

***CP* asymmetries in B^0 decays in the presence of flavor-changing neutral currents**

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We study in detail models with vectorlike quarks, with special emphasis on their implications for *CP* asymmetries in B^0 decays. In this class of models there are deviations from unitarity in the Cabibbo-Kobayashi-Maskawa matrix and flavor-changing neutral currents which, although naturally suppressed, may have important consequences. We show that even a relatively small contribution of *Z*-mediated flavor-changing neutral currents to B^0 - \bar{B}^0 mixing can lead to significant departures from the predictions of the standard model for *CP* asymmetries in B^0 decays.

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I. INTRODUCTION

The measurement of *CP* asymmetries in B^0 decays [1] provides an opportunity to test various aspects of the standard model (SM), including the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and the standard KM mechanism of *CP* violation. The simplest extension of the standard model where deviations from unitarity of the CKM matrix naturally arise consists of introducing extra quarks which are isosinglets but which mix with the standard quarks. Isosinglet quarks have been suggested in a variety of models, including E_6 grand-unified theories and some of the superstring-inspired models. The addition of isosinglet quarks to the SM provides [2] a possible connection between *CP* breaking at a high-energy scale and the observed *CP* violation at low energies and furthermore it gives a simple solution to the strong *CP* problem [3,4].

Some of the features of models with isosinglet quarks and their implications for *CP* violation have been analyzed by Branco and Lavoura [5] and by Nir and Silverman [6]. The present work complements these two previous analyses.

The paper is organized as follows. In the next section we briefly describe the model, identifying the new *CP*-violating phases which arise when both $Q = -\frac{1}{3}$ and $\frac{2}{3}$ isosinglet quarks are present and also show how deviations from unitarity and flavor-changing neutral currents (FCNC's) are closely related and both naturally suppressed in the model. In Sec. III we advocate the use of rephasing-invariant parametrizations which are especially convenient for models with isosinglet quarks. We

give two examples, one with one down-type vectorlike quark, and another with one down-type and one up-type vectorlike quark. This section of the paper is logically independent of the other sections and therefore it may be skipped by the reader not interested in the question of how to parametrize the CKM matrix. In Sec. IV we study B^0 - \bar{B}^0 mixing and *CP* asymmetries in B^0 decays. For simplicity, we consider the case of one down-type vectorlike quark (1DVLQ), but we will show that the analysis continues to be valid for an arbitrary number of down-type vectorlike quarks. Nir and Silverman have analyzed [6] in detail *CP* asymmetries in the 1DVLQ model under the assumption that B^0 - \bar{B}^0 mixing is dominated by *Z* exchange tree diagrams. We will do the analysis so that it can be applied to the general case, including the one where the *Z* exchange and the SM box diagram contributions to B^0 - \bar{B}^0 mixing are of comparable strength. We point out that if one takes into account the recent upper limit [7] on $B^0 \rightarrow \mu^+ \mu^- X$ decays, then B_d - \bar{B}_d mixing can still be dominated by the *Z*-mediated FCNC's, while in the case of B_s - \bar{B}_s mixing *Z* exchange can at most compete with the SM box diagram. We show that even a relatively small contribution by the *Z* exchange diagrams to B^0 - \bar{B}^0 mixing can drastically modify the predictions of the SM for *CP* asymmetries in B^0 decays. Finally in Sec. V we present our conclusions.

II. THE MODEL

We will assume the standard $SU(2) \times U(1)$ gauge theory, with the addition of N_d charge $-\frac{1}{3}$ and N_u charge $\frac{2}{3}$ isosinglet quarks. The quark field content of the

model will be denoted as

$$\begin{aligned}
& (u^0 d^0)_{Li}, \quad i=1, \dots, n, \\
& D_{Lp}^0, \quad p=1, \dots, N_d, \\
& U_{Lq}^0, \quad q=1, \dots, N_u, \\
& D_{R\alpha}^0, \quad \alpha=1, \dots, n+N_d, \\
& U_{R\beta}^0, \quad \beta=1, \dots, n+N_u.
\end{aligned} \quad (1)$$

The quark mass terms are

$$\begin{aligned}
\mathcal{L}_M = & \bar{u}_{Li}^0 (m_u)_{i\beta} U_{R\beta}^0 + \bar{U}_{Lq}^0 (M_U)_{q\beta} U_{R\beta}^0 \\
& + \bar{d}_{Li}^0 (m_d)_{i\alpha} D_{R\alpha}^0 + D_{Lp}^0 (M_D)_{p\alpha} D_{R\alpha}^0.
\end{aligned} \quad (2)$$

The dimensions of the four mass matrices are readily inferred from the index range convention of Eq. (1). Although most of our considerations apply to arbitrary n , N_d , N_u , we will, for simplicity, take $n=3$, $N_d=N_u=1$. The weak gauge currents can be written

$$\begin{aligned}
\mathcal{L}_g = & \mathcal{L}_W + \mathcal{L}_Z, \\
\mathcal{L}_W = & \frac{g}{\sqrt{2}} \bar{u}_{L\alpha} V_{\alpha\beta}^{\text{CKM}} \gamma_\mu d_{L\beta} W^\mu, \\
\mathcal{L}_Z = & \frac{g}{2 \cos\theta_W} [z_{\alpha\beta}^u \bar{u}_{L\alpha} \gamma^\mu u_{L\beta} - z_{\alpha\beta}^d \bar{d}_{L\alpha} \gamma^\mu d_{L\beta} \\
& - \sin^2\theta_W J_{\text{em}}^\mu] Z_\mu \quad (\alpha, \beta=1, \dots, 4),
\end{aligned} \quad (3)$$

where u_α, d_β are mass eigenstates and

$$V_{\alpha\beta}^{\text{CKM}} = \sum_{i=1}^3 U_{i\alpha}^* W_{i\beta}, \quad (4a)$$

$$z_{\alpha\beta}^u = \delta_{\alpha\beta} - U_{4\alpha}^* U_{4\beta}, \quad (4b)$$

$$z_{\alpha\beta}^d = \delta_{\alpha\beta} - W_{4\alpha}^* W_{4\beta}, \quad (4c)$$

where U and W denote the matrices which relate the weak and mass eigenstates:

$$\begin{bmatrix} u_i^0 \\ U^0 \end{bmatrix}_L = U \begin{bmatrix} u_i \\ T \end{bmatrix}_L, \quad \begin{bmatrix} d_i^0 \\ D^0 \end{bmatrix}_L = W \begin{bmatrix} d_i \\ B \end{bmatrix}_L. \quad (5)$$

Because of the presence of the vectorlike quarks there are flavor-changing neutral currents which are closely connected to the deviations from unitarity in V^{CKM} . Indeed, using the unitarity of U and W , one readily obtains

$$(VV^\dagger)_{\alpha\beta} = z_{\alpha\beta}^u, \quad (6a)$$

$$(V^\dagger V)_{\alpha\beta} = z_{\alpha\beta}^d \quad (V \equiv V^{\text{CKM}}). \quad (6b)$$

An attractive feature of models with vectorlike quarks is the fact that although deviations from unitarity in V^{CKM} and FCNC's arise, they are related through Eqs. (6) and are both suppressed in the standard quark sector by the ratio of standard quark masses to the vectorlike quark masses. This can be readily seen by making an approximate diagonalization of the quark mass matrices. By choosing an appropriate weak basis one can put, without loss of generality, the quark mass matrices in the form

$$\mathcal{M}_d = \begin{bmatrix} m_d \\ M_D \end{bmatrix} = \begin{bmatrix} G_d & J_d \\ 0 & \hat{M}_d \end{bmatrix}, \quad (7a)$$

$$\mathcal{M}_u = \begin{bmatrix} m_u \\ M_u \end{bmatrix} = \begin{bmatrix} \hat{G}_u & J_u \\ 0 & \hat{M}_u \end{bmatrix}, \quad (7b)$$

where \hat{G}_u, \hat{M}_u , and \hat{M}_d are diagonal real positive matrices of dimension n, N_u , and N_d . The matrix G_d is n dimensional and complex, while J_d and J_u are $(n \times N_d)$ and $(n \times N_u)$ complex matrices. Through a phase redefinition, one can eliminate N_d and N_u phases from J_d and J_u , respectively. It is convenient to write in block form the unitary matrices W and U which diagonalize $\mathcal{M}_d \mathcal{M}_d^\dagger$, and $\mathcal{M}_u \mathcal{M}_u^\dagger$, respectively:

$$W = \begin{bmatrix} A_d \\ B_d \end{bmatrix} = \begin{bmatrix} K_d & R_d \\ S_d & T_d \end{bmatrix}, \quad (8)$$

with analogous expressions for U . Let m be the mass scale of (G_d, J_d) and M be the mass scale of \hat{M}_d . Since G_d, J_d are $\Delta I = \frac{1}{2}$ mass terms while \hat{M}_d is a $\Delta I = 0$ mass term, it is natural to assume $M \gg m$. One can then find an approximate solution for W :

$$\begin{aligned}
T_d & \approx I_{N_d}, \quad S_d \approx -(\hat{M}_d^{-1} J_d) K_d, \\
R_d & \approx J_d \hat{M}_d^{-1},
\end{aligned} \quad (9)$$

while K_d is, up to $O(m^2/M^2)$, the unitary matrix which diagonalizes $G_d G_d^\dagger$. Analogous expressions obviously apply to U . The V^{CKM} matrix is then given by

$$V^{\text{CKM}} = (A_u^\dagger A_d) = \begin{bmatrix} K_u^\dagger K_d & K_u^\dagger J_d \hat{M}_d^{-1} \\ \hat{M}_u^{-1} J_u^\dagger K_d & \hat{M}_u^{-1} J_u^\dagger J_d \hat{M}_d^{-1} \end{bmatrix}. \quad (10)$$

Using unitarity of W, U one readily obtains

$$\begin{aligned}
(VV^\dagger)_{ij} & = \delta_{ij} - [J_u M_u^{-2} J_u^\dagger]_{ij}, \\
(V^\dagger V)_{ij} & = \delta_{ij} - [K_d^\dagger J_d M_d^{-2} J_d^\dagger K_d]_{ij},
\end{aligned} \quad (11)$$

where we have taken into account the fact that we have chosen to work in the weak basis where \hat{G}_u is diagonal and therefore $K_u \simeq I_3$. Since J_u and J_d are $O(m)$ it is clear from Eqs. (11) that deviations from unitarity and FCNC for the standard quarks will be suppressed by the ratio m^2/M^2 .

III. REPHASING-INVARIANT PARAMETRIZATION OF V^{CKM}

Since the CKM matrix is no longer unitary, it is less obvious to find the number of independent CP -violating phases for arbitrary n, N_d , and N_u . In Ref. [5] Branco and Lavoura have studied the restrictions that CP invariance imposes on the quark masses of Eq. (1). This was done by constructing the most general CP transformation which leaves invariant the charged and neutral current interactions. They obtained for the number of CP restrictions

$$N_c = \frac{1}{2}(n-1)[(n-2) + 2(N_d + N_u)]. \quad (12)$$

This corresponds in general to the number of independent CP -violating phases N_ϕ which appear in V^{CKM} . At this point it is worth noting that although the expression for V^{CKM} given by Eq. (10) is only approximate, it contains the correct number of physical phases: namely,

$$\begin{aligned} K_d &\rightarrow \frac{1}{2}(n-1)(n-2), \\ J_d &\rightarrow (n-1)N_d, \\ J_u &\rightarrow (n-1)N_u. \end{aligned} \quad (13)$$

We turn now to the question of finding an exact parametrization of V^{CKM} for models with vectorlike quarks. There are two different approaches to the problem: one, more traditional, parametrizes V^{CKM} through Euler angles and phases; the other uses rephasing-invariant quantities [8] to parametrize V^{CKM} . In the case where there are only isosinglet quarks of a given charge (e.g., $N_u=0$, $N_d=\text{arbitrary}$) the V^{CKM} matrix consists of the first n lines of an $(n+N_d)$ -dimensional unitary matrix, and the problem amounts to finding a parametrization of this unitary matrix where $\frac{1}{2}(n-1)[(n-2)+2N_d]$ physical phases appear in the first n lines. This problem was solved in Ref. [5], where an explicit parametrization through Euler angles and phases was given. At this point, it is worth mentioning that, for more than one vectorlike quark, the ‘‘standard’’ parametrization [9] of V^{CKM} does not have the above property and therefore it is not adequate. Although the solution presented in Ref. [5] is mathematically correct, parametrizations through Euler angles and phases are not the most convenient, especially when isosinglet quarks are present. Therefore we will propose here the use of rephasing-invariant parametrizations and analyze the two simplest cases: namely, $(N_d=1, N_u=0)$ and $(N_d=N_u=1)$.

A. The case $N_d=1, N_u=0$

In this case there are three phase variables and six angle variables. We propose the following choice.

Phase variables:

$$\begin{aligned} \varphi_1 &= \arg(V_{11}V_{23}V_{13}^*V_{21}^*), \\ \varphi_2 &= \arg(V_{11}V_{33}V_{13}^*V_{31}^*), \\ \varphi_3 &= \arg(V_{23}V_{32}V_{22}^*V_{33}^*). \end{aligned} \quad (14a)$$

Angle variables:

$$|V_{11}|, |V_{21}|, |V_{31}|, |V_{23}|, |V_{13}|, |V_{32}|. \quad (14b)$$

We have chosen a complete set of variables containing quantities that are either already measured or likely to be directly measured in the future. Indeed it will be seen that ϕ_1, ϕ_2, ϕ_3 correspond to the angles γ, α, β_s , respectively, which appear in the unitarity quadrangles of the 1DVLQ model (see Figs. 1 and 2).

We will show next that one can obtain the remaining elements of V^{CKM} from the input data of Eqs. (14). In order to facilitate our task, we work in the weak basis where m_u is diagonal and real. Let us consider the uni-

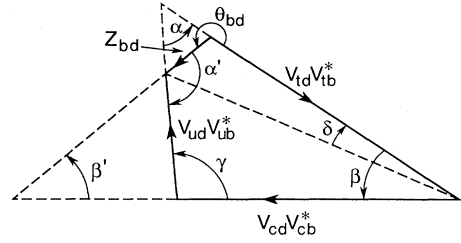


FIG. 1. The unitarity quadrangle in the B_d sector, corresponding to the first of Eqs. (26).

tary matrix W defined by Eq. (5), whose first three lines constitute V^{CKM} :

$$W = \begin{bmatrix} V^{\text{CKM}} \\ W_{4i} \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} & V_{uB} \\ V_{cd} & V_{cs} & V_{cb} & V_{cB} \\ V_{td} & V_{ts} & V_{tb} & V_{tB} \\ W_{41} & W_{42} & W_{43} & W_{44} \end{bmatrix}. \quad (15)$$

Without loss of generality, one can choose the quark phases so that the second row and the third column are real. Then $\varphi_1, \varphi_2, \varphi_3$ fix the arguments of V_{11}, V_{31}, V_{32} , respectively. Normalization of the first column ($|W_{41}|^2 = 1 - \sum_{i=1}^3 |V_{i1}|^2$) gives us $|W_{41}|$. Then orthogonality of the first and third columns together with normalization of the third column give us $\arg(W_{41}), |V_{33}|, |W_{43}|$. At this stage, the first and the third columns are completely determined and in the second column $|V_{32}|, \arg V_{32}, \arg V_{23}$ are also known. The remaining elements in the second column can be determined from orthogonality of the second column to the first and third, together with normalization of the second column. We have omitted the usual ambiguities [8,10] which arise in reconstructing the CKM matrix from input data. Note that our parametrization is such that for angle variables we have only used the moduli of V^{CKM} connecting the standard quarks. Therefore V^{CKM} can be reconstructed without directly measuring the coupling of the isovector quark B to the standard quarks. We have considered the case $N_d=1$. The extension to $N_d > 1$ is straightforward. However, there are some special features which only hold for $N_d=1$. For example, for $N_d=1$, it can easily be verified that one can choose the quark field phases in such a way that the couplings z_{ab}^d are all real. In general this is not possible for $N_d > 1$.

We have presented a rephasing-invariant parametrization of V^{CKM} where invariant phases and moduli were

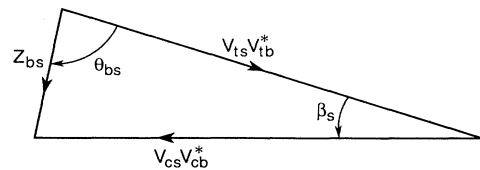


FIG. 2. Unitarity in the B_s sector. We have neglected $|V_{us}V_{ub}|$ and exaggerated $|z_{bd}|$ to show the angles.

used. In the standard model, one can also parametrize V^{CKM} using only independent moduli [11]. One may ask whether that parametrization is also possible in the presence of isovector quarks. We will show that it is only possible for $N_d=1$. The number of independent moduli (N_m) is

$$N_m = n(n + N_d - 1), \quad N_d \geq 1, \quad (16)$$

while the number of angles is

$$N_a = \frac{1}{2}n[(n + N_d - 1) + N_d]. \quad (17)$$

Taking into account that the number of independent phases N_ϕ is given by Eq. (12), with $N_u=0$, one obtains

$$N_p = N_\phi + N_a = N_m + (N_d - 1)(n - 1). \quad (18)$$

Therefore for $N_d > 1$ the number of parameters exceeds the number of independent moduli, and a parametrization through moduli is no longer possible.

B. The case $N_d = N_u = 1$

We consider now the case where there are both isovector quarks of charge $-\frac{1}{3}$ and of charge $\frac{2}{3}$. The parametrization of V^{CKM} is less obvious in this case, since there no longer exists a weak basis where either the up or down quark mass matrices are diagonal. It is convenient to introduce the auxiliary matrix X defined by

$$X = \begin{bmatrix} V^{\text{CKM}} & B_u^\dagger \\ B_d & 0 \end{bmatrix}, \quad (19)$$

where $V^{\text{CKM}} = A_d^\dagger A_d$, and A_u, A_d, B_u, B_d were defined in Eq. (8). For the moment n, N_d, N_u are arbitrary. X is an $(n + N_d + N_u)$ -dimensional matrix and it can readily be verified that X is unitary. The fact that V^{CKM} is a submatrix of a unitary matrix obviously facilitates the task of finding an appropriate parametrization. We will specialize now to the case $N_d = N_u = 1$. There are five phase variables and nine angle variables. Our choice is the following.

Invariant phases:

$$\begin{aligned} \varphi_1 &= \arg(V_{11} V_{23} V_{13}^* V_{21}^*), \\ \varphi_2 &= \arg(V_{11} V_{33} V_{13}^* V_{31}^*), \\ \varphi_3 &= \arg(V_{32} V_{23} V_{33}^* V_{22}^*), \\ \varphi_4 &= \arg(V_{11} V_{43} V_{13}^* V_{41}^*), \\ \varphi_5 &= \arg(V_{12} V_{21} V_{11}^* V_{22}^*). \end{aligned} \quad (20)$$

Moduli:

$$\begin{aligned} &|V_{11}|, |V_{12}|, |V_{21}|, |V_{22}|, |V_{13}|, \\ &|V_{23}|, |V_{33}|, |V_{31}|, |V_{43}|. \end{aligned}$$

It can be readily seen that these input parameters enable one to reconstruct the CKM matrix using unitarity of the auxiliary matrix X .

IV. CP ASYMMETRIES IN B^0 DECAYS

In this section we study B - \bar{B} mixing and CP asymmetries in B^0 decays in models with vectorlike quarks. In Ref. [6], Nir and Silverman have studied these symmetries under the assumption that Z -mediated FCNC's give the dominant contribution to B - \bar{B} mixing. We will consider here the more general case where nonstandard contributions compete with the standard box diagram at inducing B^0 - \bar{B}^0 mixing. The relevance of this analysis stems from the fact that even a relatively small contribution from new physics can produce significant departures from the SM predictions for the CP asymmetries in B^0 decays.

This section is organized as follows. First we present a general analysis of CP asymmetries in B^0 decays when new physics is added to the mixing matrix. Then we particularize to the model with charge $-\frac{1}{3}$ vectorlike quarks. It turns out that it is sufficient to consider the case where there is only one such quark, since CP asymmetries cannot distinguish $N_d=1$ from $N_d > 1$.

Let us assume that the off-diagonal element of B_q - \bar{B}_q is changed by a factor Δ_{qb} as a result of a new contribution from physics beyond the SM:

$$M_{12} = M_{12}^{(0)} \Delta_{qb} \quad (q = d, s), \quad (21)$$

where $M_{12}^{(0)}$ is the box diagram contribution. We will assume that all amplitudes contributing to the decay have the same CKM phase and furthermore that $\Gamma_{12}^{(0)} \ll M_{12}^{(0)}$. In this case the CP asymmetry is given by

$$a \equiv \frac{\Gamma(B^0 \rightarrow f) - \Gamma(\bar{B}^0 \rightarrow f)}{\Gamma(B^0 \rightarrow f) + \Gamma(\bar{B}^0 \rightarrow f)} \simeq -\sin(\Delta Mt) \sin\phi, \quad (22)$$

where

$$\begin{aligned} \phi &= \phi^{(0)} + \arg\Delta_{bq}, \quad \Delta_{bq} = \Delta_{qb}^*, \\ \phi^{(0)} &= \arg \left[\left[\frac{q}{p} \right]^{(0)} \frac{\bar{A}(f)}{A(f)} \right], \\ \left[\frac{q}{p} \right]^{(0)} &= \left[\frac{M_{12}^{(0)*}}{M_{12}^{(0)}} \right]^{1/2}. \end{aligned} \quad (23)$$

The index (0) denotes the contributions arising within the three generation SM, and $A(f), \bar{A}(f)$ stand for the decay amplitudes from the initial state $|B^0\rangle, |\bar{B}^0\rangle$ to a CP eigenstate $|f\rangle$. From Eq. (23) it follows that there are two possible sources which may change the SM prediction.

(i) The presence of the phase of Δ_{bq} , which determines the deviation from the box diagram contribution $\phi^{(0)}$. It is possible to incorporate different new physics contributions for B_d and B_s , if $\arg\Delta_{bd} \neq \arg\Delta_{bs}$.

(ii) Although the expression for $\phi^{(0)}$ is the one given by the SM, the actual numerical value of $\phi^{(0)}$ may differ from the SM prediction. This is due to the fact that models beyond the SM allow in general for a different range of the CKM matrix elements.

In Table I we establish our notation by explicitly giving ϕ for various final states. For comparison, the standard model values are also shown.

TABLE I. The predicted values for the angles ϕ_{iq} . The values shown are for CP even final states. Thus $\phi_{1d} = -2\beta = -\phi_{\psi K_S}$. By definition $\alpha = \arg(-V_{td}V_{tb}^*/V_{ud}V_{ub}^*)$; $\beta = \arg(-V_{cd}V_{cb}^*/V_{td}V_{tb}^*)$; $\gamma = \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$; $\alpha' = \arg(V_{ud}V_{ub}^*/z_{bd})$; $\beta' = \arg(z_{bd}/V_{cd}V_{cb}^*)$; and $\beta_s = \arg(-V_{cs}V_{cb}^*/V_{ts}V_{tb}^*)$. See Figs. 1 and 2.

Initial state	Quark subprocess	Final state	ϕ	Standard model	Beyond standard model
B_d	$\bar{b} \rightarrow \bar{c}c\bar{s}$	ψK_S	ϕ_{1d}	-2β	$-2\beta + \arg\Delta_{bd}$
	$\bar{b} \rightarrow \bar{c}c\bar{d}$	$D^+ D^-$	ϕ_{2d}	-2β	$-2\beta + \arg\Delta_{bd}$
	$\bar{b} \rightarrow \bar{u}u\bar{d}$	$\pi^+ \pi^-$	ϕ_{3d}	2α	$2\alpha + \arg\Delta_{bd}$
B_s	$\bar{b} \rightarrow \bar{c}c\bar{s}$	$D_s^+ D_s^-$	ϕ_{1s}	$-2\beta_s$	$-2\beta_s + \arg\Delta_{bs}$
	$\bar{b} \rightarrow \bar{c}c\bar{d}$	ψK_S	ϕ_{2s}	$-2\beta_s$	$-2\beta_s + \arg\Delta_{bs}$
	$\bar{b} \rightarrow \bar{u}u\bar{d}$	ρK_S	ϕ_{3s}	$-2\gamma - 2\beta_s$	$-2\gamma - 2\beta_s + \arg\Delta_{bs}$

We turn now to the detailed analysis of models with charge $-\frac{1}{3}$ vectorlike quarks. The new contribution to the $\Delta B=2$ effective Hamiltonian arises from Z exchange tree graphs and one readily obtains

$$\Delta_{bq} = 1 + r_q e^{2i\theta_{bq}},$$

$$r_q = \frac{1}{v|\bar{E}(x_t)|} \left| \frac{z_{bq}}{V_{tb}V_{tq}^*} \right|^2, \quad (24)$$

$$\theta_{bq} = \arg \left[\frac{z_{bq}}{V_{tq}V_{tb}^*} \right],$$

where $v = \alpha/4\pi \sin^2\theta_W$ and $\bar{E}(x_t = (m_t/m_W)^2)$ is an Inami-Lim function for the top quark box diagram. Note that $v\bar{E}(x_t) = -0.0046$ for $m_t = 140$ GeV. We assume the same QCD correction factor for both the box diagram and the Z exchange diagram. This should be a good approximation since QCD corrections above the scale of M_Z are negligible. From Eq. (24) one readily obtains

$$\arg\Delta_{bq} = \arctan \left[\frac{r_q \sin 2\theta_{bq}}{1 + r_q \cos 2\theta_{bq}} \right], \quad (25)$$

$$|\Delta_{bq}| = (1 + r_q^2 + 2r_q \cos 2\theta_{bq})^{1/2}.$$

There are two distinct cases of interest in the study of CP asymmetries: (a) Z exchange and box diagrams give comparable contributions to $B_q - \bar{B}_q$ mixing; (b) Z exchange gives the dominant contribution to $B_q - \bar{B}_q$ mixing. These two cases are distinguishable by the value of the parameter r_q : (a) $r_q \approx 1$; (b) $r_q \gg 1$. Case (b) was studied in Ref. [6]. Therefore our emphasis will be on case (a); we find that even a relatively small contribution from Z exchange to $B_q - \bar{B}_q$ mixing can imply very significant departures from the SM predictions for CP asymmetries.

In order to derive the numerical predictions for the CP asymmetries, one has to take into account the unitarity constraints. The relevant ones for our purposes are

$$V_{tb}^* V_{td} + V_{cb}^* V_{cd} + V_{ub}^* V_{ud} = z_{bd}, \quad (26)$$

$$V_{tb}^* V_{ts} + V_{cb}^* V_{cs} + V_{ub}^* V_{us} = z_{bs},$$

which lead to the unitarity quadrangles of Figs. 1 and 2. In order to determine the angle ϕ for the various final states (Table I), one has to know the values of the angles α , β , and β_s shown in Figs. 1 and 2. One readily obtains

$$\cos(\alpha - \delta) = \frac{|V_{ub}V_{ud}|^2 + (|V_{tb}V_{td}|^2 - 2|V_{tb}V_{td}||z_{bd}|\cos\theta_{bd} + |z_{bd}|^2) - |V_{cb}V_{cd}|^2}{2|V_{ub}V_{ud}|(|V_{tb}V_{td}|^2 - 2|V_{tb}V_{td}||z_{bd}|\cos\theta_{bd} + |z_{bd}|^2)^{1/2}}, \quad (27)$$

$$\cos(\beta + \delta) = \frac{|V_{cb}V_{cd}|^2 + (|V_{tb}V_{td}|^2 - 2|V_{tb}V_{td}||z_{bd}|\cos\theta_{bd} + |z_{bd}|^2) - |V_{ub}V_{ud}|^2}{2|V_{cb}V_{cd}|(|V_{tb}V_{td}|^2 - 2|V_{tb}V_{td}||z_{bd}|\cos\theta_{bd} + |z_{bd}|^2)^{1/2}},$$

$$\delta = \arg \frac{V_{tb}^* V_{td}}{V_{tb}^* V_{td} - z_{bd}} = \arctan \frac{|z_{bd}|\sin\theta_{bd}}{|V_{tb}V_{td}| - |z_{bd}|\cos\theta_{bd}},$$

$$\cos\beta_s = \frac{|V_{tb}V_{ts}|^2 + |V_{cb}V_{cs}|^2 - |z_{bs}|^2}{2|V_{tb}V_{ts}||V_{cb}V_{cs}|^2},$$

where we have neglected $|V_{us}V_{ub}|$. When $z_{bq}=0$, $\delta=0$, one recovers the SM expressions giving α , β , and β_s in terms of the sides of the SM unitarity triangles. We turn now to the experimental constraints on z_{bq} . A recent experimental search [7] for the decays $B \rightarrow X\mu^+\mu^-$ by the UA1 Collaboration [7] has led to the upper bound

$$B(B \rightarrow X\mu^+\mu^-) \leq 5.0 \times 10^{-5}. \quad (28)$$

At this point, it is worth noting that for $m_t \approx 150$ GeV and within the context of the SM, the above branching ratio is predicted [13] to be $B(B^0 \rightarrow X\mu^+\mu^-) = (6-8) \times 10^{-6}$; therefore, one order of magnitude smaller than the UA1 bound. From Eq. (28) one derives the bounds

$$\left| \frac{z_{bd}}{V_{cb}} \right| \leq 0.029, \quad \left| \frac{z_{bs}}{V_{cb}} \right| \leq 0.029. \quad (29)$$

These bounds on z_{bq} are almost an order of magnitude stricter than the bounds considered in Ref. [6]. If one writes Eqs. (26) as

$$\frac{V_{tb}^* V_{tq}}{V_{cb}^*} = \frac{z_{bq}}{V_{cb}^*} - V_{cq} - \frac{V_{ub}^* V_{uq}}{V_{cb}^*}, \quad q = d, s \quad (30)$$

and takes into account the experimental constraints

$$\begin{aligned} 0.9734 \leq |V_{ud}| \leq 0.9754, \quad 0.2173 \leq |V_{us}| \leq 0.2219, \\ 0.187 \leq |V_{cd}| \leq 0.221, \quad 0.07 \leq \left| \frac{V_{ub}}{V_{cb}} \right| \leq 0.13, \quad (31) \\ |V_{cs}| \geq 0.8, \end{aligned}$$

one readily obtains

$$\left| \frac{V_{tb}^* V_{td}}{V_{cb}^*} \right| \geq 0.031, \quad \left| \frac{V_{tb}^* V_{ts}}{V_{cb}^*} \right| \geq 0.73. \quad (32)$$

Combining Eqs. (29) and (32) one finally gets

$$\left| \frac{z_{bd}}{V_{tb}^* V_{td}} \right| \leq 0.93, \quad \left| \frac{z_{bs}}{V_{tb}^* V_{ts}} \right| \leq 0.04. \quad (33)$$

Now the condition for Z exchange to give the dominant contribution to $B_q - \bar{B}_q$ mixing is that $|z_{bq}/V_{tb}^* V_{tq}| \geq 0.07$ for $m_t = 140$ GeV. One therefore concludes that in the case of B_d the dominant contribution to the mixing may arise from Z exchange, while in the case of B_s , Z exchange can at most compete with the box diagram contribution. This completes our analysis of the unitarity constraints in the model.

We consider now $B_d - \bar{B}_d$ mixing which is given by

$$\begin{aligned} x_d = \frac{\Delta M_d}{\Gamma} = \frac{G_F^2}{6\pi^2} \tau_B \eta_{\text{QCD}} M_B B_B F_B^2 M_W^2 \\ \times |\bar{E}(x_t)| |V_{td} V_{tb}|^2 |\Delta_{bd}|, \quad (34) \end{aligned}$$

where we have followed standard notation. For $\sqrt{B_B F_B^2}$ we will use the range suggested in a recent review by

Buras and Harlander [12]:

$$160 \text{ MeV} \leq \sqrt{B_B F_B^2} \leq 240 \text{ MeV} \quad (35)$$

following recent lattice calculations in the static limit. For η_{QCD} we will use here the value $\eta_{\text{QCD}} = 0.55$, which is consistent with the renormalization used in obtaining Eq. (35), and for τ_B we will take $\tau_B = 1.28 \pm 0.06$ ps, which is the recent world average for τ_{B^0} , including the results from the CERN e^+e^- collider LEP [13]. Equation (34) fixes for us the experimentally allowed range for the product $|V_{td} V_{tb}| |\Delta_{bd}|^{1/2}$, given by

$$\begin{aligned} |V_{td} V_{tb}| |\Delta_{bd}|^{1/2} = \left[\frac{6\pi^2}{G_F^2 \eta_{\text{QCD}} M_W^2 M_B} \right]^{1/2} \\ \times \left[\frac{x_d^{1/2}}{\tau_B^{1/2} B_B^{1/2} F_B} \right] \frac{1}{\sqrt{|\bar{E}(x_t)|}}. \quad (36) \end{aligned}$$

The terms in the first set of square brackets are taken as exact. In the second set of square brackets, $\tau_B, B_B^{1/2} F_B$ are constrained to be within the indicated ranges, while x_d is within the range implied by the experimental results of ARGUS, CLEO, and LEP [14]:

$$x_d = 0.67 \pm 0.10. \quad (37)$$

We are now in a position to evaluate CP asymmetries in the model, taking into account all the experimental and theoretical constraints [15]. At this point it should be obvious that the number of DVLQ's is irrelevant to the discussion. None of the input in this section is dependent on the value of $N_d \geq 1$.

For given values of r_q and θ_{bq} , one obtains $\arg \Delta_{bq}$, $|\Delta_{bq}|$ from Eqs. (25) and then Eq. (36) fixes the allowed range of $|V_{td} V_{tb}|$.

In Fig. 3 we present our main result. Recall that one of the most important predictions of the SM is the sign of some of the CP asymmetries in the B^0 decays. In particular, $\sin(\phi_{\psi K_S}) = -\sin(\phi_{1d})$ is predicted to be positive and in fact [16] for $m_t \geq 120$ GeV and $F_B > 170$ MeV,

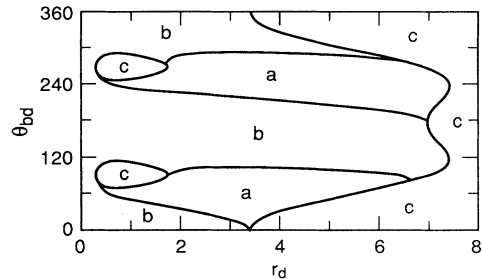


FIG. 3. The regions of r_d, θ_{bd} space where (a) The model predicts $\sin(\phi_{1d})$ to be positive, thus contradicting the standard model result, (b) the model predicts $\sin(\phi_{1d})$ to be negative, and (c) the unitarity quadrangle does not close. For these values of r_d, θ_{bd} Eq. (26) is not consistent with Eqs. (25) and (36). For this figure the values $m_t = 140$ GeV, $|V_{ud}| = 0.9744$, $|V_{ub}|/|V_{cb}| = 0.1$, $|V_{cd}| = 0.204$, and $\sqrt{B_B F_B^2} = 0.2$ GeV were used.

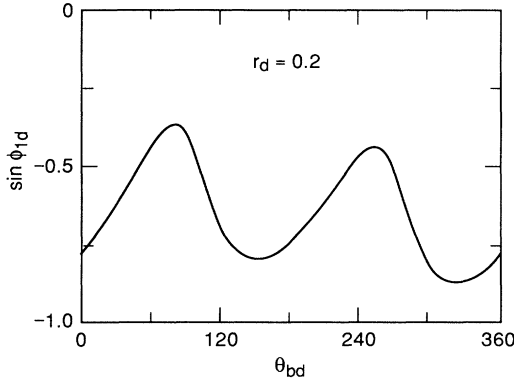


FIG. 4. $\sin(\phi_{1d})$ as a function of θ_{bd} , for $r_d=0.2$. The values of the other parameters are as in Fig. 3. Even for this low value of r_d , the DVLQ model value can differ significantly from the SM prediction, which is $\sin(\phi_{1d}^{\text{SM}})=-0.68$ for the parameter values indicated.

$a(\psi K_S) \geq 0.26$. This is no longer true in the presence of DVLQ's. In Fig. 3 we indicate the region in r_d, θ_{bd} space where the asymmetry $a(\psi K_S)$ has a sign opposite to the one predicted by the SM. It is seen from Fig. 3 that a positive sign for $\sin(\phi_{1d})$ can be obtained even for a relatively small value of r_q .

In Figs. 4–9 we give the values of various CP asymmetries as a function θ_{bd} , for various values of r_d . For comparison, we also give the prediction of the SM for the same choice of $m_t, B_B F_B^2, \tau_B, x_d, |V_{qd}|, |V_{qb}|$ ($q=c,t$). We have used $m_t=140$ GeV, and the central values of the ranges indicated above for the other parameters. We choose values of r_d ranging from 0.2 to 2.5. It is clear from the figures that even a relatively small contribution of the Z exchange diagrams (i.e., $r_q < 1$) to $B_q-\bar{B}_q$ mixing can lead to substantial deviations from the SM predictions. For example, in Fig. 5, $\sin(\phi_{1d})$ is plotted in terms of θ_{bd} for $r_d=0.6$, and one sees that for some regions of θ_{bd} , a_{1d} can have a sign opposite to the one predicted by

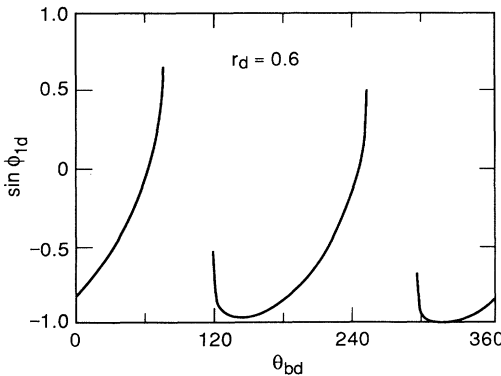


FIG. 5. The same as in Fig. 4, but with $r_d=0.6$. For this value of r_d , $\sin(\phi_{1d})$ can take on large positive values. Compare with Fig. 3.

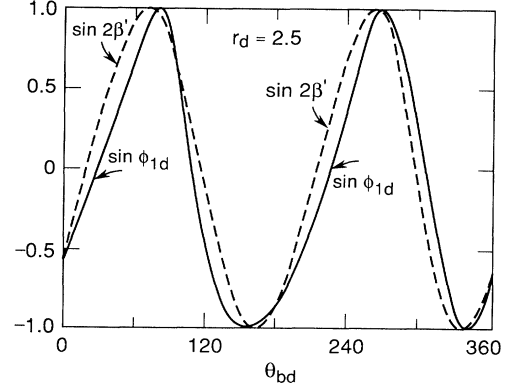


FIG. 6. $\sin(\phi_{1d})$ and $\sin 2\beta'$ for $r_d=2.5$. For this larger value of r_d these functions are almost the same.

the standard model. For large values of r_d , which corresponds to dominance of the Z exchange diagrams, one recovers the results presented in Ref. [6], that is, that $\phi_{1d} \rightarrow 2\beta', \phi_{3d} \rightarrow -2\alpha'$ (see Figs. 6 and 9).

We point out that these results are essentially independent of constraints from the value of ϵ_K of the $K^0-\bar{K}^0$ system. The large deviations we predict for the values of the asymmetries as compared to the SM are due to the fact that they no longer measure $\sin(-2\beta)$ and $\sin(2\alpha)$, but $\sin(-2\beta + \arg \Delta_{bd})$ and $\sin(2\alpha + \arg \Delta_{bd})$. In fact, the values of α and β implied by the parameter space used in our calculations leading to Figs. 3–9 never differ significantly from the values allowed in the SM.

Finally we consider briefly the influence of the Z-mediated FCNC's on the CP asymmetries of the B_s^0 decays. The SM predicts $\beta_s \simeq 0$, and because of the experimental restriction Eq. (29), this result holds in the DVLQ model also. However, a significant value for the angle ϕ_{1s} is not ruled out in the DVLQ model. From Eqs. (24) and (33) one deduces the maximum value of r_s possible:

$$r_s \leq 0.35 \quad \text{for } m_t = 140 \text{ GeV} . \quad (38)$$

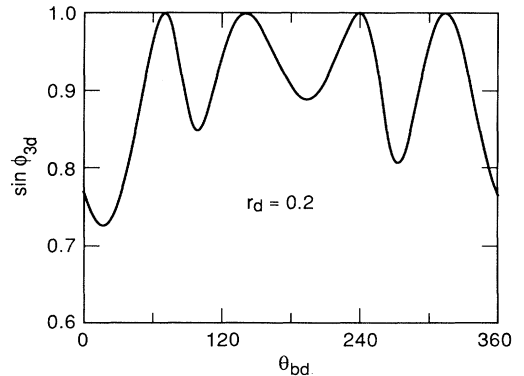
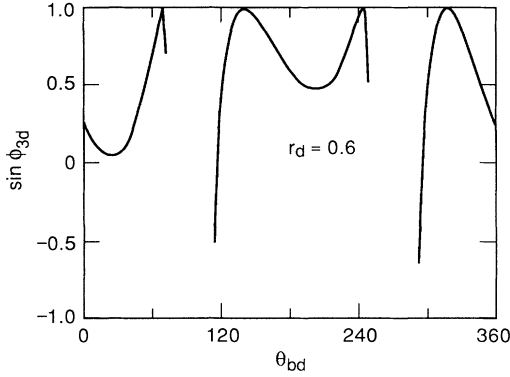
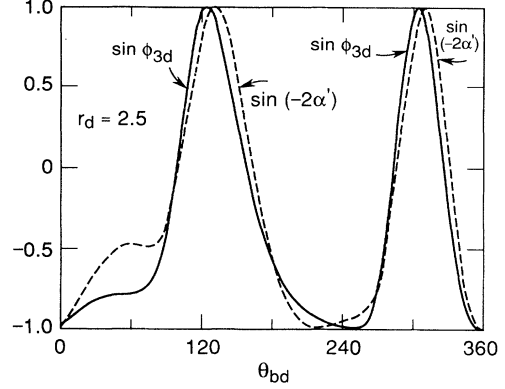


FIG. 7. $\sin(\phi_{3d})$ as a function of θ_{bd} , for $r_d=0.2$. Again all other parameter values are as in Fig. 3. The SM prediction is $\sin(\phi_{3d}^{\text{SM}})=0.99$.

FIG. 8. The same as Fig. 7 but with $r_d = 0.6$.FIG. 9. $\sin(\phi_{3d})$ and $\sin(-2\alpha')$ for $r_d = 2.5$.

One readily verifies that for $r_q < 1$, the maximum value of $\arg\Delta_{bq}$ is given by

$$(\arg\Delta_{bq})_{\max} = \arctan \left[\frac{r_q}{\sqrt{1-r_q^2}} \right]. \quad (39)$$

Therefore $\arg\Delta_{bs} \leq 21^\circ$. As the measured angle is $\phi_{1s} = -2\beta_s + \arg\Delta_{bs}$, strong deviations from the SM prediction are again possible.

V. CONCLUSIONS

We have analyzed some of the main features of a minimal extension of the SM where vectorlike quarks are introduced. We show that in these models deviations from unitarity and FCNC's are related and both are naturally suppressed. We advocate the use of rephasing-invariant parametrizations of the CKM matrix and give two examples for $N_d = 1, N_u = 0$ and for $N_d = 1, N_u = 1$.

Special emphasis was given to the consequences of the model for CP asymmetries in B^0 decays. It was shown that even a small contribution of the Z exchange diagrams to $B^0-\bar{B}^0$ mixing can lead to drastic deviations from the SM predictions for CP asymmetries in B^0 decays.

After this work was essentially completed we received

a paper by Soares and Wolfenstein [17] where the authors examine the implications of new physics on CP asymmetries in B^0 decays. We thank Soares for having given us a copy of the paper. We have also noticed a recent paper by Silverman [18] whose content partially overlaps with our Sec. IV. However, Silverman only considers the case of FCNC Z exchange dominating $B_d^0-\bar{B}_d^0$ mixing. As previously emphasized, the main point in Sec. IV is that even a small contribution of Z exchange to $B^0-\bar{B}^0$ mixing can lead to predictions for CP asymmetries in B^0 decays drastically different from those of the standard model.

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