### Nucleon structure as a background for determination of fundamental parameters

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(Received 8 February 1993)

We consider deep inelastic (quasi)elastic lepton-nucleon scattering and investigate the possibilities of eliminating or suppressing theoretical uncertainties induced by the nucleon structure in measuring the standard model parameters or in searching for new physics. On the basis of rather general assumptions about nucleon structure we have obtained new relations between cross sections and neutral current parameters which are weakly dependent on the nucleon structure. We also investigate the dependence of the QCD  $\Lambda$  parameter extracted from the data on the unknown large-scale nucleon structure and propose a modification of the conventional QCD predictions in which the dependence on this uncertainty factor is suppressed.

PACS number(s): 13.60.Hb, 12.38.Aw, 13.40.Fn, 25.30.-c

#### I. INTRODUCTION

Precision measurements of the standard model (SM) and the QCD parameters as well as the search for new physics manifestations require special efforts for elimination of the factors poorly controlled theoretically. In extracting fundamental parameters from the data these factors bring uncertainties and play the role of some specific background. Further we will investigate deep inelastic (DIS) and (quasi)elastic lN scattering, considering these processes as a source of important information on fundamental parameters and new physics manifestations. In this case the main uncertainties come from the nucleon structure characterized by the structure functions (SF's) or form factors of the nucleon (FNN). Neither of them has been calculated on the basis of the first principles of the theory yet. Therefore the values of the fundamental parameters extracted from the data may have large systematic errors due to the uncertainties in the theoretical description of the nucleon structure.

Below we investigate the possibilities of elimination or, at least, suppression of these uncertainties.

This paper consists of two parts devoted to the electroweak sector of the SM and QCD sectors, respectively.

Considering the electroweak sector (in Sec. II) we search for the combinations of the lN scattering cross sections in which nucleon structure dependence can be eliminated on the basis of a rather general hypothesis. A well-known example is the Paschos-Wolfenstein relation [1]

$$R_1(v) = \frac{\sigma_{\mathrm{NC}}(vN^{I=0}) - \sigma_{\mathrm{NC}}(\overline{v}N^{I=0})}{\sigma_{\mathrm{CC}}(vN^{I=0}) - \sigma_{\mathrm{CC}}(\overline{v}N^{I=0})} = \rho^2(\frac{1}{2} - \sin^2\theta_W) \ .$$

Here  $\sigma_{\rm NC}$  and  $\sigma_{\rm CC}$  are the neutral and charged current cross sections of the deep inelastic scattering  $v(\bar{v})$  on an isoscalar target  $N^{I=0}$ . Despite the fact that the cross sections depend on the SF's, this dependence disappeared from the specific combination and the right-hand side of the relation is expressed only in terms of the SM parameters  $\rho$  and  $\sin^2 \theta_W$ . In Sec. II B we obtain new relations of this kind for deep inelastic and (quasi)elastic  $v(\bar{v})$  and longitudinally polarized  $e_{L,R}^{\pm}$  or  $\mu_{L,R}^{\pm}$  scattering on nonpolarized nucleons and nuclei.

Considering the QCD sector (in Sec. III) we investigate the dependence of the extracted value of the well-known  $\Lambda$  parameter on specific uncertainties induced by the large-scale nucleon structure. It enters into conventional QCD predictions for DIS through the initial condition SF  $f^{IC}(x)$  of  $Q^2$ -evolution equations. At present  $f^{IC}(x)$ cannot be calculated within QCD and for practical purposes it is necessary to employ some parametrizations. It may give rise to the dependence of the  $\Lambda$  parameter extracted from the data on a special choice of the parametrization. We propose modified predictions of QCD for DIS for which this dependence is suppressed.

#### II. ELECTROWEAK SECTOR

#### A. Effective Lagrangian and IN scattering cross sections

We start with the standard model neutral current (NC) Lagrangian extended to include neutral extra gauge bosons  $Z_i$ , representing possible tree-level new physics:

$$L_{\rm NC} = e A^{\mu} J_{\mu}^{\rm EM} + g_z Z^{\mu} J_{\mu}^{Z} + \sum_i g_i Z_i^{\mu} J_{\mu}^i . \qquad (1)$$

The fields Z and  $Z_i$  are mass eigenstates. Z is the lightest boson identified with that observed experimentally. The neutral currents can be written in the general form

$$J^{i}_{\mu} = \sum_{i} \left\{ \epsilon^{i}_{Lf} \overline{f}_{L} \gamma_{\mu} f_{L} + \epsilon^{i}_{Rf} \overline{f}_{R} \gamma_{\mu} f_{R} \right\} .$$
<sup>(2)</sup>

The chiral constants  $\epsilon_{L,R}$  are given in the Appendix for two special cases: for the exact SM without extra bosons  $Z_i$  and for the superstring-inspired extension of the SM with one extra Z' boson [2,3].

When studying lN scattering it is convenient to follow the effective NC Lagrangian [4]

$$\mathcal{L}_{\mathrm{NC}}^{\mathrm{eff}} = -\frac{4G}{\sqrt{2}} \{ \overline{l}_L \gamma_\mu l_L J_L^{(l)\mu} + \overline{l}_R \gamma_\mu l_R J_R^{(l)\mu} \}$$
(3)

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which is obtained from the initial Lagrangian (1). Here  $[l_{L,R} = \frac{1}{2}(1 \mp \gamma_5)l, l = \nu, e, \mu]$ . Effective hadron currents (i = L, R) are introduced:

$$J_i^{(l)\mu} = \sum_q \left\{ E_{iL}^{lg}(Q^2) \overline{q}_L \gamma^\mu q_L + E_{iR}^{lg}(Q^2) \overline{q}_R \gamma^\mu q_R \right\}$$
(4)

$$+ \frac{1}{2} \{ \alpha_i^l(Q^2) V^{3\mu} + \beta_i^l(Q^2) A^{3\mu} + \gamma_i^l(Q^2) V^{0\mu} + \delta_i^l(Q^2) A^{0\mu} \} .$$
 (5)

Here  $V^3$  and  $V^0$  are the isovector and isoscalar vector currents:  $A^3$  and  $A^0$  are the isovector and isoscalar axial-vector currents. We will use both forms of effective hadronic currents.

The effective chiral constants  $E_{ij}^{lg}$  and effective isotopical constants  $\alpha_i^l, \beta_i^l, \gamma_i^l, \delta_i^l$  have absorbed photon and Zboson propagators and depend upon  $Q^2$ . These constants contain the SM parameters and possible new physics in neutral current interactions. Their specific forms for the previously mentioned case of the extra Z' boson are given in the Appendix. With the accuracy of the improved Born approximation these effective constants absorb leading electroweak corrections as well.

Therefore to measure the SM parameters  $(\sin^2 \theta_W, \rho)$ and to investigate new physics manifestations in NC *lN*scattering one should extract these effective constants from the data separating them from the nucleon structure. The latter is a source of theoretical uncertainty and is represented by a distribution function (DF) in DIS and form factors of the nucleon (FFN) in (quasi)elastic scattering.

We will consider a way of separating effective chiral and isotopic constants from the nucleon structure within a reasonable approximation. We accept the QCD-parton picture for DIS and isospin invariance of strong interactions and CVC (conservation of vector current) for (quasi)elastic scattering.

On the basis of the effective Lagrangian (3) we can ex-

press the cross sections of **DIS** and (quasi)elastic *IN* scattering in terms of effective  $E_{ij}^{lq}$  and  $\alpha_i^l$ ,  $\beta_i^l \gamma_i^l$ ,  $\delta_i^l$  constants. They take a rather universal form for deep inelastic scattering,

$$\frac{d^2 \sigma^{\rm NC}}{dx \, dy}(l_L N) = \varphi_N(E^l_{(L)L}, E^l_{(L)R}) , \qquad (6)$$

$$\frac{d^2 \sigma^{\text{NC}}}{dx \, dy} (\bar{l}_R N) = \varphi_N(E^l_{(L)R}, E^l_{(L)L}) , \qquad (7)$$

$$\frac{d^2 \sigma^{\rm NC}}{dx \, dy} (e_L^+ N) = \varphi_N(E_{(R)L}^e, E_{(R)R}^e) , \qquad (8)$$

$$\frac{d^2\sigma^{\rm NC}}{dx\,dy}(e_R^-N) = \varphi_N(E^e_{(R)R}, E^e_{(R)L}) , \qquad (9)$$

$$\frac{d^2 \sigma^{\text{CC}}}{dx \, dy} [v(\overline{v})N]$$
  
=  $\sigma_0 x \{ f_{s(\overline{s})} + f_{d(\overline{d})} + [f_{\overline{u}(u)} + f_{\overline{c}(c)}](1-y)^2 \}, \quad (10)$ 

and (quasi)elastic scattering

$$\frac{d\sigma}{dQ^2} [l_L(\bar{l}_R)N \rightarrow l_L(\bar{l}_R)N] = \varphi_{\pm}(F_{VL(N)}^l, F_{ML(N)}^l, F_{AL(N)}^l),$$
(11)

$$\frac{d\sigma}{dQ^2} [\bar{l}_L(l_R)N \to \bar{l}_L(l_R)N] = \varphi_{\pm}(F_{VR(N)}^l, F_{MR(N)}^l, F_{AR(N)}^l),$$
(12)

$$\frac{d\sigma}{dQ^2} [\nu n(\bar{\nu}p) \rightarrow e_L^- p(e_R^+ n)] = \frac{d\sigma}{dQ^2} [e_L^- p(e_R^+ n) \rightarrow \nu_n(\bar{\nu}p)]$$
$$= \cos^2\theta_c \varphi_{\pm}(F_V^{CC}, F_M^{CC}, F_A^{CC}) .$$
(13)

The following universal functions are introduced:

$$\varphi_{N}(\epsilon_{L},\epsilon_{R}) = \sigma_{0}x \sum_{q} \left\{ f_{q}^{N}(x,Q^{2}) \left[ |\epsilon_{L}(q)|^{2} + (1-y)^{2} |\epsilon_{R}(q)|^{2} \right] + f_{\overline{q}}^{N}(x,Q^{2}) \left[ |\epsilon_{R}(q)|^{2} + (1-y)^{2} |\epsilon_{L}(q)|^{2} \right] \right\},$$
(14)

$$\varphi_{\pm}(F_{V},F_{M},F_{A}) = \frac{G^{2}}{\pi} \left\{ \left[ \frac{F_{V} \pm F_{A}}{2} \right]^{2} + (1-y)^{2} \left[ \frac{F_{V} \mp F_{A}}{2} \right]^{2} + \frac{My}{4E} (F_{A}^{2} - F_{V}^{2}) + \frac{y}{2} EF_{M} \left[ (1-y) \frac{E}{2M} F_{M} + y(F_{V} + \frac{1}{4}F_{M} \mp F_{A}) \pm 2F_{A} \right] \right\},$$
(15)

where  $\sigma_0 = 2G^2 ME / \pi$ ;  $f_q^N(x,Q^2)$ ,  $f_{\bar{q}}^N(x,Q^2)$  are the distribution functions (DF's) of quarks and antiquarks in the nucleon; N = p, n; M is the nucleon mass; and  $F_{V,M,A}(Q^2)$  are the FFN. E is the initial lepton energy in the lab system,  $Q^2 = 2MExy = Sxy$ .

FFN  $F_{k(N)}^{l}(Q^{2})$  and  $F_{k}^{CC}(Q^{2})$  are determined from the matrix elements:

$$\langle p, n | J_{(i)}^{l\mu} | p, n \rangle = \overline{u}(p_2) \left[ F_{Vi}^l(p, n) \gamma^{\mu} - \frac{\sigma^{\mu\nu} q_{\nu}}{2M} F_{Mi(p, n)}^l - \gamma^{\mu} \gamma^5 F_{Ai(p, n)}^l \right] u(p_1) ,$$

$$\langle p | J_{\mu}^{CC} | n \rangle = \overline{u}(p_2) \left[ F_{V}^{CC} \gamma^{\mu} - \frac{\sigma^{\mu\nu} q_{\nu}}{2M} F_{M}^{CC} - \gamma^{\mu} \gamma^5 F_{A}^{CC} \right] u(p_1) .$$

$$(16)$$

Following the isotopic symmetry of strong interactions and the CVC hypothesis, one can put down

$$F_{V,M}^{\rm CC} = F_{1,2}^p - F_{1,2}^n , \qquad (17)$$

$$F_{Vi(p,n)}^{l}(Q^{2}) = r_{+i}^{l} F_{1}^{p}(Q^{2}) - r_{-i}^{l} F_{1}^{n}(Q^{2}) , \quad i = L, R , \quad (18)$$
  
$$F_{Mi(p,n)}^{l}(Q^{2}) = r_{+i}^{l} F_{2}^{p}(Q^{2}) - r_{-i}^{l} F_{2}^{n}(Q^{2}) ,$$

$$F^{l}_{Ai(p,n)}(Q^{2}) = \pm \beta^{l}_{i} F^{CC}_{A}(Q^{2}) + \delta^{l}_{i} F^{0}_{A}(Q^{2}) , \qquad (19)$$

where  $r_{\pm i}^{l} = \frac{1}{2}(\alpha_{i}^{l} \pm 3\gamma_{i}^{l})$  and  $F_{1,2}^{p,n}$  are the electromagnetic FFN. According to the definition,  $G_{M}^{p,n} = F_{1}^{p,n} + F_{2}^{p,n}$  is the magnetic FFN; the isoscalar axial-vector FFN  $F_{A}^{0}$  is, in the matrix element,

$$\langle N | A^0_{\mu} | N \rangle = \overline{u}(p_2) \gamma_{\mu} \gamma^5 u(p_1) F^0_A(Q^2) . \qquad (20)$$

# **B.** Factorization and elimination of the nucleon structure dependence

We are searching for the combinations of cross sections for different processes where the dependence upon the structure functions and form factors of the nucleon characterizing its structure are canceled.

Let us begin with deep inelastic lN scattering and introduce the quantities

$$\Delta_{\pm L,R}^{l} = \frac{d^{2} \sigma^{\mathrm{NC}}}{dx \, dy} (l_{L,R} n) \pm \frac{d^{2} \sigma^{\mathrm{NC}}}{dx \, dy} (l_{L,R} p) , \qquad (21)$$

$$\overline{\Delta}_{\pm L,R}^{l} = \frac{d^{2}\sigma^{\rm NC}}{dx \, dy} (\overline{l}_{R,L}n) \pm \frac{d^{2}\sigma^{\rm NC}}{dx \, dy} (\overline{l}_{R,L}p) , \qquad (22)$$

$$\Delta_{\pm} = \frac{d^2 \sigma^{\rm CC}}{dx \, dy} (vn) \pm \frac{d^2 \sigma^{\rm CC}}{dx \, dy} (vp)$$
$$= \frac{d^2 \sigma^{\rm CC}}{dx \, dy} (e_L^- p) \pm \frac{d^2 \sigma^{\rm CC}}{dx \, dy} (e_L^- n) , \qquad (23)$$

$$\overline{\Delta}_{\pm} = \frac{d^2 \sigma^{\rm CC}}{dx \, dy} (\overline{v}n) \pm \frac{d^2 \sigma^{\rm CC}}{dx \, dy} (\overline{v}p) = \frac{d^2 \sigma^{\rm CC}}{dx \, dy} (e_R^+ p) \pm \frac{d^2 \sigma^{\rm CC}}{dx \, dy} (e_R^+ n) .$$
(24)

They can also be expressed in terms of the cross sections of scattering on nuclei:

$$\begin{split} \Delta_{+L,R}^{l} &= \frac{d^{2} \sigma^{\text{NC}}}{dx \, dy} (l_{L,R} N), \ \overline{\Delta}_{+L,R}^{l} = \frac{d^{2} \sigma^{\text{NC}}}{dx \, dy} (l_{R,L} N) \ , \\ \Delta_{+} &= \frac{d^{2} \sigma^{\text{CC}}}{dx \, dy} (l_{L} N), \quad \overline{\Delta}_{+} = \frac{d^{2} \sigma^{\text{CC}}}{dx \, dy} (l_{R} N) \ , \\ \Delta_{-L,R}^{l} &= \frac{1}{\beta} \left[ \frac{1}{A_{1}} \frac{d^{2} \sigma^{\text{NC}}}{dx \, dy} (l_{L,R} A_{1}) \right. \\ &\left. - \frac{1}{A_{2}} \frac{d^{2} \sigma^{\text{NC}}}{dx \, dy} (l_{L,R} A_{2}) \right] \ , \\ \overline{\Delta}_{-L,R}^{l} &= \frac{1}{\beta} \left[ \frac{1}{A_{1}} \frac{d^{2} \sigma^{\text{NC}}}{dx \, dy} (\overline{l}_{R,L} A_{2}) \right] \ , \\ \left. - \frac{1}{A_{2}} \frac{d^{2} \sigma^{\text{NC}}}{dx \, dy} (\overline{l}_{R,L} A_{2}) \right] \ , \\ \Delta_{-} &= \frac{1}{\beta} \left[ \frac{1}{A_{1}} \frac{d^{2} \sigma^{\text{CC}}}{dx \, dy} (v A_{1}) - \frac{1}{A_{2}} \frac{d^{2} \sigma^{\text{CC}}}{dx \, dy} (v A_{2}) \right] \\ &= \frac{1}{\beta} \left[ \frac{1}{A_{2}} \frac{d^{2} \sigma^{\text{CC}}}{dx \, dy} (e_{R}^{+} A_{2}) - \frac{1}{A_{1}} \frac{d^{2} \sigma^{\text{CC}}}{dx \, dy} (e_{R}^{+} A_{1}) \right] \ , \end{split}$$

where N is the isoscalar target,  $A_1, A_2$  are the nuclei with different isospin and atomic weights  $A_1, A_2, \beta = n_1/A_1 - n_2/A_2$ , and  $n_{1,2}$  is the number of neutrons in the nucleus  $A_{1,2}$ .

Using formulas (6)-(10) and (14), we find

$$(\overline{\Delta})_{-L}^{l} = \sigma_{0} f(x, Q^{2}) x \{ |E_{(L)L,R}^{ld}|^{2} - |E_{(L)L,R}^{lu}|^{2} + (1-y)^{2} (|E_{(L)R,L}^{ld}|^{2} - |E_{(L)R,L}^{lu}|^{2}) \},$$

$$(25)$$

$$\Delta'_{-R} = \sigma_0 f(x, Q^2) x \left\{ |E_{(R)R,L}^{ld}|^2 - |E_{(R)R,L}^{lu}|^2 + (1-y)^2 (|E_{(R)L,R}^{ld}|^2 - |E_{(R)L,R}^{lu}|^2) \right\},$$
(26)

$$\Delta_{-} = \sigma_0 x f(x, Q^2) , \quad \overline{\Delta}_{-} = -\sigma_0 x f(x, Q^2) (1 - y)^2 , \qquad (27)$$

$$\Delta_{-i}^{l} + \overline{\Delta}_{-i}^{l} = -\sigma_{0} x f(x, Q^{2}) [1 + (1 - y)^{2}]_{\frac{1}{2}} (\alpha_{i}^{l} \gamma_{i}^{l} + \beta_{i}^{l} \delta_{i}^{l}) , \qquad (28)$$

$$\Delta_{-i}^{l} - \overline{\Delta}_{-i}^{l} = e_{i}\sigma_{0}xf(x,Q^{2})[1 - (1 - y)^{2}]_{\frac{1}{2}}(\alpha_{i}^{l}\delta_{i}^{l} + \gamma_{i}^{l}\beta_{i}^{l}), \qquad (29)$$

$$\Delta_{-}\pm\bar{\Delta}_{-}=\sigma_{0}xf(x,Q^{2})[1\mp(1-y)^{2}], \qquad (30)$$

$$\Delta_{+i}^{l} + \overline{\Delta}_{+i}^{l} = \sigma_{0} x h_{1}(x, Q^{2}) [1 + (1 - y)^{2})]_{\frac{1}{4}}^{1} [(\alpha_{i}^{l})^{2} + (\beta_{i}^{l})^{2} + (\gamma_{i}^{l})^{2} + (\delta_{i}^{l})^{2}], \qquad (31)$$

$$\Delta_{+i}^{l} - \overline{\Delta}_{+i}^{l} = -e_{i}\sigma_{0}xh_{2}(x,Q^{2})[1 - (1 - y)^{2}]_{\frac{1}{2}}(\alpha_{i}^{l}\beta_{i}^{l} + \gamma_{i}^{l}\delta_{i}^{l}), \qquad (32)$$

$$\Delta_{+} \pm \overline{\Delta}_{+} = \sigma_{0} x h_{1,2} (x, Q^{2}) [1 \pm (1-y)^{2}], \qquad (33)$$

where

$$f(x,Q^2) = f_u^p(x,Q^2) - f_d^p(x,Q^2) , \qquad (34)$$

$$h_{1,2}(x,Q^2) = f_u^{p+n}(x,Q^2) + f_c^{p+n}(x,Q^2) \pm f_{\overline{c}}^{p+n}(x,Q^2) \pm f_{\overline{u}}^{p+n}(x,Q^2) , \qquad (35)$$

and  $f_p^p, f_q^n$  are the DF of quarks in the proton and neutron, respectively,  $f_q^{p+n} \equiv f_q^p + f_q^n, e_L = -1, e_R = 1$ . The dependence upon the nucleon structure is accumulated in f and  $h_i$ , which are common factors in formulas (26)–(33).

Now we obtain similar relations for (quasi)elastic IN scattering. Let us introduce the differences

$$\mathcal{N}_{L(p,n)}^{l} = \frac{d\sigma}{dQ^{2}}(l_{L}p, n \to l_{L}p, n) - \frac{d\sigma}{dQ^{2}}(\overline{l}_{R}p, n \to \overline{l}_{R}p, n) ,$$
(36)

$$\mathcal{N}_{R(p,n)}^{e} = \frac{d\sigma}{dQ^{2}} (e_{L}^{+}p, n \rightarrow e_{L}^{+}p, n) - \frac{d\sigma}{dQ^{2}} (e_{R}^{-}p, n \rightarrow e_{R}^{-}p, n) , \qquad (37)$$

$$\mathcal{C} = \frac{d\sigma}{dQ^2} (\nu n \to e_L^- p) - \frac{d\sigma}{dQ^2} (\bar{\nu} p \to e_R^+ n)$$
(38)

$$=\frac{d\sigma}{dQ^2}(e_L^- p \to \nu n) - \frac{d\sigma}{dQ^2}(e_R^+ n \to \overline{\nu} p) . \qquad (39)$$

Because of the instability of the free neutron it is impossible to observe the direct ln (quasi)elastic scattering process. Instead we can investigate lepton scattering on a neutron inside the nucleus, the simplest of which is a deuteron. In the proper kinematic domain with a slow spectator nucleon and large enough square momentum  $Q^2$  transferred from the lepton to the active nucleon [5] we can adopt the impulse approximation. Then cross sections of the  $ld \rightarrow l'pn, nn, pp$  processes can be written down as an incoherent sum of ln and lp (quasi)elastic scattering cross sections:

$$d_{L}^{l} \equiv \mathcal{N}_{L(p)}^{l} + \mathcal{N}_{L(n)}^{l} = \frac{d\sigma}{dQ^{2}} (l_{L}d \rightarrow l_{L}np) - \frac{d\sigma}{dQ^{2}} (\overline{l}_{R}d \rightarrow \overline{l}_{R}np) , \qquad (40)$$

$$d_{R}^{l} \equiv \mathcal{N}_{R(p)}^{l} + \mathcal{N}_{R(n)}^{l} = \frac{d\sigma}{dQ^{2}} (e_{L}^{+} d \rightarrow e_{L}^{+} np) - \frac{d\sigma}{dQ^{2}} (e_{R}^{-} d \rightarrow e_{R}^{-} np) , \qquad (41)$$

$$\mathcal{C} = \frac{d\sigma}{dQ^2} (vd \to e_L^- pp) - \frac{d\sigma}{dQ^2} (\bar{v}d \to e_R^+ nn)$$
$$= \frac{d\sigma}{dQ^2} (e_L^- d \to vnn) - \frac{d\sigma}{dQ^2} (e_R^+ d \to \bar{v}pp) .$$
(42)

Following formulas (11)-(13), and (15), we obtain

$$\mathcal{N}_{i(p,n)}^{l} = \omega(E,Q^{2})(F_{V_{i}(p,n)}^{l} + F_{M_{i}(p,n)}^{l})F_{A_{i}(p,n)}^{l}, \quad (43)$$

$$\mathcal{C} = \cos^2 \theta_c \omega(E, Q^2) (F_V^{\rm CC} + F_M^{\rm CC}) F_A^{\rm CC} , \qquad (44)$$

$$\omega(E,Q^2) = \frac{G^2}{\pi} \frac{Q^2}{ME} \left[ 1 - \frac{Q^2}{4ME} \right].$$

Lets us use the scaling law for the FFN:

$$\frac{G_M^p}{\mu_p} \approx \frac{G_M^n}{\mu_n} , \qquad (45)$$

$$F_A^0 \approx \frac{\lambda}{2} F_A^{\rm CC} , \qquad (46)$$

where  $\mu_p = 2.79$ ,  $\mu_n = -1.91$  are the magnetic moments of the proton and neutron.

Relation (45) is well known and valid with a high accuracy in a wide interval of  $Q^2$ . The scaling law for the axial FFN (46) is less reliable. It may have some grounds, for instance, in QCD, based on local duality [6] or a dipole extrapolation of the results of perturbative calculations [7]. However, the experimental status of this relation is not quite clear. The normalization constant  $\lambda$  is calculated or taken from experiment. In the nonrelativistic quark SU(6) model  $\lambda=0.6$ . Some difference in values of  $\lambda$  in different approaches as well as deviation from scaling law (46) do not lead to a noticeable influence on the effects under discussion. This is explained by the small contribution of  $\delta F_A^0$  to the initial formula (19) due to the small value of the parameter  $\delta$ . One should remember that in the SM at the tree level  $\delta=0$ .

On the basis of formulas (17), (19), (45), and (46) we transform (43) into

$$\mathcal{N}_{i(p,n)}^{l} = \frac{1}{4}\omega(E,Q^{2})(F_{V}^{\text{CC}} + F_{M}^{\text{CC}})F_{A}^{\text{CC}}(3\gamma_{i}^{l}\mu \pm \alpha_{i}^{l})(\lambda\delta_{i}^{l} \pm \beta_{i}^{l}) , \qquad (47)$$

$$d_i^l = \frac{1}{2}\omega(E,Q^2)(F_V^{\rm CC} + F_M^{\rm CC})F_A^{\rm CC}(\alpha_i^l\beta_i^l + 3\gamma_i^l\delta_i^l\lambda\mu) , \qquad (48)$$

where  $\mu = (\mu_p + \mu_n) / (\mu_p - \mu_n)$ .

Formulas (26)-(33), (44), (47), and (48) are initial ones for obtaining the relations independent of DF and FFN. From the cross section combinations  $\Delta, \mathcal{N}, \mathcal{C}$  one should choose two combinations so that DF and FFN would be canceled if the combinations are divided by one another. For example, in the ratio  $\Delta_{-L}^l/\Delta_-$  the DF  $f(x,Q^2)$  is canceled. Thus, one can easily obtain all the relations of this kind from formulas (26)-(33), (44), (47), and (48). We do not write them down in the general form, confining ourselves to the approximation  $Q^2 \ll M_Z^2 \approx 10^4 \text{ GeV}^2$  in the SM limit. If the given approximation is inapplicable, e.g., in experiments at the DESY *ep* collider HERA where  $Q^2 \approx 10^{3-5} \text{ GeV}^2$ , one should use the formulas in the general form.

Since electron scattering is due to both weak interaction W and electromagnetic interaction  $I_{\rm EM}$ , the structure of the relevant formulas is  $I_{\rm EM}^2 + I_{\rm EM}W + W^2$ . In the  $Q^2$  region considered, the leading term is the one with the maximum power of  $I_{\rm EM}$ . As to the neutral current parameters and the dependence upon the Z'-boson contributions, they all are included in W. So below we give the relations that do not involve the dominating term  $I_{\rm EM}^2$  which is of little physical interest. In the formulas we shall only retain the leading term of the  $I_{\rm EM}$  W type corresponding to electroweak interference. In this case the accuracy is worse by no more than 1-2%, for  $Q^2 < 200 \text{ GeV}^2$ . In the given approximation the desired relations have the following form.

For  $\nu N$  scattering,

$$\frac{\overline{(\overline{\Delta})}_{-L}}{\Delta_{-}} = -(1-y)^{2} \frac{\overline{(\overline{\Delta})}_{-L}}{\overline{\Delta}_{-}}$$
$$= \frac{1}{6} \rho^{2} X_{W} \{ [1+(1-y)^{2}] \\ \times (1-2X_{W}) \pm [1-(1-y)^{2}] \}, \quad (49)$$

$$\frac{\Delta_{-L}^{\nu}}{\Delta_{-L}^{\nu}} = \frac{(1 - X_W)(1 - y)^2 - X_W}{(1 - X_W) - X_W(1 - y)^2} , \qquad (50)$$

$$\frac{\Delta_{+L}^{\nu} + \overline{\Delta}_{+L}^{\nu}}{\Delta_{+} + \overline{\Delta}_{+}} = \frac{\rho^2}{2} (1 - 2X_W + \frac{20}{9} X_W^2) , \qquad (51)$$

$$\frac{\Delta_{-L}^{\nu} + \overline{\Delta}_{-L}^{\nu}}{\Delta_{-} - \overline{\Delta}_{-}} = \frac{\rho^2}{3} X_W (1 - 2X_W) , \qquad (52)$$

$$\frac{\Delta_{+L}^{\nu} - \overline{\Delta}_{+L}^{\nu}}{\Delta_{+} - \overline{\Delta}_{+}} = \frac{\rho^{2}}{2} (1 - 2X_{W}) , \qquad (53)$$

$$\frac{\Delta_{-L}^{\nu} - \overline{\Delta}_{-L}^{\nu}}{\Delta_{-} + \overline{\Delta}_{-}} = \frac{\rho^2}{3} X_{W} , \qquad (54)$$

$$\frac{\mathcal{N}_{L(p,n)}^{\nu}}{\mathcal{C}} = \frac{\rho^2}{4\cos^2\theta_C} \{1 - 2X_W(1\pm\mu)\}, \qquad (55)$$

$$\frac{\mathcal{N}_{L(p)}^{\nu} + \mathcal{N}_{L(n)}^{\nu}}{\mathcal{O}} = \frac{\rho^2}{\cos^2\theta_C} \left(\frac{1}{2} - X_W\right) \,. \tag{56}$$

For eN scattering (i = L, R),

$$\frac{\Delta_{\pm i}^{e} - \Delta_{\pm i}^{e}}{\Delta_{\pm} \mp \overline{\Delta}_{\pm}} = \frac{e_{i}}{1 \mp 2} \chi \frac{M^{2}}{Q^{2}} \rho \epsilon_{i}^{z^{0}}(e) , \qquad (57)$$

$$\frac{\mathcal{N}_{i(p,n)}^{e}}{\mathcal{C}} = -\frac{\chi(1\pm\mu)}{2\cos^{2}\theta_{C}}\frac{M^{2}}{Q^{2}}\rho\epsilon_{i}^{z^{0}}(e) , \qquad (58)$$

$$\frac{d_i^e}{\mathcal{C}} = -\frac{\chi}{\cos^2\theta_C} \frac{M^2}{Q^2} \rho \epsilon_i^{z^0}(e) , \qquad (59)$$

$$R_{\rm NC}^{e} = \frac{\mathcal{N}_{L(p,n)}^{e}}{\mathcal{N}_{R(p,n)}^{e}} = \frac{d_{L}^{e}}{d_{R}^{e}} = -\frac{\Delta_{\pm L}^{e} - \overline{\Delta}_{\pm L}^{e}}{\Delta_{\pm R}^{e} - \overline{\Delta}_{\pm R}^{e}}$$
$$= \frac{\epsilon_{L}^{z^{0}}(e)}{\epsilon_{R}^{z^{0}}(e)} = \frac{2X_{W} - 1}{2X_{W}} . \tag{60}$$

The double differential cross sections  $d^2\sigma/dx dy$ , entering into  $\Delta$ , (22)-(33) can be replaced in formulas (57) and (58) by the differential cross sections  $d\sigma/dQ^2$ , and in formulas (50)-(53), and (60) they can be replaced both by  $d\sigma/dy$ ,  $d\sigma/dQ^2$  and by the total cross sections  $\Delta\sigma$  taken in any region of variables x,y. According to the definition,

$$\Delta \sigma^a = \int_{X_{\min}}^{X_{\max}} dx \, \int_{Y_{\min}}^{Y_{\max}} dy \frac{d^2 \sigma^a}{dx \, dy} \, , \quad a = \text{NC, CC} \, . \tag{61}$$

Relations (49)-(60) can be used for extraction of the SM parameters  $\rho$  and  $X_W = \sin^2 \theta_W$  from the experimental data on deep inelastic and (quasi)elastic  ${}^{(-)}N$  and eN scattering.

Specific difficulties of vN experiments associated with

relative normalization of v and  $\overline{v}$  beam flows make it preferable to use relations without combinations of cross sections from different beams, i.e., combinations of vNand  $\overline{v}N$  scattering cross sections. These are relations (49)  $\Delta_{-L}^{v}/\Delta_{-}$  and  $\overline{\Delta}_{-L}^{v}/\overline{\Delta}_{-}$ . Their disadvantage is that they contain differential cross sections  $d^{2}\sigma/dx \, dy$  and, consequently, require data on  $\stackrel{(-)}{v}N$  scattering in a narrow-band beam (NBB). This limits the statistics for extraction of the parameters  $(\rho, X_W)$ . The situation is quite the opposite for relations (51)–(56), among which there is the Paschos-Wolfenstein relation [formula (53)]. They are formulated for total cross sections, which is much more favorable for gathering statistics, but these are combina-

tavorable for gathering statistics, but these are combinations of  $\nu N$  and  $\overline{\nu}N$  scattering cross sections. As a result, there are uncertainties related to different normalization of  $\nu$  and  $\overline{\nu}$  beams. Among relations (57)–(60) for eN scattering we would like to single out the last three relations for  $\mathcal{R}^{\ell}$  (60)

like to single out the last three relations for  $\mathcal{R}_{NC}^{e}$  (60). Their definition does not include the cross sections for charged current eN scattering, which is a rare process occurring only due to weak interaction. These relations do not involve the parameter  $\rho$ . As a result, the extraction of the remaining parameter  $X_W$  from the data becomes more reliable. We also notice that  $\mathcal{R}_{NC}^{e}$  is very sensitive to  $X_W$ :

$$K = \frac{d}{dX_W} \ln \mathcal{R}_{\rm NC}^e = \frac{1}{X_W (1 - 2X_W)} \approx 8 \quad \text{for } X_W = 0.23 \; .$$
(62)

So the error  $\Delta X_W$  induced by the error  $\Delta \mathcal{R}_{NC}^e$  in the measurement of  $\mathcal{R}_{NC}^e$  is suppressed by a large factor K = 8:

$$\Delta X_W = \frac{1}{8} \frac{\Delta \mathcal{R}_{\rm NC}^e}{\mathcal{R}_{\rm NC}^e} . \tag{63}$$

The sensitivity of other relations, including the Paschos-Wolfenstein relation, is much lower and does not exceed K = 1.

The relations we have obtained are subject to radiative corrections. We do not consider this problem since it requires special effort. We expect the radiative corrections will not destroy the obtained relations. The reason is based on the similarity in the structure of all obtained relations including the Paschos-Wolfenstein relation. For the latter it was shown [8] that because of this special structure the radiative corrections are effectively selfcanceled and can be neglected.

#### **III. QCD SECTOR**

## A. On model dependence of the QCD predictions for deep inelastic scattering

Now we turn to the sector of the SM and consider QCD predictions for deep inelastic *lN* scattering. Unlike the electroweak sector, the subject under consideration is the nucleon structure represented by the DF. The uncertainty factor is a long-scale nucleon structure poorly controlled in QCD.

QCD governs the  $Q^2$  evolution of the DF within the

renormalization group improved perturbative theory, formulated in terms of the running coupling constant  $\alpha_s(Q^2)$ . Its  $Q^2$  dependence is usually parametrized by the well-known  $\Lambda$  parameter. We will investigate the problem of extraction of this parameter from the experimental data on the deep inelastic *lN* scattering structure functions (SF's)  $F_k(x,Q^2)$ . Here we consider the nonsinglet SF  $F_3(x,Q^2)$  and the nonsinglet component of SF  $F_{1,2}^{NS}$ . The former can be directly measured in deep inelastic  $v(\bar{v})N$  scattering. The case of the singlet component of the SF is much more complicated and is not considered in our approach. In the leading order of the perturbative theory QCD predicts that SF's obey the equation

$$\langle f(s) \rangle_n = \langle f^{\rm IC}(0) \rangle_n e^{d_n s}$$
 (64)

Here we use the notation  $f(x, Q^2) = F_3, F_{1,2}^{NS}$ :

$$s = \frac{2}{\beta_0} \ln \frac{\alpha_s(Q^2/\Lambda^2)}{\alpha_s(Q_0^2/\Lambda^2)}$$
(65)

 $Q_0^2$  is an arbitrary reference point;  $\beta_0$  and  $d_n$  in the leading order of the perturbation theory are

$$\beta_0 = \frac{1}{16\pi^2} (11 - \frac{2}{3}n_f) , \qquad (66)$$

$$d_n = \frac{1}{24\pi^2} \left[ 1 - \frac{2}{n(n+1)} + 4 \sum_{j=2}^n \frac{1}{j} \right], \qquad (67)$$

and  $n_f$  is number of quark flavors.

The moments of the SF are defined as

$$\langle f(s) \rangle_n = \int_0^1 dx \; x^n f(x, Q^2) \; .$$
 (68)

To solve the evolution equation (64) it is necessary to know the initial conditions  $f^{IC}(x,0)$  which are a SF at the reference point  $Q^2 = Q_0^2$ .

This function depends on the large-scale nucleon structure and presently cannot be calculated within QCD in a reliable way. Therefore to make an advance in the solution of the evolution equation one may try to use some models for  $f^{IC}$  calculations. This results in model dependence of any further analyses including the extraction of the  $\Lambda$  parameter from the data.

Another way to cope with the problem is the extraction of  $f^{IC}$  directly from the data. In this case the problem is converted into another one. In fact, for calculation of moments according to formula (68) one needs to restore  $f^{IC}$  in the whole interval  $x \in [0,1]$  whereas the data points cover only the middle limited part of this interval and do not approach the end points x=0 and x=1. The tails of  $f^{IC}$  remain unconstrained by the data and  $\langle f^{IC}(0) \rangle_n$  becomes dependent on the extrapolation to the end points. This dependence is more essential for the  $x \rightarrow 1$  tail of the  $f^{IC}(x,0)$ . The dependence on the  $x \rightarrow 0$  tail is reduced due to the damping factor  $x^{n-1}$  in the definition (68) of  $\langle f^{IC}(0) \rangle_n$ .

In our approach we reformulate QCD predictions (64) in terms of new objects [instead of the ordinary moments (68)] which contain a new damping factor suppressing contributions from both domains  $x \rightarrow 1$  and  $x \rightarrow 0$ .

In this way we obtain QCD predictions mostly on the x domains covered by the experimental points. Thus the value of the parameter  $\Lambda$  extracted from the data originates mostly from the data points, not by the special form of parametrization used for analytical representation of these data in the formulas. This is because the dependence on parametrization comes only from domains in the vicinity of the crucial points x = 1,0 unconstrained by the data points.

#### **B.** Modified QCD predictions

Let us consider Eq. (68) at continuum values of n and differentiate it m times with respect to n. The result is

$$\frac{d^m \langle f(s) \rangle_n}{dn^m} \equiv \langle f(s) \rangle_n^{(m)} = \int_0^1 dx \ x^n \ln^m x f(x,s) \ . \tag{69}$$

The object  $\langle f(s) \rangle_n^{(m)}$  possesses the property we are looking for. It contains the damping factor  $\chi_n^m(x) = x^n \ln^m x$ which suppresses the contribution from vicinity of the end points x = 0, 1. The suppression becomes stronger with larger m and n. The function  $\chi_n^m$  has a maximum at point  $x_0 = e^{-m/(n-1)}$  and the dominant contribution to the integral (69) comes from the interval around point  $x_0$ .

To obtain QCD predictions for the objects  $\langle f \rangle_n^{(m)}$  we differentiate both sides of Eq. (64):

$$\langle f(s) \rangle_n^{(m)} = \sum_{k=0}^m C_m^k \langle f^{\rm IC}(0) \rangle_n^{(m)} (e^{d_n s})^{(m-k)} .$$
 (70)

 $C_m^k$  are the binomial coefficients.

This equation does not give a solution of the problem we consider, because it contains  $\langle f^{\text{IC}} \rangle_n^{(m)}$  with low k values including k=0. Our goal is to exclude terms with a low k(k < l) and obtain predictions localized in the region excluding end points x=0,1. The degree of localization will be controlled by the value of the parameter l.

Let us consider the system of l algebraic equations obtained from the original one (70) at different values of  $m = l, \ldots, 2l$ :

$$\langle f(s) \rangle_{n}^{(l)} = \sum_{k=0}^{l} C_{l}^{k} \langle f^{\text{IC}}(0) \rangle_{n}^{(k)} (e^{d_{n}s})^{(l-k)}$$
  

$$\vdots \qquad \vdots \qquad \vdots \qquad (71)$$
  

$$\langle f(s) \rangle_{n}^{(2l-1)} = \sum_{k=0}^{2l-1} C_{2l-1}^{k} \langle f^{\text{IC}}(0) \rangle_{n}^{(k)} (e^{d_{n}s})^{(2l-1-k)} .$$

Let us express  $\langle f^{IC}(0) \rangle_n^{(k)}$  with k < l in terms of those with  $k \ge l$ :

$$\langle f^{\rm IC}(0) \rangle_n^{(k < l)} = \sum_{i=l}^{2l-1} \{ a_n^i(s) \langle f^{\rm IC}(0) \rangle_n^{(i)} + b_n^i(s) \langle f(s) \rangle_n^{(i)} \} .$$
(72)

 $a_n^i$  and  $b_n^i$  are calculable functions. Substituting expression (72) into expression (70) with m = 2l we obtain

$$\sum_{k=0}^{l} r_{n,k}^{(l)}(s) \langle f(s) \rangle_{n}^{(l+k)} = \sum_{k=0}^{l} q_{n,k}^{(l)}(s) \langle f^{\text{IC}}(0) \rangle_{n}^{(l+k)} .$$
(73)

Functions  $r_{n,k}^{(l)}(s)$  and  $q_{n,k}^{(l)}(s)$  have a rather complicated

analytical form and can be found in our paper [9] at several values of l, n, and k.

Equation (73) is a solution of the initial problem. It contains  $\langle f \rangle_n^k$  only at  $k \ge l$  which are weakly dependent on the contribution from the vicinity of the end points x = 0, 1.

It is useful to transform Eq. (73) into the integral form

$$\int_{0}^{1} dx \{ R_{n}^{l}(x,s) f(x,s) - Q_{n}^{l} f^{\text{IC}}(x,0) \} = 0 , \qquad (74)$$

where

$$R_{n}^{l}(s) = \sum_{k=0}^{l} r_{n,k}^{(l)}(s) x^{n} \ln^{l+k} x , \qquad (75)$$

$$Q_n^l(s) = \sum_{k=0}^l q_{n,k}^{(l)}(s) x^n \ln^{l+k} x \quad .$$
(76)

It is easy to show that these functions are mainly located within the interval  $e^{-2l/n} \le x \le e^{-l/n}$ . The localization becomes more perfect at larger values of l and n and for sufficiently large ones we write down approximate sum rules for the nonsinglet SF f(x,s):

$$\int_{x_{\min}}^{x_{\max}} dx \{ R_n^l(x,s) f(x,s) - Q_n^l(x,s) f(x,0) \} \approx 0 , \quad (77)$$

where  $x_{\min}^2 = x_{\max} = e^{-l/n}$ .

Equations (75)-(77) can be applied to the deep inelastic *lN* scattering data analyses and extraction of the  $\Lambda$  parameter. These equations give a basis for the procedure which is weakly dependent on SF parametrization.

We expect that perturbative next-to-leading QCD corrections and nonperturbative twist corrections do not essentially modify the result. It is well known that these corrections become large only in the regions near the end points x = 0, 1. Therefore their contributions to Eqs. (75)–(77) are suppressed as any contributions from those regions. For the same reason we can neglect contributions from the singlet component of the SF which are essentially only near x = 0.

In principle, contributions of these corrections can be taken into account starting not from the leading order equation (64), but from the system of next-to-leading order equations with singlet components of the SF involved. However the procedure we proposed here becomes much more complicated and we do not consider this case.

Equations (75)-(77) can be applied to the analysis of lN scattering data and extraction of the  $\Lambda$  parameter value. In our paper [9] we analyzed the data on this basis [10]. General conclusions are the following. The dependence of the extracted value of  $\Lambda$  on a specific choice of the SF parametrization is essentially suppressed. This statement has been verified directly for several different parametrizations. However at the same time the error  $\Delta\Lambda$  in the definition of  $\Lambda$  increases. For the data [10] it is about  $\Delta\Lambda/\Lambda \approx 0.7-1$ . Therefore more precise data are necessary to obtain a sensible result for the extracted value of the parameter  $\Lambda$  in the framework of the proposed approach.

In Eqs. (75)-(77) and the extraction of  $\Lambda$  we have suppressed the dependence on the SF parametrization. The latter is a theoretically uncontrollable factor connected with the large-scale nucleon structure. Instead, we obtained a large error  $\Delta \Lambda$ , which demands much precise *lN* scattering data than presently available. The situation looks as if we have converted a theoretical problem into an experimental one.

It is very essential to point out that in our approach  $\Delta\Lambda$  is a representation of the real constraints which experimental data impose on the  $\Lambda$  parameter.

A and  $\Delta\Lambda$  weakly depend on the hypothesis involved in the extraction procedure. The situation is different when extraction is based on conventional QCD predictions (64). The error in this case is smaller  $\Delta\Lambda/\Lambda \approx 0.2-0.5$ , but the dependence on SF parametrization is uncontrollable. Therefore, the reliability of this result may be questionable.

#### ACKNOWLEDGMENTS

We would like to thank F. Buccella, P. Herczeg, B. Z. Kopeliovich, and A. V. Radyushkin for useful discussions.

#### APPENDIX

Here we give the chiral constants taking into account the contribution of the superstring inspired Z' boson as a possible representative of beyond the SM physics. They are

$$\epsilon_i^{(1)}(f) = \epsilon^{z0}(f)\cos\theta + \frac{g_{z'}}{g_z}\epsilon^{z'}(f)\sin\theta , \quad i = L, R , \qquad (A1)$$

$$\epsilon_i^{(2)}(f) = \epsilon^{z'}(f) \cos\theta - \frac{g_z}{g_{z'}} \epsilon^{z0}(f) \sin\theta , \qquad (A2)$$

$$\epsilon^{z0}(f) = T_{3L}(f) - X_W Q^{\text{EM}}(f) . \tag{A3}$$

Here  $X_W = \sin^2 \vartheta_W$ ,  $T_{3L}(f)$  and  $Q^{\text{EM}}(f)$  are the third component of the weak isospin and the electric charge of the fermion f. The chiral constants  $\epsilon^{z'}$  are given in Table I.

Angle  $\theta_{E_6}$ , involved in their definition, characterizes the scheme of the  $E_6$  gauge symmetry breaking and is a free parameter of the theory. The relation between the coupling constants  $g_z$  and  $g_{z'}$  also depends upon symmetry breaking. The following result of the renormalization group analysis is known in the general case [2]:

$$\left[\frac{g_{z'}}{g_z}\right]^2 \leq \frac{5}{3}X_W \; .$$

. .

TABLE I. Chiral constants  $\epsilon_{L,R}^{z'}$ , parametrizing fermion-Z'-boson interactions. [Here  $\xi = (1/2\sqrt{10})\sin\theta_{E_6}$ ,  $\lambda = (1/2\sqrt{6})\cos\theta_{E_6}$ .]

Fermion	$\epsilon_L^{z'}$	$\epsilon_{R}^{z'}$
ν	$\lambda + 3\xi$	$5\xi - \lambda$
е	$\lambda + 3\xi$	$\xi - \lambda$
u	$\lambda - \xi$	$\xi - \lambda$
d	$\lambda - \xi$	$-\lambda -3\xi$

The effective current constants have the form

$$E_{(l)j}^{lq} = \frac{\chi}{2} \frac{M^2}{Q^2} \left[ Q^{\text{EM}}(l) Q^{\text{EM}}(q) + \frac{Q^2}{X_W(1 - X_W)} \left\{ \frac{\epsilon_i^{(1)}(l)\epsilon_j^{(1)}(q)}{M_1^2 + Q^2} + \left[ \frac{g_z}{g_{z'}} \right]^2 \frac{\epsilon_i^{(2)}(l)\epsilon_j^{(2)}(q)}{M_2^2 + Q^2} \right\} \right],$$

$$\alpha_{(i)}^{l}(Q^2) = \chi \frac{M^2}{Q^2} Q^{\text{EM}}(l) + a_{(i)}^{l}(Q^2) \alpha^Z + b_{(i)}^{l}(Q^2) \alpha^{Z'},$$

$$\beta_{(i)}^{l}(Q^2) = a_{(i)}^{l}(Q^2) \beta^Z + b_{(i)}^{l}(Q^2) \beta^{Z'},$$

$$\gamma_{(i)}^{l}(Q^2) = \frac{\chi}{3} \frac{M^2}{Q^2} Q^{\text{EM}}(l) + a_{(i)}^{l}(Q^2) \gamma^Z + b_{(i)}^{l}(Q^2) \gamma^{Z'},$$

$$\delta_{(i)}^{l}(Q^2) = a_{(i)}^{l}(Q^2) \delta^Z + b_{(i)}^{l}(Q^2) \delta^{Z'},$$

$$(A4)$$

$$a_i^{l}(Q^2) = \frac{\chi}{1 - X_W} \left[ \epsilon_i^{(1)}(l) \cos\theta \frac{M_W^2}{M_1^2 + Q^2} - \left[ \frac{g_z}{g_{z'}} \right] \epsilon_i^{(2)}(l) \sin\theta \frac{M_W^2}{M_2^2 + Q^2} \right],$$

$$(A6)$$

$$b_{i}^{l}(Q^{2}) = \frac{2}{1 - X_{W}} \left[ \frac{g_{z'}}{g_{z}} \right]^{2} \left[ \left[ \frac{g_{z}}{g_{z'}} \right] \epsilon_{i}^{(1)}(l) \sin\theta \frac{M_{W}^{2}}{M_{1}^{2} + Q^{2}} + \epsilon_{i}^{(2)}(l) \cos\theta \frac{M_{W}^{2}}{M_{2}^{2} + Q^{2}} \right],$$
(A6)

where  $\chi = 2\pi \alpha \sqrt{2}/GM^2 \approx 0.6 \times 10^4$ . In the tree approximation  $\alpha^z = 1 - 2X_W$ ,  $\beta^z = 1$ ,  $\gamma^z = -\frac{2}{3}X_W$ ,  $\delta^z = 0$ ,  $\alpha^{z'} = -\beta^{z'} = -\gamma^{z'} = 2\sin\theta_{E_6}/\sqrt{10}$ ,  $\delta^{z'} = 2\cos\theta_{E_6}/\sqrt{2}$ . Mixing angle  $\theta$  is  $\tan^2\theta = (M_0^2 - M_1^2)/(M_2^2 - M_0^2)$ , where  $M_0 = M_W/\cos\theta_W$ .

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