PHYSICAL REVIEW D VOLUME 48, NUMBER 3 1 AUGUST 1993

Inelastic channels in WW scattering

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(Received 26 February 1993)

If the electroweak symmetry-breaking sector becomes strongly interacting at high energies, it can be probed through longitudinal W scattering. We present a model with many inelastic channels in the $W_L W_L$ scattering process, corresponding to the production of heavy fermion pairs. These heavy fermions affect the elastic scattering of W_L's by propagating in loops, greatly reducing the amplitudes in some charge channels. We conclude that the symmetry-breaking sector cannot be fully explored by using, for example, the $W_L^\pm W_L^\pm$ mode alone, even when no resonance is present; all $W_L W_L \to W_L W_L$ scattering modes must be measured.

PACS number(s): 14.80.Er, 11.15.Ex, 12.15.Ji

I. INTRODUCTION

One of the major goals of the Superconducting Super Collider (SSC) is to explore the physics which spontaneously breaks the electroweak symmetry of the standard model. This symmetry breaking gives rise to Goldstone bosons, which become the longitudinal components W_L of the massive vector gauge bosons. (We use W to denote either the $W^\pm$ or Z^0 boson.) Consequently, the symmetry-breaking sector can be directly probed by scattering longitudinal W's.

At energies $s \gg M_W^2$, the $W_L W_L$ scattering amplitudes, by virtue of the equivalence theorem, become approximately equal to the scattering amplitudes of the Goldstone bosons of the broken symmetry [1–3]. If we assume the existence of a custodial SU(2) symmetry, the pattern of symmetry breaking is $SU(2)_L \times SU(2)_R \to SU(2)_V$ and the Goldstone boson scattering amplitudes are given by

$$\begin{aligned}
&\mathcal{M}(Z_L^0 Z_L^0 \to W_L^- W_L^+) = A(s,t,u) , \\
&\mathcal{M}(W_L^- W_L^+ \to Z_L^0 Z_L^0) = A(s,t,u) , \\
&\mathcal{M}(W_L^- W_L^+ \to W_L^- W_L^+) = A(s,t,u) + A(t,s,u) , \\
&\mathcal{M}(Z_L^0 Z_L^0 \to Z_L^0 Z_L^0) = A(s,t,u) + A(t,s,u) + A(u,t,s) , \\
&\mathcal{M}(W_L^\pm Z_L^0 \to W_L^\pm Z_L^0) = A(t,s,u) , \\
&\mathcal{M}(W_L^\pm W_L^\pm \to W_L^\pm W_L^\pm) = A(t,s,u) + A(u,t,s) .
\end{aligned} \tag{1.1}$$

In an energy expansion of the amplitudes, the lowest-order term is uniquely determined to be

$$A(s,t,u) = \frac{s}{f^2} + \cdots , \tag{1.2}$$

where $f = 250$ GeV is the scale of symmetry breaking.

Higher-order terms in the energy expansion of the amplitude (1.2) are sensitive to the specific form of the symmetry-breaking mechanism. For example, if there is a light Higgs boson, the next term in the expansion is $s^2/M_H^2 f^2$, signaling the approach to a resonance in the $W_L W_L$ scattering amplitudes at $s \sim M_H^2$. On the other hand, if there are no light resonances, the low-energy result (1.2) will continue to hold at higher energies. This amplitude grows with increasing center-of-mass energy, becoming strong at around 1 TeV. [Of course, corrections to Eq. (1.2) must eventually become important, because the lowest-order term violates partial-wave unitarity, i.e., $|\mathrm{Re}\, a_0^0| > \frac{1}{2}$, at about $2\sqrt{2\pi} f \sim 1.3$ TeV.] This enhanced $W_L W_L$ scattering amplitude indicates the presence of new strong interactions at or above 1 TeV. It has been claimed that the energy and luminosity of the SSC are large enough that, whatever form the new interactions take, they would be observable in $W_L W_L$ two-body interactions via leptonic decays of W_L's [4]. In particular, the like-sign mode $W_L^\pm W_L^\pm$ is a favorite candidate for observing these new strong interactions because the standard model background for this mode is small [5]. This prediction of strong $W_L W_L$ scattering in the absence of a light resonance is called the "no-lose theorem."

In this paper, we explore the possibility that there are inelastic channels in the $W_L W_L$ scattering process [6,7], and the implications of these inelastic processes for the no-lose theorem. In this scenario, the W_L's would scatter not only into a $W_L W_L$ final state but also into other final states. Even if the production of these other particles is not directly observed, they would affect the *elastic* scattering of W_L's by propagating in loops. These loops necessarily contribute to the imaginary part of the elastic scattering amplitude, which is related by the optical theorem to the total cross section. The loops also contribute to the real part of the elastic amplitude, interfering with the Born contribution.

If the number of inelastic channels is large, these loop effects will be strong, and may significantly reduce the signal in some charge channels, e.g., the $W_L^\pm W_L^\pm$ channels, making it difficult to observe. To exemplify this possibility, we examine in this paper a model in which the

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inelastic channels take the form of heavy fermions, which are pair produced in $W_L W_L$ scattering. Unlike the case of a Higgs boson, production of these particles will not produce resonances in the elastic $W_L W_L$ scattering amplitudes. They alter the amplitudes through loop effects, however, and lead to behavior markedly different from the low-energy result (1.2) at scales above the threshold for fermion pair production but below 1 TeV. (New physics must enter at around 1 TeV, because the amplitudes violate unitarity not far above this scale.) The lesson is that to be certain of detecting the symmetry-breaking sector it will be necessary to measure scattering in all the final state $W_L W_L$ modes.

II. THE LARGE-N FERMION MODEL

The scattering of longitudinal W bosons can be studied within the framework of a nonlinear σ model. This is a minimalist approach because the model contains only the Goldstone bosons π of the spontaneously broken symmetry, parametrized by the matrix

$$\Sigma=\exp\left[\frac{i\tau_a\pi_a}{f}\right], \quad a=1,2,3 , \tag{2.1}$$

where τ_a are the Pauli matrices. By the equivalence theorem, the Goldstone bosons π correspond to the longitudinal degrees of freedom W_L of the vector gauge bosons.

To study the effects of inelastic channels in $W_L W_L$ scattering, we couple the Goldstone bosons to N degenerate fermion doublets ψ^j with mass $m=gv$. The effects of these fermions will be most important when the Yukawa coupling g is large. To capture this, we will not calculate the amplitudes perturbatively in g, but rather in a $1/N$ expansion. The results will be valid for arbitrary Yukawa coupling g, i.e., for all values of the fermion mass m. (Were we to calculate perturbatively in the Yukawa coupling, the real part of the loop correction would contribute through interference with the tree-level amplitude, but the imaginary part would be higher order. In the large-N approach, the imaginary part of the loop correction contributes in leading order, and will be important above the threshold for production of fermions.)

The Lagrangian for our model [7] is

$$\mathcal{L}=\frac{Nv_0^2}{4}\mathrm{Tr}(\partial_\mu\Sigma\partial^\mu\Sigma^\dagger) + \sum_{j=1}^{N}[\bar\psi^j i\partial\!\!\!/\psi^j-g_0v_0(\bar\psi_L^j\Sigma\psi_R^j+\bar\psi_R^j\Sigma^\dagger\psi_L^j)] , \tag{2.2}$$

with

$$\Sigma=\exp\left[\frac{i\tau_a\pi_a^0}{\sqrt{N}v_0}\right],$$

$$\psi_L^j=\tfrac{1}{2}(1-\gamma_5)\psi^j ,$$

$$\psi_R^j=\tfrac{1}{2}(1+\gamma_5)\psi^j .$$

The Feynman rules are obtained by expanding Σ in terms of the Goldstone fields π_a^0. The first term in the Lagrangian (2.2) yields vertices involving only Goldstone bosons:

$$\mathrm{Tr}(\partial_\mu\Sigma\partial^\mu\Sigma^\dagger)=\frac{2}{Nv_0^2}(\partial_\mu\pi_a^0)(\partial^\mu\pi_a^0)+\frac{2}{3N^2v_0^4}[\pi_a^0(\partial_\mu\pi_a^0)\pi_b^0(\partial^\mu\pi_b^0)-\pi_a^0\pi_a^0(\partial_\mu\pi_b^0)(\partial^\mu\pi_b^0)]+\cdots . \tag{2.3}$$

The last term in Eq. (2.2) yields fermion–Goldstone-boson vertices:

$$\bar\psi_L^j\Sigma\psi_R^j+\bar\psi_R^j\Sigma^\dagger\psi_L^j=\bar\psi^j\left[1+\frac{i\tau_a\pi_a^0}{N^{1/2}v_0}\gamma_5-\frac{\pi_a^0\pi_a^0}{2Nv_0^2}-\frac{i(\tau_a\pi_a^0)(\pi_b^0\pi_b^0)}{6N^{3/2}v_0^3}\gamma_5+\frac{(\pi_a^0\pi_a^0)(\pi_b^0\pi_b^0)}{24N^2v_0^4}+\cdots\right]\psi^j . \tag{2.4}$$

We will use these vertices to compute Green functions to leading order in $1/N$, holding the parameters g_0 and v_0 fixed as $N\rightarrow\infty$.

The Lagrangian (2.2) is *not* the most general one with global $SU(2)_L\times SU(2)_R$ chiral symmetry; we have omitted a possible derivative coupling of the form

$$\kappa_L\bar\psi_L(\Sigma i\partial\!\!\!/\Sigma^\dagger)\psi_L+\kappa_R\bar\psi_R(\Sigma^\dagger i\partial\!\!\!/\Sigma)\psi_R .$$

(If parity is conserved, then $\kappa_L=\kappa_R$.) We also have not included any four-derivative terms involving Σ. To leading order in $1/N$, no such terms are needed to absorb divergences; all divergences due to fermion loops can be absorbed into the bare parameters in (2.2).

To leading order in $1/N$, the only corrections to the Green functions come from fermion loops. There are no leading order corrections to the fermion self-energy, so the inverse fermion propagator remains that of a free field:

$$\Gamma^{(2)}_{\psi\bar\psi}(p)=p\!\!\!/-m \tag{2.5}$$

with mass

$$m=g_0v_0 . \tag{2.6}$$

There is no fermion wave function renormalization to leading order in $1/N$.

Fermion loops contribute a divergence to the Goldstone boson self-energy. Regularizing the fermion loop integral in $d=(4-\epsilon)$ dimensions, we find

$$\Gamma^{(2)}_{\pi_a^0\pi_b^0}(p^2)=\left[\left[1+\frac{g_0^2n(\epsilon)}{2\pi^2\epsilon m^\epsilon}-\frac{g_0^2}{4\pi^2}\right]p^2+O(p^4)\right]\delta_{ab} , \tag{2.7}$$

where $n(\epsilon)=1+\frac{1}{2}\epsilon(1-\gamma+\ln 4\pi)$ and $\gamma=0.577\,215\ldots$. The two-point function (2.7) vanishes at $p^2=0$; the Goldstone boson remains massless. The Goldstone boson wave function renormalization

$$\pi_a^0 = Z_\pi^{1/2}\pi_a\ , \quad v_0 = Z_\pi^{1/2} v\ , \quad \Gamma_{\pi_0}^{(n)} = Z_\pi^{-n/2}\Gamma_\pi^{(n)} \tag{2.8}$$

is chosen to be

$$Z_\pi^{-1} = 1 + \frac{g_0^2 n(\epsilon)}{2\pi^2 \epsilon m^\epsilon} - \frac{g_0^2}{4\pi^2}\ , \tag{2.9}$$

so that the renormalized Goldstone propagator has unit on-shell residue:

$$\left.\frac{d\Gamma^{(2)}_{\pi_a\pi_b}}{dp^2}\right|_{p^2=0} = \delta_{ab}\ . \tag{2.10}$$

We can rewrite Eq. (2.9) as

$$Z_\pi = 1 - \frac{m^{2-\epsilon} n(\epsilon)}{2\pi^2 v^2 \epsilon} + \frac{m^2}{4\pi^2 v^2} \tag{2.11}$$

using $Z_\pi^{-1} = v^2/v_0^2$ and $g_0 = m/v_0$.

The wave function renormalization (2.8) is the only renormalization necessary in this model to leading order in $1/N$. In particular, the divergences in the Goldstone boson four-point functions (1.1) due to fermion loops are precisely absorbed by the Goldstone boson wave function renormalization. (If Goldstone boson loops were not suppressed by a factor of $1/N$, additional four-derivative counterterms would be needed.) Adding up all the fermion loop contributions to the four-point functions, we obtain the renormalized amplitude

$$A(s,t,u) = \frac{1}{N}\left\{\frac{s}{v^2} - \frac{m^2}{4\pi^2 v^4} s F_2(s) - \frac{m^4}{4\pi^2 v^4}[F_4(s,t) + F_4(s,u) - F_4(t,u)]\right\}, \tag{2.12}$$

where the functions $F_2(s)$ and $F_4(s,t)$ are defined by

$$\begin{aligned} F_2(s) &= \int_0^1 dx \ln\left[1 - \frac{s}{m^2}x(1-x) - i\varepsilon\right], \\ F_4(s,t) &= \int_0^1 dx \left[x^2 - x + \frac{m^2(s+t)}{st}\right]^{-1} \\ &\quad \times \left\{\ln\left[1 - \frac{s}{m^2}x(1-x) - i\varepsilon\right] + \ln\left[1 - \frac{t}{m^2}x(1-x) - i\varepsilon\right]\right\}. \end{aligned} \tag{2.13}$$

The integral $F_2(s)$ is given by

$$\begin{aligned} F_2(s<0) &= -2 + \left[1 - \frac{4m^2}{s}\right]^{1/2} \ln\left[\frac{\sqrt{4m^2-s} + \sqrt{-s}}{\sqrt{4m^2-s} - \sqrt{-s}}\right], \\ F_2(0<s<4m^2) &= -2 + 2\left[-1 + \frac{4m^2}{s}\right]^{1/2} \arctan\left[\frac{s}{4m^2-s}\right]^{1/2}, \\ F_2(s>4m^2) &= -2 + \left[1 - \frac{4m^2}{s}\right]^{1/2}\left[\ln\left[\frac{\sqrt{s} + \sqrt{s-4m^2}}{\sqrt{s} - \sqrt{s-4m^2}}\right] - i\pi\right], \end{aligned} \tag{2.14}$$

and the integral $F_4(s,t)$ can be written [8] in terms of Spence functions as

$$\begin{aligned} F_4(s,t) &= 2\left[1 - \frac{4m^2(s+t)}{st}\right]^{-1/2} \\ &\quad \times \left[\mathrm{Sp}\left[\frac{x_+}{x_+ - y_+(s)} + i\sigma_s\varepsilon\right] + \mathrm{Sp}\left[\frac{x_+}{x_+ - y_-(s)} - i\sigma_s\varepsilon\right] - \mathrm{Sp}\left[\frac{-x_-}{x_+ - y_+(s)} + i\sigma_s\varepsilon\right]\right. \\ &\quad - \mathrm{Sp}\left[\frac{-x_-}{x_+ - y_-(s)} - i\sigma_s\varepsilon\right] + \mathrm{Sp}\left[\frac{x_+}{x_+ - y_+(t)} + i\sigma_t\varepsilon\right] + \mathrm{Sp}\left[\frac{x_+}{x_+ - y_-(t)} - i\sigma_t\varepsilon\right] \\ &\quad \left. - \mathrm{Sp}\left[\frac{-x_-}{x_+ - y_+(t)} + i\sigma_t\varepsilon\right] - \mathrm{Sp}\left[\frac{-x_-}{x_+ - y_-(t)} - i\sigma_t\varepsilon\right] + 2\pi i\Theta(st)\ln\left[\frac{x_+}{-x_-}\right]\right], \end{aligned} \tag{2.15}$$

where

$$x_{\pm}=\frac{1}{2}\pm\left[\frac{1}{4}-\frac{m^2(s+t)}{st}\right]^{1/2},$$
$$y_{\pm}(s)=\frac{1}{2}\pm\left[\frac{1}{4}-\frac{m^2}{s}\right]^{1/2}, \quad (2.16)$$

and σ_s denotes the sign of s.

Now we fix N to a finite value, the number of fermion doublets in the model. Then we set the scale v equal to $f/\sqrt{N}$, where $f=250$ GeV is the scale of symmetry breaking, to obtain

$$A(s,t,u)=\frac{s}{f^2}-\frac{Nm^2}{4\pi^2 f^4}sF_2(s) - \frac{Nm^4}{4\pi^2 f^4}[F_4(s,t)+F_4(s,u)-F_4(t,u)] \ . \quad (2.17)$$

In the limit $s \ll m^2$, the amplitude $A(s,t,u)$ approaches s/f^2, in accord with the low-energy result (1.2). The $W_L W_L$ scattering amplitudes are obtain by substituting Eq. (2.17) into Eq. (1.1). The partial waves are defined by

$$a_J^I=\frac{1}{64\pi}\int_{-1}^{1} d(\cos\theta)P_J(\cos\theta)T(I) \ , \quad (2.18)$$

where the isospin amplitudes $T(I)$ are given by

$$T(0)=3A(s,t,u)+A(t,s,u)+A(u,t,s) \ ,$$
$$T(1)=A(t,s,u)-A(u,t,s) \ , \quad (2.19)$$
$$T(2)=A(t,s,u)+A(u,t,s) \ .$$

The model we have just presented depends on only two parameters: the number of degenerate fermion doublets N and the fermion mass m. In Sec. III, we will examine the behavior of this model in the heavy fermion limit. In Sec. IV, we will consider the model with a large number of somewhat lighter fermions ($m \sim 250$ GeV), in order to study the effect of inelastic channels on elastic $W_L W_L$ scattering in the TeV region.

III. THE HEAVY FERMION LIMIT

In this section, we will examine the effect of very heavy fermions on elastic $W_L W_L$ scattering at energies *below* the threshold for fermion pair production. The heavy fermion limit is obtained by letting the Yukawa coupling become large, holding the symmetry-breaking scale fixed at $f=250$ GeV. This procedure is legitimate in our model, since we have calculated the scattering amplitudes nonperturbatively in the Yukawa coupling (to leading order in $1/N$).

We consider a model with $N=3$ species of degenerate fermion doublets of mass $m=1$ TeV. This corresponds to one additional generation of quark doublets, due to the color degeneracy. The nonvanishing $J=0$ and $J=1$ partial waves of the elastic $W_L W_L$ scattering amplitudes are shown by the solid lines in Fig. 1. These are to be compared with the partial waves of the low-energy amplitude $A(s,t,u)=s/f^2$, which are shown by the dotted lines. We note that the isospin $I=0$ and $I=1$ partial waves increase somewhat faster with energy due to the heavy fermions, violating unitarity at a lower scale. For $N=3$, the a_0^0 partial wave exceeds $\frac{1}{2}$ at around 1.2 TeV (with $|a_0^0|<1$ up to about 1.6 TeV); unitarity is violated at an even lower scale for larger N. On the other hand, the isospin $I=2$ partial wave, which alone contributes to the $W_L^{\pm} W_L^{\pm}$ scattering amplitude, is greatly reduced at high energies by heavy fermion loop effects.

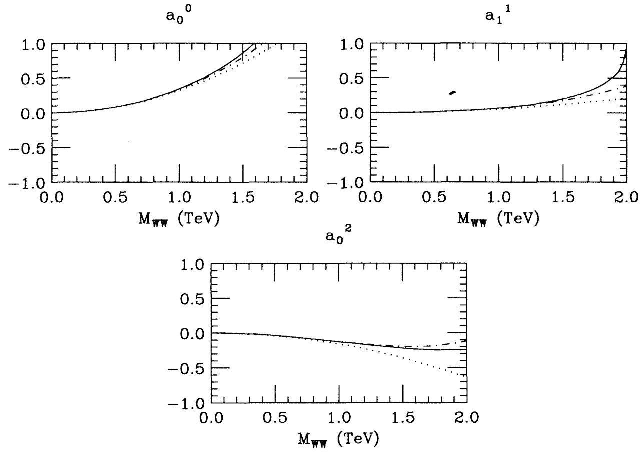

FIG. 1. The partial waves a_J^I with isospin I and angular momentum J of the elastic $W_L W_L$ scattering amplitudes for three fermion doublets with $m=1$ TeV (solid lines), with $m=\infty$ (dot-dashed lines), and in the absence of heavy fermions (dotted lines).

The effect of taking the fermion mass even larger can be seen by noting that the amplitude (2.17) has a well defined limit as $m \to \infty$, namely,

$$A(s,t,u)\to\frac{s}{f^2}-\frac{N}{48\pi^2}\frac{s^2}{f^4}+\frac{N}{48\pi^2}\frac{t^2+u^2}{f^4} \ , \quad m\to\infty \ . \quad (3.1)$$

The fermions do not completely decouple as their mass increases, but leave behind mass-independent contributions of $O(s^2)$. The partial waves of the amplitudes in this infinite mass limit (with $N=3$) are shown in Fig. 1 by the dot-dashed lines.

The $O(s^2)$ contributions to the scattering amplitudes (3.1) correspond to four-derivative terms in the low-energy effective Lagrangian [9,10]:

$$\mathcal{L}_4=\frac{L_1}{16\pi^2}\mathrm{Tr}(\partial_\mu\Sigma^\dagger\partial^\mu\Sigma)\mathrm{Tr}(\partial_\nu\Sigma^\dagger\partial^\nu\Sigma) + \frac{L_2}{16\pi^2}\mathrm{Tr}(\partial_\mu\Sigma^\dagger\partial_\nu\Sigma)\mathrm{Tr}(\partial^\mu\Sigma^\dagger\partial^\nu\Sigma) \ . \quad (3.2)$$

Very heavy fermions induce four-derivative terms with coefficients [11,12]

$$L_1=-\frac{N}{24} \ , \quad L_2=\frac{N}{12} \ . \quad (3.3)$$

These four-derivative terms are a linear combination of those induced by an isoscalar spin-zero resonance and those induced by an isovector spin-one resonance [11,13].

A heavy scalar particle with mass m_S and width $\Gamma_S = 3g^2 m_S^3/32\pi f^2$ induces four-derivative terms with coefficients

$$L_1 = \frac{64\pi^3 f^4 \Gamma_S}{3m_S^5}\ , \quad L_2 = 0\ . \tag{3.4}$$

(A standard model Higgs boson corresponds to $g=1$.) A heavy vector meson with mass m_V and width Γ_V induces four-derivative terms with coefficients

$$L_1 = -\frac{192\pi^3 f^4 \Gamma_V}{m_V^5}\ , \quad L_2 = \frac{192\pi^3 f^4 \Gamma_V}{m_V^5}\ . \tag{3.5}$$

Thus N heavy fermion doublets induce $O(s^2)$ terms in the scattering amplitude equivalent to those induced by the combination of a scalar particle with $m_S^5 = 512\pi^3 f^4 \Gamma_S/N$ and a vector particle with $m_V^5 = 2304\pi^3 f^4 \Gamma_V/N$. In the model with three degenerate very heavy fermion doublets, the four-derivative terms are equivalent to those of a scalar resonance with $m_S \sim \Gamma_S \sim 2$ TeV together with a scaled-QCD ρ with $m_V \sim 2$ TeV and $\Gamma_V \sim 0.4$ TeV. At an energy scale much less than the masses of the fermions and the scalar and vector resonances, it is not possible to differentiate these two symmetry-breaking mechanisms. Thus, it is important to probe longitudinal W interactions in the TeV region.

IV. INELASTIC CHANNELS IN $W_L W_L$ SCATTERING

The primary goal of this paper is to examine the effect of inelastic channels on elastic $W_L W_L$ scattering. In Sec. II, we presented a model containing additional species of heavy fermions, which can be produced in $W_L W_L$ scattering. The $W_L W_L$ scattering only becomes inelastic above the threshold for fermion pair production, however, so we do not want the fermions to be too heavy. In this section, we will choose the fermion mass to be $m = 250$ GeV so that $W_L W_L$ scattering in this model will be inelastic in an energy range accessible to the SSC ($\sqrt{S} = 40$ TeV).

The greater the number of additional fermion species, the more pronounced will be their effect on elastic $W_L W_L$ scattering. There is an experimental constraint on the number N of additional fermion doublets, however, due to their contribution to Δr [14], or equivalently, to the S parameter [15]. For N degenerate heavy fermion doublets, the shift in Δr is

$$\Delta r = \frac{\alpha N}{12\pi \sin^2\theta_W}\ , \tag{4.1}$$

where θ_W is the weak mixing angle defined in the (α, G_F, M_Z) scheme. Equivalently, the shift in the S parameter is [16]

$$S = \frac{N}{6\pi}\ . \tag{4.2}$$

The bounds $\Delta r \lesssim 0.015$ or $S \lesssim 1$ require $N \lesssim 16$. In this section, we will choose $N = 15$, corresponding to five additional generations of degenerate quark doublets, due to the color degeneracy.

In Fig. 2, we show the nonvanishing $J=0$ and $J=1$ partial waves of elastic $W_L W_L$ scattering amplitudes for the model with $N=15$ and $m=250$ GeV. This figure also shows the partial waves for the low-energy result $A(s,t,u) = s/f^2$. The magnitudes of the a_0^0 partial waves in the model with 15 heavy fermion doublets and in the low-energy model are similar, and both remain unitary ($|a_0^0| < 1$) below 1.5 TeV. However, the heavy fermions greatly suppress the a_1^1 and a_0^2 partial waves relative to the low-energy result.

In Fig. 3, we show the constituent cross sections for $W_L W_L$ scattering. The contributions to the cross sections due to the real (solid lines) and imaginary (dashed lines) parts of the amplitudes are plotted separately. The total cross section is given by the sum of the two contributions. We also show (dotted lines) the cross sections for the low-energy result.

We see from Fig. 3 that the cross sections for scattering into the $W_L^+ W_L^-$ and $Z_L^0 Z_L^0$ modes are somewhat increased in the heavy fermion model relative to the low-energy result, because of enhancement from the loop contribution to the scattering amplitudes. The cross sections for the $W_L^+ W_L^-$ and $Z_L^0 Z_L^0$ modes in the TeV region are almost entirely due to the imaginary forward part of the amplitude, the effect becoming stronger for higher M_{WW}. The imaginary forward part of the amplitude is related by the optical theorem to the total cross section, which is large due to the presence of many inelastic channels [17].

The cross sections for the $W_L^+ Z_L^0$ and $W_L^+ W_L^+$ modes, however, are greatly reduced in the heavy fermion model relative to the low-energy result. The $W_L^+ W_L^+$ cross section at $M_{WW} = 1.5$ TeV is down by about a factor of 10 from the low-energy result, and the $W_L^+ Z_L^0$ mode is reduced by a similar factor. This occurs because the tree level amplitude is largely canceled by the real part of the loop amplitude, and the imaginary part of the amplitude is zero for $W_L^+ W_L^+$ and small for $W_L^+ Z_L^0$.

The event rates for scattering processes at the SSC are

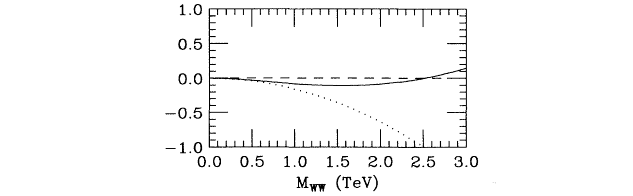

FIG. 2. The partial waves a_J^I of the elastic $W_L W_L$ scattering amplitudes for 15 fermion doublets with $m = 250$ GeV (solid lines = real part, dashed lines = imaginary part), and in the absence of heavy fermions (dotted lines = real part, no imaginary part).

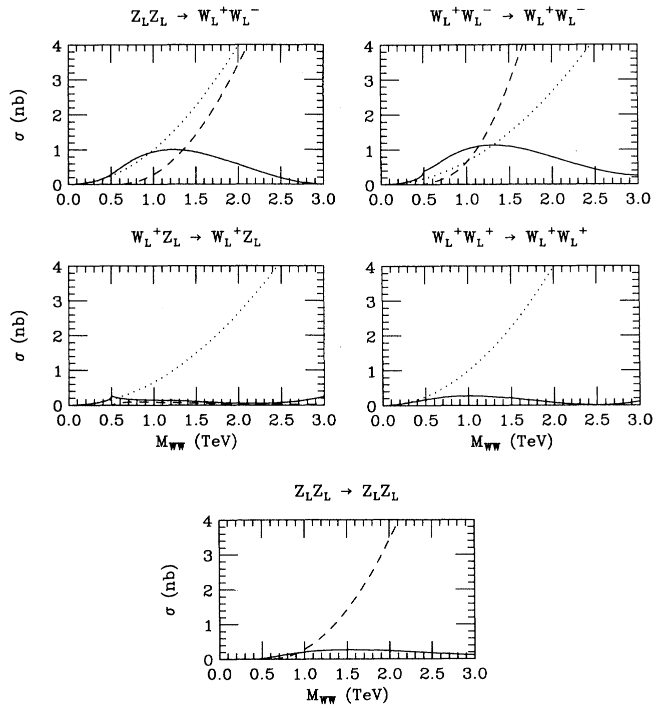

FIG. 3. Constituent cross sections in the model with 15 fermion doublets of mass 250 GeV. The solid (dashed) lines show the contributions due to the real (imaginary) part of the amplitude. The dotted lines show the constituent cross sections in the model with no heavy fermions.

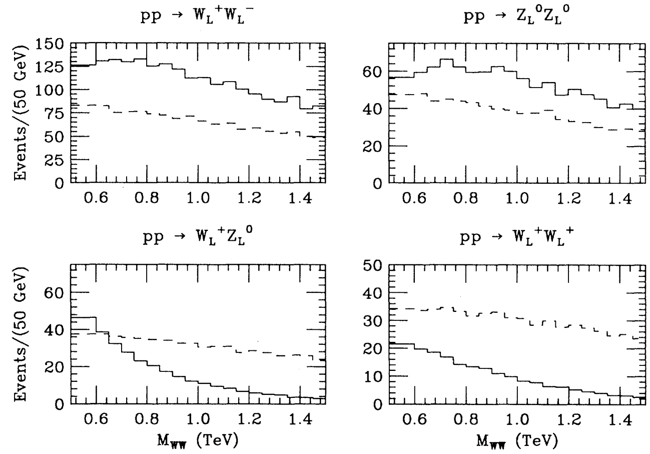

FIG. 4. Event rates in the model with 15 fermion doublets of mass 250 GeV. The solid lines show the invariant mass M_{WW} distributions of this model. The dotted lines show the event rates in the model with no heavy fermions.

obtained by folding these constituent cross-sections with the parton luminosities. Our purpose here is not to study the phenomenology of the heavy fermion model in detail, but to illustrate the differences between this model and the low-energy model [18]. In Fig. 4, we show the production rates of longitudinal W pairs in one SSC year (with 10^4 pb^{-1}) for various charge modes as a function of the invariant mass M_{WW}. As we anticipated, the $W_L^+W_L^+$ and $W_L^+Z_L^0$ event rates in the heavy fermion model are greatly reduced in the TeV region. The $W_L^+W_L^-$ and $Z_L^0Z_L^0$ event rates are increased, but by less than a factor of 2 for $M_{WW} \lesssim 1.5$ TeV. In Fig. 4, the branching ratio of the W boson decay has not been included. Each decay product of the W's is required to have a minimum transverse momentum of 20 GeV within rapidity ± 2.5. The event rate is calculated using the effective-W approximation. The parton distribution function used is the leading order set, fit SL, of Morfin and Tung [19]. The scale used in evaluating the parton distribution function in conjunction with the effective-W method is M_W.

As seen from Fig. 2, the a_0^0 partial wave of the heavy fermion model does not differ significantly from that of the low-energy model up to the scale (~ 1.5 TeV) at which they both violate unitarity. Beyond 1.5 TeV, $|a_0^0|$ exceeds 1, at which point new physics must enter to unitarize the physical amplitudes. In the literature, various methods, such as saturating or unitarizing the partial waves, have been used to suggest what might happen when the partial waves of the low-energy model violate perturbative unitarity. Using these methods, we expect the a_0^0 partial waves to have roughly the same behavior in both the heavy fermion model and the low-energy model. However, the a_1^1 and a_0^2 partial waves in the heavy fermion model remain small relative to those in the low-energy model. This implies that, for $M_{WW} \lesssim 3$ TeV, the $W_L^\pm W_L^\pm$ and $W_L^\pm Z_L^0$ event rates predicted by this model will always be much smaller than the corresponding event rates in the low-energy model.

V. CONCLUDING REMARKS

In this paper, we have examined the effects of inelastic channels in the $W_L W_L$ scattering process in one specific model. In this model, containing heavy fermion doublets, the rate of elastic $W_L W_L$ scattering at energies above the threshold for fermion pair production differs significantly from the low-energy result. In particular, we found a large suppression of the $W_L^\pm W_L^\pm$ mode. This implies that, even when the symmetry-breaking sector does not contain any resonances, the $W_L^\pm W_L^\pm \to W_L^\pm W_L^\pm$ interaction does not necessarily become strong as it does in the low-energy model. A lesson to be drawn from this model is that all charge modes of the $W_L W_L \to W_L W_L$ process need to be observed to be sure of detecting the symmetry-breaking sector.

ACKNOWLEDGMENTS

It is a pleasure to thank G. L. Kane for asking the questions which stimulated this work, and for many fruitful discussions. We are also grateful to J. Bagger, J. Bjorken, Gordon Feldman, B. Grinstein, C. Im, G. Ladinsky, S. Meshkov, S. Mrenna, F. Paige, and E. Poppitz for discussions. The work of S.G.N. has been supported by the National Science Foundation under Grant No. PHY-90-96198. The work of C.P.Y. was funded in part by TNRLC Grant No. RGFY9240.

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