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Fermi theory of nuclear β decay and heavy neutrino searches

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We examine the data analysis of nuclear β decay of ^{3}H, ^{14}C, ^{35}S, and ^{63}Ni, using a relativistic Fermi function for a Hulthen screened field. This Fermi function does not differ much from that used by Hime, Jelley, and Simpson in the cases of light nuclei, but in the cases of sulfur and nickel reduces significantly the claimed spectral excess and, hence, the mixing of the heavy neutrino. We also discuss radiative corrections to ^{35}S data.

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Recent controversy regarding the existence of a heavy neutrino in nuclear β decay has attracted much attention [1–7]. At issue is the reported spectral excess in β decay which is interpreted by authors of Refs. [1,3,4] as the evidence of a 17-keV neutrino with a 0.8% to 1.6% mixing. Since the claimed mixing is of the order 1% and since many factors are involved in the data analysis, corrections of the same order ($>0.1\%$) should be addressed and fully investigated.

The dimensionless Fermi function is an important factor affecting β decay. It plays an important role in the decay process, and its screening is important especially in the case of heavy nuclei or low energies [8]. In this note we examine the effect of an alternative Fermi function on different decay processes in the energy regions of interest relevant to the reported heavy-neutrino signal [9]. In particular, we use a relativistic Fermi function obtained by Durand [8] based on an exact solution to the Klein-Gordon equation with a Hulthen potential for the screened field and compare it to the screened Fermi functions employed by Hime, Jelley, and Simpson [1,3,4]. It is found that while the two different approaches do not differ significantly in the cases of ^{3}H and ^{14}C, our results reduce the spectral excess by about 0.3% and 1% for ^{35}S and ^{63}Ni, respectively. This undoubtedly calls for further investigation regarding theoretical uncertainties which are larger than 0.1% in order to fully understand the issue of heavy-neutrino mixing. The correction in this Rapid Communication makes the excess signal in the ^{63}Ni data of Ref. [4] totally vanish.

The screened Fermi function was first studied by Rose [10] using the WKB method. Further calculations have been done since then, based both on exactly solvable models [8,11] and on approximation methods [12,13]. In allowed nuclear β decay, one can write the decay spectra, assuming only a zero mass neutrino, as

$$\frac{dN(Z,E)}{dE} \sim pEF(Z,E)R(E)C(E)\sum\{P_n(Q-T-\varepsilon_n)^2\}\ , \tag{1}$$

where p, E, T, and Q are the electron momentum, total energy, kinetic energy, and its maximum allowed kinetic energy, and Z is the atomic number for the daughter nucleus. The function $R(E)$ is for radiative corrections which we will briefly discuss toward the end. The shape correction $C(E)$ is usually not large [14]. P_n is the probability for a final state n with energy ε_n. For the purpose of this paper we take $n=1$, $P_n=1$, and $\varepsilon_n \sim 0$. We shall concentrate on $F(Z,E)$. It affects the data analysis in the Kuri plot,

$$K=\left[\frac{dN(Z,E)/dE}{pEF(Z,E)R(E)C(E)}\right]^{1/2}\ , \tag{2}$$

and the shape function defined as

$$S=\frac{N_{\text{expt}}}{N_{\text{theory}}(M_\nu=0)}\ . \tag{3}$$

If the neutrino is truly massless, the Kurie plot should be a linear function of $Q-T$ following Eqs. (1) and (2). The shape function in (3) should be unity when proper normalization is taken for a fixed end point Q. Any exotic feature will certainly give rise to a deviation from linearity. Therefore the Fermi function $F(Z,E)$ is crucial when the deviation of concern is of order 1% or less, as is the reported heavy neutrino mixing.

In Refs. [1,3,4], to account for the electron shielding, the authors employed an energy shift V_0 in the unscreened Fermi function of Coulomb interaction between the emitted β particle and the charge Z of the daughter nucleus, according to Rose's approximation [10]. The unscreened Fermi function is of the form

$$F^{us}(Z,E)=F_0(Z,E)f(\beta)\ , \tag{4}$$

where the relativistic factor $F(\beta)$ is a polynomial of β $(=v/c)$ and $F_0(Z,E)=X/(1-e^{-X})$, $X=2\pi Z\alpha/\beta$, is the nonrelativistic limit of the Fermi function, where α is the fine structure constant. The coefficients in the polynomial vary in different decay processes according to a fit to the results of radial solutions of the Dirac equation using a pure Coulomb potential [12].

In order to provide a more detailed study on this issue from a *different* point of view, we adopt the relativistic Fermi function in Ref. [8], obtained by solving the exact Klein-Gordon equation with the Hulthen potential:

$$V(r)=\frac{Z\alpha\lambda\exp(-\lambda r)}{1-\exp(-\lambda r)}\ , \tag{5}$$

where the screening strength is denoted by λ. The screened Fermi function $F^s(Z,E)$ is defined as the value of the quantity $|u(r)/pr|^2$ at the nuclear surface $r=R$, where $u(r)$ is the radial wave function. It is given as

$$F^s(Z,E)=(\lambda R)^{2\sigma-2}\left|\frac{\Gamma(\sigma+i\nu)\Gamma(\sigma-i\xi)}{\Gamma(2\sigma)\Gamma(1-2i\kappa)}\right|^2\ , \tag{6}$$

where σ, ν, ξ, and κ are dimensionless and are functions of p, E, and λ [15,16]. This function may be more reliable in the cases of heavy nuclei and low energy β decay of light nuclei [8], both cases being of much interest to us. Similar results were also obtained by Eman [11,16].

The screening strength λ in (6) is determined using the energy shift V_0 from the approximation of the Hulthen potential at small r:

$$V(r)\rightarrow\frac{Z\alpha}{r}-\frac{1}{2}Z\alpha\lambda \quad \text{and} \quad \lambda=\frac{2V_0}{Z\alpha}\ . \tag{7}$$

This is based on the fact that when the screening is weak $(\lambda\rightarrow 0)$, the potential $V(r)$ is essentially the Coulomb potential plus a shift V_0, as Rose pointed out. We then compare Fermi function (6) with Eq. (4) with the energy shift $E\rightarrow E-V_0$, using

$$G(Z,E)=pEF(Z,E)\ . \tag{8}$$

In both approaches, the energy shift takes the value [8]

$$V_0=1.45Z^{4/3}\alpha^2 m_e\ , \tag{9}$$

for all nuclei except ^{3}H, whose V_0 can be specifically determined [11,13,17–19]. For a detailed estimation of V_0 the reader is referred to Refs. [16–19]. We draw the ratio of two different $G(Z,E)$ functions for ^{3}H and ^{14}C in Fig. 1, normalized near 4 keV for ^{3}H and around the end point for ^{14}C. The difference in the two ratios is small, at the level of a few$\times 10^{-4}$. The results for ^{35}S and ^{63}Ni are given in Fig. 2. They show a significant difference between the two $G(Z,E)$ functions in these nuclei: about 0.3% and 1% for sulfur and nickel, respectively. The correction due to the relativistic screened Fermi function from the Hulthen-Klein-Gordon equation is larger in the region of lower energy and smaller toward the end point, compared with the correction of Eq. (4) with the potential shift V_0. Therefore, the use of the Fermi function (6) should reduce the reported spectral excess by about 0.3%

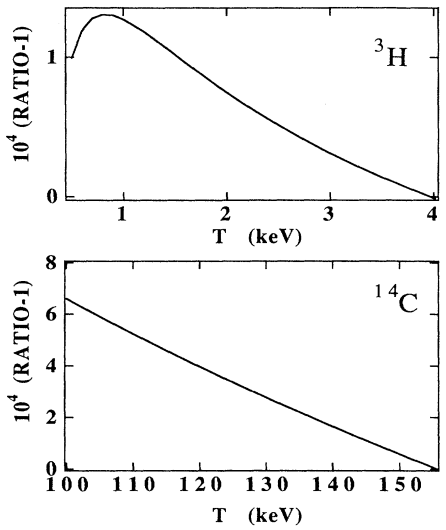

FIG. 1. The ratio of the two $G(Z,E)$ functions—see Eq. (8) *and* text—for ^{3}H and ^{14}C in their respective regions of interest. The ratio has been normalized such that the size is 1 near 4 keV for ^{3}H and near the end point for ^{14}C.

and 1% for sulfur and nickel β decay, respectively. To study the impact of V_0, we vary its value by $\pm 50\%$ in the case of sulfur. As shown in Fig. 3, the ratio of the two $G(Z,E)$ functions changes only slightly its energy dependence on V_0 when normalized around the end point Q. Therefore the reduction of the spectral excess in the energy region below $Q-17$ keV will remain reasonably unaffected regardless of the exact choice of the potential shift. Thus screening does not play an important role in the difference between the two Fermi functions.

Clearly for tritium the difference between the two Fer-

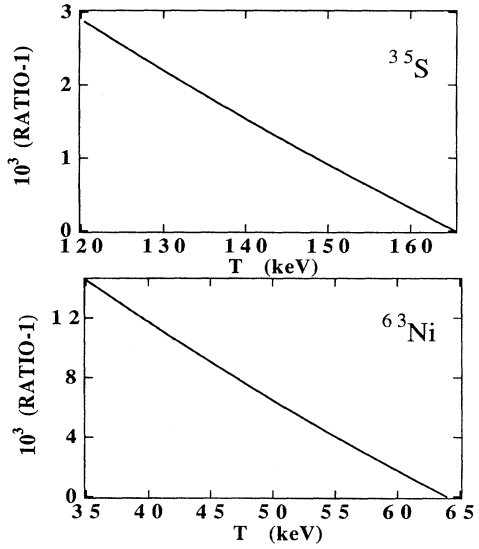

FIG. 2. The ratio of the two $G(Z,E)$ functions for ^{35}S and ^{63}Ni in their respective regions of interest. The ratio has been normalized such that the size is 1 near the end point.

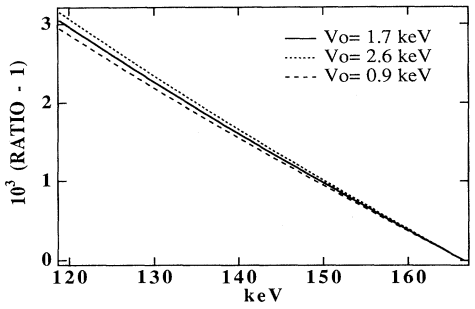

FIG. 3. The ratio of the two $G(Z,E)$ functions for ^{35}S with different values of V_0 (V_0 and $V_0 \pm 50\%$), all normalized such that the size is 1 near the end point.

mi functions is negligible. This is expected since both the atomic and nuclear structures of ^{3}H are relatively simple and hence the Coulomb interaction is much simpler; and although the screening is more significant, it is more understood, thanks in part to the last 20 years of tritium end-point experiments. However, the case involving heavy nuclei is of a more complex nature. To be specific, we apply the correction shown in Fig. 2 to ^{63}Ni β-decay data. We simulate the spectrum assuming a 17-keV neutrino of 1% mixing. The statistical accuracy is in accordance with the Hime-Jelley ^{63}Ni data [4], about 0.6 million events per keV for electron kinetic energies near 50 keV. The result is shown in Fig. 4(a), along with the ratio (dashed line) of the two different $G(Z,E)$ for the case of ^{63}Ni, as given in Fig. 2. This ratio, applied to the data, represents a correction due to the screened Fermi function from the Hulthen-Klein-Gordon equation. The deviation of the simulated data from the dashed line is shown in Fig. 4(b) to be consistent with zero within errors. The spectral excess is reduced to a level no different than the case of a pure zero-mass neutrino. It is thus natural to compare the differences among various models. The striking feature manifested in our calculation, which pushes the heavy-neutrino mixing to a lower value, certainly calls for further investigation.

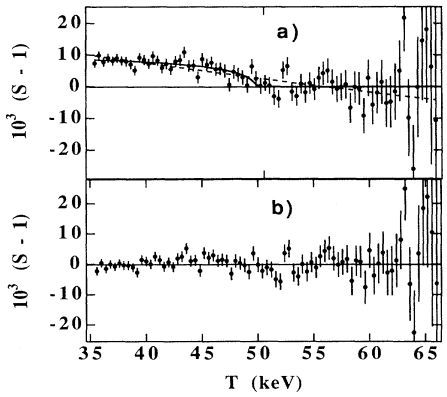

FIG. 4. (a) The simulated ^{63}Ni data (points with error), assuming a 17-keV neutrino with 1% mixing which is represented by the solid curve. The zero line represents the null theory. The dashed line is the ratio of Hulthen-Klein-Gordon $G(Z,E)$ function to that of authors in Refs. [1,3,4]. The ratio has been normalized to fit the ^{63}Ni simulated data. (b) The simulated ^{63}Ni data after we apply the correction due to the screened Fermi function from the Hulthen-Klein-Gordon equation; it is statistically consistent with zero [$\chi^2 = 80$ for 77 degrees of freedom (39% C.L.) compared with 68 for the 17-keV neutrino (75% C.L.)].

Since screening does not contribute much to our findings one may argue that the absence of spin in the Klein-Gordon case is behind the difference between the two Fermi functions. But also since spin is summed over in the Dirac case, one would expect that it does not play an important role either. This seems to be supported by the fact that the spin did not make a difference in the tritium case as shown in Fig. 1. However, we acknowledge that the effect of spin is not exactly known. On the other hand one can speculate that it is the treatment of the potential in the two different approaches that is making a major difference. This issue deserves further investigation in its own right and is beyond the scope of our paper [20].

It is worthwhile to notice that the difference between the two Fermi functions is linearly dependent on T, as shown in Figs. 1 and 2. It coincides with the usual type of shape correction factor $C(E)$ which contains a piece proportional to $Q-T$ [14]. If one includes such an overall piece in the data analysis (as done by a number of experiments; e.g., see Ref. [2]), the correction due to the screened Fermi function from the Hulthen-Klein-Gordon equation and other effects may be absorbed. Since the analyses of both ^{35}S measurements of Guelph and Oxford and ^{63}Ni measurement of Oxford [3,4] do not contain $C(E)$, the impact of different Fermi functions should be carefully examined.

The radiative correction $R(E)$ ($=1+\Sigma\Delta R_i$) in Eq. (1) also affects the overall spectra of β decay, especially at high energies. The lowest-order term is energy dependent,

$$\Delta R_1 = \frac{\alpha}{2\pi} f(E,Q) , \tag{10}$$

while the higher-order terms do not show significant energy dependence [17,21]. Figure 5 shows ΔR_1 for both tritium and sulfur in their respective energy ranges of in-

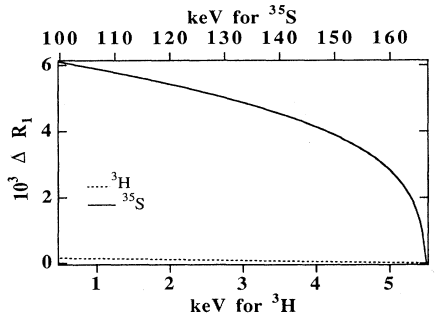

FIG. 5. The energy dependence of the radiative correction ΔR_1 (see text) for sulfur and tritium, normalized to be zero at energies of 167 keV and 6 keV, respectively.

terest. This correction for ^{3}H is rather insignificant as expected. However in the case of ^{35}S, it provides an energy-dependent change of about 0.6% between 100 to 167 keV. Clearly this correction is important and should be considered in analyzing the ^{35}S data [22]; it suppresses the higher-energy events compared to the lower-energy events and hence one obtains extra mixing if $R(E)$ is omitted from the fit. The author of Ref. [4] attributed to $R(E)$ only about 0.08% reduction in mixing (i.e., changes the mixing by 9%), for $\sim$0.15% relative change in the shape of ^{35}S data over the energy region of interest. Our calculation [6,21], shown in Fig. 5, agrees with that of Ref. [17] for various Q values. Our calculation gives a 0.4% relative change in the region between 120 to 166 keV. This is a factor of 2.7 larger than that of Ref. [4]; therefore, the mixing should be reduced by 0.21% (i.e., changing the mixing by 25% instead of 9%). We agree with Ref. [4] on the effect of $R(E)$ but we disagree on the size of $R(E)$ and hence on the amount of the reduction of the mixing. The extra mixing obtained by Hime and Jelley [3,4] for ^{35}S translates into a significant increase in χ^2 due to the neglect of $R(E)$ in the fit. For further details on this issue the reader is directed to our Monte Carlo studies of Ref. [23] in which we found that the absence of $R(E)$ from the fitted theory (while it is present in the simulated data) produce a 1% mixing compared to the 0.8% input mixing of the Monte Carlo simulation. In addition, it is found that the difference in endpoint energy between the null and positive fits (ΔQ) is 40 eV when $R(E)$ is present in both data and fit and 53 eV when $R(E)$ is present in data but absent in the fit [24]. The 13-eV discrepancy in ΔQ was also pointed out by Piilonen and Abashian [25] as a compensation for the neglect of $R(E)$ in the fit. This compensation however does not work for energy points above 164 keV. Instead, for an absolute minimum χ^2 extra mixing is favored rather than shifted end-point energy by a few units of χ^2 coming from the points above 164 keV. Clearly the neglect of $R(E)$ reduces both the mixing and significance of the Hime-Jelley findings [26].

In conclusion, we have demonstrated that the 17-keV neutrino signal can be reduced to such a level that the very existence of the massive particle becomes uncertain. The reduction comes from different sources in data analysis, such as the model-dependent screening Fermi function, overall shape corrections, radiative corrections, and even adjustment of maximum allowed decay energy. Even though the Fermi function used in this note is from a simple Hulthen model and the electron spins have been neglected, together with the discussion on several other crucial factors, it serves the purpose of providing a different viewpoint and casting a different light on the issue of heavy neutrino searches.

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[9] The region of interest is taken to be that for which a 17-keV signal has been looked for experimentally. That corresponds to about the first 3 to 4 keV in ^{3}H, 100 to 154 keV for ^{14}C, 120 to 167 keV for ^{35}S, and 35 to 67 keV for ^{63}Ni.

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[15] The arguments of the screened Fermi function (6) read (see Ref. [8]) $\sigma=\frac{1}{2}+\frac{1}{2}[1-(4b/\lambda^2)]^{1/2}$, $\xi=\lambda^{-1}[p^2+a+b]^{1/2}+p/\lambda$, $\nu=\lambda^{-1}[p^2+a+b]^{1/2}-p/\lambda$, $\kappa=p/\lambda$, where $a=2EZ\alpha\lambda$ and $b=(Z\alpha\lambda)^2$. The units here are such that $\hbar=c=1$. The complex Γ functions were evaluated using MATHEMATICA on a Mac IIci computer. To verify the validity of the results, we separated the complex Γ function into real and imaginary parts and calculate each individually (using a Vax code). The MATHEMATICA results were confirmed.

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reference confirmed our conclusion that the symbol κ in Eq. (12) in the Durand paper [8] is a typo and should read λ instead, as is shown in Eq. (6) of this paper. This is also apparent from dimensional considerations alone.

[17] See, e.g., H. Behrens and W. Buhring, *Electron Radial Wave Functions and Nuclear Beta-Decay* (Clarendon, Oxford, 1982).

[18] For ^{3}H we used $V_0=35$ eV (see Ref. [6]). For other nuclei the value 1.45 in Eq. (9) is not a well-known quantity; it varies between 1 and 2 as a function of Z (e.g., see Refs. [8,16,17]).

[19] See, e.g., I. G. Kaplan *et al.*, Phys. Lett. **161B**, 389 (1985); J. Lindhard and P. G. Hansen, Phys. Rev. Lett. **57**, 965 (1986).

[20] The origin of the difference between the two considered Fermi functions is important beyond the issue of the 17-keV neutrino. For example, the rate of superallowed β decays (ft value) exhibits some quadratic Z dependence from which an extrapolation to $Z=0$ gives a value of 3070±2.5 s [see Fig. 1 of D. H. Wilkinson, Phys. Lett. B **241**, 317 (1990)]. Our correction above will certainly change the Z dependence (our crude estimate is that it will more nearly linearize the dependence). It will not change the extrapolation to $Z=0$ much and hence not affect the underlying Fermi constant, since our effect vanishes as $Z\rightarrow 0$.

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[22] The published ^{35}S data by Hime and Jelley Ref. [3] did not include this radiative correction, which reduces their mixing and significance; see Ref. [6].

[23] M. Bahran and G. R. Kalfleisch, this issue, Phys. Rev. D **47**, R754 (1993).

[24] A downward shift δQ in the end-point energy Q causes the shape specturm to be divided by $(1-\delta Q/[Q-T])^2$, bringing up the shape spectrum near Q.

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[26] The significance of the 17-keV claim is based on the difference in χ^2 between the positive (p) and the null (n) fits for the same data set. This in turn is a function of the ratio between the null and the positive theory. Most (if not all) of the χ^2 is coming from the region of kinetic energies above $Q-m_\nu$, in which the ratio of the two theories is proportional to $[1/(1-M)](1-\Delta Q/[Q-T])^2$, where M is the heavy neutrino mixing and $\Delta Q=Q_p-Q_n$. Clearly with radiative corrections missing from the Hime-Jelley data we have 13-eV uncertainty in ΔQ due to a 25% uncertainty in M. This will translate into a very significant uncertainty in the χ^2 findings. In fact, it turns out that a 10-eV difference in ΔQ would cause a change of 30 units of χ^2 for the Hime-Jelley data (see Ref. [23]).