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Z', new fermions, and flavor-changing processes: Constraints on E_6 models from $\mu \to eee$

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We study a new class of flavor-changing interactions, which can arise in models based on extended gauge groups (rank >4) when new charged fermions are present together with a new neutral gauge boson. We discuss the cases in which the flavor-changing couplings in the new neutral current coupled to the Z' are theoretically expected to be large, implying that the observed suppression of neutral flavor-changing transitions must be provided by heavy Z' masses together with small Z-Z' mixing angles. Concentrating on E_6 models, we show how the tight experimental limit on $\mu \to eee$ implies serious constraints on the Z' mass and mixing angle. We conclude that if the value of the flavor-changing parameters is assumed to lie in a theoretically natural range, in most cases the presence of a Z' much lighter than 1 TeV is unlikely.

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I. INTRODUCTION

The large set of accurate measurements performed during the last few years has established that the standard electroweak theory provides an excellent description of the particle physics phenomena up to about 100 GeV. It should also be stressed that the present theory accommodates in a satisfactory way the whole spectrum of known particles, and only two states, the top quark and the Higgs scalar that are necessary for the consistency of the model, have not been discovered yet. Another feature of the standard model (SM) that explains in a satisfactory way a large set of experimental limits on rare processes, is the Glashow-Iliopoulos-Maiani suppression of flavor-changing neutral currents (FCNC's) in the quark sector, together with the absence of lepton-flavor-violating (LFV) currents. All these features are quite peculiar to the SM and, in general, most of its possible extensions predict a larger spectrum of states as well as larger rates for FCNC processes. Clearly the direct detection of any new unconventional particle would be a major breakthrough towards the identification of a new and more fundamental theory, but at the same time it is not unlikely that some hints of the existence of new physics will come from the observation of rare processes at rates larger than what is expected in the standard theory.

The aim of this paper is to analyze a new class of FCNC interactions that are generally present in most of the extensions of the SM which predict one (or more) additional neutral gauge boson Z_1 together with new charged fermions, and that are induced by Z_1 interactions. In general, the Z_1 is expected to be mixed with the standard Z_0 and, as a consequence, both the resulting mass eigenstates, which we will denote as Z and Z', will have flavor-changing couplings to the known charged fermions.

In particular, we will concentrate on LFV interactions, which are strictly forbidden in the SM, and we will show that they could provide a clear signature for this kind of new physics. In turn, the existing limits on LFV processes imply serious constraints for several interesting models. To stress the power of these constraints we will apply them to a class of E_6 grand unified theories (GUT's), which is a well-known example of theories where additional fermions and new neutral gauge bosons are simultaneously present, and provides a good frame for illustrating the kind of effects that we want to explore. In an attempt to quantify the constraints, we will assume for the relevant E_6 LFV couplings a range of values that we believe is theoretically natural, and we will then turn the existing limits on LFV decays into limits on the Z' parameters. Though model dependent, our bounds turn out to be much tighter than the present limits obtained from direct searches at colliders [1] or as derived from the analysis of other Z' indirect effects [2–4].

The interesting possibility of violating the conservation of the lepton family number as a consequence of Z_1 interactions was already briefly analyzed in Ref. [5] for a general class of extended electroweak models. It was shown that LFV decays of the standard neutral gauge boson such as $Z \to l_i l_j$ ($i \neq j$, hereafter understood) can be induced by the presence of a Z_1 (i) if a sizable mixing with the Z_1 is present, and (ii) if the new neutral gauge bosons do not couple universally to the fermion generations. While the first condition is a natural feature of extended gauge models, it could seem that the second one is somewhat more difficult to realize. However, we will show that both these conditions are naturally satisfied in extended gauge models that, like E_6, also predict new charged fermions. This was already noted in a recent paper [2], where the consequences of the simultaneous presence of new neutral gauge bosons and new fermions were analyzed, and a general formalism for taking into account their combined effects was outlined.

In models such as E_6, where several new neutral leptons are present and a quite general form for the corre-

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sponding mass matrices and mixing patterns is possible, LFV processes such as $Z \rightarrow l_i l_j$ arise naturally due to the contributions of loop diagrams [6]. A $Z_0 l_i l_j$ vertex for the gauge eigenstate Z_0 boson can, however, already appear at the tree level, due to the presence of new exotic charged leptons that could be mixed with the standard ones. A general discussion of these kinds of flavor-changing vertices is given, e.g., in Ref. [7], while the particular case of E_6 models is analyzed in Ref. [8]. In general the Z_0 flavor-changing vertices are strongly suppressed as the ratio between the masses of the known and of the new fermions. The interest of considering flavor-changing Z_1 interactions stems from the fact that in some class of models no suppression factors are expected for the $Z_1 f_i f_j$ vertices. Then, in the context of these models, the new interactions due to the additional gauge boson could well represent the main source of flavor-changing transitions.

In Sec. II we will first review the essential formalism for dealing with gauge boson and fermion mixing effects [2] and we will also discuss the theoretical expectations for the various flavor-changing parameters. In Sec. III we will concentrate on E_6 models. We will confront the theoretical expression for the decay $\mu \rightarrow eee$ with the extremely stringent experimental limit

$$B(\mu^+ \rightarrow e^+ e^+ e^-) < 1 \times 10^{-12}$$

obtained by the SINDRUM Collaboration [9], and we will derive constraints on the mass of the new gauge boson and on the $Z_0 Z_1$ mixing angle as a function of the LFV parameters. Finally, in Sec. IV we will draw our conclusions.

II. Z' AND NEW FERMION EFFECTS. FORMALISM

We will now review the formalism for describing the combined effects due to the presence of a new neutral gauge boson, and of new fermions that could be mixed with the known ones. We will closely follow the presentation given in Ref. [2] concentrating mainly on the charged fermions sector and on flavor-changing effects, and we refer to Ref. [2] for a more general discussion. We will assume a low-energy gauge group of the form $\mathcal{G}_{SM} \times U_1(1)$, where

$$\mathcal{G}_{SM} = SU(2)_L \times U(1)_Y \times SU(3)_C$$

is the usual SM gauge group. Then, in the gauge basis, the corresponding neutral current Lagrangian reads

$$-\mathcal{L}_{NC} = e J^\mu_{em} A_\mu + g_0 J^\mu_0 Z_{0\mu} + g_1 J^\mu_1 Z_{1\mu} \; . \tag{2.1}$$

In (2.1) Z_0 is the SM neutral gauge boson, which couples with a strength

$$g_0 = (4\sqrt{2} G_F M^2_{Z_0})^{1/2} \tag{2.2}$$

to the usual combination of the neutral isospin and electromagnetic currents:

$$J^\mu_0 = J^\mu_3 - s^2_W J^\mu_{em} \tag{2.3}$$

where $s^2_W = \sin^2\theta_W$ is the weak mixing angle. The new Z_1 corresponds to the additional $U_1(1)$ factor, and couples to the new J_1 current with a strength g_1. In particular, if we assume that the $U_1(1)$ originates at low energy from a GUT based on a *simple* group, then the $q_1(f)$ charges of the fermions are fixed by the gauge group. Normalizing the new generator Q_1 to the hypercharge axis $Y/2$ and assuming a similar renormalization-group evolution for the two Abelian couplings g_1 and g_Y down to the electroweak scale, the coupling strength of the new interaction can be written as

$$g_1 \simeq g_0 s_W \; . \tag{2.4}$$

In general, after spontaneous symmetry breaking the Z_0-Z_1 mass matrix turns out to be nondiagonal. The Z and Z' gauge boson mass eigenstates then correspond to two orthogonal combinations of Z_0 and Z_1 that we will parametrize in term of an angle ϕ. As a result, the currents that couple with strength g_0 to the physical Z and Z' are

$$\begin{bmatrix} J^\mu_Z \\ J^\mu_{Z'} \end{bmatrix} = \begin{bmatrix} c_\phi & s_\phi \\ -s_\phi & c_\phi \end{bmatrix} \begin{bmatrix} J^\mu_0 \\ s_W J^\mu_1 \end{bmatrix} . \tag{2.5}$$

We see that the main effects of the presence of the new gauge boson are the additional contribution to NC amplitudes, described by the third term in the Lagrangian (2.1), and the mixing between the J_0 and J_1 currents in (2.5). We will not discuss additional indirect effects, as, e.g., the shifts induced by Z_0-Z_1 mixing in the value of the weak angle s_W and in the overall coupling strength g_0 when expressed as a function of the Z mass [2–4], since they are irrelevant for the present analysis.

We now discuss the form of the two currents J_0 and J_1 and, in particular, the effects that could originate from the presence of additional charged fermions. We will henceforth denote as *new,* possible degrees of freedom in addition to the standard 15 *known* fermions per generation. Since no new fermions have been directly observed yet, the new states must be rather heavy, and $m_{new} \gtrsim M_Z/2$ can be taken as the present model-independent limit on the new masses. Accordingly, we will denote the corresponding mass eigenstates as *heavy,* while the known mass eigenstates will be labeled as *light.* In the presence of additional fermions, the light mass eigenstates will correspond to superpositions of the known and new states. Conservation of the electric and color charges forbids a mixing between gauge eigenstates with different $U(1)_{em}$ and $SU(3)_c$ quantum numbers, and, in turn, this implies that the electromagnetic and color currents of the light mass eigenstates are not modified in the presence of the new states. However, the neutral isospin generator T_3 and the new generator Q_1 are spontaneously broken, then a mixing between gauge eigenstates with different t_3 and q_1 eigenvalues is allowed, and as a result the couplings of the light mass eigenstates to the Z_0 and Z_1 will be affected.

Since in the gauge currents chirality is also conserved, it is convenient to group the fermions with the same electric charge and chirality $\alpha = L, R$ in a column vector of the known ($\mathcal{K}$) and new ($\mathcal{N}$) gauge eigenstates $\Psi_\alpha^0=(\Psi_{\mathcal{K}}^0,\Psi_{\mathcal{N}}^0)_\alpha^T$. The relation between the gauge eigenstates Ψ_α^0 and the corresponding light and heavy mass eigenstates $\Psi_\alpha=(\Psi_l,\Psi_h)_\alpha^T$ is then given by a unitary transformation

$$\begin{bmatrix}\Psi_{\mathcal{K}}^0\\ \Psi_{\mathcal{N}}^0\end{bmatrix}_\alpha = U_\alpha \begin{bmatrix}\Psi_l\\ \Psi_h\end{bmatrix}_\alpha ,$$

where (2.6)

$$U_\alpha=\begin{bmatrix}A & G\\ F & H\end{bmatrix}_\alpha , \quad \alpha=L,R .$$

The submatrices A and F describe the overlap of the light eigenstates with the known and the new states, respectively, and from the unitarity of U we have

$$A^\dagger A+F^\dagger F=AA^\dagger+GG^\dagger=I . \tag{2.7}$$

Note that we have not introduced an extra index to label the electric charge; nevertheless we will treat Ψ_α^0 and Ψ_α as vectors corresponding to a definite value of $q_{\rm em}$.

In terms of the fermion mass eigenstates the neutral current corresponding to a (broken) generator $\mathcal{Q}$ reads

$$J_{\mathcal{Q}}^\mu=\sum_{\alpha=L,R}\overline{\Psi}_\alpha\gamma^\mu U_\alpha^\dagger \mathcal{Q}_\alpha U_\alpha \Psi_\alpha , \tag{2.8}$$

where $\mathcal{Q}_\alpha$ represents a generic diagonal matrix of the charges for the chiral fermions. Here we have to consider only the mixing effects in J_3 appearing in (2.3) and in J_1, since the term proportional to $J_{\rm em}$ in J_0 is not modified by fermion mixing. Hence in (2.8) $\mathcal{Q}=T_3$, Q_1 and the elements of the corresponding matrices are given by the eigenvalues t_3 and q_1.

From (2.8) we see that if in one subspace of states with equal electric charge and chirality the matrix $\mathcal{Q}_\alpha$ is proportional to the identity, then $U_\alpha^\dagger \mathcal{Q}_\alpha U_\alpha=\mathcal{Q}_\alpha$ and for these fermions the corresponding current is not modified in going to the base of the mass eigenstates. In the SM for example, for a given electric charge and chirality the t_3 eigenvalues of the fermions are indeed the same, and this implies in particular the absence (at the tree level) of FCNC's. In models with new fermions in contrast, the diagonal matrices $\mathcal{Q}_\alpha$ have the general form $\mathcal{Q}_\alpha={\rm diag}(\mathcal{Q}_\alpha^{\mathcal{K}},\mathcal{Q}_\alpha^{\mathcal{N}})$ and do not commute with U.

To put in evidence the indirect effects of fermion mixings in the couplings of the light mass eigenstates, we now project (2.8) on the light components Ψ_l, obtaining

$$J_{l\mathcal{Q}}^\mu=\sum_{\alpha=L,R}\overline{\Psi}_{l\alpha}\gamma^\mu(A_\alpha^\dagger \mathcal{Q}_\alpha^{\mathcal{K}} A_\alpha+F_\alpha^\dagger \mathcal{Q}_\alpha^{\mathcal{N}} F_\alpha)\Psi_{l\alpha} . \tag{2.9}$$

This equation is quite general, and describes the effects of fermion mixings in the neutral currents of light states for a wide class of models. If the gauge group is generation independent, all the known states appearing in one vector Ψ_α^0 have the same eigenvalues with respect to the generators of the gauge symmetry, and hence we have $\mathcal{Q}_\alpha^{\mathcal{K}}=q_\alpha^{\mathcal{K}}I$ with $q_\alpha^{\mathcal{K}}=t_3(f_\alpha^{\mathcal{K}}),q_1(f_\alpha^{\mathcal{K}})$. The same is not true, in general, for the new states; however, if we consider the particular case when the mixing is with only one type of new fermions with the same $q_\alpha^{\mathcal{N}}$ charges, then we have $\mathcal{Q}_\alpha^{\mathcal{N}}=q_\alpha^{\mathcal{N}}I$ as well. Under these conditions, and by means of the unitarity relation (2.7), (2.9) reduces to the simple form

$$J_{l\mathcal{Q}}^\mu=\sum_{\alpha=L,R}\overline{\Psi}_{l\alpha}\gamma^\mu[q_\alpha^{\mathcal{K}}I+(q_\alpha^{\mathcal{N}}-q_\alpha^{\mathcal{K}})F_\alpha^\dagger F_\alpha]\Psi_{l\alpha} . \tag{2.10}$$

We will restrict ourselves to this equation, which is general enough for describing the mixing with the additional charged fermions in E_6, and we refer to Ref. [2] for a discussion of more general cases.

A few consideration are now in order. In (2.10) the first term $q_\alpha^{\mathcal{K}}I$ inside the square brackets gives the couplings of a particular light fermion in the absence of mixing effects. The second term represents the modifications due to fermion mixings. The matrix $F^\dagger F$ appearing in this term is, in general, not diagonal, and while the magnitude of the diagonal elements will affect the strength of the flavor diagonal couplings of the mass eigenstates, the off-diagonal terms will induce FCNC's. Clearly whenever the coefficient $(q_\alpha^{\mathcal{N}}-q_\alpha^{\mathcal{K}})$ vanishes, the mixing effects are absent. When $t_3(f_\alpha^{\mathcal{N}})\neq t_3(f_\alpha^{\mathcal{K}})$ the J_0 current is modified, and then the existing low-energy and on-resonance NC data as well as the current limits on rare processes can be used to constrain directly the corresponding elements of $F^\dagger F$. Model-independent limits on the diagonal elements $(F^\dagger F)_{ii}$ affecting Z_0 interactions were first given in Ref. [7] and subsequently updated in Ref. [10], and the most recent limits in the frame of E_6 models can be found in Ref. [2]. Bounds on the off-diagonal terms $(F^\dagger F)_{i\neq j}$ have been given in Ref. [7] as well. All these limits turn out to be very stringent, usually at the level of 1% or better. In contrast, if $t_3(f_\alpha^{\mathcal{N}})=t_3(f_\alpha^{\mathcal{K}})$ the J_0 current is not modified, but since, in general, we still have $q_1(f_\alpha^{\mathcal{N}})\neq q_1(f_\alpha^{\mathcal{K}})$, sizable effects could indeed be present in J_1. We stress that the present experimental data cannot be used to set limits on these mixings, since they only affect a new hypothetical interaction, and we can only rely on theoretical speculations to estimate their magnitude.

According to these considerations it is clear that from a phenomenological point of view it is convenient to classify possible new fermions in terms of their transformation properties under $SU(2)_L$. Since we are only interested in fermions with conventional electric charges, the new states must be singlets or doublets of weak isospin. A rather heterodox exception is that of a gauge triplet of fermions [11], but we will not consider this possibility here. According to the nomenclature in use [2,7,10], we denote the particles with unconventional isospin assignments (left-handed singlets or right-handed doublets) as *exotic* fermions. All the standard fermions, as well as all the new states that have conventional $SU(2)_L$ assignments, are referred to as *ordinary*. For example, mirror fermions, having opposite $SU(2)_L$ assignments from those of the known fermions, are exotic. Sequential fermions are simply repetition of the new fermions. They could be present in a complete new family or as components of large fermion representations and are clearly classified as ordinary. In this paper we will mainly concentrate on vector multiplets of new fermions for which the L and R

components have the same $SU(2)_L$ transformation properties, and hence always contain both ordinary and exotic states. From the previous discussion, we see that while ordinary-exotic fermion mixings are tightly constrained due to the effects induced in the J_0 current, no limits can be given for the ordinary-ordinary mixings, since they affect only J_1.

This classification is also very convenient for discussing the possible form of the fermion mass matrices and the expected magnitude of the mixings between the known and the new fermions. To give an example, let us introduce for each fermion family a vector gauge singlet of new fermions $(X^0_{EL},X^0_{OR})_i$ (E=exotic, O=ordinary, $i=1,2,3$) with the same electric and color charges than the known fermions $(f^0_{OL},f^0_{OR})_i$. Then in the gauge eigenstate basis the mass term reads

$$\mathcal{L}_{\rm mass}=(\bar f^0_O,\bar X^0_E)_L\mathcal{M}\begin{bmatrix} f^0_O \\ X^0_O\end{bmatrix}_R+\text{H.c.}\ , \tag{2.11}$$

where, e.g., $f^0=(f^0_1,f^0_2,f^0_3)^T$, etc. The nondiagonal mass matrix $\mathcal{M}$ takes the form

$$\mathcal{M}=\begin{bmatrix} D & D' \\ S' & S\end{bmatrix}\ , \tag{2.12}$$

where D and D' are 3×3 matrices generated by vacuum expectation values (VEV's) of doublets multiplied by Yukawa couplings, while S and S' are generated by VEV's of singlets. As a general rule, while the mass terms which couple ordinary L fermions to ordinary R fermions (or exotic L fermions to exotic R fermions) arise from VEV's of Higgs doublets, the entries which couple ordinary fermions to the exotic ones are generated by VEV's of singlets. Then, in general, Higgs singlets are responsible for the large masses of new heavy fermions in vector multiplets and, in most cases, also contribute to the mass of the new heavy gauge boson; hence it is natural to assume $S,S'\sim\Lambda\gg D,D'$.

The diagonal mass matrix M is obtained via a biunitary transformation acting on the L and R sectors:

$$\begin{aligned} M^2&=U_L^{O\text{-}E}(\mathcal{M}\mathcal{M}^\dagger)U_L^{O\text{-}E\dagger} \\ &=U_R^{O\text{-}O}(\mathcal{M}^\dagger\mathcal{M})U_R^{O\text{-}O\dagger}\ . \end{aligned} \tag{2.13}$$

Since $D/\Lambda,D'/\Lambda\sim\varepsilon\ll 1$, the order of magnitude of the different entries in $\mathcal{M}\mathcal{M}^\dagger$ and $\mathcal{M}^\dagger\mathcal{M}$ is

$$\mathcal{M}\mathcal{M}^\dagger\sim\Lambda^2\begin{bmatrix}\varepsilon^2 & \varepsilon \\ \varepsilon & 1\end{bmatrix}\ , \tag{2.14}$$

$$\mathcal{M}^\dagger\mathcal{M}\sim\begin{bmatrix}\Lambda^2 & \Lambda^2 \\ \Lambda^2 & \Lambda^2\end{bmatrix}\ . \tag{2.15}$$

Given the form of $\mathcal{M}\mathcal{M}^\dagger$ in (2.14) and keeping in mind the expression (2.6) for the matrices U, we see that for the matrix describing the ordinary-exotic mixings in (2.13) it is natural to expect that the submatrices F and G would acquire an overall suppression factor ε, of the order of the ratio of the light to heavy mass scale. In contrast, since all the entries in (2.15) are of the same order of magnitude, such a suppression is not present for the ordinary-ordinary F and G mixing terms. Now, since it is precisely $F^\dagger F$ in (2.10) which affects the flavor diagonal couplings and also induces FCNC's, the suppression of the ordinary-exotic mixings explains in a natural way the nonobservations of these effects in the Z_0 interactions. On the other hand, for the ordinary-ordinary mixings there is no reason to expect the elements of $F^\dagger F$ to be particularly small and, accordingly, flavor-changing processes can be expected to occur at a sizable rate in Z_1 interactions.

Written explicitly, the flavor diagonal chiral couplings of a light f fermion to the Z_0 and Z_1 gauge bosons are

$$\begin{aligned} \varepsilon^f_{0\alpha}&=t_3(f_\alpha)-s^2_W q_{\rm em}(f) \\ &\quad+[t_3(f^{\mathcal{N}}_\alpha)-t_3(f^{\kappa}_\alpha)](F^\dagger_\alpha F_\alpha)_{ff}\ , \end{aligned} \tag{2.16}$$

$$\varepsilon^f_{1\alpha}=q_1(f_\alpha)+[q_1(f^{\mathcal{N}}_\alpha)-q_1(f^{\mathcal{K}}_\alpha)](F^\dagger_\alpha F_\alpha)_{ff},\quad \alpha=L,R\ .$$

while the $f_i f_j$ flavor-changing couplings read

$$\begin{aligned} \kappa^{ij}_{0\alpha}&=[t_3(f^{\mathcal{N}}_\alpha)-t_3(f^{\mathcal{K}}_\alpha)](F^\dagger_\alpha F_\alpha)_{ij}\ , \\ \kappa^{ij}_{1\alpha}&=[q_1(f^{\mathcal{N}}_\alpha)-q_1(f^{\mathcal{K}}_\alpha)](F^\dagger_\alpha F_\alpha)_{ij},\quad \alpha=L,R\ . \end{aligned} \tag{2.17}$$

The corresponding couplings to the physical Z and Z' bosons that we will denote as ε^f_α, ε'^f_α and $\kappa^{ij}_\alpha,\kappa'^{ij}_\alpha$ can be readily obtained via the transformation (2.5). For the flavor-changing couplings we have, for example,

$$\begin{aligned} \kappa^{ij}_\alpha&=c_\phi\kappa^{ij}_{0\alpha}+s_\phi s_w\kappa^{ij}_{1\alpha}\ , \\ \kappa'^{ij}_\alpha&=-s_\phi\kappa^{ij}_{0\alpha}+c_\phi s_w\kappa^{ij}_{1\alpha},\quad \alpha=L,R\ , \end{aligned} \tag{2.18}$$

and analogous expressions hold for ε^f_α and ε'^f_α too. In terms of the flavor nondiagonal couplings (2.18), the FCNC Lagrangian for the light f^i and f^j fermions in the mass eigenstate basis finally reads

$$-\mathcal{L}^{ij}_{\rm FC}=g_0\sum_{\alpha+L,R}(\bar f^i_\alpha\gamma^\mu\kappa^{ij}_\alpha f^j_\alpha Z_\mu+\bar f^i_\alpha\gamma^\mu\kappa'^{ij}_\alpha f^j_\alpha Z'_\mu)\ . \tag{2.19}$$

From the first equation in (2.18) we see that even in the case when only ordinary-ordinary mixing effects are present, and hence $\kappa^{ij}_{0\alpha}=0$, the Z boson can still mediate flavor-changing transitions, suppressed now by a factor proportional to the Z_0-Z_1 mixing [5]. However, for several models the existing limits on ϕ are rather stringent: $|\phi|\lesssim 0.02$ [2–4] and then we can expect that if the Z' is not too heavy, FCNC processes would be mainly induced by Z' exchange.

III. CONSTRAINTS ON E_6 MODELS FROM $\mu\to eee$

In the previous section we have shown that in the presence of new charged fermions the new neutral current J_1 could induce sizable flavor-changing transitions either via Z' interactions, or as a consequence of a nonvanishing Z_0-Z_1 mixing. This kind of new physics would manifest itself in affecting the rates for several processes which are forbidden or highly suppressed in the SM. In the quark sector it could enhance the branchings for the leptonic decays of mesons such as K^0, D^0, $B^0 \rightarrow l^+ l^-$, and it would also affect the neutral meson mixings and mass differences. In the lepton sector it would induce several LFV neutrinoless τ decay modes such as $\tau \rightarrow eee$, $\mu\mu\mu$, μee, $e\mu\mu$, $\mu\pi$, $\mu\rho$ which are all constrained at the level of $\lesssim few \times 10^{-5}$ [12], but, in particular, it would also give rise to the decay $\mu \rightarrow eee$ for which the existing limit [9] is much more stringent:

$$B(\mu^+ \rightarrow e^+ e^+ e^-) < 1.10 \times 10^{-12} \quad (\text{at } 90\% \text{ C.L.}) . \tag{3.1}$$

Given the LFV Lagrangian (2.19), the expression for this decay rate relative to the charged current decay $\mu \rightarrow \nu e \bar{\nu}$ is

$$\frac{B(\mu \rightarrow eee)}{B(\mu \rightarrow \nu e \bar{\nu})} = 2[3(\varepsilon_R^2 + \varepsilon_L^2)(\kappa_R^2 + \kappa_L^2) + (\varepsilon_R^2 - \varepsilon_L^2)(\kappa_R^2 - \kappa_L^2)] + 2\left[\frac{M_Z^2}{M_{Z'}^2}\right]^2 [3(\varepsilon_R'^2 + \varepsilon_L'^2)(\kappa_R'^2 + \kappa_L'^2) + (\varepsilon_R'^2 - \varepsilon_L'^2)(\kappa_R'^2 - \kappa_L'^2)]$$
$$+ 4\frac{M_Z^2}{M_{Z'}^2}[3(\varepsilon_R \varepsilon_R' + \varepsilon_L \varepsilon_L')(\kappa_R \kappa_R' + \kappa_L \kappa_L') + (\varepsilon_R \varepsilon_R' - \varepsilon_L \varepsilon_L')(\kappa_R \kappa_R' - \kappa_L \kappa_L')] , \tag{3.2}$$

where for $\kappa_{R,L}$ and $\kappa'_{R,L}$ defined in (2.18) we have dropped the indices $i = e$ and $j = \mu$, and $\varepsilon_{R,L}$ and $\varepsilon'_{R,L}$ refer to the electron couplings.

As a first result, by confronting (3.1) and (3.2) we can derive the limits on the $Z\mu e$ LFV vertices. Assuming that the Z' is completely decoupled from the low-energy physics ($M_{Z'} \rightarrow \infty$ and $\phi \rightarrow 0$) and taking $s_W^2 = 0.23$, we obtain

$$|\kappa_L^{e\mu}| < 1.1 \times 10^{-6} , \qquad |\kappa_R^{e\mu}| < 1.2 \times 10^{-6} . \tag{3.3}$$

As we have discussed, if these couplings do originate from some kind of mixing of the electron and muon with heavy exotic leptons, we expect them to be strongly suppressed, e.g., by a factor of the order $m_\mu^2 / M_Z^2 \sim 10^{-6}$ or smaller, and then we see that the existence of FCNC's induced by ordinary-exotic mixings does not conflict with the stringent limits in (3.3).

Now, in order to study the effects of the Z' flavor-changing vertices, we first need to fix the q_1 charges of the fermions, which in turn determine the coefficient of the flavor-changing mixings. This can be done by choosing a specific GUT, and we will carry out our analysis in the frame of E_6. Since E_6 has rank 6, while the SM gauge group $\mathcal{G}_{SM}$ has rank 4, the breaking of E_6 to the SM will lead to extra Z's. We will consider the possibility that either E_6 breaks directly to rank 5, or that one of the two extra Z''s is heavy enough so that its effects on the low-energy physics are negligible, and in these cases the formalism developed in the previous section can be straightforwardly applied. We will choose the embedding of $\mathcal{G}_{SM}$ into E_6 trough the maximal subalgebra chain

$$E_6 \rightarrow U(1)_\psi \times SO(10) \rightarrow U(1)_\chi \times SU(5) \rightarrow \mathcal{G}_{SM} ,$$

then an effective extra $U_1(1)$ could arise at low energy as a combination of the $U(1)_\psi$ and $U(1)_\chi$ factors. We will parametrize this combination in terms of an angle β, and this will define an entire class of Z' models in which each fermion f is coupled to the new boson through the effective charge

$$q_1(f) = q_\psi(f) \sin\beta + q_\chi(f) \cos\beta . \tag{3.4}$$

Particular cases that are commonly studied in the literature [2–4,13] correspond to $\sin\beta = -(\frac{5}{8})^{1/2}$, 0, 1 and are respectively denoted Z_η, Z_χ, and Z_ψ models. Z_ψ occurs in $E_6 \rightarrow SO(10)$, while Z_η occurs in superstring models when E_6 directly breaks down to rank 5. As we will see this model plays a peculiar role in the present analysis, since it completely evades the kind of constraints that we are investigating. Finally, a Z_χ boson occurs in $SO(10) \rightarrow SU(5)$ and couples to the conventional fermions in the same way as the Z' present in SO(10) GUT's; however, since SO(10) does not contain additional charged fermions, the kind of flavor-changing effects that we are studying here is absent. In contrast, new charged quarks and leptons are present in E_6. In the GUT's based on this gauge group the fermions are assigned to the fundamental **27** representation that contains, beyond the standard 15 degrees of freedom, 12 additional states for each generation, among which we have a vector doublet of new leptons $(NE^-)_L^T, (E^+ N^c)_L^T$ on which we will now concentrate.

The chiral couplings of the leptons to the Z_1 and the coefficient of the LFV term, are determined by the q_ψ and q_χ charges of the new and known states, which are

$$\begin{aligned} q_\psi(E_L) &= -q_\psi(E_R) = -\tfrac{1}{3}(\tfrac{5}{2})^{1/2} , \\ q_\chi(E_L) &= q_\chi(E_R) = -\tfrac{1}{3}(\tfrac{3}{2})^{1/2} , \\ q_\psi(e_L) &= -q_\psi(e_R) = \tfrac{1}{6}(\tfrac{5}{2})^{1/2} , \\ q_\chi(e_L) &= 3q_\chi(e_R) = \tfrac{1}{2}(\tfrac{3}{2})^{1/2} . \end{aligned} \tag{3.5}$$

With respect to the $SU(2)_L$ transformation properties, the E_L^+ heavy leptons are exotic, and then the mixing of their CP conjugate states E_R^- with e_R, μ_R, and τ_R are constrained by Z_0 interactions. From (3.3) we have, for example

$$(F_R^\dagger F_R)_{e\mu} < 2.4 \times 10^{-6} , \tag{3.6}$$

while the 90% C.L. limits on the flavor diagonal mixings given in Ref. [2] are, respectively,

$$(F_R^\dagger F_R)_{ee} < 1.3 \times 10^{-2} , \tag{3.7}$$

$$(F_R^\dagger F_R)_{\mu\mu} < 1.1 \times 10^{-2} . \tag{3.8}$$

Because of the tight bound (3.6) it is reasonable to neglect the LFV couplings in the R sector and (conservatively) set $\kappa_R^{e\mu}=\kappa_R'^{e\mu}=0$ in (3.2). According to (3.7), it is also justified to neglect the effects of the fermion mixings in the flavor diagonal couplings of the R electrons.

In contrast, the E_L^- leptons are ordinary, and no bounds exist on their mixing with the light leptons. We will still neglect the diagonal term $(F_L^\dagger F_L)_{ee}$ since it is reasonable to expect that if this term were so large as to spoil the approximation $\varepsilon_{1\alpha}^f \simeq q_1(f_\alpha)$ the value of the off-diagonal term $(F_L^\dagger F_L)_{e\mu}$ would also be large, possibly leading to even stronger limits then the ones derived here.

Because of the approximations made, for each value of the parameter β in (3.4) the branching ratio (3.2) depends of the values of M_Z', ϕ, and $\mathcal{F}_{e\mu}\equiv(F_L^\dagger F_L)_{e\mu}$. However, it is easy to see that since the gauge boson mixing effects in the diagonal electron couplings are in any case very small, being $|\phi|\lesssim 0.02$ [2–4], the relevant variables are actually only two, namely, $\mathcal{F}_{e\mu}(M_Z^2/M_{Z'}^2)$ and $\mathcal{F}_{e\mu}\phi$. Moreover, once the Higgs sector of the model is specified, $M_{Z'}$ and ϕ are no longer independent quantities. For example, an approximate relation that holds for small mixings and when $M_{Z'}(\gg M_Z)$ originates from a large Higgs singlet VEV [3] reads

$$\phi \simeq -\frac{M_Z^2}{M_{Z'}^2} s_W \frac{\sum_i t_3^i q_1^i |\langle \phi^i \rangle|^2}{\sum_i t_3^{i^2} |\langle \phi^i \rangle|^2} , \tag{3.9}$$

and in this case the branching ratio (3.2) is, in practice, only a function of $\mathcal{F}_{e\mu}(M_Z^2/M_{Z'}^2)$.

As in the SM for the Cabibbo-Kobayashi-Maskawa (CKM) matrix, in E_6 we also do not have a clue for predicting the values of the fermion mixing parameters. Without attempting to push too far an analogy between the mixings we are interested in and the CKM matrix, we will merely note that both these cases involve mixings among ordinary fermions, and that we do not expect in the present case any additional suppression factor. We also note that all the CKM matrix elements are $>10^{-3}$ and that, in particular, the mixing between the first and the second generation is rather large. We will then assume that the LFV term $\mathcal{F}_{e\mu}$ lies in the range 10^{-2}–10^{-4}. Under this assumption the presence of a too light Z' as well as a too large amount of Z_0-Z_1 mixing will clearly conflict with the limit (3.1). The bounds that can be derived in this way are indeed very strong, but obviously they cannot replace the direct [1] or indirect [2–4] limits on the Z' parameters, since for very small values of the LFV Z' couplings ($\mathcal{F}_{e\mu}\lesssim 10^{-6}$) they would in fact be weaker. We will nevertheless present our constraints in the form of numerical limits on $M_{Z'}$ and ϕ, since, in doing so, the strength of the arguments that have been discussed here is put in clear evidence.

Our results are collected in Figs. 1 and 2. Figure 1 shows the bounds on M_Z', the thick solid line depicts the bounds obtained for $\mathcal{F}_{e\mu}=10^{-2}$ and by setting the gauge boson mixing angle ϕ to zero. The decay $\mu\rightarrow eee$ is due only to Z' exchange in this case. The limits for different values of $\mathcal{F}_{e\mu}$ can be read off this line as well, by assuming for the vertical axis units of GeV $[100\mathcal{F}_{e\mu}]^{1/2}$. The thick dashed line, drawn here for convenience, shows the bounds corresponding to $\phi=0$ and $\mathcal{F}_{e\mu}=10^{-3}$, vertical

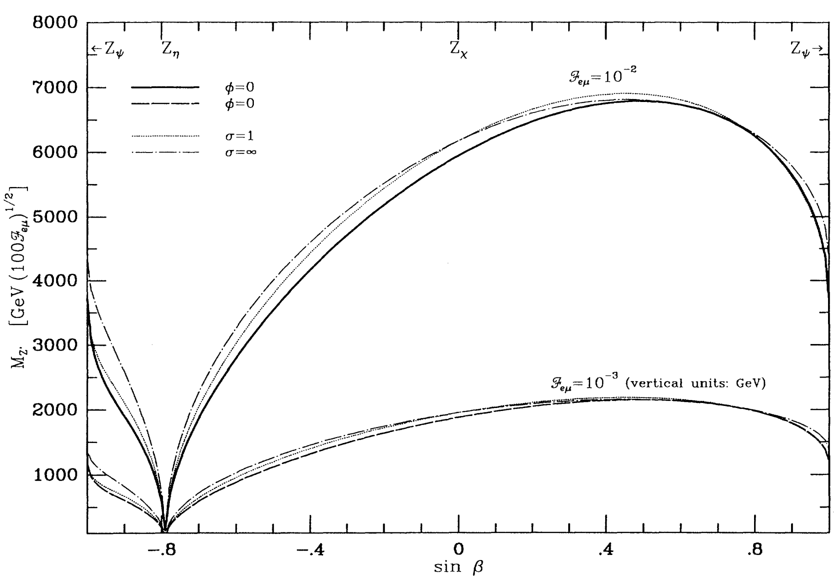

FIG. 1. Limits on $M_{Z'}$ for a general E_6 neutral gauge boson, as a function of sinβ and for different values of the lepton-flavor-violating term $\mathcal{F}_{e\mu}\equiv(F_L^\dagger F_L)_{e\mu}$. The thick solid line is obtained by setting the Z_0-Z_1 mixing angle ϕ to zero, and assuming $\mathcal{F}_{e\mu}=10^{-2}$. The limits for different values of $\mathcal{F}_{e\mu}$ can be read off this line by assuming for the vertical axis units of GeV$(100\mathcal{F}_{e\mu})^{1/2}$. The thick dashed line depicts the limits corresponding to $\mathcal{F}_{e\mu}=10^{-3}$ (vertical units in GeV). The bounds obtained by allowing for a nonvanishing Z_0-Z_1 mixing, consistent with the values of M_Z' when a minimal Higgs sector is assumed, are also shown. The dotted lines correspond to equal VEV's of the two Higgs doublets present in the model, i.e., $\sigma\equiv\bar{v}/v=1$ while the dot-dashed lines correspond to $\sigma=\infty$.

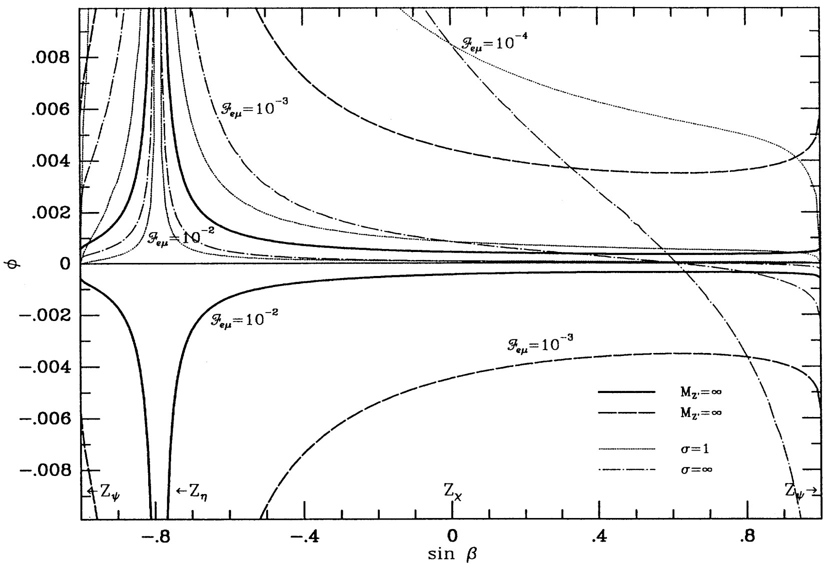

FIG. 2. Limits on the Z_0-Z_1 mixing angle ϕ for a general Z_1 from E_6, as a function of sinβ and for different values of the lepton-flavor-violating term $\mathcal{F}_{e\mu}\equiv(F_L^\dagger F_L)_{e\mu}$. The thick solid and dashed lines are obtained in the limit $M_{Z'}\to\infty$ assuming $\mathcal{F}_{e\mu}=10^{-2}$ and 10^{-3}, respectively. Limits for different values of $\mathcal{F}_{e\mu}$ can be obtained from these lines by rescaling the vertical units by $(10^2\mathcal{F}_{e\mu})^{-1}$ and $(10^3\mathcal{F}_{e\mu})^{-1}$, respectively. The dotted ($\sigma=1$) and dot-dashed ($\sigma=\infty$) lines show the limits obtained for a finite Z' mass and assuming a minimal Higgs sector. In this case the bounds are tighter and are essentially determined by the corresponding limits on $M_{Z'}$ through Eq. (3.9). Also the limits corresponding to $\mathcal{F}_{e\mu}=10^{-4}$ are shown in this case.

units are again in GeV in this case. We see that for $\mathcal{F}_{e\mu}>10^{-3}$ a Z' below 1 TeV would be excluded for most of the values of β. Also, it is clear that $\mathcal{F}_{e\mu}\simeq 10^{-4}$ still leads to significant bounds, being $M_{Z'}$ constrained to values $\gtrsim 400$ GeV for a large part of the sinβ axis.

To study the possible effects on these results of a nonvanishing mixing angle ϕ, i.e., when both the Z' and Z bosons contribute to the decay, we have used (3.9) assuming two doublets of Higgs fields h_{N^c} and h_N with VEV's $\bar{v}$ and v. Since $\bar{v}$ and v give mass respectively to the t and b quarks, $\sigma\equiv\bar{v}^2/v^2>1$ is theoretically preferred. The bounds on $M_{Z'}$ obtained by allowing for a Z_0-Z_1 mixing consistent with this minimal Higgs sector are shown Fig. 1 by the dotted and dot-dashed lines, which correspond to $\sigma=1$ and ∞, respectively. It is apparent that by allowing for a nonvanishing value of ϕ, the limits on the Z' mass are only slightly affected.

Figure 2 depicts the constraints on the Z_0-Z_1 mixing angle ϕ. The solid line shows the bounds obtained by assuming $\mathcal{F}_{e\mu}=10^{-2}$ and taking the limit $M_{Z'}\to\infty$. In this case the decay $\mu\to eee$ is mediated only by the Z boson, and is due to the mixing between the Z_0 and the Z_1. The dotted line shows the bounds for $\mathcal{F}_{e\mu}=10^{-3}$ in the same limit. The limits for different choices of $\mathcal{F}_{e\mu}$ are easily obtained from the solid [dashed] lines by rescaling the vertical units by $(10^2\mathcal{F}_{e\mu})^{-1}$ $[(10^3\mathcal{F}_{e\mu})^{-1}]$.

The dotted ($\sigma=1$) and dot-dashed lines ($\sigma=\infty$) enclose the regions of the limits obtained assuming a minimal Higgs sector. In this case the value of $M_{Z'}$ is finite and consistent, according to (3.9), with the values of ϕ at the bound. We see that with this additional constraint significant limits are found for $\mathcal{F}_{e\mu}=10^{-4}$ as well.

From Fig. 2 it is apparent that for a minimal Higgs sector the limits on ϕ are significantly tighter than in the $M_{Z'}\to\infty$ limit, showing that (3.1) in first place gives direct constraints on the Z' mass, while the bounds on the Z_0-Z_1 mixing obtained independently of $M_{Z'}$ are weaker. We note that this behavior is opposite to what is encountered in deriving limits on the Z' parameters from precise electroweak data [2–4], where in fact the best bounds on the Z' mass are obtained from the tight limits on ϕ implied by the measurements at the CERN Large Electron-Positron (LEP) Collider.

From Figs. 1 and 2 it is apparent that for the η model, corresponding to sin$\beta=-(\frac{5}{8})^{1/2}$, both the Z' mass and the Z_0-Z_1 mixing angle are not constrained by the present analysis. This is due to the fact that in this model, for the ordinary-ordinary fermion mixings, in addition to $t_3^{\mathcal{H}}=t_3^{\mathcal{N}}$ we also have $q_\eta^{\mathcal{H}}=q_\eta^{\mathcal{N}}$, implying that both the coefficients of the $F_L^\dagger F_L$ term in the J_3 and in the J_1 currents vanish. This also happens in the quark sector and in the neutral sector; hence, the unsuppressed flavor-changing vertices are completely absent for the Z_η boson, and this ensures that no other processes can be found for implementing these kinds of constraints for the η model. The reason for this can be understood by considering the decomposition

$$E_6\to SU(6)\times SU(2)_I\ ,$$

where SU(6) contains the SM group, while the $SU(2)_I$ is "inert" in the sense that I_{3I} does not contribute to the $Q_{\rm em}$ generator [14,15]. I_{3I} corresponds to $\beta=\arctan(\frac{3}{5})^{1/2}$ in (3.4) and is orthogonal to $Q_\eta(\beta=\arctan[-(\frac{5}{3})^{1/2}])$ which is then contained in the SU(6) factor as well. The fermions in the **27** of E_6 with the same SM quantum numbers (q_{em}, t_3, color) form multiplets (singlets and doublets) of $SU(2)_I$ and clearly these multiplets also carry definite values of the Q_η charge. All the ordinary fermions with the same color and electric charges, being members of the same $SU(2)_I$ multiplet, also have the same q_η, and this implies the absence of both the diagonal and the flavor-changing ordinary-ordinary mixing effects. In contrast, ordinary-exotic fermion mixing could still give rise to FCNC's in the η model, but as already discussed the corresponding transitions are expected to be largely suppressed, and do not imply any useful constraint.

According to this discussion, if an additional Z' with a mass of a few hundreds GeV is found together with new fermions that could fit in the **27** of E_6, the observed absence of unsuppressed FCNC's would suggest that it could most probably be a Z_η.

IV. CONCLUSIONS

We have carried out an analysis of models that predict a new neutral gauge boson and new charged fermions from the point of view of FCNC processes. We have argued that in most of these models unsuppressed flavor-changing couplings of the light fermions to the new Z' can be present as a consequence of a mixing between the known and the new charged fermions. By assuming that these flavor-changing vertices should not be unnaturally small, we have inferred that the observed suppression of FCNC processes can still be explained in a natural way if the new gauge boson is sufficiently heavy and almost unmixed with the standard Z. Also we have attempted a semiquantitative analysis of this kind of new physics in the frame of E_6 models, by confronting the theoretical expectations for the LFV effects with the extremely stringent limits on the $\mu\to eee$ decay mode. Our conclusions are that the existence of Z' bosons from E_6 much lighter than 1 TeV is unlikely, with the noticeable exception of the superstring-inspired η model which completely evades our constraints. At the same time, our analysis suggests that the observation of FCNC processes at rates larger than the SM predictions could be interpreted as a hint for the simultaneous presence of additional gauge bosons and new charged fermions. Indeed, these new states could manifest themselves indirectly via this kind of flavor-changing effects well before they are directly produced.

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