs M^2, where $f_P(M^2)$ is the RHS of Eq. (5) for $m_d - m_u = 3$ MeV and (a) $\gamma = -2.5 \times 10^{-3}$, (b) $\gamma = -6 \times 10^{-3}$. The numbers at the curves correspond to the value of $s_{D^+} - s_{D^0}$ (in GeV^2). The dashed horizontal lines are the boundaries set by Eq. (9).

As for the D mass difference, the corresponding sum rule in Eq. (5) is contributed both by $m_d - m_u$ and $\langle \bar{d}d \rangle - \langle \bar{u}u \rangle$. In Fig. 3 we show the numerical results for $m_d - m_u = 3$ MeV and two different values of $\gamma = \langle \bar{d}d \rangle / \langle \bar{u}u \rangle - 1$. One can see that the value $\gamma = -2.5 \times 10^{-3}$ is consistent with the $(m_{D^+} - m_{D^0})_{\rm hadr}$ from Eq. (9), while a higher value $\gamma = -6 \times 10^{-3}$ is definitely excluded.

In summary, we conclude from our numerical analysis of the sum rules for isospin mass splittings in D and D^* mesons that $m_d - m_u = 3 \pm 1$ MeV and $\gamma = -(2.5 \pm 1) \times 10^{-3}$. The result for $m_d - m_u$ is consistent with the earlier estimates and together with the relation $(m_u + m_d)\langle \bar{u}u \rangle = -F_\pi^2 m_\pi^2$ excludes the option $m_u = 0$ advocated in Ref. [12]. Our estimate for the condensate difference parameter γ is significantly smaller than usually adopted and supports the value obtained in Ref. [11] from the isospin splittings in baryons.[1] The much larger value of γ found in Ref. [6] arises mainly from the terms nonanalytic in quark mass from one-loop correction in the chiral perturbation theory. Such nonanalytic in m_q terms do not appear in the QCD sum rule calculation of D meson isospin splittings. So, if the value of γ found in Ref. [6] were correct, it would mean that higher order terms in the operator product expansion and in α_s are essential in this calculation.

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