Note on Weinberg operators in the standard model

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I demonstrate the vanishing of the dimension-six Weinberg operator in the standard Kobayashi-Maskawa model of *CP* violation in the absence of QCD corrections. I then argue that dimension-eight operators do appear, and estimate their contribution to the neutron electric dipole moment.

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I. INTRODUCTION

Since Weinberg [1] pointed out that gluonic operators of dimension $d \geq 6$ can contribute significantly to the neutron electronic dipole moment (NEDM), there has been a renewal of interest in the subject. The Weinberg operator and related operators have been calculated in most models of *CP* violation [2]. However, save for the work of Bigi and Uraltsev [3], these operators have not been studied in the standard Kobayashi-Maskawa (KM) model. This is understandable considering the small expected size of these effects. However, as the limit on the NEDM marches downward [4,5], it is important to have an accurate prediction for the KM model.

In this paper I will take a step in that direction by showing that in the absence of QCD corrections, Weinberg operators in the standard model must be at least of dimension eight. Although on dimensional grounds such operators are suppressed by two additional powers of a "large" mass compared to the original dimension-six Weinberg operator, due to the enhancement of dimension-eight operators in the QCD evolution [6–8] such operators may still give an important contribution to the NEDM.

II. THE STANDARD MODEL

In the KM model, all *CP*-violation amplitudes may be written as

$$M=\sum_{u_1,u_2,d_1,d_2}\Phi^{u_2d_2}_{u_1d_1}M^{d_1d_2}_{u_1u_2},$$

where

$$\Phi^{u_2d_2}_{u_1d_1}=\mathrm{Im}(V_{u_1d_1}V^*_{u_2d_1}V_{u_2d_2}V^*_{u_1d_2}) .$$

Because the KM angles have been factored out, the partial amplitudes M depend on the quarks only through their masses. Φ has two important properties which follow from unitarity of the Cabibbo-KM (CKM) matrix [9,10]: it is antisymmetric in (u_1,u_2) and (d_1,d_2), and the sum over any index, e.g., u_1, vanishes. For three families (which is all that I will consider), one may write [9]

$$\Phi^{u_2d_2}_{u_1d_1}=J\sum_{\gamma k}\epsilon_{u_1u_2\gamma}\epsilon d_1d_2k .$$

These properties of Φ, which are often called the Glashow-Iliopoulos-Maiani (GIM) mechanism, force any *CP*-violating partial amplitude $M^{d_1d_2}_{u_1u_2}$ to depend on four quark masses and to be antisymmetric under the exchange of up or down quark masses. This antisymmetry leads to cancellations between the contributions of different quarks, and is responsible for suppressing most *CP*-violating effects in the KM model [11]. As an example of this, Shabalin [12] was able to show that the quark electric dipole moment vanishes to two loops in the KM model. In the same spirit, I will show that the dimension-six Weinberg operator, which may also be considered the chromoelectric dipole moment of the gluon [2], vanishes to three-loop order in the KM model.[1]

In order to do this, I will study the structure of the partial amplitudes. Most important will be the behavior under exchange of quark masses, but Dirac matrix structure will also be important because it determines the possible terms which can appear in the amplitude. To simplify the language, I will adopt a few shorthand expressions. A term with an odd or even number of γ matrices will be called simply odd or even. An expression will be called left handed if it can be written as[2] $\mathcal{O}L$. "Down" and "up" will generally refer to families of quarks; when they refer to particular flavors, the meaning will be clear from context. As an example, in $(\not{P}-m_{u_1})L$, $\not{P}L$ is odd and left handed and m_{u_1} is an up-type quark.

In the conventional approach to calculating the coefficients of Weinberg operators, one inserts a number of gluons and expands in the gluon momenta, keeping

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[1]It may be helpful in the following to keep in mind that the properties of Φ allow one to write

$$M=\frac{1}{4}\sum_{\substack{u_1<u_2\\ d_1<d_2}}\Phi^{u_2d_2}_{u_1d_1}(M^{d_1d_2}_{u_1u_2}-M^{d_1d_2}_{u_2u_1}-M^{d_2d_1}_{u_1u_2}+M^{d_2d_1}_{u_2u_1}) . \quad (1)$$

[2]L and R denote the left- and right-handed projectors $(1\mp\gamma_5)/2$.

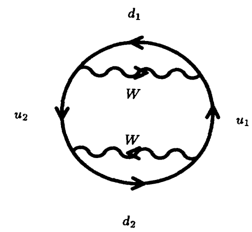

FIG. 1. One possible topology involving two electroweak bosons. Solid lines represent quark propagators in the background field, wavy lines electroweak bosons (W, Φ).

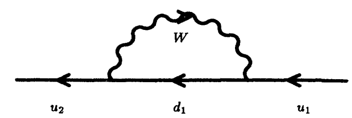

FIG. 2. Diagrams which generate F.

only the first few terms in that expansion. This becomes complicated for more than one gluon. A convenient way to organize this expansion is to work in the background field picture and to adopt the operator formalism of Novikov, Shifman, Vainshtein, and Zakharov [13] (NSVZ). This approach has a long history dating from Schwinger [14]. For my purpose it suffices to note that the covariant derivative $D_\mu = \partial_\mu - igA_\mu(x)$ (A_μ is the background gauge field in the fundamental representation) is raised to the level of an operator, $P_\mu = iD_\mu$. Then $[P_\mu, P_\nu] = igG_{\mu\nu}$, with $G_{\mu\nu}$ the field strength, and the quark propagator is written as

$$\langle 0|Tq^a(x)\bar{q}^b(y)|0\rangle$$
$$= \left\langle x,a \left| \frac{i}{\not{P}-m} \right| y,b \right\rangle$$
$$= \left\langle x,a \left| \frac{i}{P^2+(g/2)\sigma G - m^2}(\not{P}+m) \right| y,b \right\rangle . \quad (2)$$

Here $\sigma G \equiv \sigma_{\mu\nu}G^{\mu\nu}$, and a and b are color indices which I will suppress from now on. This representation of the propagator has the advantage that apart from noncommutativity of P, all the familiar Feynman rules still apply. By replacing P_μ with $p_\mu + gA_\mu$ and expanding in A_μ, one recovers the usual perturbative expansion.

In the background field picture, Weinberg operators are obtained by expanding an amplitude M in powers of G and its covariant derivatives, which is done by expanding the propagators. It is important to realize that once a desired power of G has been obtained, P_μ can be treated as commuting, because any noncommutativity will be of a higher order in G and can be ignored. This fact will be needed to establish the behavior of M under the exchange of quark masses.

Weinberg operators of the standard model are at least a three-loop effect because there must be four W vertices, and thus two W loops in addition to the one quark loop. The relevant diagram is shown in Fig. 1. A second possible diagram, with the W and up quark lines interchanged, is proportional to $|V_{u_1d_1}|^2|V_{u_2d_2}|^2$ and thus cannot contribute to CP-violating effects.[3]

The amplitude which corresponds to Fig. 1 is

$$M^{d_1d_2}_{u_2u_1} = \mathrm{tr}\left\langle x \left| \frac{1}{\not{P}-m_{u_2}} F^{d_1}_{u_2u_1} \frac{1}{\not{P}-m_{u_1}} F^{d_2}_{u_1u_2} \right| x \right\rangle , \quad (3)$$

where the operator F corresponds to the diagram of Fig. 2 and is given (in the unitary gauge) by

$$F^{d_1}_{u_2u_1}(\not{P}) = i\left[\frac{ig}{2\sqrt{2}}\right]^2 \int \frac{d^4l}{(2\pi)^4} \frac{\gamma_\mu L(\not{P}+\not{l}+m_{d_1})\gamma_\nu L}{(l+P)^2+(g/2)\sigma G - m^2_{d_1}} D^{\mu\nu}_W(l) . \quad (4)$$

F can be expanded as[4]

$$F(\not{P}) = \sum_i F_i \mathcal{O}_i \quad (5)$$
$$= \Sigma_0 + F_2(\not{P}^2)\{\not{P}, \sigma G\}L + F_3(\not{P}^2)D^\mu G_{\mu\nu}\gamma^\nu L + O(G^2) . \quad (6)$$

Here F_i are covariant form factors and $\mathcal{O}_i$ are operators. F_i and $\mathcal{O}_i$ are independent of the external masses (quarks); F_i depends on the internal mass as $m^2_{d_1}$, and $\mathcal{O}_i$ has no mass dependence at all. The index i labels the order of the terms in the background field strength.

It is necessary to first establish the properties of F. In the unitary gauge, because the quark-W vertices are $\gamma_\mu L$, it is naively clear that F is odd, purely left handed, and independent of the external masses. However, because this is a nonrenormalizable gauge, the reader might wonder whether this is an artifact of the gauge choice. In a generic gauge, the unphysical charged Higgs boson will also contribute, coupling to quarks with a vertex (omitting the coupling and numerical factors) $V = (m_u R - m_d L)/M_W$. There is also the possibility that

[3]There is of course another diagram with the roles of the up and down quarks reversed. The treatment of that diagram is exactly the same.

[4]A note of caution: The matrix elements of $F(\not{P})$ will of course also depend on G. This will induce corrections higher order in G–there will be, e.g., $O(G^3)$ terms coming from F_2. However, the aim is to make the symmetry properties clear; if it can be shown that terms without at least an *explicit* G dependence at some order vanish, the higher-order corrections will not matter.

renormalization counterterms will depend on the external masses.

To resolve the first issue, rewrite the Higgs vertex as

$$V=[-(\boldsymbol{P}-m_u)R+L(\boldsymbol{P}-m_d)]/M_W .$$

It is then clear that the external mass terms will cancel against the external propagators when inserted into M. The residues of these cancellations will not contribute to the final result because the resulting partial amplitude depends on at most three quark masses and will vanish in the sum. Thus only the $R(\boldsymbol{P}-m_d)$ part of the vertex is relevant. Performing a similar rearrangement for $\overline{V}$, it follows that the only relevant contribution from the Higgs loop has the form $R(\boldsymbol{P}-m_d)(N)(\boldsymbol{P}-m_d)L$, with N depending only on m_d. There will be no external mass dependence and the R,L projectors ensure that the result will be odd and left handed.

There still remains the question of the renormalization counterterms. Note that these will affect only the potentially divergent part of F. Terms in F containing nonrenormalizable operators such as the anticommutator $\{\boldsymbol{P},\sigma G\}$, for which there can be no counterterms, are not affected. Shabalin [12,15] and others [16] have shown that the renormalized Σ takes the form[5]

$$\Sigma_{u_2u_1}=(\boldsymbol{P}-m_{u_2})\overline{\Sigma}_{u_2u_1}(\boldsymbol{P}-m_{u_1}) , \tag{7}$$

with

$$\overline{\Sigma}=F_0^{(1)}(\boldsymbol{P}^2)(\boldsymbol{P}R+m_{u_2}R+m_{u_1}L)+F_0^{(2)}(\boldsymbol{P}^2)\boldsymbol{P}L . \tag{8}$$

Here $F_0^{(1)}$ and $F_0^{(2)}$ are symmetric in the external masses. It follows from these arguments that apart from Σ, all the terms in the expansion of F are left-handed, odd, and independent of the external masses.

Insertion of the expansion (6) for F into Eq. (3) gives the result

$$M_{u_1u_2}^{d_1d_2}=\mathrm{tr}\langle x|\overline{\Sigma}_{u_2u_1}^{d_1}\Sigma_{u_1u_2}^{d_2}|x\rangle+\mathrm{tr}\langle x|\overline{\Sigma}_{u_2u_1}^{d_1}\hat{F}^{d_2}|x\rangle+\mathrm{tr}\langle x|\hat{F}^{d_1}\overline{\Sigma}_{u_1u_2}^{d_2}|x\rangle+\mathrm{tr}\left\langle x\left|\frac{1}{\boldsymbol{P}-m_{u_2}}\hat{F}^{d_1}\frac{1}{\boldsymbol{P}-m_{u_1}}\hat{F}^{d_2}\right|x\right\rangle , \tag{9}$$

with $\hat{F}=F-\Sigma_0$ [so F is $O(G^2)$]. Consider the second and third terms in Eq. (3). Because of the Dirac structure of $\hat{F}$, only the $\boldsymbol{P}R$ part of $\overline{\Sigma}$ will survive the trace. But the coefficient of $\boldsymbol{P}R$ is symmetric in the external masses and $\hat{F}$ is independent of them, so the second and third terms will not contribute to the total amplitude. Finally, it is only a matter of some algebra to show that $\mathrm{tr}(\overline{\Sigma}\Sigma)$ is symmetric in the external masses.

Consequently, only the last term in (9) can contribute to the CP-violating amplitude. The Dirac structure of $\hat{F}$ allows me to write it as

$$\mathrm{tr}\left\langle x\left|\frac{1}{\boldsymbol{P}^2-m_{u_2}^2}\boldsymbol{P}\hat{F}^{d_1}\frac{1}{\boldsymbol{P}^2-m_{u_1}^2}\boldsymbol{P}\hat{F}^{d_2}L\right|x\right\rangle . \tag{10}$$

After substitution of $\hat{F}=\sum_{2\le i}F_i\mathcal{O}_i$ and use of the fact that the F_i commute with the propagators and with each other, this becomes

$$\sum_{2<i,j}\mathrm{tr}\left\langle x\left|F_i^{d_1}F_j^{d_2}\frac{1}{\boldsymbol{P}^2-m_{u_2}^2}\boldsymbol{P}\mathcal{O}_i\frac{1}{\boldsymbol{P}^2-m_{u_1}^2}\boldsymbol{P}\mathcal{O}_jL\right|x\right\rangle . \tag{11}$$

The only dependence on m_{u_1} and m_{u_2} in this expression is in the propagators. In the expansion of the propagators in powers of G, only terms where the two propagators are expanded to different orders will contribute. Consequently, the leading term will be

$$-\sum_{i,j}\mathrm{tr}\left\langle x\left|F_i^{d_1}F_j^{d_2}\left[\frac{1}{P^2-m_{u_2}^2}\frac{g}{2}\sigma G\frac{1}{P^2-m_{u_2}^2}\boldsymbol{P}\mathcal{O}_i\frac{1}{P^2-m_{u_1}^2}\boldsymbol{P}\mathcal{O}_j+\frac{1}{P^2-m_{u_2}^2}\boldsymbol{P}\mathcal{O}_i\frac{1}{P^2-m_{u_1}^2}\frac{g}{2}\sigma G\frac{1}{P^2-m_{u_1}^2}\boldsymbol{P}\mathcal{O}_j\right]\right|x\right\rangle , \tag{12}$$

which is at least of order G^3. To this point, I have only made use of the symmetry under the exchange $m_{u_1}\leftrightarrow m_{u_2}$. But there is also the exchange $m_{d_1}\leftrightarrow m_{d_2}$ to consider. Once a given order in G has been obtained, the $\mathcal{O}_i$ will commute. Thus, antisymmetrization will eliminate terms with identical F_i's. It follows that the lowest-dimension term which survives will be $\sigma G\mathcal{O}_2\mathcal{O}_3$, which is of dimension seven. However, there is no gauge- and Lorentz-invariant dimension-seven operator containing only gluons, so the first term to contribute will in fact be of dimension eight. This completes the proof that Weinberg operators can first arise at dimension eight.

III. CONCLUSION

I have shown that in the absence of QCD corrections, the coefficient C_6 of the dimension-six Weinberg operator vanishes in the KM model. This extends earlier work by Bigi and Uraltsev [3], who argued that the coefficient

[5]Of course, these authors calculated Σ in the conventional way for $A_\mu=0$, but by gauge invariance the substitution $p\to P$ will only produce a correction of $O(G)$, which will be absorbed in the other terms.

should be suppressed. They went on to the estimate the size of four-loop QCD contributions to the coefficient, finding $C_6 \simeq JG_F^2 m_c^2$. Performing a similar estimate for the three-loop coefficients C_8 of the dimension-eight operators, I find $C_8 \simeq JG_F^2 m_c^2 / m_u^2$. Taking m_u to be current mass would be an error because QCD corrections will become important long before such a small scale is reached. For numerical estimates it is better to try to take this into account by choosing m_u to be a constituent mass ≈ 300 MeV.

Finally, it is interesting to compare the contributions of C_6 and C_8 to the NEDM. Using QCD sum rules, Bigi and Uraltsev estimated that C_6 produced a contribution to the NEDM $d_6 \simeq 10^{-34}$ e cm. In order to take advantage of their estimate, I use "naive dimensional analysis" [17] (NDA) to relate the hadronic matrix elements of O_6 and O_8. Although NDA probably does not give a reliable estimate of the actual matrix elements [3,2,18], it appears to be reliable for estimating the *ratios* of the matrix elements [2]. With this caveat, I find $d_8 \simeq (\zeta_8/\zeta_6) m_\chi^2 / m_u^2 d_6$, where $m_\chi = 2\pi F_\pi \approx 1.2$ GeV, and ζ_8 and ζ_6 are QCD evolution factors whose ratio is ≈ 10 [7,2]. Thus, $d_8 \simeq 24\, d_6 \simeq 10^{-33}$ e cm.

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