Self-consistent dynamics of a light composite scalar Higgs boson

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We apply a simple approximate relativistic amplitude-unitarization generalization of nonrelativistic Schrödinger-equation dynamics to the scattering of longitudinal mass-degenerate W and Z gauge bosons. The strong energy dependence of our amplitude near the WW threshold then makes possible a nonperturbative self-consistent nonelementary neutral Higgs scalar bound state (H) just below this threshold. We must, however, include a constant term approximating high-energy inelastic effects in addition to H exchange. Everything, including the H mass, can then be determined in terms of the small phenomenological WWH coupling and W mass, which serves to set the energy scale of the problem; this is the same number of arbitrary parameters as in the underlying electroweak theory. The partial-wave amplitude containing the H is then in approximate agreement at zero energy with the one given by the perturbative crossing-symmetric H-pole-only tree-graph amplitude. We find unacceptable zero-energy disagreement, however, if, instead of an inelasticity term, we insert a subtraction constant approximating the effect of short-range high-mass exchanges to obtain our H. Similar self-consistent H bound-state solutions can also arise near the $t\bar{t}$ threshold in $t\bar{t}$ scattering with H exchange.

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There is a recurrent interest in the possibility that the electroweak-symmetry-breaking Higgs scalar (H) may be a bound system rather than an elementary particle [1]. Recently, Sivers and Uretsky [2], building on an earlier preliminary exploration by Lee, Quigg, and Thacker [3], found a self-consistent bootstrap [4] neutral scalar resonance H in the unitarized-amplitude scattering of mass-degenerate $W^{\pm},Z^0$ weak-isospin triplets near the WW threshold. Additional short-range high-mass exchanges approximated by a large subtraction constant were also required. A related calculation with two arbitrary constants was carried out by Hikasa and Igi [5].

In our Padé-approximant amplitude-unitarization scheme we find that there are, in fact, two simple ways in which we can go beyond elastic unitarity and H exchange in order to guarantee a self-consistent H. One possibility is to introduce a term which approximates the presence of high-energy inelastic effects. We then find that we can approximately reproduce both the phenomenological perturbation-theory couplings and the zero-energy amplitude given by a crossing-symmetric tree-graph approximation.

An alternative possibility, which resembles the Sivers-Uretsky scheme, is to introduce a subtraction term which approximates simple high-mass exchange. Here we find that, while we can again reproduce the phenomenological perturbation-theory couplings, the zero-energy amplitude is then many times larger than the corresponding perturbative tree-graph approximation. Given the general success of such electroweak approximations at low energies, this difference is simply too extreme to be acceptable.

Standard fundamental electroweak theory has at least two arbitrary parameters, e.g., a dimensionless coupling constant and a mass to set the energy scale of the problem. In our scheme everything is determined in terms of the W mass and the WWH coupling ($\propto\sqrt{\beta}$ below).

In our dynamics we follow Refs. [2,3] in retaining only the longitudinal components of the W and Z vector bosons so that we effectively have a spinless problem. Strictly speaking, this is a high-energy approximation which requires energies $\gg$ the W mass for its *a priori* justification. However, we shall see that the self-consistency of our dynamics relies predominantly on the rapid energy dependence of our amplitude near a channel threshold, which permits a bound state close to such a threshold. This and other basic dynamical features should not be affected much by the introduction of transverse W and Z, although the actual numerical values of our H parameters would be modified somewhat.

Our actual dynamics is based on a simple relativistic generalization of ordinary nonrelativistic Schrödinger-equation dynamics for two-body scattering [6], and is basically an amplitude-unitarization procedure. Suppose $\tilde{q}$ and $\tilde{q}'$ are the initial and final center-of-mass three-momenta, and s is the kinetic energy for nonrelativistic scattering with the Yukawa-potential superposition

$$V(r)=-\gamma\sum_m g_m e^{-mr}/r\ , \tag{1}$$

corresponding to the exchange of particles of mass m. The amplitude A then has a branch point at $q^2=0$. If we take the corresponding cut to run from $s=0$ to ∞ in the s plane, and apply the Cauchy integral formula to $(A-\gamma W)$, we obtain the dispersion relation

$$A(s,t)=\gamma W(t)+\int_0^\infty ds'\,\mathrm{Im}\,A(s',t)/\pi(s'-s) \tag{2}$$

for any fixed value of the momentum transfer $t=-(\tilde{q}'-\tilde{q})^2$. Here $\gamma W(t)=\gamma\sum_m g_m/(m^2-t)$ is the first Born approximation (Fourier transform) of $V(r)$ and is the (constant) large-s limit of A for fixed t. Equation (2) can be easily verified to the second Born approximation, but continues to be valid to all orders.

For elastic scattering, S-matrix unitarity gives

$$4\pi \operatorname{Im} A(s,t)=\rho(s)\int d^2\hat{q}''A^*(s,t_1)A(s,t_2)\ , \qquad (3)$$

where $\rho(s)=q$ is the usual nonrelativistic phase-space factor, $t_1=-(\vec{q}''-\vec{q})^2$, $t_2=-(\vec{q}'-\vec{q}'')^2$, $\vec{q}''$ is the intermediate-state momentum, and $d^2\hat{q}''$ the infinitesimal solid-angle element for the $\vec{q}''$ direction. Exchange potentials, corresponding to the interchange of particles in a collision, can be reduced in the usual way to effective direct potentials by introducing appropriately symmetrized amplitudes.

Equations (2) and (3) together constitute a nonlinear integral equation for A completely equivalent to the Schrödinger equation with the same V. But they can be readily generalized to the relativistic case [6], where A now becomes the invariant amplitude, s and t the usual Mandelstam variables, and ρ the relativistic phase-space factor; for two equal-mass (μ) particles, for example,

$$\rho(s)=[(s-4\mu^2)/s]^{1/2}\theta(s-4\mu^2)\ , \qquad (4)$$

where $\theta(x)=1$ for $x\geq 0$ and $\theta(x)=0$ for $x<0$. With spinless t-channel exchange, the direct potential W continues to depend only on t. Relativistic exchange potentials arising from spinless Mandelstam u-channel exchange likewise depend only on u. As in the nonrelativistic case, they can be reduced to effective direct potentials by again introducing appropriately symmetrized amplitudes. Higher-spin energy-dependent exchanges will be discussed later.

Unlike the Bethe-Salpeter and other similar equations, Eqs. (2) and (3) do not give spurious singularities and/or inelastic contributions. They can be solved, at least formally, by iteration, starting, e.g., with $A=\gamma W$ as a first approximation. We then basically get an expansion in γ, which takes on the form

$$A_l(s)=\gamma W_l(s)+\gamma^2 B_l(s)+\cdots \qquad (5)$$

in the lth partial wave, where γW and $\gamma^2 B$ can be represented by the "unitarity" diagrams of Figs. 1 and 2 for two-body W,Z scattering; it is understood that we must also add in the corresponding diagrams with crossed lines. Figure 2 is given by Eqs. (2) and (3) with $A\rightarrow\gamma W$ on the right-hand side of Eq. (3). We can readily extend our scheme to include the effect of inelastic processes by adding graphs with additional intermediate states.

A truncated version of the expansion (5) no longer satisfies unitarity. We can restore unitarity, and achieve much more rapid convergence, by rearranging the series of Eq. (5) into $[N,N]$ [7] or $[1,N]$ [8] Padé approximants, where

$$[N,M]=\frac{\gamma n_1+\cdots+\gamma^N n_N}{1+\gamma d_1+\cdots+\gamma^M d_M}\ , \qquad (6)$$

with the n_i and d_i chosen such that an expansion of Eq. (6) in powers of γ agrees exactly with the expansion of Eq. (5) up to the γ^{N+M} term [6–8]. We shall restrict ourselves to the $[1,1]$ approximant, which then gives

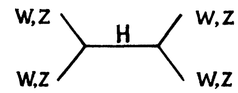

FIG. 1. Two-body W,Z scattering with H exchange giving γW.

$$A_l(s)\simeq\gamma W_l/(1-\gamma B_l/W_l)\ . \qquad (7)$$

This not only satisfies unitarity, but is exact for factorizable models with $A_l\simeq\sum_n\gamma^n uK^n v$, which are often reasonable approximations for Eq. (5) [9].

Treating the approximately mass-degenerate $W^{\pm},Z^0$ as a weak-isospin (I) triplet with average mass $m_Z\simeq m_W\simeq 85$ GeV/c^2, Fig. 1 gives an $I=0$ contribution

$$\gamma W=\beta m_H^2[1/(m_H^2-t)+1/(m_H^2-u)]+R\ , \qquad (8)$$

where m_i is the mass of particle $i\,(=H,W,\ldots)$ and the $1/(m_H^2-u)$ "exchange-potential" term comes from crossing the two final-state lines in Fig. 1; we have added in a term R to represent all other exchanges. Standard electroweak phenomenological perturbation theory would give

$$\beta=(m_H c^2/246\ \text{GeV})^2/16\pi$$

with our amplitude normalization. If θ is the center-of-mass scattering angle, the $l=0$ projection $W_0=\frac{1}{2}\int_{-1}^{1}W d\cos\theta$ gives, with $\mu=m_W$,

$$\gamma W_0(s)\simeq 4\beta m_H^2/(2m_H^2-4m_W^2+s)+c\ , \qquad (9)$$

where we have made the small-x approximation $\ln[(1+x)/(1-x)]\simeq 2x$, which is accurate for small

$$x=(s-4m_W^2)/(2m_H^2+s-4m_W^2)\ ,$$

and adequate for higher energies; the remainder c vanishes if we do not make any further corrections or additions (from R) to γW_0.

If we use the fact that $B_0(s)$ has a branch point at $s=4m_W^2$, with a corresponding cut in the s plane running from $s=4m_W^2$ to ∞, and apply the Cauchy integral formula, the resulting dispersion relation for $B_0(s)$ gives

$$\gamma^2 B_0(s)=\int_0^{\infty}ds'[\rho(s')|\gamma W_0(s')|^2/\pi(s'-s)]+K(s)\ , \qquad (10)$$

where we have used Fig. 2 or Eq. (3) with $A\rightarrow\gamma W$ on the right-hand side. Here K represents all other corrections and contributions to $\gamma^2 B_0$ (coming from other singularities in the s plane). It can usually be dropped in a reasonable first approximation if $\gamma^2 B_0$ is given by Fig. 2 alone.

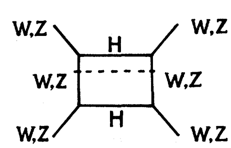

FIG. 2. Unitarity diagram for two-body W,Z scattering giving $\gamma^2 B$.

Equation (7) can now be readily seen to give $A_0(s)\to\gamma W_0(s)$ at the $s=4m_W^2-2m_H^2$ pole of Eq. (9).

A neutral scalar H manifests itself as a pole contribution

$$A_{0H}(s)=3\beta m_H^2/(m_H^2-s) \tag{11}$$

to the $I=0$ amplitude A_0, with an extra imaginary part in the denominator if it is a resonance above the WW threshold. If it is a nonelementary composite state, it would arise from Eq. (7) if

$$\mathrm{Re}\,A_0^{-1}(m_H^2)=0 \tag{12}$$

with pole residue

$$\frac{1}{3\beta m_H^2}=-\left[\frac{\partial}{\partial s}\mathrm{Re}\,A_0^{-1}(s)\right]_{s=m_H^2}. \tag{13}$$

We do not find any sensible self-consistent solution of Eqs. (7)–(13) with $c=0$ and $K=0$. We find, however, that a solution is possible with a large enough $K\neq 0$ in Eq. (10).

A $K\simeq a=\text{const}\neq 0$ term in Eq. (10) would represent the contribution of high-energy inelastic effects associated with the production of two or more particles, which mostly have s thresholds (s_I) well above the WW threshold at $s=4m_W^2$. In effect we would then be adding in graphs similar to Fig. 2 but with higher-mass vertical-line intermediate states. This is equivalent to inserting an effective factor $[1+r\theta(s-s_I)]$ into Eq. (4) and then absorbing the $r\theta$ contribution into K in Eq. (10). Such a K would then have a slow s dependence with large s_I. With $c=0$, Eq. (7) then gives, for $s\le 4m_W^2$,

$$\mathrm{Re}\,A_0^{-1}(s)=1/\gamma W_0-[K+\beta^2 J(s)]/(\gamma W_0)^2-(4m_W^2/s-1)^{1/2}, \tag{14}$$

where

$$\pi(\beta/\gamma W_0)^2 J=[2+2m_W^2/m_H^2+sm_W^2/m_H^2(m_H^2-2m_W^2)](1-2m_W^2/m_H^2)^{-1/2}\ln[m_H/m_W\sqrt{2}+(m_H^2/2m_W^2-1)^{1/2}]-1-s/2(m_H^2-2m_W^2)+2(4m_W^2/s-1)^{1/2}\tan^{-1}[(4m_W^2/s-1)^{1/2}],$$

which has a Taylor expansion in $(4m_W^2-s)$ around $s=4m_W^2$, since $\tan^{-1}0=0$ here. Using Eq. (9) with $c=0$, Eqs. (12) and (13) then give

$$(1-m_H^2/4m_W^2)^{-1/2}=(2m_W^2/3\beta m_H^2)+[5\sqrt{2}\ln(1+\sqrt{2})+2]/\pi+O[(1-m_H^2/4m_W^2)^{1/2}], \tag{15}$$

which immediately gives a self-consistent m_H near $2m_W$ for any given $\beta\ll 1$, including the phenomenological

$$\beta=[m_H c^2/(246\text{ GeV})]^2/16\pi\simeq 0.0095$$

of standard electroweak perturbation theory, where it is in principle an arbitrary parameter. From Eqs. (12) and (14) the inelastic-effect constant $K\simeq a$ must then have the approximate value

$$K\simeq(\beta m_W^2/2m_H^2)-\beta^2[3\sqrt{2}\ln(1+\sqrt{2})-2]/2\pi \tag{16}$$

if we are to have such a self-consistent H. The strong energy dependence of our amplitude near the WW-ZZ threshold arising from the last term of Eq. (14), and leading to the large first term of Eq. (15), also plays a crucial dynamical role in making possible this self-consistent H.

With a small (m_Z-m_W) mass splitting, which introduces an additional arbitrary parameter in our scheme, we replace Eq. (4) by

$$\rho(s)=[2(s-4m_W^2)^{1/2}\theta(s-4m_W^2)+(s-4m_Z^2)^{1/2}\theta(s-4m_Z^2)]/3\sqrt{s} \tag{17}$$

and keep $m_W\simeq m_Z$ elsewhere in first approximation. We then find that there are two possible self-consistent H solutions for $\beta\ll 1$, corresponding to two slightly different values of K. In one, H is immediately below the WW threshold, and in the other, between the WW and ZZ thresholds. The latter would have $m_W\to m_Z$ and an extra factor of $\frac{1}{3}$ in the last and first terms of Eqs. (14) and (15), respectively, and give a peak in the $WW\to WW$ cross section.

Returning to $m_W=m_Z$ and turning next to $s=0$, we find that, for the $\beta\simeq 0.0095$ of standard electroweak perturbation theory, Eq. (14) now gives an $I=0$, $l=0$ amplitude within 13% of the one given by the perturbative crossing-symmetric pole-only tree-graph amplitudes A^I of Ref. [2], which have the form

$$A^0=\beta m_H^2[3/(m_H^2-s)+1/(m_H^2-t)+1/(m_H^2-u)], \tag{18}$$

$$A^1=\beta m_H^2[1/(m_H^2-t)-1/(m_H^2-u)], \tag{19}$$

$$A^2=\beta m_H^2[1/(m_H^2-t)+1/(m_H^2-u)], \tag{20}$$

and incorporate the poles of Eqs. (8) and (11); (Ref. [3] also includes constant terms of the same order which will not affect our general conclusions). There is, of course, no reason why Eqs. (14) and (18) should agree exactly in a nonperturbative scheme such as the one we are using, and even a perturbation expansion would generate comparable corrections to Eq. (18). The general success of electroweak perturbation theory at low energies, however, would lead us to expect the two amplitudes to have at least approximate agreement at $s=0$. In fact, we find that this agreement improves if we make the more realistic finite-Λ $K\simeq a\Lambda/(\Lambda-s)$ approximation for our inelastic effects, as long as Λ is large enough to permit the above type of self-consistent H in the first place. In the $\beta\to 0$ limit the agreement becomes exact, with $m_H\to 2m_W$, if we have $\Lambda=28m_W^2+\lambda(\beta)$, where $\lambda(\beta)\to 0$, but $\lambda(\beta)/\beta \to \infty$; an example would be $\lambda \propto \sqrt{\beta}$.

An alternative to a strong-inelasticity $K\neq 0$, $c=0$ scheme is one with large heavy-mass exchanges, which we approximate by taking a constant $R\neq 0$ in Eq. (8) or a constant $c\neq 0$ in Eq. (9); this would arise if we had, e.g., an additional exchange of the same form as the H exchange of Fig. 1 or Eq. (8), but with $m_H\to\infty$. The integral of Eq. (10) will then diverge unless we replace it by a subtracted dispersion relation, which is equivalent to taking

$$K(s)=K_0-\int_0^\infty ds'\rho(s')|\gamma W_0(s')|^2/\pi(s'+s_0) \qquad (21)$$

in Eq. (10), and combining the integrals of Eqs. (10) and (21) to obtain convergence. Any additional constant contribution (of order γ^2) arising in Eq. (2) or (5) can then be absorbed in γW_0 to obtain a modified constant c in Eq. (9), leading in turn to a modified expansion (5) and Padé approximant (7). The constants K_0 and s_0 are then chosen so as to prevent the appearance of double poles in the resulting $1/A_0$ obtained from Eq. (7). This amounts to taking $K_0=0$ and

$$s_0=2m_H^2-4m_W^2+4\beta m_H^2/c$$

in Eq. (21), and leads to

$$\gamma W_0(s)/A_0(s)=1-\int_0^\infty ds'\frac{\rho(s')\gamma W_0(s')[\gamma W_0(s')-c]}{\pi(s'-s)[\gamma W_0(s)-c]}\ , \qquad (22)$$

which is equivalent to the purely elastic N/D equations of Ref. [2], at least if the "left-hand cut" is approximated by the $s=4m_W^2-2m_H^2$ pole of Eq. (9). Equation (22) can be readily seen to give $A_0(s)=\gamma W_0(s)$ at this pole.

With Eq. (22) we now find that Eqs. (12) and (13) do lead to a self-consistent H near the WW-ZZ threshold for any given $\beta\ll 1$, including the phenomenological $\beta\simeq 0.0095$ of standard electroweak perturbation theory, but only if

$$c\simeq\pi/2\sqrt{2}\ln(1+\sqrt{2})\ .$$

The strong energy dependence of our amplitude near the WW-ZZ threshold again plays a crucial role in making possible this self-consistent H.

Turning next to $s=0$, however, we find that Eq. (22) now gives an $I=0$ A_0 value ($\simeq\pi/2$) many times larger than the value (0.0665) given by Eq. (18). While some difference between these values might again be expected in a nonperturbative scheme such as the one we are using, this difference is simply too extreme to be acceptable, given the general phenomenological success of electroweak perturbation theory at low energies; it is therefore also not likely to be ameliorated by the introduction of transverse W and Z. It is possible, however, that it may not arise with a better treatment of high-mass exchanges which properly takes into account the energy dependences we might expect with higher-spin exchanges. With both inelastic effects and $c\neq 0$ we continue to have an $I=0$, $l=0$, $s=0$ amplitude which is larger than the one given by Eq. (18), although the situation is not as bad as it is in the complete absence of inelasticity.

If we extend our scheme to include particles with mass $>m_W$, such as the t quark, and consider, e.g., $t\bar{t}$ scattering, we find, as before, that Eqs. (12)–(14) can give a self-consistent H near the $t\bar{t}$ threshold with $\beta\to\beta'\ll 1$ and an appropriate value for a constant K representing high-energy inelastic effects; since the t is an isoscalar, we must replace $3\beta\to 2\beta'$ in Eq. (11) if $\beta\to\beta'$ in Eq. (9), remembering that the $1/(m_H^2-u)$ "exchange-potential" term is now absent in the equivalent of Eq. (8). We have ignored the effect of the spin of the t but its inclusion should not change any of our basic conclusions. The effect of the WW and ZZ channels is also relatively small and was neglected. At $s=0$ we find that our modified Eqs. (9) and (10) now give an $l=0$ amplitude (7), e.g., with $m_t=150$ GeV, which is roughly equal to the one given by the crossing-symmetric pole-only tree-graph amplitude of Eq. (18) with $\beta\to 2\beta'$, $3\to 1$ and no $1/(m_H^2-u)$ term, although the agreement is not quite as close as it was for W,Z scattering. There is again no reason for the agreement to be exact with our nonperturbative scheme.

We have not as yet explored the possibility of higher-spin resonances, which may perhaps lead to $\beta\ll 1$ H solutions requiring a smaller inelasticity contribution. The exchange of such states in the t and u channels would mean an energy-dependent potential, since we would have extra s-dependent factors multiplying the t and u poles of the type encountered in Eq. (8). In the nonrelativistic case such potentials are dealt with without introducing any divergences by allowing the g_m coefficients in Eq. (1) to vary explicitly with energy. For any given fixed energy $\bar{s}$ we then have the same equations (2) and (3) as before, but with the replacements

$$W(t)\to W(t,\bar{s})=\sum_m g_m(\bar{s})/(m^2-t)\ ,$$

$$A(s,t)\to A(s,t,\bar{s}),\quad A(s',t)\to A(s',t,\bar{s})\ ,$$

and

$$A(s,t_i)\to A(s,t_i,\bar{s})$$

for $i=1,2$ [6]. The actual physical amplitude is then $A(s,t,\bar{s})$ at $s=\bar{s}$. The generalization to the relativistic case is otherwise the same as before, with s,t,u becoming again the usual Mandelstam variables, and $\rho(s)$ the relativistic phase-space factor [6], which reduces to Eq. (4) for equal-mass scattering.

Eventually we must go beyond the longitudinal W,Z approximation and include the full spin complications of the W and Z and any other particles we might bring into our model. We must also construct more explicit models for the inelastic effects which played such an important role in Eqs. (14)–(16), and go beyond the kind of simplified [1,1] Padé approximant we used in Eq. (7). These improvements may require considerably more care in dealing properly with the singularities of the resulting equations than was necessary with Eq. (7). Considerable care must also be exercised in making any kind of discrete or lattice approximations, since these could easily introduce large errors in view of the rapid energy dependence of our amplitudes near two-body thresholds. However, we do not expect our main conclusions, which relied so critically on this latter already-present qualitative feature, to be significantly affected by any improvements we might make in our calculations.

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