Low-energy grand unification with SU(16)

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We study the possibility of achieving a low unification scale in a grand unification scheme based on the gauge group SU(16). Baryon number symmetry being an explicit local gauge symmetry here, the gauge-boson-mediated proton decay is absent. We present in detail a number of symmetry-breaking patterns and the Higgs field representations giving rise to the desired symmetry breakings and identify one chain giving low-energy unification. These Higgs field representations are constructed in such a way that Higgs-boson-mediated proton decay is absent. At the end we indicate the very rich low-energy physics obtainable from this model which includes quark-lepton ununified symmetry and chiral color symmetry. In brief some phenomenological implications are also studied.

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I. INTRODUCTION

Grand unified theories (GUT's) [1] offer the possibility of a simple, but unified description of strong and electroweak interactions. Typically in these models at some high energy all the interactions arise out of a single Lagrangian which is locally invariant under the gauge transformations of a single simple Lie group called the unification group. A large spectrum of GUT's are proposed in the literature which is broadly classified by the unification group. In the minimal SU(5) model all interactions unite at a single step at an energy around 10^{14} GeV, therefore predicting the absence of any new physics between the standard electroweak breaking scale (M_Z) and the unification scale (M_U) while the SO(10) model admits intermediate breakings of symmetry. On the other hand there are models which are inspired by superstring theories; one of them postulates the exceptional group E_6 as the unifying group. This specific model predicts at least 12 exotic fermions on top of the 15 standard fermions. All these theoretically very attractive models have at least one common prediction, namely, the decay of proton.

There has been a desperate search by the experimentalists to see the signature of proton decay for the last decade and a half. Contrary to theoretical beliefs proton decay has not been discovered. At present the lower limit of the half-life of a proton is a whopping 10^{32} years. The nonobservation of proton decay has made all the above models a little less attractive.

At this juncture one interesting possibility is that unification is achieved at a low-energy scale which means that the big desert of particle physics between the electroweak scale and the unification scale becomes small but other experimental constraints including that on the lifetime of proton remains satisfied. A grand unification scheme based on SU(16) as the unification group offers such a possibility. It is worth noting here that due to the low unification scale such models are free from the problems of grand unified monopoles [3].

Earlier works on low-energy unification in GUT's considered SU(15) as the unification group [4]. Here we extend the idea to the left-right-symmetric version of such a theory. We show that retaining all the good features of SU(15) we can also incorporate left-right symmetry in intermediate stages. Unlike the SU(15) GUT here lepton number is a local gauge symmetry which may survive to a low-energy scale. A right-handed neutrino can be accommodated naturally as all the fermions transform in the fundamental representation of SU(16).

This paper is structured as following. In Secs. II and IV we give the symmetry-breaking chains and the Higgs field representations required for the breakings of symmetry. In Sec. III we present some mathematical preliminaries useful for the calculation of the mass scales. In Sec. V we calculate the mass scales using the renormalization group equations. In Sec. VI we study some phenomenological consequences of this model and in Sec. VII we state the conclusions. In the Appendix we give some group theoretic essentials.

II. SYMMETRY BREAKING AND HIGGS FIELDS

To achieve low-energy unification we propose a number of symmetry-breaking chains. At the level of highest symmetry the theory is invariant under the gauge group SU(16). At and above this level the coupling constant is that of the group SU(16). With the decrease in energy, the group goes through a number of symmetry breaking phases, and the theory becomes least symmetric at the present energies with the residual symmetry of SU(3) color and the symmetry of electromagnetic interactions. It is noteworthy that the baryon number symmetry remains exact up to a very-low-energy scale of a few TeV. This makes the proton stable in the sense that the gauge-boson-mediated proton decay is absent. Interestingly the completely ununified symmetry group of the quarks and leptons also appears at a low-energy scale together with the chiral color symmetry. The appearance of this group at a comparatively low scale makes this model worthy of phenomenological studies [10]. In this section we illustrate one chain in some detail to draw attention to the underlying group theoretic points. In Sec. IV we shall consider more possible chains before finally going to calculate the symmetry breaking scales.

Breaking chain 1

Here at first we give the breaking chains that can give rise to the standard model groups $SU(3)_C \times SU(2)_L \times U(1)_Y$. We note here that there can be in general a number of chains of descent to the standard model:

$$
\begin{aligned}
SU(16) &\xrightarrow{M_U} G[SU(12)\times SU(4)^l] \\
&\xrightarrow{M_1} G_1[SU(6)_L\times SU(6)_R\times U(1)_B\times SU(4)^l] \\
&\xrightarrow{M_2} G_2[SU(3)_L\times SU(2)_L^q\times SU(6)_R\times U(1)_B\times SU(4)^l] \\
&\xrightarrow{M_3} G_3[SU(3)_L\times SU(2)_L^q\times SU(3)_R\times U(1)_R^q\times U(1)_B\times SU(2)_L^l\times SU(2)_R^l\times U(1)^{\text{lep}}] \\
&\xrightarrow{M_4} G_4[SU(3)_L\times SU(2)_L^q\times SU(2)_L^l\times SU(3)_R\times U(1)_R\times U(1)_B\times U(1)^l] \\
&\xrightarrow{M_5} G_5[SU(3)_c\times SU(2)_L\times U(1)_B\times U(1)_h] \\
&\xrightarrow{M_6} G_6[SU(3)_c\times SU(2)_L\times U(1)_Y] \\
&\xrightarrow{M_Z} G_7[SU(3)_c\times U(1)_{\text{em}}] \ .
\end{aligned}
$$

Here the superscript q or l denotes that quarks or leptons have a nontrivial transformation law under these groups and the subscripts L and R mean so for the left- and right-handed fermions. The subscript c stands for the color gauge group of QCD.

In a previous paper [5] we have shown that in an SU(15) GUT the effect of Higgs bosons plays a significant role in the evolution of the coupling constants with increasing energy and hence on the values of the mass scales. This is due to the presence of high-dimensional Higgs fields required to obtain the desired symmetry-breaking pattern. The influence of the Higgs fields on the evolution of coupling constants can be so serious that they can alter the symmetry-breaking pattern altogether. In the SU(16) GUT the symmetry-breaking pattern is very similar to that of its SU(15) counterpart. So in the SU(16) or in the SU(15) GUT's the Higgs effects must be taken seriously. Here we shall consider the Higgs fields required to obtain the breaking chain and their contribution in the renormalization group equations in detail.

The Higgs structure is similar to that we proposed for SU(15) GUT. We denote 1^n as the totally antisymmetric nth-rank tensor and 1^n1^m as the representation which has m and n vertical boxes in the first and second columns of its Young's tableau. For the transition from the group G_1 to group G_2 the G_2 singlet component of the Higgs field should acquire vacuum expectation value. Turning to the specific case of SU(16) we note that at the scale M_U the breaking can be achieved by giving the vacuum expectation value to the $SU(12)\times SU(4)$ singlet component of 1^4. Using the exact same procedure we see that breaking at the scale M_1 can be done by $1^{14}1$, which leaves $U(1)_B$ unbroken. At the scale M_2 the breaking of $SU(6)_L$ to its special maximal algebra requires a somewhat large dimensional Higgs field representation. We use the 14 144-dimensional Higgs field $1^{14}1^2$ to break this group. As a passing comment we note here that this Higgs field will contribute significantly to the β functions of the renormalization group equations and make its presence strongly felt in the determination of the mass scales. The group $SU(4)^l$ can be broken by a Higgs field which transforms as a 15-plet under $SU(4)^l$ and which is contained in **255** under SU(16). At the stage M_3 the breaking of $SU(6)_R$ to $SU(3)_R\times U(1)_R$ is a bit complicated. **255** breaks $SU(6)_R$ to $SU(3)\times SU(3)\times U(1)_R$ and, subsequently the two $SU(2)_L$ groups of the quark and leptonic sectors respectively are glued by $1^{14}1^2$. The breaking of the lepton number local gauge symmetry $U(1)^{\text{lep}}$ can be achieved by either **16** or the two index symmetric Higgs field of dimension 136. In the first case it carries a lepton number of one unit and in the second case it carries that of two units. We shall see that the choice of specific Higgs field shall give interesting difference of physics in the context of neutrino oscillations. At the scale M_5 the breaking is done by the 1^4 Higgs field, which is 1820-dimensional. The baryon number is broken by either 1^5 or 1^6. In both the cases we get interesting physics. As an example in the first case we get processes where baryon number changes by 3 units and in the second case it changes by 2 units. It is well known that to give masses to the fermions, the vacuum expectation value has to be given to the component $(1,2,-\frac{1}{2})$ which is contained in either 1^2 or 11. These Higgs-field representations are summarized in Table I.

TABLE I. Higgs fields required for the breaking chain 1.

SU(16)	G	G_1	G_2	G_3	G_4	G_5	G_6
1820							
255	(143,1)						
14144	(4212,1)	(189,1,0,1)					
255	(1,15)	(1,1,0,15)	(1,1,0,1,15)				
255	(143,1)	(1,35,0,1)	(1,1,35,0,1)				
14144	(4212,1)	(1,189,0,1)	(1,1,189,0,1)				
136	(1,10)	(1,1,0,10)	(1,1,1,0,10)	$\left[1,1,1,0,0,1,3,\frac{2}{\sqrt{8}}\right]$			
16	(1,4)	(1,1,0,4)	(1,1,1,0,4)	$\left[1,1,1,0,0,1,2,\frac{1}{\sqrt{8}}\right]$			
1820	(66,6)	$(6,\bar{6},0,6)$	$(3,2,\bar{6},0,6)$	$\left[3,2,\bar{3},-\frac{1}{\sqrt{12}},0,2,2,0\right]$	$\left[3,2,\bar{3},-\frac{1}{\sqrt{12}},0,2,\frac{1}{\sqrt{12}}\right]$		
4368(1^5)	(220,6)	$\left[1,\overline{20},\frac{3}{2\sqrt{6}},6\right]$	$\left[1,1,\overline{20},\frac{3}{2\sqrt{6}},6\right]$	$\left[1,1,1,\frac{1}{\sqrt{12}},\frac{3}{2\sqrt{6}},1,1,-\frac{2}{\sqrt{8}}\right]$	$\left[1,1,1,\frac{1}{\sqrt{12}},\frac{3}{2\sqrt{6}},1,-\frac{2}{\sqrt{12}}\right]$	$\left[1,1,\frac{3}{2\sqrt{6}},-\frac{1}{\sqrt{24}}\right]$	
136	(78,1)	$(6,\bar{6},0,1)$	$(3,2,\bar{6},0,1)$	$\left[3,2,\bar{3},\frac{1}{\sqrt{12}},0,1,1,0\right]$	$\left[3,2,\bar{3},\frac{1}{12},0,1,0\right]$	$\left[1,2,0,\frac{1}{\sqrt{24}}\right]$	$\left[1,2,-\frac{1}{2}\sqrt{\frac{3}{20}}\right]$

Let us now turn our attention to the group theoretic transformation properties of the fermions under the different symmetry groups in the symmetry breaking scheme. A minimal left-right-symmetric theory should have at least one right-handed neutrino (ν_R) on top of the standard twelve quarks, which includes three left-handed doublets and six right-handed singlets under the weak interaction gauge group $SU(2)_L$, and three leptons, namely one left-handed doublet and one right-handed singlet. At grand unification energies and above these sixteen fermions should transform under some representation of the unification group. As a passing comment here we state that this requirement makes SU(16) a very natural choice of the unification gauge group, which has a 16-dimensional fundamental representation. In the model the fermions transform under the fundamental representation of SU(16). Now as the energy becomes lower the symmetry breakings occur and the transformation properties of the fermions change with each symmetry breaking taking place. In the following we summarize these transformation properties. We use the notation that (m,n) is a representation which transforms under the semisimple group $SU(M)\times SU(N)$ as an m plate under the former group and as an n-plate under the later group:

$$
\begin{aligned}
SU(16)\rightarrow\quad & 16\\
G\rightarrow\quad & (12,1)+(1,4)\\
G_1\rightarrow\quad & (1,\bar{6},n,1)+(6,1,-n,1)+(1,1,0,4)\\
G_2\rightarrow\quad & (1,1,\bar{6},n,1)+(3,2,1,-n,1)\\
& +(1,1,1,0,4)\\
G_3\rightarrow\quad & (1,1,\bar{3},p,n,1,1,0)+(1,1,\bar{3},-p,n,1,1,0)\\
& +(3,2,1,0,-n,1,1,0)+(1,1,1,0,0,1,2,m)\\
& +(1,1,1,0,0,2,1,-m)\\
G_4\rightarrow\quad & (1,1,\bar{3},p,n,0)+(1,1,\bar{3},-p,n,0)\\
& +(3,2,1,0,-n,0)+(1,1,1,0,0,-2\sqrt{\tfrac{2}{3}}m)\\
& +(1,2,1,0,0,\sqrt{\tfrac{2}{3}}m)+(1,1,1,0,0,0)\\
G_5\rightarrow\quad & (\bar{3},1,n,n)+(\bar{3},1,n,-n)+(3,2,-n,0)\\
& +(1,2,0,n)+(1,1,0,-2n)+(1,1,0,0)\\
G_6\rightarrow\quad & (\bar{3},1,-\tfrac{2}{3}K)+(\bar{3},1,\tfrac{1}{3}K)\\
& +(3,2,\tfrac{1}{6}K)+(1,1,K)+(1,1,-\tfrac{1}{2}K)\\
& +(1,1,0)^{\sim}\,.
\end{aligned}
\tag{1}
$$

Here the U(1) normalizations are defined in terms of

$$
n=\frac{1}{2\sqrt{6}}\,,\quad m=\frac{1}{2\sqrt{2}}\,,\quad p=\frac{1}{2\sqrt{3}}\,,\quad K=\sqrt{\tfrac{3}{20}}\,.
$$

We know that in the electroweak breaking scale M_Z the generators of electromagnetic symmetry group $U(1)_{em}$ arise as a linear combination of the generator of the U(1) part of the weak isospin group $SU(2)_L$ and that of the weak hypercharge $U(1)_Y$ by the equation

$$
Q=T_L^3+Y\,. \tag{2}
$$

Let us call this equation the U(1) *matching condition* at the scale M_Z. Similarly, at the various symmetry breaking scales in the above breaking chain we have used different matching conditions for the groups. These matching conditions are stated below.

At the scale M_4 the lepton-number symmetry breaks as the generator of $U(1)^{lep}$ and the diagonal generator of

$SU(2)^l_R$ mix with each other in the following way to generate the group $U(1)^l$:

$$Y^l=\sqrt{\tfrac{1}{3}}T^3_{2^l_R}+\sqrt{\tfrac{2}{3}}Y^{\text{lep}} . \tag{3}$$

At the scale M_5 $U(1)_R$ and $U(1)^l$ break to make $U(1)_h$:

$$Y_h=\sqrt{\tfrac{1}{2}}Y_R+\sqrt{\tfrac{1}{2}}Y^l . \tag{4}$$

At the scale M_6, the baryon number ceases to be a local gauge symmetry and conventional hypercharge appears from the linear combination of $U(1)_B$ and $U(1)_h$:

$$Y=-\sqrt{\tfrac{1}{10}}Y_B-\sqrt{\tfrac{9}{10}}Y_h . \tag{5}$$

III. MATHEMATICAL PRELIMINARIES

In this section we briefly touch on two more mathematically involved topics. To begin with we note that the generators of SU(16) and that of the standard model groups cannot be normalized in the same way. We proceed further in the section by giving a short discussion of the process of calculating the contribution of the Higgs fields to the β functions. Let us fix all generators of SU(16) so they are normalized to 2. In that case at the standard model energies the generators of $SU(3)_C$ and $SU(2)_L$ automatically become the generators of SU(16). In contrast the generators of $U(1)_Y$ are normalized to $\frac{1}{2}$. So in the renormalization group equations we have to multiply the β function corresponding to the $U(1)_Y$ group by the appropriate factor of 4. Similarly it is easy to see that all other U(1) groups in the symmetry breaking chain have to be multiplied by 4. Turning to the non-Abelian groups it can be checked that the group $SU(2)^q_L$ in all stages is normalized to $\frac{3}{2}$; hence, to treat it at par with all other groups one has to multiply the β function corresponding to this by a factor of $\frac{4}{3}$. $SU(3)_L$ and $SU(3)_R$ in all the stages are normalized to 1; hence, one finds the aforesaid factor to be 2. To complete the discussion on the normalization factors we note that all other groups are normalized to $\frac{1}{2}$, hence the relevant factor is 4.

At this point let us turn our attention to the expression of the β function $[b_i(N)]$ for the group SU(N):

$$b(N)=-\frac{1}{(4\pi)^2}[\tfrac{11}{3}N-\tfrac{1}{6}T-\tfrac{4}{3}n_f] . \tag{6}$$

For U(1) groups N vanishes. Here n_f denotes the number of families of fermions and $T(R)$ denotes the contribution of the Higgs fields which transform nontrivially under the group under consideration. To calculate T we have followed the following sum rule [9].

Suppose R_i and r_i ($i=1,2,\ldots$) are different representations of a group SU(N), which when vectorically multiplied satisfies the relation

$$R_1\times R_2=\sum_{i=1} r_i . \tag{7}$$

Also, for the representation of dimension r, the contribution to the renormalization group equation is $T(R)$. Then,

$$T(R_1\times R_2)=R_2T(R_1)+R_1T(R_2)=\sum_{i=1} T(r_i) . \tag{8}$$

To use these equations one uses the following information to start with:

$$T(N)=\tfrac{1}{2} ,$$

$$T(N^2-1)=N ,$$

$$T\left[\frac{N(N-1)}{2}\right]=\frac{N-2}{2} ,$$

$$T\left[\frac{N(N+1)}{2}\right]=\frac{N+2}{2} ,$$

$$T(1)=0 .$$

As an example consider 3 and $\bar{3}$ representations of SU(3). When vectorically multiplied they give

$$3\times\bar{3}=1+8 \tag{9}$$

so, using the sum rule,

$$T(8)=3T(3)+3T(\bar{3})-T(1)=3 . \tag{10}$$

IV. OTHER PATHS TO THE STANDARD MODEL

We have already noted that there can be a number of paths to the standard model groups starting from the unification group SU(16). Let us consider here two typical chains of descent. In the first case (chain 2) here we shall break the U(1) groups as low as possible. It is in a sense one extreme case as the β function coefficients for the U(1) groups are very small in magnitude compared to those of the other groups (the eigenvalues of the Casimir operator vanish).

Breaking chain 2

$$SU(16)\xrightarrow{M_U}G[SU(12)\times SU(4)^l]$$

$$\xrightarrow{M_1}G_1[SU(6)_L\times SU(6)_R\times U(1)_B\times SU(4)^l]$$

$$\xrightarrow{M_2}G_2[SU(3)_L\times SU(2)^q_L\times SU(6)_R\times U(1)_B\times SU(4)^l]$$

$$\xrightarrow{M_3}G_3[SU(3)_L\times SU(2)^q_L\times SU(3)_R\times U(1)^q_R\times U(1)_B\times SU(2)^l_L\times SU(2)^l_R\times U(1)^{\text{lep}}]$$

$$\overset{M_4}{\rightarrow} G_4[\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L^q \times \mathrm{U}(1)_R^q \times \mathrm{U}(1)_B \times \mathrm{SU}(2)_L^l \times \mathrm{SU}(2)_R^l \times \mathrm{U}(1)^{\mathrm{lep}}]$$
$$\overset{M_5}{\rightarrow} G_5[\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L^q \times \mathrm{SU}(2)_L^l \times \mathrm{U}(1)_R^q \times \mathrm{U}(1)_B \times \mathrm{U}(1)_R^l \times \mathrm{U}(1)^{\mathrm{lep}}]$$
$$\overset{M_6}{\rightarrow} G_6[\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y]$$
$$\overset{M_Z}{\rightarrow} G_7[\mathrm{SU}(3)_c \times \mathrm{U}(1)_{\mathrm{em}}] .$$

Note that all the U(1) groups remain ununified up to the scale M_6. At that scale they merge together to give the familiar hypercharge of the standard model. At the scale M_6 the matching condition is

$$Y = -\sqrt{\tfrac{2}{20}}\,Y_B - \sqrt{\tfrac{9}{20}}\,Y_R^q - \sqrt{\tfrac{3}{20}}\,Y_R^l - \sqrt{\tfrac{6}{20}}\,Y^{\mathrm{lep}} . \tag{11}$$

Another interesting possibility is to break SU(16) via the left-right symmetric group of Pati and Salam [2]. The low-energy phenomenology of the Pati-Salam group is widely studied. So it will be interesting to see how low the intermediate scales can come down to so we can make some concrete predictions of the model in view of the oncoming experiments. Hence in the second chain that is discussed here (chain 3) the left-right symmetric group $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ will be kept as low as possible.

Breaking chain 3

$$\mathrm{SU}(16) \overset{M_U}{\rightarrow} G[\mathrm{SU}(12) \times \mathrm{SU}(4)^l]$$
$$\overset{M_1}{\rightarrow} G_1[\mathrm{SU}(6)_L \times \mathrm{SU}(6)_R \times \mathrm{U}(1)_B \times \mathrm{SU}(4)^l]$$
$$\overset{M_2}{\rightarrow} G_2[\mathrm{SU}(3)_L \times \mathrm{SU}(2)_L^q \times \mathrm{SU}(6)_R \times \mathrm{U}(1)_B \times \mathrm{SU}(4)^l]$$
$$\overset{M_3}{\rightarrow} G_3[\mathrm{SU}(3)_L \times \mathrm{SU}(2)_L^q \times \mathrm{SU}(3)_R \times \mathrm{SU}(2)_R^q \times \mathrm{U}(1)_B \times \mathrm{SU}(4)_L]$$
$$\overset{M_4}{\rightarrow} G_4[\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L^q \times \mathrm{SU}(2)_R^q \times \mathrm{U}(1)_B \times \mathrm{SU}(2)_L^l \times \mathrm{SU}(2)_R^l \times \mathrm{U}(1)^{\mathrm{lep}}]$$
$$\overset{M_5}{\rightarrow} G_5[\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L^{q+l} \times \mathrm{SU}(2)_R^{q+l} \times \mathrm{U}(1)_{B-L}]$$
$$\overset{M_6}{\rightarrow} G_6[\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y]$$
$$\overset{M_Z}{\rightarrow} G_7[\mathrm{SU}(3)_c \times \mathrm{U}(1)_{\mathrm{em}}] .$$

We notice that $\mathrm{U}(1)_{B-L}$ group is formed at the scale M_5 when baryon number symmetry and lepton number symmetry are broken together at the same scale. The matching condition is

$$Y_{B-L} = \sqrt{\tfrac{1}{4}}\,Y_B - \sqrt{\tfrac{3}{4}}\,Y^{\mathrm{lep}} . \tag{12}$$

At the scale M_6 the generator of the group $\mathrm{U}(1)_{B-L}$ and the diagonal generator of the right-handed SU(2) group form a linear combination to generate the conventional hypercharge:

$$Y = -\sqrt{\tfrac{9}{10}}\,T_R^3 - \sqrt{\tfrac{1}{10}}\,Y_{B-L} . \tag{13}$$

Applying principles similar to those we used for calculating the Higgs structure for the first breaking chain, we can calculate the Higgs fields required to break SU(16) in the fashion of chain 1. The essential difference in chain 2 is that the breaking of $\mathrm{SU}(2)_R^l$ to $\mathrm{U}(1)_R^l$, which can be conveniently done by **255** which has a component (1,15) under G, and at the scale M_6 the four U(1) groups are glued by a combination of Higgs fields 1^4 and 1^3. The breaking chain 3 is much more symmetric and simpler too. The Higgs fields that we require to break the chain are also less complicated. To break the left-right symmetry group we need the Higgs fields $(1,1,3,-\sqrt{\tfrac{3}{8}})$, (1,2,2,0), (1,2,2,0), and $(1,1,3,\sqrt{\tfrac{3}{8}})$ which can be easily embedded in the group G in the representations (143,6), (1,15), and (78,1) and hence in SU(16). The details of the Higgs fields required for the chains 2 and 3 are given in Tables II and III.

V. MASS SCALES

To evaluate the mass scales we use the standard procedure of evolving the couplings with energy [7]. The energy dependences of the couplings are completely determined by the particle content of the theory and their couplings inside the loop diagrams of the gauge bosons. This is expressed by the renormalization group equation. The

TABLE II. Higgs fields required for the breaking chain 2.

SU(16)	G	G_1	G_2	G_3	G_4	G_5	G_6
1820							
255	(143,1)						
14144	(4212,1)	(189,1,0,1)					
255	(1,15)	(1,1,0,15)	(1,1,0,1,15)				
255	(143,1)	(1,35,0,1)	(1,1,35,0,1)				
14144	(4212,1)	(1,189,0,1)	(1,1,189,0,1)				
1820	(495,1)	$(15,\overline{15},0,1)$	$(6,1,\overline{15},0,1)$	$(6,1,\bar{6},0,0,1,0,0)$			
255	(1,15)	(1,1,0,15)	(1,1,1,0,15)	(1,1,1,0,0,1,3,0)	(1,1,0,0,1,3,0)		
1820	(66,6)	$(6,\bar{6},0,6)$	$(3,2,\bar{6},0,6)$	$\left[3,2,\bar{3},\frac{1}{\sqrt{12}},0,2,2,0\right]$	$\left[1,2,\frac{1}{\sqrt{12}},0,2,2,0\right]$	$\left[1,2,2,\frac{1}{\sqrt{12}},0,-\frac{1}{2},0\right]$	
1820	(220,1)	$\left[1,\overline{20},\frac{3}{2\sqrt{6}},1\right]$	$\left[1,1,\overline{20},\frac{3}{2\sqrt{6}},1\right]$	$\left[1,1,1,-\frac{1}{\sqrt{12}},\frac{3}{2\sqrt{6}},1,1,0\right]$	$\left[1,1,-\frac{1}{\sqrt{12}},\frac{3}{2\sqrt{6}},1,1,0\right]$	$\left[1,1,1,-\frac{1}{\sqrt{24}},\frac{3}{2\sqrt{6}},0,0\right]$	
560	(66,4)	$(6,\bar{6},0,4)$	$(3,2,\bar{6},0,4)$	$\left[3,2,\bar{3},-\frac{1}{\sqrt{12}},0,2,1,\frac{1}{\sqrt{8}}\right]$	$\left[1,2,-\frac{1}{\sqrt{12}},0,2,1,\frac{1}{\sqrt{8}}\right]$	$\left[1,2,2,-\frac{1}{\sqrt{12}},0,0,\frac{1}{\sqrt{8}}\right]$	
136	(78,1)	$(6,\bar{6},0,1)$	$(3,2,\bar{6},0,1)$	$\left[3,2,\bar{3},\frac{1}{\sqrt{12}},0,1,1,0\right]$	$\left[3,2,\bar{3},\frac{1}{12},0,1,0\right]$	$\left[1,2,0,\frac{1}{\sqrt{24}}\right]$	$\left[1,2,-\frac{1}{2}\sqrt{\frac{3}{20}}\right]$

one-loop RG equation is given by the equation

$$\mu\frac{d}{d\mu}\alpha(\mu)=2b\alpha^2(\mu) , \tag{14}$$

where

$$\alpha=\frac{g^2}{4\pi} . \tag{15}$$

The β-function coefficients are already defined. Now, using the above information and the matching conditions given with each symmetry-breaking chain, one can relate the SU(16) coupling constant $\alpha_{SU(16)}$ with the standard model couplings α_{3c}, α_{2L}, and α_{1Y} at the scale of the mass of the Z particle M_Z. At this point let us remember that there are three quark doublets and one leptonic doublet under the group $SU(2)_L$ in the standard model; hence, in the evolution of coupling α_{2L} the quark and leptonic groups $SU(2)_L^q$ and $SU(2)_L^l$ do not contribute equally to the standard model group $SU(2)_L$; instead, they contribute with a relative factor 3.

TABLE III. Higgs fields required for the breaking chain 3.

SU(16)	G	G_1	G_2	G_3	G_4	G_5	G_6
1820							
255	(143,1)						
14144	(4212,1)	(189,1,0,1)					
14144	(4212,1)	(1,189,0,1)	(1,1,189,0,1)				
255	(1,15)	(1,1,0,15)	(1,1,1,0,15)	(1,1,1,1,0,15)			
1820	(495,1)	$(15,\overline{15},0,1)$	$(6,1,\overline{15},0,1)$	$(6,1,\bar{6},1,0,1)$			
1820	(66,6)	$(6,\bar{6},0,6)$	$(3,2,\bar{6},0,6)$	$(3,2,\bar{3},2,0,6)$	$(1,2,\bar{2},0,2,2,0)$		
$1^2 1^2$	(572,4)	$\left[70,1,\frac{3}{2\sqrt{6}},4\right]$	$\left[1,2,1,-\frac{3}{2\sqrt{6}},4\right]$	$\left[1,2,1,1,\frac{3}{2\sqrt{6}},4\right]$	$\left[1,2,1,-\frac{3}{2\sqrt{6}},2,1,-\frac{1}{\sqrt{8}}\right]$		
$1^{11} 1^3$	(143,6)	(1,35,0,6)	(1,1,35,0,6)	(1,1,1,3,0,6)	$\left[1,1,3,0,1,1,\frac{2}{\sqrt{8}}\right]$	$\left[1,1,3,-\sqrt{\frac{3}{8}}\right]$	
136	(78,1)	$(6,\bar{6},0,1)$	$(3,2,\bar{6},0,1)$	$\left[3,2,\bar{3},\frac{1}{\sqrt{12}},0,1,1,0\right]$	$\left[3,2,\bar{3},\frac{1}{12},0,1,0\right]$	$\left[1,2,0,\frac{1}{\sqrt{24}}\right]$	$\left[1,2,-\frac{1}{2}\sqrt{\frac{3}{20}}\right]$
$1^{14} 1$	(1,15)	(1,1,0,15)	(1,1,1,0,15)	(1,1,1,1,0,15)	(1,1,1,0,2,2,0)	(1,2,2,0)	$\left[1,2,-\frac{1}{2}\sqrt{\frac{3}{20}}\right]$
$1^{11} 1^3$	(143,6)	(35,1,0,6)	(1,3,1,1,0,6)	(1,1,1,3,0,6)	$\left[1,1,0,1,1,\frac{2}{\sqrt{8}}\right]$	$\left[1,1,3,-\sqrt{\frac{3}{8}}\right]$	$\left[1,2,-\sqrt{\frac{3}{20}}\right]$

Chain 1

$$
\begin{aligned}
g_{3c}^{-2}(M_Z)={}&g_{\mathrm{SU}(16)}^{-2}(M_U)+2b_{12}M_{U1}+(b_{6L}+b_{6R})M_{12}+(b_{3L}+b_{6R})M_{23}\\
&+(b_{3L}+b_{3R})M_{34}+(b_{3L}+b_{3R})M_{45}+2b_{3c}M_{56}+2b_{3c}M_{6Z}\ ,\\
g_{2L}^{-2}(M_Z)={}&g_{\mathrm{SU}(16)}^{-2}(M_U)+(\tfrac{3}{2}b_{12}+\tfrac{1}{2}b_4^l)M_{U1}+(\tfrac{3}{2}b_{6L}+\tfrac{1}{2}b_4^l)M_{12}\\
&+(\tfrac{3}{2}b_{2L}^q+\tfrac{1}{2}b_4^l)M_{23}+(\tfrac{3}{2}b_{2L}^q+\tfrac{1}{2}b_{2L}^l)M_{34}+(\tfrac{3}{2}b_{2L}^q+\tfrac{1}{2}b_{2L}^l)M_{45}+2b_{2L}M_{56}+2b_{2L}M_{6Z}\ ,\\
g_{1Y}^{-2}(M_Z)={}&g_{\mathrm{SU}(16)}^{-2}(M_U)+(\tfrac{11}{10}b_{12}+\tfrac{9}{10}b_4^l)M_{U1}+(\tfrac{9}{10}b_{6R}+\tfrac{1}{5}b_{1B}+\tfrac{9}{10}b_4^l)M_{12}\\
&+(\tfrac{9}{10}b_{6R}+\tfrac{1}{5}b_{1B}+\tfrac{9}{10}b_4^l)M_{23}+(\tfrac{9}{10}b_{1R}+\tfrac{1}{5}b_{1B}+\tfrac{6}{10}b_1^{\mathrm{lep}}+\tfrac{3}{10}b_{2R}^l+)M_{34}\\
&+(\tfrac{9}{10}b_{1R}^q+\tfrac{1}{5}b_{1B}+\tfrac{9}{10}b_1^l)M_{45}+(\tfrac{9}{5}b_{1h}+\tfrac{1}{5}b_{1B})M_{56}+2b_{1Y}M_{6Z}\ .
\end{aligned}
\tag{16}
$$

Chain 2

$$
\begin{aligned}
g_{3c}^{-2}(M_Z)={}&g_{\mathrm{SU}(16)}^{-2}(M_U)+2b_{12}M_{U1}+(b_{6L}+b_{6R})M_{12}+(b_{3L}+b_{6R})M_{23}\\
&+(b_{3L}+b_{3R})M_{34}+2b_{3c}M_{45}+2b_{3c}M_{56}+2b_{3c}M_{6Z}\ ,\\
g_{2L}^{-2}(M_Z)={}&g_{\mathrm{SU}(16)}^{-2}(M_U)+(\tfrac{3}{2}b_{12}+\tfrac{1}{2}b_4^l)M_{U1}+(\tfrac{3}{2}b_{6L}+\tfrac{1}{2}b_4^l)M_{12}\\
&+(\tfrac{3}{2}b_{2L}^q+\tfrac{1}{2}b_4^l)M_{23}+(\tfrac{3}{2}b_{2L}^q+\tfrac{1}{2}b_{2L}^l)M_{34}+(\tfrac{3}{2}b_{2L}^q+\tfrac{1}{2}b_{2L}^l)M_{45}+(\tfrac{3}{2}b_{2L}^q+\tfrac{1}{2}b_{2L}^l)M_{56}+2b_{2L}M_{6Z}\ ,\\
g_{1Y}^{-2}(M_Z)={}&g_{\mathrm{SU}(16)}^{-2}(M_U)+(\tfrac{11}{10}b_{12}+\tfrac{9}{10}b_4^l)M_{U1}+(\tfrac{9}{10}b_{6R}+\tfrac{1}{5}b_{1B}+\tfrac{9}{10}b_4^l)M_{12}\\
&+(\tfrac{9}{10}b_{6R}+\tfrac{1}{5}b_{1B}+\tfrac{9}{10}b_4^l)M_{23}+(\tfrac{9}{10}b_{1R}^q+\tfrac{1}{5}b_{1B}+\tfrac{3}{10}b_{2R}^l+\tfrac{6}{10}b_1^{\mathrm{lep}})M_{34}\\
&+(\tfrac{9}{10}b_{1R}^q+\tfrac{1}{5}b_{1B}+\tfrac{3}{10}b_{2R}^l+\tfrac{6}{10}b_1^{\mathrm{lep}})M_{45}+(\tfrac{9}{10}b_{1R}+\tfrac{1}{5}b_{1B}+\tfrac{3}{10}b_{1R}^l+\tfrac{6}{10}b_1^{\mathrm{lep}})M_{56}+2b_{1Y}M_{6Z}\ .
\end{aligned}
\tag{17}
$$

Chain 3

$$
\begin{aligned}
g_{3c}^{-2}(M_Z)={}&g_{\mathrm{SU}(16)}^{-2}(M_U)+2b_{12}M_{U1}+(b_{6L}+b_{6R})M_{12}+(b_{3L}+b_{6R})M_{23}\\
&+(b_{3L}+b_{3R})M_{34}+2b_{3c}M_{45}+2b_{3c}M_{56}+2b_{3c}m_{6Z}\ ,\\
g_{2L}^{-2}(M_Z)={}&g_{\mathrm{SU}(16)}^{-2}(M_U)+(\tfrac{3}{2}b_{12}+\tfrac{1}{2}b_4^l)M_{U1}+(\tfrac{3}{2}b_{6L}+\tfrac{1}{2}b_4^l)M_{12}\\
&+(\tfrac{3}{2}b_{2L}^q+\tfrac{1}{2}b_4^l)M_{23}+(\tfrac{3}{2}b_{2L}^q+\tfrac{1}{2}b_4^l)M_{34}+(\tfrac{3}{2}b_{2L}^q+\tfrac{1}{2}b_{2L}^l)M_{45}+2b_{2L}^{q+l}M_{56}+2b_{2L}M_{6Z}\ ,\\
g_{1Y}^{-2}(M_Z)={}&g_{\mathrm{SU}(16)}^{-2}(M_U)+(\tfrac{11}{10}b_{12}+\tfrac{9}{10}b_4^l)M_{U1}+(\tfrac{9}{10}b_{6R}+\tfrac{1}{5}b_{1B}+\tfrac{9}{10}b_4^l)M_{12}+(\tfrac{9}{10}b_{6R}+\tfrac{1}{5}b_{1B}+\tfrac{9}{10}b_4^l)M_{23}\\
&+(\tfrac{9}{10}b_{2R}^q+\tfrac{1}{5}b_{1B}+\tfrac{9}{10}b_4^l)M_{34}+(\tfrac{9}{10}b_{2R}^q+\tfrac{1}{5}b_{1B}+\tfrac{3}{10}b_{2R}^l+\tfrac{6}{10}b_1^{\mathrm{lep}})M_{45}+(\tfrac{9}{5}b_{2R}^{q+l}+\tfrac{1}{5}b_{1(B-L)})M_{56}+2b_{1Y}M_{6Z}\ .
\end{aligned}
\tag{18}
$$

Here M_{ij} is defined as $\ln(M_i/M_j)$. As a comment we note that generally one would expect that the coefficients of b_{2R}^{q+l} and $b_{1(B-L)}$ to be $\frac{6}{5}$ and $\frac{4}{5}$, respectively, as it appears in the SO(10) model. It is worth noting that these factors are dependent on the normalization of the U(1) part of $\mathrm{SU}(2)_R^{q+l}$ and that of the group $\mathrm{U}(1)_{B-L}$. This point will be elaborated further in the Appendix.

To calculate the mass scales we also have to know the numerical values of the β-function coefficients. To know them one has to know the contribution of the Higgs scalars to the β functions (T). In Tables IV–VI we give these values.

Now with the quantities $g_{1Y}^{-2}(M_Z)$, $g_{2L}^{-2}(M_Z)$, and $g_{3c}^{-2}(M_Z)$ at hand one can construct two different linear combinations with them to form the experimentally measured quantities at the energy scale M_Z. It is easy to check that the following relations hold between them:

$$
\begin{aligned}
&\sin^2(\theta_W)=\tfrac{3}{8}-\tfrac{5}{8}e^2(g_{1Y}^{-2}-g_{2L}^{-2})\ ,\\
&1-\frac{8}{3}\frac{\alpha}{\alpha_s}=e^2(g_{2L}^{-2}+\tfrac{5}{3}g_{1Y}^{-2}-\tfrac{8}{3}g_{3c}^{-2})\ .
\end{aligned}
\tag{19}
$$

From the present experimental measurements at the CERN e^+e^- collider LEP the values of $\sin^2(\theta_W)$ and α_s have been very accurately measured. We use for our purpose the following values [11] for them and for the U(1) coupling α at the scale M_Z:

$$
\begin{aligned}
&\sin^2(\theta_W)=0.2336\pm0.0018\ ,\\
&\alpha_s=0.108\pm0.005\ ,\\
&\alpha=\frac{1}{128.8}\ .
\end{aligned}
\tag{20}
$$

TABLE IV. Contributions of the Higgs scalar to the renormalization group (RG) equation at various energy scales in chain 1.

G	G_1	G_2	G_3	G_4	G_5	G_6
$[12]=1492$	$[6_L]=69$	$[3_L]=42$	$[3_L]=15$	$[3_L]=9$	$[3_c]=0$	$[3_c]=0$
$[4^l]=293$	$[6_R]=93$	$[2^q_L]=63$	$[2^q_L]=22.5$	$[2^q_L]=13.5$	$[2_L]=0.5$	$[2_L]=0.5$
	$[1_B]=45$	$[6_R]=93$	$[3_R]=15$	$[3_R]=9$	$[1_B]=0.375$	$[1_Y]=0.075$
	$[4^l]=63$	$[1_B]=45$	$[1^q_R]=7.58$	$[1^q_R]=3.16$	$[1_h]=0.083$	
		$[4^l]=63$	$[1_B]=0.375$	$[1_B]=0.375$		
			$[2^l_L]=18$	$[2^l_L]=9$		
			$[1^{\text{lep}}]=2$	$[1_l]=3.16$		

Having this information at hand one can straightaway go to calculate the mass scales of symmetry breaking. (See Tables VII and VIII.)

Let us discuss the calculation of the first chain in some detail. Let us now assume that $M_4=M_3=M_A$. This means that the groups $SU(6)_L$, $SU(6)_R$, and $SU(4)^l$ happen to break at the same scale. Similarly let us also assume that $M_4=M_5=M_B$. Now using the values of the $T(R)$'s from Table II and solving for M_{U1} and M_{B6} in terms of the other variables one gets

$$M_{U1}=-0.28-0.10M_{1A}-0.10M_{6Z}+0.04M_{AB}\ ,$$
$$M_{B6}=19.80-4.81M_{1A}-2.93M_{6Z}-0.21M_{AB}\ . \tag{21}$$

As the symmetry breaking at M_U occurs before it happens at M_1, M_{U1} is at least positive. So from the first equation one infers that for a specific set of values of the other parameters in the right-hand side there is a minimum value to M_{AB}. Varying the parameters of the equations one gets the following subset of the solution set allowed by the equations. Taking M_Z to be around 91 GeV one can also calculate the unification scale and the scale M_6 where the completely ununified symmetry of the quarks and leptons and the chiral color symmetry is broken. We note that as the parameter M_{AB} increases, i.e., as the separation between the scale M_A and the scale M_B increases, the scale M_B comes down. In a similar way let us determine the breaking scales that we may get from the solution of chain 2. Here we keep the U(1) groups as low as possible in the hope that it will give rise to distinct phenomenology at low energy. To begin with let us keep $M_2=M_3=M_A$ and $M_4=M_5=M_B$. The solutions are

$$M_{U1}=-0.70-0.01M_{1A}-0.02M_{6Z}+0.01M_{AB}\ ,$$
$$M_{B6}=5.35-0.44M_{1A}-0.78M_{6Z}-0.81M_{AB}\ . \tag{22}$$

To keep M_{U1} positive we have to have M_{AB} larger than 70. This pushes the unification scale beyond the Planck scale and hence makes the breaking chain uninteresting.

The third option that we have considered here is to come to the low-energy groups via the left-right symmetric Pati-Salam group in chain 3. The solutions in this case are

$$M_{U1}=-0.99+0.13M_{1A}+0.01M_{56}+0.02M_{A5}\ ,$$
$$M_{6Z}=6.84+2.64M_{1A}-0.22M_{56}-0.1M_{A5}\ . \tag{23}$$

The solution set of these equations is interesting, though low-energy unification is not possible here. Let us at first state a sample solution set. It is obvious from the equations that to keep M_{U1} positive one needs a rather larger value of M_{1A}, which on the other hand pushes M_{6Z} up. The minimum value of M_{1A} is around 7.6, which gives a minimum value of the unification scale of around 10^{17} GeV. In a previous paper [6] we have shown that with the precisely measured value of $\sin^2(\theta_W)$ that is available now left-right symmetry at the low energy coming from a grand unified scenario is ruled out. This analysis comes as a confirmation of that result and it shows that even having the number of parameters that we have in the form of a number of breaking stages, left-right symmetry cannot come down to a low energy for any choice of the parameter space.

VI. PHENOMENOLOGY

A. Proton decay

Having the mass scales and Higgs structures in hand we proceed in this paper to discuss the issue of proton decay now. In all the breaking chains that we have considered here, the quark-lepton unification is broken at the

TABLE V. Contributions of the Higgs scalar to the RG equation at various energy scales in chain 2.

G	G_1	G_2	G_3	G_4	G_5	G_6
$[12]=1459$	$[6_L]=111$	$[3_L]=103$	$[3_L]=36$	$[3_c]=0$	$[3_c]=0$	$[3_c]=0$
$[4^l]=109$	$[6_R]=120$	$[2^q_L]=99$	$[2^q_L]=31.5$	$[2^q_L]=3.5$	$[2^q_L]=2.5$	$[2_L]=0.5$
	$[1_B]=7.5$	$[6_R]=102$	$[3_R]=36$	$[1^q_R]=1.25$	$[2^l_L]=2$	$[1_Y]=0.075$
	$[4^l]=62$	$[1_B]=7.5$	$[1^q_R]=7.58$	$[1_B]=0.375$	$[1^q_R]=0.583$	
		$[4^l]=78$	$[1_B]=0.375$	$[2^l_L]=3$	$[1_B]=0.375$	
			$[2^l_L]=27$	$[2^l_R]=4$	$[1^l_R]=1$	
			$[1^l_R]=20$	$[1^{\text{lep}}]=0.5$	$[1^{\text{lep}}]=12.5$	
			$[1^{\text{lep}}]=4.5$			

TABLE VI. Contributions of the Higgs scalar to the RG equation at various energy scales in chain 3.

G	G_1	G_2	G_3	G_4	G_5	G_6
$[12]=1831$	$[6_L]=171$	$[3_L]=79.5$	$[3_L]=57$	$[3_c]=0$	$[3_c]=0$	$[3_c]=0$
$[4^l]=646$	$[6_R]=105$	$[2_L^q]=71$	$[2_L^q]=77$	$[2_L^q]=8$	$[2_L^{q+l}]=4$	$[2_L]=1.5$
	$[1_B]=105$	$[6_R]=117$	$[3_R]=54$	$[2_R^q]=7$	$[2_R^{q+l}]=4$	$[1_Y]=0.225$
	$[4^l]=149$	$[1_B]=3$	$[2_R^q]=75$	$[1_B]=1.5$	$[1_{(B-L)}]=2.25$	
		$[4^l]=89$	$[1_B]=3$	$[2_L^l]=6$		
			$[4^l]=51$	$[2_R^l]=5$		
				$[1^{\text{lep}}]=3.5$		

scale M_U while the quark-antiquark unification is broken at the scale M_1. As a result the leptoquark gauge bosons (X_μ) will acquire mass at the scale M_U while the diquark gauge bosons (Y_μ) acquire mass at the scale M_1. Under the group G_1 their transformation properties are

$$X_\mu \Longrightarrow (6,1,-B,\bar{4})+(1,\bar{6},B,\bar{4}) +(\bar{6},1,B,4)+(1,6,-B,4)\ ,$$

$$Y_\mu \Longrightarrow (6,6,-2B,1)+(\bar{6},\bar{6},2B,1)\ ,$$

where

$$B=\frac{1}{2\sqrt{6}}\ . \tag{24}$$

Now, $U(1)_B$ being an explicit local gauge symmetry of the model, X_μ and Y_μ contain different "baryon numbers" and hence cannot mix directly to form an SU(16) invariant operator.

The mixing can be induced indirectly through the term $D_\mu\phi_a D^\mu\phi_b$, where D_μ is the covariant derivative of the SU(16) invariant theory. $D_\mu\phi_a D^\mu\phi_b$ will contain a term $X_\mu\phi_a X^\mu\phi_b$. When ϕ_a and ϕ_b acquire vacuum expectation values the mixing between X_μ and Y^μ occurs. But this can occur only at the scale M_6; hence, the amplitude is suppressed by a factor of $O(M_5M_6/M_1^2M_2^2)$.

To see how the gauge bosons couple to the Higgs fields we note that all the gauge bosons at the SU(16) level transform under the 224-dimensional adjoint representation. We also note the following tensor product at the SU(16) level:

$$\mathbf{224}\times\mathbf{224}=\mathbf{1}+\mathbf{224}+\mathbf{224}+\mathbf{14\,175}+\mathbf{10\,800} +\mathbf{12\,376}+\mathbf{12\,376}\ . \tag{25}$$

Being the product of two self-conjugate representations, all the terms in the right-hand side are self-conjugate, and therefore couple only to self-conjugate representations. From Table I one sees that the Higgs field which can induce a baryon-number-violating effect is 1^5, which is 4368 dimensional.

The only self-conjugate combination made up with 1^5's is $\langle 4368\rangle\langle\overline{4368}\rangle$, which again carries no baryon number and, hence, does not give rise to any baryon-number-violating process [8].

To see the Higgs-field-mediated proton decay at first we note that the fermions are in the 16-dimensional fundamental representation. To give mass to the fermions the coupling of the form $\bar{\psi}_L^c\psi_L\phi$ must exist. The minimum-dimensional Higgs field which can do the job is **120**. This field can give rise to Higgs-mediated proton decay if 1^6 breaks the baryon number due to the presence of the term $\langle 1^6\rangle\langle 1^6\rangle\langle 1^2\rangle\langle 1^2\rangle$ in the Lagrangian. In that case we can choose **136** to give mass to the fermions. In our choice 1^5 breaks the baryon number, hence it does not couple to **120**. Hence, there is no Higgs-mediated proton decay.

B. N-$\bar{N}$ oscillations

Let us consider the SU(16) level operator $\langle 1^5\rangle\langle 1^5\rangle\langle 1^5\rangle\langle 16\rangle$. This forms a singlet under SU(16) and hence it is allowed in the Lagrangian. This term gives rise to $\Delta B=3$ processes. If instead we choose **136** to break the lepton number symmetry, then this process vanishes.

In the preceding subsection we noted that if 1^6 breaks the baryon number symmetry then one has to choose **136** to give mass to the fermions; here we note that then the term $\langle 1^{14}1^2\rangle\langle 136\rangle\langle 136\rangle\langle 1^6\rangle\langle 1^6\rangle$ will be allowed in the Lagrangian, which may give rise to $\Delta B=3$ processes. As the term is of dimension 5 it will be suppressed by M_U. With 1^2 we can construct the SU(16) level operator $\langle 1^5\rangle\langle 1^5\rangle\langle 1^4\rangle\langle 1^2\rangle$ which can break the baryon number by two units and hence give rise to gauge-boson-mediated N-$\bar{N}$ oscillations. To see the Higgs-field-mediated processes we note that if 120-dimensional Higgs field couples to the fermions and 1^6 breaks the baryon number then

TABLE VII. Mass scales from chain 1.

M_{AB}	M_{1A}	M_{6Z}	M_{B6}	M_{U1}	M_B	M_U
7	0	0	18.4	0	10^9	10^{12}
9.5	1	0	12.9	0	10^8	10^{12}
10.75	1.5	0	10.3	0	10^7	10^{11}
12	2	0	8.7	0	10^6	10^{11}
14.5	3	0	2.3	0	10^4	10^{11}

TABLE VIII. Mass scales from chain 3.

M_{1A}	M_{A5}	M_{56}	M_{U1}	M_{6Z}	M_6	M_U
7.61	0	0	0	26.92	10^{13}	10^{17}
6.84	5	0	0	24.89	10^{12}	10^{18}
6.07	10	0	0	21.86	10^{11}	10^{18}
5.30	15	0	0	19.35	10^{10}	10^{19}
4.53	20	0	0	16.79	10^9	10^{19}

the operator $\langle\overline{120}\rangle\langle\overline{120}\rangle\langle\overline{120}\rangle\langle 1^6\rangle$ can give rise to Higgs-field-mediated N-$\bar{N}$ oscillations.

VII. CONCLUSION

In this paper we have seen that there exists one possible breaking chain in a grand unified theory based on the group SU(16), where a unification scale of the order of 10^{11} GeV is possible. There exists a very-low-energy scale (M_B) which may be almost anywhere between the unification scale and the electroweak scale, where completely ununified symmetry of quarks and leptons may exist together with chiral color symmetry. The scale M_B decreases when the separation between the scale M_A and the scale M_B is increased. Qualitatively we understand it in the following way. The β-function coefficients can be looked into as the slope of the lines if one plots the inverse coupling constants with respect to energy. It can be easily checked that, since at the SU(16) level all the fermions transform under the fundamental representation of the group and in the other levels they transform in a more complicated way under the various groups in the intermediate stages, all the groups cannot be normalized in the same way. To compensate for the mismatch in the normalizations, the β-function coefficient has to be multiplied by appropriate factors. Because of this the slopes of the curves representing the inverse couplings also get multiplied by the appropriate factors and the couplings get united earlier giving rise to low-energy unification.

We have also seen that this model satisfies the experimental constraints coming from proton decay experiments in the sense that proton decay is suppressed. We have shown that there exists at least one choice of the Higgs sector where there is no Higgs-mediated proton decay either.

For some specific choice of the Higgs fields there may exist interesting physical consequences such as the N-$\bar{N}$ oscillation. There is also the possibility of having the see-saw mechanism to give Majorana mass to the neutrinos and this also may have observable consequences.

Last but not the least we emphasize again that there exists very rich low-energy physics coming from this model, hence keeping in mind the forthcoming high-energy experiments at the Superconducting Super Collider and CERN Large Hadron Collider and other places this model is worthy of further investigation.

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APPENDIX

1. SU(16) tensor products

$$\begin{aligned}
&\mathbf{16}\times\mathbf{16}=\mathbf{120}_a+\mathbf{136}_s\ ,\\
&\overline{\mathbf{16}}\times\mathbf{16}=\mathbf{1}+\mathbf{255}\ ,\\
&\mathbf{16}\times\mathbf{120}=\mathbf{560}_a+\mathbf{1360}\ ,\\
&\overline{\mathbf{120}}\times\mathbf{120}=\mathbf{1}+\mathbf{255}+\mathbf{14\,144}\ ,\\
&\overline{\mathbf{136}}\times\mathbf{136}=\mathbf{1}+\mathbf{255}+\mathbf{18\,240}\ ,\\
&\mathbf{560}_a\times\mathbf{16}=\mathbf{1820}_a+\mathbf{7140}\ ,\\
&\mathbf{1820}_a\times\mathbf{16}=\mathbf{4368}_a+\mathbf{24\,752}\ .
\end{aligned}\tag{A1}$$

2. SU(16) branching rules

$$\begin{aligned}
&\mathrm{SU}(16)\Longrightarrow\mathrm{SU}(12)\times\mathrm{SU}(4)\ ,\\
&\mathbf{16}=(12,1)+(1,4)\ ,\\
&\mathbf{136}=(78,1)+(12,4)+(1,10)\ ,\\
&\mathbf{120}=(66,1)+(12,4)+(1,6)\ ,\\
&\mathbf{255}=(143,1)+(12,\bar{4})+(\overline{12},4)+(1,15)+(1,1)\ ,\\
&\mathbf{560}=(220,1)+(66,4)+(12,6)+(1,\bar{4})\ ,\\
&\mathbf{1820}=(495,1)+(220,4)+(66,6)+(12,\bar{4})+(1,1)\ ,\\
&\mathbf{14\,144}=(1,1)+(1,35)+(12,\bar{4})+(12,\overline{20})+(\overline{12},4)\\
&\qquad+(\overline{12},20)+(\overline{66},6)+(66,\bar{6})+(143,1)\\
&\qquad+(143,15)+(70,\bar{4})+(\overline{780},4)+(4212,1)\ .
\end{aligned}\tag{A2}$$

3. SU(12) tensor products

$$\begin{aligned}
&\mathbf{12}\times\mathbf{12}=\mathbf{66}_a+\mathbf{78}_s\ ,\\
&\overline{\mathbf{12}}\times\mathbf{12}=\mathbf{1}+\mathbf{143}\ ,\\
&\mathbf{12}\times\mathbf{66}=\mathbf{220}_a+\mathbf{572}\ ,\\
&\overline{\mathbf{78}}\times\mathbf{78}=\mathbf{1}+\mathbf{143}+\mathbf{5940}\ ,\\
&\overline{\mathbf{66}}\times\mathbf{66}=\mathbf{1}+\mathbf{143}+\mathbf{4212}\ ,\\
&\mathbf{220}_a\times\mathbf{12}=\mathbf{495}+\mathbf{2145}\ ,\\
&\mathbf{495}_a\times\mathbf{12}=\mathbf{792}+\mathbf{5148}\ .
\end{aligned}\tag{A3}$$

4. SU(12) branching rules

$$\begin{aligned}
&\mathrm{SU}(12)\Longrightarrow\mathrm{SU}(6)\times\mathrm{SU}(6)\times\mathrm{U}(1)\ ,\\
&\mathbf{12}=(6,1,-B)+(1,\bar{6},B)\ ,\\
&\mathbf{66}=(15,1,-2B)+(1,\overline{15},2B)+(6,\bar{6},0)\ ,\\
&\mathbf{78}=(21,1,-2B)+(1,\overline{21},2B)+(6,\bar{6},0)\ ,
\end{aligned}$$

$$143=(35,1,0)+(\overline{6},\overline{6},2B)+(6,6,-2B)+(1,1,0)+(1,35,0)\,,$$
$$220=(20,1,-3B)+(1,\overline{20},3B)+(6,\overline{15},B)+(15,\overline{6},-B)\,,$$
$$495=(15,1,-4B)+(20,\overline{6},-2B)+(15,\overline{15},0)+(6,\overline{20},2B)+(1,\overline{15},4B)\,,$$
$$792=(\overline{6},1,-5B)+(15,\overline{6},-3B)+(20,\overline{15},-B)+(15,\overline{20},B)+(6,\overline{15},3B)+(1,6,5B)\,,$$
$$572=(70,1,-3B)+(15,\overline{6},-B)+(6,\overline{15},B)+(21,\overline{6},-B)+(6,\overline{21},B)+(1,\overline{70},3B)\,,$$
$$4212=(189,1,0)+(15,15,-4B)+(6,6,-2B)+(84,6,-2B)+(\overline{15},\overline{15},4B)$$
$$+(1,35,0)+(1,189,0)+(\overline{6},\overline{84},2B)+(\overline{84},\overline{6},2B)+(6,84,-2B)+(1,1,0)+(35,1,0)+(35,35,0)+(\overline{6},\overline{6},2B)\,. \quad \text{(A4)}$$

5. SU(6) branching rules

$$SU(6)\Longrightarrow SU(3)\times SU(2)\,,$$
$$6=(3,2)\,,$$
$$15=(6,1)+(\overline{3},3)\,,$$
$$20=(1,4)+(8,2)\,, \quad \text{(A5)}$$
$$21=(\overline{3},1)+(6,3)\,,$$
$$35=(1,3)+(8,1)+(8,3)\,,$$
$$70=(1,2)+(8,4)+(8,2)+(10,2)\,.$$

6. Normalization of $U(1)_{(B-L)}$ and $SU(2)_R^{+I}$

Consider chain 3. Under the group G_6 the 16 fermions transform as

$$(\overline{3},1,2,1/\sqrt{24})(3,2,1,-1/\sqrt{24})$$
$$+(1,1,2-3/\sqrt{24})+(1,2,1,3/\sqrt{24})\,.$$

The T_{2R}^3 parts of the right-handed SU(2) group are to be taken as $\pm 1/\sqrt{24}$ so as to get the correct U(1) charges at the standard model level. $U(1)_{B-L}$ is normalized to 2 while the U(1) generator of the right-handed SU(2) is normalized to $\frac{8}{24}$, i.e., $\frac{1}{3}$. Taking this factor of 6 in the relative normalization one can easily get the familiar matching conditions of the SO(10) model.

7. Anomaly cancellation and mass scales

A 16-dimensional fundamental representation of SU(16) is not anomaly-free. To get the cancellation of anomaly one has to introduce mirror fermions. But these fermions will not alter the values of the mass scales obtained here. This is because in the two equations used for $\sin^2(\theta_W)$ and $1-\frac{8}{3}\alpha/\alpha_s$ respectively the fermion contributions to the β-function coefficients cancel.

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