Spin structure of constituent quarks

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We point out that in chiral quark models constituent quarks can have a nontrivial spin structure. The constituent quarks are quasiparticles, surrounded by a cloud of quark-antiquark pairs, with the angular momentum of the cloud polarized opposite to the valence quark spin. We discuss baryon magnetic moments and axial-vector current matrix elements in this context.

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In the last few years a great deal of interest has been focused on the question how the spin of the proton is shared among its constituents. Naive nonrelativistic quark models (NQM's) reproduce magnetic moments remarkably well. In such models the proton spin is simply the sum of valence quark spins. On the other hand, the European Muon Collaboration (EMC) experiment [1] together with information on semileptonic hyperon decays indicates that only a fraction of the proton spin is carried by the spin of the quarks, in apparent disagreement with the NQM. In terms of the fractions Δq of the proton spin S_μ carried by quark $q=u,d,s$ in the proton, i.e., $S_\mu \Delta q = \langle P,S|\bar{q}\gamma_\mu\gamma_5 q|P,S\rangle$, this analysis gives

$$\begin{aligned}\Delta u &= 0.78\pm0.08 \ ,\\ \Delta d &= -0.50\pm0.08 \ ,\\ \Delta s &= -0.16\pm0.08 \ .\end{aligned} \tag{1}$$

Hence the total spin sum is $g_A^0=\Delta u+\Delta d+\Delta s=0.12\pm0.17$, whereas the NQM predicts $g_A^0=1$.

In a recent article Karl [2] suggested that there is no strong contradiction between octet-baryon magnetic moments and axial current matrix elements, even in a constituent quark picture, if one is only willing to accept that the constituent quarks themselves have a nontrivial spin structure. The purpose of this Brief Report is to show that such a picture arises indeed naturally in a chiral quark model [4,5]. In such a model constituent quarks are quasiparticles dressed by polarization clouds of quark-antiquark pairs (and gluons; see also Ref. [6]). The angular momentum of the cloud is polarized opposite to the valence quark spin, so that axial-vector current matrix elements are effectively reduced from their NQM values.

In his analysis Karl starts from a set of generalized Sehgal equations [3] which connect the baryon magnetic moments with the spin fractions Δq in the proton. For instance

$$\Delta u = [u_\uparrow - u_\downarrow + \bar{u}_\uparrow - \bar{u}_\downarrow]a_u \ , \tag{2}$$

where $u_\uparrow$ is the number of u quarks, $\bar{u}_\uparrow$ the number of u antiquarks with spin parallel to the proton spin, etc., and a_u is the one-body axial current matrix element for a u quark in the proton. Analogous expressions hold for d and s quarks. In the naive nonrelativistic quark model one has $u_\uparrow=\frac{5}{3}$, $u_\downarrow=\frac{1}{3}$, $d_\uparrow=\frac{1}{3}$, $d_\downarrow=\frac{2}{3}$, all other quark spin distributions are equal to zero. Furthermore, $a_u=a_d=1$ and, hence, $\Delta u=\frac{4}{3}$, $\Delta d=-\frac{1}{3}$, and $\Delta s=0$ in this model. Under the assumption that all quarks and antiquarks of a given flavor are in a $J=\frac{1}{2}$ mode of some confining potential or cavity, the baryon magnetic moments can be expressed as linear combinations of the Δq's. For example the proton magnetic moment reads

$$\mu(P)=\mu_u\Delta u+\mu_d\Delta d+\mu_s\Delta s \ . \tag{3}$$

The μ_q's are defined as the quark magnetic moments per unit axial charge:

$$\mu_q=\left[\frac{1-\lambda_q}{1+\lambda_q}\right]\frac{\tilde{\mu}_q}{a_q} \ , \tag{4}$$

where

$$\lambda_q=\frac{\bar{q}_\uparrow-\bar{q}_\downarrow}{q_\uparrow-q_\downarrow}$$

is the ratio of antiquark to quark polarizations and $\tilde{\mu}_q$ is the magnetic moment of the quark q in the proton. These effective moments incorporate contributions not only from valence quarks but also from quark-antiquark pairs. A best fit [2] to baryon magnetic moments and to the empirical axial-vector coupling constant $g_A\equiv g_A^3=\Delta u-\Delta d=1.26$ from neutron β decay is achieved with[1]

$$\mu_u=2.42, \quad \mu_d=-1.21, \quad \text{and } \mu_s=-0.71 \ , \tag{5}$$

together with

$$g_A^0=0.27\pm0.23, \quad g_A^8=\Delta u+\Delta d-2\Delta s=0.86\pm0.05 \ . \tag{6}$$

The errors reflect the fact that g_A^0 is rather poorly determined by the fit, whereas g_A^8 follows with much higher accuracy. In the naive NQM the corresponding values would be $g_A^0=g_A^8=1$ and $g_A=\frac{5}{3}$ instead. Solving Eq. (6)

[1]All magnetic moments are given in units of nuclear magnetons, $e/2M$, with the proton mass $M=938$ MeV.

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together with the empirical $g_A = 1.26$ gives

$$\Delta u = 0.86 \pm 0.12 \; , \quad \Delta d = -0.40 \pm 0.12 \; , \quad \Delta s = -0.20 \pm 0.05 \; . \tag{7}$$

These values are not far from those of Eq. (1) based on semileptonic decays and the EMC data. In particular, g_A^0 from the fit is not incompatible with these data. However, g_A^8 of Eq. (6) is too large as compared with the empirical $g_A^8 = 0.60 \pm 0.05$ from an overall SU(3) fit to semileptonic hyperon decays.

Reference [2] asked for constituent quark models which correspond to Eq. (5) and (6). We now show that a chiral quark model which combines spontaneous chiral symmetry breaking with the axial U(1) anomaly does indeed yield values for effective magnetic moments and axial current matrix elements of constituent quarks that are roughly consistent with (5) and (6). Our approach is based on the three-flavor version of the Nambu–Jona-Lasinio (NJL) model [4]. It starts from the following effective Lagrangian:

$$\mathcal{L}_{\rm NJL} = \bar{\psi}(i\partial^\mu \gamma_\mu - \hat{m})\psi + \mathcal{L}_4 + \mathcal{L}_6 \; , \tag{8}$$

with $\psi = (u,d,s)^t$ and the current quark mass matrix $\hat{m} = \mathrm{diag}(m_u^0, m_d^0, m_s^0)$. The local four-quark interaction $\mathcal{L}_4$ is symmetric under the chiral $\mathrm{U}(3)_L \times \mathrm{U}(3)_R$ group:

$$\mathcal{L}_4 = G_S \left[\left[\bar{\psi}\frac{\lambda_a}{2}\psi \right]^2 + \left[\bar{\psi} i\gamma_5 \frac{\lambda_a}{2}\psi \right]^2 \right] - G_V \left[\left[\bar{\psi}\gamma_\mu \frac{\lambda_a}{2}\psi \right]^2 + \left[\bar{\psi}\gamma_\mu\gamma_5 \frac{\lambda_a}{2}\psi \right]^2 \right] . \tag{9}$$

Here λ_a with $a = 0, 1, \ldots, 8$ are the standard U(3) flavor matrices including the singlet $\lambda_0 = \sqrt{2/3}\,\mathrm{diag}(1,1,1)$. The interaction (9) incorporates scalar-pseudoscalar $(P+S)$ and vector–axial-vector $(V+A)$ interactions with coupling strengths G_S and G_V of dimension (length)^2. For our purposes we use a "minimal" model which starts from a local effective interaction between quark color currents, $-G_C(\bar{\psi}\gamma_\mu t^a \psi)^2$, where t^a $(a = 1, \ldots, 8)$ are the SU(3) color generators. By Fierz rearrangement one finds $\mathcal{L}_4$ of Eq. (9) with $G_S = 2G_V = \frac{8}{9}G_C$.

In nature the axial $\mathrm{U}(1)_A$ symmetry is broken dynamically, presumably by instantons. A minimal effective interaction, first suggested by 't Hooft [7], which selectively breaks $\mathrm{U}(1)_A$ but leaves the remaining $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times \mathrm{U}(1)_V$ untouched, is a six-quark interaction in the form of a 3×3 determinant in flavor space:

$$\mathcal{L}_6 = G_D \{ \det[\bar{q}_i(1+\gamma_5)q_j] + \det[\bar{q}_i(1-\gamma_5)q_j] \} \; . \tag{10}$$

The effective Lagrangian (8) with the interaction $\mathcal{L}_4 + \mathcal{L}_6$ has been used extensively in the mean field approximation to study a variety of low energy, nonperturbative phenomena. A cutoff Λ of order 1 GeV is employed to regularize momentum space (loop) integrals. The physical picture behind this model is that strong interactions between quarks operate at low momenta, i.e., for quark momenta smaller than Λ, whereas they are "turned off" for momenta larger than Λ.

For sufficiently strong coupling G_S the vacuum undergoes spontaneous chiral symmetry breaking. Quark condensates $\langle \bar{u}u \rangle$, $\langle \bar{d}d \rangle$, and $\langle \bar{s}s \rangle$ develop. Current quarks turn into constituent quarks with large dynamical masses determined by a set of gap equations. For example, the constituent u quark has a mass $m_u = m_u^0 - G_S\langle \bar{u}u \rangle - G_D\langle \bar{d}d \rangle\langle \bar{s}s \rangle$. The constituent quarks are quasiparticles. Their strong interaction surrounds the valence quarks by quark-antiquark polarization clouds, so that the constituent quarks have nontrivial form factors. Spontaneous chiral symmetry breaking implies the existence of pseudoscalar Goldstone bosons. In the chiral limit $m_u^0 = m_d^0 = m_s^0 = 0$ and with $\mathcal{L}_6 = 0$, the whole pseudoscalar nonet of π's, K's, η_0 and η_8 is massless. In the NJL model these modes emerge as explicit solutions of the quark-antiquark Bethe-Salpeter equation [4]. Dynamical $\mathrm{U}(1)_A$ symmetry breaking by $\mathcal{L}_6$ of Eq. (10) gives the singlet η_0 a nonzero mass. Furthermore, explicit breaking of chiral $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ by bare quark masses $m_s^0 > m_{u,d}^0 > 0$ moves all masses of the pseudoscalar nonet to their physical values. In particular, the η-η' system is well reproduced including its mixing angle which comes out typically around $\theta_P \approx -10°$.

Consider now the singlet axial current $j_\mu^5 = \bar{\psi}\gamma_\mu\gamma_5\psi$. In QCD this current has the anomalous divergence $\partial^\mu j_\mu^5 = (3g^2/8\pi^2)\mathbf{E}\cdot\mathbf{B}$ (for $N_f = 3$), where $\mathbf{E}$ and $\mathbf{B}$ are the color electric and magnetic gluon fields. In the model (8) this $\mathrm{U}(1)_A$ anomaly is reexpressed through the six-point interaction (10). In fact this interaction is constructed such that it simulates instanton effects at the mean field level. Higher order effects are likely to unbalance the simple flavor structure of the four-point interaction $\mathcal{L}_4$, for instance by rescaling the coupling strength in front of the flavor singlet V and A terms in (9). Symmetry considerations certainly permit additional terms

$$\delta L_4 = -\delta G_A \left[\bar{\psi}\gamma_\mu\gamma_5 \frac{\lambda_0}{2}\psi \right]^2 - \delta G_V \left[\bar{\psi}\gamma_\mu \frac{\lambda_0}{2}\psi \right]^2 \tag{11}$$

in the effective four-point interaction. The fact that the masses of the ω and ρ mesons are almost degenerate indicates that δG_V is small and we ignore it. There is no reason, however, to ignore δG_A. Our aim is now to explore the combined effects of δG_A and G_D on the spin structure of constituent quarks, subject to constraints imposed by known properties of the η-η' system.

Let us now match our chiral quark model with the phenomenological analysis [2]. The constituent quark currents in this model have a nonzero antiquark content. Hence their effective magnetic moments μ_q^* correspond to $(1-\lambda_q)\bar{\mu}_q$ in Eq. (4). Similarly, the constituent quark spin matrix elements Σ_q correspond to $(1+\lambda_q)a_q$ in Eq. (4). They are given by $\Sigma_q = \langle q | j_0^5 | q \rangle$ with the singlet axial current $j_\mu^5 = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s$. The quark magnetic moments per unit axial charge of Eq. (5) should then be compared with

$$\mu_q = \mu_q^* / \Sigma_q \tag{12}$$

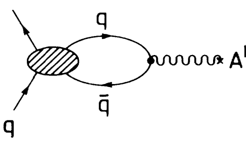

FIG. 1. Screening of quark axial-vector currents by the quark-antiquark polarization cloud.

in our model.

Previous NJL model calculations [5] have demonstrated that the anomalous magnetic moments of constituent quarks are negligibly small (less than 10^{-2} magnetons) once the constraint of small ω-ρ mass splitting is imposed. Hence the μ_q^* are given by their Dirac values in terms of the constituent quark masses. In proper nucleon magneton units $e/2M$, we have

$$\mu_u^* = \frac{2}{3}\frac{M}{m_u}, \quad \mu_d^* = -\frac{1}{3}\frac{M}{m_d}, \quad \mu_s^* = -\frac{1}{3}\frac{M}{m_s} . \tag{13}$$

Next we discuss the constituent quark spin matrix elements Σ_q. In the limiting case $G_V = \delta G_A = 0$, i.e., in the absence of any axial-vector interactions, we have $\Sigma_u = \Sigma_d = \Sigma_s = 1$ and $\Delta u = \frac{4}{3}$, $\Delta d = -\frac{1}{3}$, $\Delta s = 0$ as in the naive quark model. However, screening effects induced by the $V+A$ and $\delta\mathcal{L}_4$ interactions have an influence on Σ_q as illustrated in Fig. 1. A probing axial field now sees a $q\bar{q}$ polarization cloud that contributes to the constituent quark axial current. This screening effect reduces Σ_q from unity. The resulting spin fractions Δu, Δd, and Δs in the proton are then expressed in terms of the constituent quark spins using standard SU(6) relations.

We now present results for μ_u, μ_d, and μ_s together with Δu, Δd, and Δs as they emerge from the quasiparticle structure of the constituent quarks in a typical NJL calculation. The parameters of the model are constrained as follows. We choose here the minimal color-current coupling which implies $G_S = 2G_V$. We require a good fit to the pseudoscalar meson spectrum, i.e., to the masses of π, K, η, and η'. The coupling constants G_D and δG_A determine the η' mass and influence the η-η' mixing angle θ. We restrict the range of δG_A by demanding $|\theta| < 20°$ in agreement with Ref. [9]. The momentum space cutoff $\Lambda \approx 0.9$ GeV is fixed by fitting the pion decay constant $f_\pi = 93$ MeV.

Such a scenario is realized, for example, when the coupling strength G_S exceeds the critical coupling $G_S^{\rm crit} = 4\pi^2/3\Lambda^2$ for spontaneous chiral symmetry breaking by a factor of about $\frac{3}{2}$. With current quark masses $m_u^0 = m_d^0 = 6$ MeV and $m_s^0 = 140$ MeV, the resulting constituent quark masses turn out to be $m_u = m_d = 470$ MeV and $m_s = 620$ MeV. With $\delta G_A/G_V = 1.3$ and G_D fixed to reproduce the η' mass one finds $\theta \approx -20°$. The constituent quarks have singlet axial current matrix elements $\Sigma_u = \Sigma_d = 0.58$ and $\Sigma_s = 0.57$. Together with $\mu_u^* = -2\mu_d^* = 1.33$ and $\mu_s^* = -0.51$ this gives

$$\mu_u = 2.30, \quad \mu_d = -1.15, \quad \text{and } \mu_s = -0.89 . \tag{14}$$

The spin fractions become

$$\Delta u = 0.94, \quad \Delta d = -0.31, \quad \Delta s = -0.06 . \tag{15}$$

These values approach those given in Eqs. (5) and (7) as extracted by Karl [2]. We have systematically varied δG_A, always keeping the constraints imposed by the pseudoscalar meson sector throughout the procedure, with the results shown in Fig. 2.

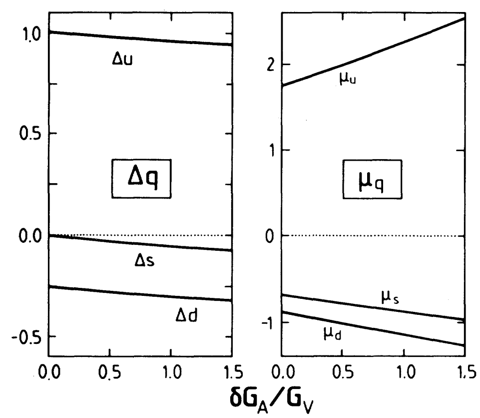

FIG. 2. Constituent quark effective magnetic moments μ_u, μ_d, and μ_s in units of $e/2M$ (dashed curves), and spin fractions $\Delta u, \Delta d, \Delta s$ in the proton (solid curves) as a function of $\delta G_A/G_V$ with $2\Lambda G_V = G_S$. The remaining model NJL parameters are constrained throughout by fitting empirical properties of the pseudoscalar mesons.

In summary, while the "spin crisis" is certainly not resolved in the present model, the calculations point to the importance of screening effects induced by the quark-antiquark polarization cloud that gives the constituent quarks a nontrivial structure. The angular momentum of the cloud is polarized opposite to the valence spin. As a consequence the singlet axial constant g_A^0 is reduced to about $\frac{1}{2}$, with the general pattern of baryon magnetic moments still reproduced. A further substantial reduction of axial constants is expected due to the relativistic motion of constituent quarks in the nucleon, as discussed, e.g., in [8].

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