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Discrete symmetry, neutrino magnetic moment, and the 17-keV neutrino

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The problem of generating large transition magnetic moments for nearly massless neutrinos in a truly three-generation case is discussed. A model to achieve the same by exploiting an octahedral symmetry is presented. The scheme also accommodates a radiatively generated mass of 17 keV for a pseudo Dirac neutrino that decays rapidly through the Majoron channel.

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Two problems in neutrino physics have attracted much attention over the past few years. The first, and relatively long-standing one, deals with the deficiency in the solar neutrino count in the Davis and Kamiokande experiments [1] and the related matter of the apparent anticorrelation between the observed solar neutrino flux and the sunspot activity [2]. The other, more recent one, is concerned with the reported signature of a 1% admixture of a 17-keV neutrino with the ν_e [3].

While the first problem can be resolved by postulating a relatively large magnetic moment for the neutrino [4], to generate the latter in realistic models is no mean task. For such an attempt normally leads to too large a value for the neutrino mass. An elegant solution to this problem was suggested by Voloshin [5] in the form of an $SU(2)_\nu$ symmetry connecting ν_L and ν_R (or ν_e and ν_μ if you are interested in transition moments) so that the mass term is a triplet while the magnetic moment term is a singlet. In the limit of exact $SU(2)_\nu$ symmetry you then have the spectacle of a nonzero μ_ν but an identically vanishing m_ν. Several models [6,7] have been constructed using this idea and some variants, but most require some amount of fine tuning. The reason lies in the phenomenological necessity of breaking the continuous non-Abelian symmetry at a scale too high to protect m_ν [8].

A way out of this imbroglio is to employ a non-Abelian discrete symmetry instead, an idea that has been richly harvested [9]. An aesthetic problem persists though in such attempts, in the form of the unequal treatment they mete out to the standard model (SM) fermions. The point to remember is that if you put all the SM ν's in the same representation, then for an odd number of generations it is the mass term that contains the singlet and not the μ_ν term [10]. Hence, for three generations the Voloshin mechanism does not work. Instead, one should attempt to construct models wherein the lowest-dimensional Higgs operators coupling to the neutrino current are antisymmetric in the generation index [7]. To achieve this in a model where the ν's lie in a representation R of the symmetry group, it is essential that the symmetric and antisymmetric parts of $R\times R$ lie in equivalent irreducible representations.

In our efforts to construct a model based on such ideas, we find that a very slight extension of the same also affords a solution to the second problem mentioned at the outset of this Brief Report. Although phenomenological considerations [11] indicate that the new find is most probably a Dirac particle and that it may be identified with the ν_τ, yet many embarrassing questions remain. Not the least of which are the questions of generating such a low scale, and, more importantly, satisfying the strict theoretical constraints emanating from cosmology [12] and primordial nucleosynthesis [13]. Although some models have been proposed [14,15], only one of these [15] makes an effort to connect the two issues that have been raised here.

For our purpose we choose the (24-element) symmetry group ($\mathcal{O}$) to be that of the octahedron, i.e., the one generated by rotations about three fourfold axes (f_i), four threefold axes (t_K), and six twofold axes (z_α) [16]. The group algebra is given by $f_i^4=e$, $t_1=f_2f_3$, $t_2=f_3f_1$, $t_3=f_1f_2$, $t_4=f_1t_2f_3$, $z_i=f_it_i^2$, $z_{i+3}=f_it_4^2$. $\mathcal{O}$ has five irreducible representations: namely,

$$\mathcal{A}_1\colon f_i=1,\quad \mathcal{A}_2\colon f_i=-1,\quad \mathcal{E}\colon f_1=\sigma_1, f_2^*=f_3=(-\sigma_1+\sqrt{3}\,\sigma_2)/2\ ,$$
$$\mathcal{F}_1\colon f_1=\exp(\pi T_1/2),\quad f_2=\exp(-\pi T_2/2),\quad f_3=\exp(-\pi T_3/2),\quad \mathcal{F}_2\colon f_i=-f_i(\mathcal{F}_1)\ ,$$

where $(T_i)_{jk}=\epsilon_{ijk}$. Note that only $\mathcal{F}_{1,2}$ are faithful representations. The Clebsch-Gordan decomposition is given by (A and S denote symmetry properties)

$$\mathcal{F}_1\times\mathcal{A}_2=\mathcal{F}_2,\quad \mathcal{E}\times\mathcal{A}_2=\mathcal{E},\quad \mathcal{F}_1\times\mathcal{E}=\mathcal{F}_1+\mathcal{F}_2\ ,$$
$$\mathcal{F}_1\times\mathcal{F}_1=(\mathcal{A}_1+\mathcal{E}+\mathcal{F}_2)^S+\mathcal{F}_1^A,\quad \mathcal{E}\times\mathcal{E}=(\mathcal{A}_1+\mathcal{E})^S+\mathcal{A}_2^A\ ,$$

the rest following trivially.

THE MODEL

To the standard model fermions we add a charge $+1$ vector singlet pair of leptons per generation. Also we introduce three right-handed neutrino fields. The new additions however are given an unconventional assignment of the total lepton number, which is conserved explicitly. The quarks are the same as in the SM and we shall not talk about them any further. The entire leptonic spectrum [under $SU(2)_L\otimes U(1)_Y\otimes\mathcal{O}\otimes U(1)_l$] is then

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$L_L(2,-\frac{1}{2},\mathcal{I}_1,1)$, $E_R(1,-1,\mathcal{I}_1,1)$, $F_{L,R}(1,1,\mathcal{I}_1,1)$, $N_{1R}(1,0,\mathcal{A}_1,-1)$, $N_{2R}(1,0,\mathcal{A}_1,-2)$, and $N_{3R}(1,0,\mathcal{A}_1,-4)$.

As for the scalar sector, apart from the $\phi(2,\frac{1}{2},\mathcal{A}_1,0)$ and $H(2,\frac{1}{2},\mathcal{E},0)$ which give masses to the SM fermions, we also have $\Sigma(1,0,\mathcal{I}_1,5)$ and $\sigma(1,0,\mathcal{A}_1,6)$ to break the lepton number and give a Majorana mass term, $\Omega(1,1,\mathcal{I}_1,7)$, $\Xi(1,1,\mathcal{A}_1,6)$, $\chi(2,\frac{3}{2},\mathcal{A}_1,0)$, and $\eta(2,\frac{1}{2},\mathcal{I}_1,2)$ that traverse in loops responsible for various radiative generations, and finally $\xi(2,\frac{1}{2},\mathcal{I}_1,-2)$ and $\zeta(2,\frac{1}{2},\mathcal{I}_1,-3)$ to give Dirac masses to the neutrinos.

The fermion mass and Yukawa terms then read

$$\begin{aligned}\mathcal{L}_{m+Y}=\tilde{m}\overline{F_L}F_R+\overline{L_L}E_R(a_1\phi+a_2H)+b_1\overline{N_{1R}}L_L\xi\\+b_2\overline{N_{2R}}L_L\zeta+c\overline{N_{2R}^c}N_{3R}\sigma+g_1\overline{F_R}L_L\chi\\+g_2\overline{L_L^c}F_L\eta^\dagger+\text{H.c.}\ ,\end{aligned}\tag{1}$$

while the Higgs potential, apart from the usual quadratic and quartic invariants, also contains the cross terms

$$\begin{aligned}\mathcal{L}_{\text{Higgs}}=\Omega^\dagger\Sigma_\eta(\lambda_1\phi+\lambda_1'H)+\lambda_2\Xi^\dagger\sigma^\dagger\Omega\Sigma\\+\lambda_3\chi^\dagger\sigma^\dagger\Xi\phi+\lambda_4\chi^\dagger\Sigma^\dagger\xi\Omega\\+\lambda_5\zeta^\dagger\sigma^\dagger\xi\Sigma+\mu_1\eta^\dagger\zeta\Sigma+\cdots\ ,\end{aligned}\tag{2}$$

where we have displayed only those terms that interest us. In all of the above, the Clebsch-Gordan coefficients are implicitly present.

The fields η,ζ,ξ are ascribed a positive (mass)^2 value so that they do not gain a vacuum expectation value (VEV) at the tree level. One good feature of our model is that we do not need to introduce a new high scale as all symmetry including $\mathcal{O}$ and the lepton number are broken at the weak scale. The tree-level VEV's are then

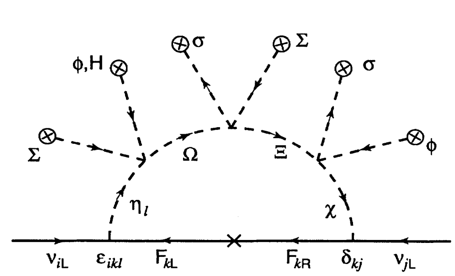

FIG. 1. Diagrams (sans photon lines) contributing to neutrino magnetic moments.

$$\begin{aligned}\langle\sigma\rangle=s,\quad \langle\Sigma\rangle=(S_1,S_2,S_3)\ ,\\ \langle\phi\rangle=v_s,\quad \langle H\rangle=(v_1,v_2)\ ,\end{aligned}\tag{3}$$

where only the $\mathcal{O}$ dependence is exhibited. Apropos the domain wall problem, it can be tackled [17] by either invoking symmetry nonrestoration in multi-Higgs models or the possible absence of high temperature phase transition in a system with large net lepton number as is the case here.

To this level then, the charged lepton mass matrix is diagonal with all three exotic particles degenerate with a mass $\tilde{m}\sim 200$ GeV. This form assures that there are no flavor-changing neutral currents (FCNC's) to the leading order. The model however cannot explain the SM fermion mass hierarchy which is to be taken care of by appropriate choice of VEV'S and Yukawa couplings. On the other hand, no Dirac masses for the neutrinos have been generated and the neutrino mass matrix is of rank 2.

A magnetic moment for the neutrino is generated through the diagram in Fig. 1 on insertion of a photon on either internal line. The contribution to μ_ν can be symbolically expressed as

$$\mu_\nu\sim\frac{2e}{16\pi^2}\frac{g_1g_2\lambda_1\lambda_2\lambda_3S^2s^2v^2}{\tilde{m}^7(x_\chi-x_\Omega)}\left[\frac{h(x_\Xi,x_\eta)-h(x_\Xi,x_\chi)}{x_\eta-x_\chi}-\frac{h(x_\Xi,x_\eta)-h(x_\Xi,x_\Omega)}{x_\eta-x_\Omega}\right]\ ,\tag{4}$$

where

$$h(x,y)=\frac{f(x)-f(y)}{x-y}\ ,$$

$f(x)=(1-x)^{-3}[(1-4x+3x^2)/2-x^2\ln x]$ and $x_\chi\equiv m_\chi^2/\tilde{m}^2$. The function $f(x)$ is monotonically decreasing with $f(0)=\frac{1}{2}$, $f(1)=\frac{1}{3}$, and $f(\infty)=0$. It should be noted that the above is only the contribution for a particular set of fields traveling in the loop. The full family dependence of μ_ν can easily by obtained by summing over all such diagrams taking into account the different masses, VEV's, and couplings. To get an order of magnitude estimate we assume that all the scalars and the F fields have mass ~ 200 GeV and that the couplings in Eq. (4) are each ~ 0.1. We then have

$$\mu_\nu\sim 10^{-11}\mu_B\tag{5}$$

and hence of the correct order of magnitude to explain the observed anticorrelation [2,4].

Normally, with the removal of the photon, this diagram would generate a mass correction for the neutrino thus requiring fine tuning. However, in the present model, this correction term is antisymmetric in the generation index and hence does not contribute at all to the neutrino Majorana mass. As pointed out right at the beginning, this is *not* a consequence of Voloshin-like symmetry. Rather, unlike in the Voloshin mechanism, here the μ_ν term is not a group invariant and hence cannot arise until after the symmetry is broken. The key to the protection of the mass lies in the structure of the theory and more particularly that of the lowest-order diagram leading to μ_ν. A look at the fermion line of Fig. 1 shows that irrespective of the scalars traversing the loop, the effective operator coupling to the neutrino current has to be antisymmetric. This result owes its origin to the fact that the mass term for the exotic fermions ($F_{L,R}$) is $\mathcal{O}$ invariant (i.e., independent of the breaking) and hence proportional to the unit matrix in the generation space. Any departure from such structure is caused only by higher-dimensional operators and shall be commented upon later.

This would have been the whole story were it not for the fields N_{iR} and the scalars ξ and ζ. Although there are no three- or four-dimensional operators leading to VEV's for them, higher-dimensional operators arising from radiative corrections do contribute to $\langle \xi_i \rangle$, etc. A typical example is the operator $\xi \Sigma^{\dagger 2} \sigma^2 \phi^\dagger$ (as in Fig. 2) resulting in

$$\langle \xi_i \rangle \sim \frac{\lambda_2 \lambda_3 \lambda_4}{16\pi^2} \frac{S^2 s^2 v}{m_\xi^2 m_{\text{loop}}^2} \sim 1 \text{ MeV} , \tag{6}$$

where m_{loop} is the typical mass of the scalars in the loop. Similar values for $\langle \zeta_i \rangle$ and $\langle \eta_i \rangle$ are also generated through such diagrams and mixings with each other. Nonzero $\langle \eta_i \rangle$ of course lead to mixings of the Sm charged leptons with the exotics, but due to the huge disparity in scales the levels of FCNC are somewhat below current experimental limits. The neutrino mass matrix, in the $(\nu_i N_1 N_2 N_3)$ basis (where ν_i represent the SM particles and all fields are of the same helicity), now reads

$$M_\nu = \begin{bmatrix} 0 & M_1^T \\ M_1 & M_2 \end{bmatrix} ,$$

$$M_1 \equiv \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ 0 & 0 & 0 \end{bmatrix} , \tag{7}$$

$$M_2 \equiv \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & M \\ 0 & M & 0 \end{bmatrix} ,$$

where $\alpha_i = b_1 \langle \xi_i \rangle$, $\beta_i = b_2 \langle \zeta_i \rangle$, and $M = cs$. M_ν, which is of rank 4, has the eigenvalues $0, 0, \pm[(G - \sqrt{G^2 - 4H})/2]^{1/2}$ and $\pm[(G + \sqrt{G^2 - 4H})/2]^{1/2}$. Here $G = M^2 + \boldsymbol{\alpha}^2 + \boldsymbol{\beta}^2$ and $H = M^2 \boldsymbol{\alpha}^2 + (\boldsymbol{\alpha} \times \boldsymbol{\beta})^2$. Note that α_i, β_i can naturally be ~ 10 keV without requiring either an artificial generation of such a scale or unnaturally small Yukawa couplings. Assuming $M \sim 250$ GeV, the neutrino spectrum then consists of three apparently Dirac particles—one superheavy, one massless, and one of mass 17 keV. The mixing of ν_{17} with ν_e is engendered by the ratios of the Dirac mass terms and easily give the required strength.

At this stage it is as well to point out that the full symmetry of M_ν is not a symmetry of the theory and hence is broken by quantum corrections. For example, the off-diagonal mass terms for the charged leptons arising out of $\langle \eta_i \rangle$ would lead to nontrivial mixing in that sector and hence to neutrino mass corrections through diagrams as in Fig. 1. However, due to the smallness of $\langle \eta_i \rangle$, these corrections are almost of the seesaw type in magnitude ($\sim 10^{-3}$ eV) and do not alter the neutrino spectrum to any significant degree. Also, higher loop diagrams generate Majorana mass terms of similar order and involving "ordinary" neutrinos. As a result of all these, the mass degeneracies are lifted and the Dirac neutrinos split into three pairs of pseudo Dirac particles. The small masses for ν_e and ν_μ that are thus generated would be adequate for a Mikheyev-Smirnov-Wolfenstein- (MSW-) type of resonance enhancement in the Sun [18]. Also the effective mass contributing to the neutrinoless double β decay [$\beta\beta_{0\nu}$], is $\lesssim \beta_1^2/M$ and though miniscule, affords an example where the effective Majorana mass for $\beta\beta_{0\nu}$ could be larger than that to be observed in Kurie plots [19].

Of course, one might wonder if diagrams analogous to those in Fig. 1, but with N_{iR} as the virtual leptons instead of $F_{L,R}$ would contribute to Majorana mass terms. For if they did, the earlier group theoretical argument leading to exact cancellations would not hold and indeed the contributions could be large. However, it is easy to see that there is no place for such apprehension. Two facts need to be noted. Firstly, there is no Dirac term involving N_{3R} and secondly, the only tree order (and hence large) Majorana mass term is of the form $(\overline{N_{2R}^c} N_{3R} + \text{H.c.})$. As a result, there can exist no one-loop diagram with N_{iR} as the internal particle(s) and contributing to the neutrino Majorana masses. This can be verified rigorously by working with the mass eigenstates instead. Such arguments obviously do not hold for complicated multiloop diagrams, but those contributions are too small to be relevant.

The Majoron (the only surviving Goldstone boson in the theory), to the leading order, is given by

$$\begin{aligned} \vartheta \sim (&6s \,\text{Im}\sigma + 5S_i \,\text{Im}\Sigma_i + 2\langle \eta_i \rangle \text{Im}\eta_i \\ &- 2\langle \zeta_i \rangle \text{Im}\zeta_i - 3\langle \xi_i \rangle \text{Im}\xi_i)/N \end{aligned} \tag{8}$$

(where N gives the normalization) and is hence primarily an $SU(2)_L$ singlet. Thus its coupling with the SM charged leptons is highly suppressed and fully in consonance with the bonds coming from Z-decay width [20] as well as astrophysical considerations [21]. However, if one considers the coupling of the ν's with the Majoron, one gets

$$G_{\nu\vartheta} \approx N^{-1} \begin{bmatrix} 0 & G_1^T \\ G_1 & 6M_2 \end{bmatrix} , \quad G_1 \equiv \begin{bmatrix} 2\alpha_1 & 2\alpha_2 & 2\alpha_3 \\ 3\beta_1 & 3\beta_2 & 3\beta_3 \\ 0 & 0 & 0 \end{bmatrix} , \tag{9}$$

which is not diagonalized simultaneously along with M_ν. This then leads to a nondiagonal ν-ϑ coupling of the order of m_ν/N and as a consequence to a very fast decay of the 17-keV neutrino which would have a lifetime $\sim 10^5$ s.

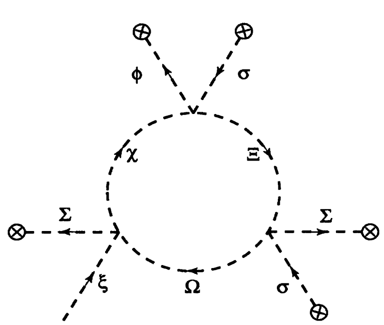

FIG. 2. Typical diagram leading to radiative generation of $\langle \xi_i \rangle$.

To conclude, we have presented a model based on a non-Abelian discrete symmetry $\mathcal{O}$ that leads to a significant amount of transition magnetic moment for nearly massless neutrinos. The model is *not* a discrete version of the Voloshin mechanism, which we have argued cannot work for the truly 3-generation case. Rather, the protection of m_ν owes its existence to the absence of any family symmetric effective scalar operator to the lowest order. The magnetic moment term itself arises on breaking the symmetry, which, being discrete, can be preserved until at least the weak scale. Higher order effects do lead to small mass corrections but these are greatly suppressed.

A simple extension of this model is shown to accommodate a pseudo Dirac 17-keV neutrino as well. The latter can be identified with the ν_τ and is generated through a cripple seesaw mechanism that keeps ν_e and ν_μ massless. However, tiny FCNC's in the charged lepton sector and multiloop diagrams together cause small mass corrections of the order of 10^{-3} eV. The ν_{17} decays very fast into a lighter neutrino and a singlet-doublet Majoron and is thus consistent with all known experiments, whether terrestrial or cosmic.

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