RAPID COMMUNICATIONS

PHYSICAL REVIEW D VOLUME 47, NUMBER 12 15 JUNE 1993

Quantum Einstein-Maxwell fields: A unified viewpoint from the loop representation

Rodolfo Gambini
Instituto de Física, Facultad de Ciencias, Tristan Narvaja 1674, Montevideo, Uruguay

Jorge Pullin
Department of Physics, University of Utah, Salt Lake City, Utah 84112
(Received 20 October 1992)

We propose a naive unification of electromagnetism and general relativity based on enlarging the gauge group of Ashtekar's new variables. We construct the connection and loop representations and analyze the space of states. In the loop representation, the wave functions depend on two loops, each of them carrying information about both gravitation and electromagnetism. We find that the Chern-Simons form and the Jones polynomial play a role in the model.

PACS number(s): 04.50.+h, 04.20.Fy, 04.60.+n

I. INTRODUCTION

The introduction of the Ashtekar new variables [1] for the treatment of canonical gravity has opened new hopes that general relativity may be nonperturbatively quantized. In particular the loop representation [2] allows us to construct for the first time physical states of quantum gravity without recurring to minisuperspace approximations [3]. Knot theory and in particular the Jones polynomial play a crucial role in the theory [4].

The main successes of this program at the quantum level have up to the present largely referred to vacuum general relativity. It is evident that it would be desirable to extend some of these results to the case in which there are matter fields present in order to make contact with physics at energies lower than those where gravity is the only dominant interaction.

Another point is that many authors [5] have argued that only by taking into account the quantum properties of the matter that form the reference frames can physical quantum observables be defined in quantum gravity. Although the construction of [5] requires very specific kinds of matter, it is an extra motivation for studying the incorporation of matter to the theory as a general issue.

In this work we will suggest that the idea of a unified theory described in terms of Ashtekar's new variables is possible and that several appealing results of the vacuum theory find very naturally their counterpart in the unified model. We will show, for instance, that knot theory still plays a crucial role and that the techniques used to find states for the theory in the vacuum case are still applicable. In summary, these ideas for quantizing gravity can lead to interesting new insights also in the case where matter fields are present and therefore are well suited for understanding the physics of particles at the energies of unification and not just pure gravity.

The idea of unifying gravity with other forces enlarging the group of Ashtekar variables is not new [6–8]. However the program outlined in this paper is less ambitious than others. We do not pretend to recover the same form of the constraints as the vacuum ones just with an enlarged group. We will see, however, that the "kinematic" constraints can actually be rewritten in this way. This will be enough to find several connections between results of the vacuum theory and the unified theory. The Hamiltonian constraint will be quite different and we will briefly outline the consequences of this. Although we will only give details for the Einstein-Maxwell case, the same ideas can be straightforwardly generalized to Einstein-Yang-Mills theories for SU(N).

II. EINSTEIN-MAXWELL THEORY IN TERMS OF A U(2) CONNECTION

We begin with a brief summary of the Einstein-Maxwell theory in terms of Ashtekar's new variables. We will assume a 3+1 foliation of spacetime has been performed, with spatial three-surfaces Σ on which all variables are defined. The variables for the gravitational part are a (densitized) triad $\tilde{E}^a_i$, which determines the spatial metric by $\tilde{\tilde{q}}^{ab} = \tilde{E}^a_i \tilde{E}^b_i$ and an SU(2) connection A^i_a as conjugate momentum. Throughout the paper we will denote densities of weight +1 with an overtilde and of weight −1 with an undertilde. For the Maxwell field the variables are the electric field $\tilde{\mathbf{e}}^a$ and the vector potential $\mathbf{a}_a$. The dynamics of the theory is pure constraint, and the constraints (with the inclusion of a cosmological constant Λ) are [9]

$$\partial_a \tilde{\mathbf{e}}^a = 0, \tag{1}$$

$$D_a \tilde{E}^a = 0, \tag{2}$$

$$i\sqrt{2}\tilde{E}^a_i F^i_{ab} - \tfrac{1}{2}\tilde{\mathbf{e}}^a \mathbf{f}_{ab} = 0, \tag{3}$$

$$\epsilon^{ijk}\tilde{E}^a_i \tilde{E}^b_j F^k_{ab} + \Lambda \det(E)^2 - \tfrac{1}{8}\det(E)^{-2}\{\tilde{E}^a_i \tilde{E}^b_i \tilde{E}^c_j \tilde{E}^d_j \underset{\sim}{\eta}_{acf} \underset{\sim}{\eta}_{bdg}[\tilde{\mathbf{e}}^f \tilde{\mathbf{e}}^g + \tilde{\mathbf{b}}^f \tilde{\mathbf{b}}^g]\} = 0, \tag{4}$$

where $\det(E)^2 = \frac{-1}{3\sqrt{2}} \underset{\sim}{\eta}_{abc} \epsilon^{ijk} \tilde{E}^a_i \tilde{E}^b_j \tilde{E}^c_k$, D_a is the covariant derivative built with the Ashtekar connection with action on SU(2)-valued quantities $D_a \lambda^i = \partial_a \lambda^i + \epsilon^{ijk} A^j_a \lambda^k$, F^i_{ab} is the field strength built from the Ashtekar connection, and $\mathbf{f}_{ab}$ is the usual Maxwell field strength built from $\mathbf{a}_a$. $\tilde{\mathbf{b}}^a$ is the usual magnetic field density $\tilde{\mathbf{b}}^a = \tilde{\eta}^{abc} \mathbf{f}_{bc}$. We will use bold lower case letters to distinguish quantities of the Maxwell field from similar ones in the Ashtekar formulation of gravity.

Equation (1) is the Gauss law of the U(1) symmetry. Equation (2) is the Gauss law of Ashtekar's formalism, stemming from the invariance under triad rotations. Equation (3) is the diffeomorphism constraint and Eq. (4) is the Hamiltonian constraint. Notice that this constraint can be made polynomial by multiplying by $\det(E)^2$. Such a rescaling is potentially dangerous in the case where $\det(E) = 0$ and we will discuss its implications later on.

We will now show how to write these equations in terms of a single set of variables. We introduce a U(2) connection

$$\mathcal{A}_a = A^i_a \sigma_i + i\, \mathbf{a}_a \mathbf{1}, \tag{5}$$

and in a similar fashion a U(2) electric field,

$$\tilde{\mathcal{E}}^a = \tilde{E}^{ai} \sigma_i + \tilde{\mathbf{e}}^a \mathbf{1}. \tag{6}$$

That is, we are taking the direct product of U(1) and SU(2) to form a U(2) symmetry. We can similarly introduce a field tensor $\mathcal{F}_{ab}$ and a magnetic field $\tilde{\mathcal{B}}^a_i$. From these we can recover the original quantities by taking traces:

$$\tilde{E}^a = \tilde{\mathcal{E}}^a - \tfrac{1}{2} \mathrm{Tr}(\tilde{\mathcal{E}}^a), \tag{7}$$

$$\tilde{e}^a = \tfrac{1}{2} \mathrm{Tr}(\tilde{\mathcal{E}}^a), \tag{8}$$

and similarly for $\tilde{\mathcal{B}}^a_i$. There is some arbitrariness in the way in which we combine the two groups into U(2). Concretely, we could perform a relative constant rescaling of the electromagnetic and the gravitational quantities (as for instance $\tilde{\mathcal{E}}^a = \tilde{E}^{ai} \sigma_i + \text{const} \times \tilde{\mathbf{e}}^a \mathbf{1}$), which we arbitrarily fix to unity. We will see the implications of this fact later on.

The introduction of these quantities allows us to rewrite the constraint equations as

$$\mathcal{D}_a \tilde{\mathcal{E}}^a = 0, \tag{9}$$

$$\mathrm{Tr}(\tilde{\mathcal{E}}^a \mathcal{F}_{ab}) = 0, \tag{10}$$

$$\begin{aligned} &\tfrac{1}{6} \underset{\sim}{\eta}_{abc} \underset{\sim}{\eta}_{edf} \mathrm{Tr}(\tilde{\mathcal{E}}^a \tilde{\mathcal{E}}^b \tilde{\mathcal{E}}^c) \mathrm{Tr}(\tilde{\mathcal{E}}^e \tilde{\mathcal{E}}^d \tilde{\mathcal{B}}^f) + \underset{\sim}{\eta}_{abc} \underset{\sim}{\eta}_{edf} \mathrm{Tr}(\tilde{\mathcal{E}}^a \tilde{\mathcal{E}}^e) \mathrm{Tr}(\tilde{\mathcal{E}}^b \tilde{\mathcal{B}}^c) \mathrm{Tr}(\tilde{\mathcal{E}}^d \tilde{\mathcal{B}}^f) \\ &\quad - \underset{\sim}{\eta}_{abc} \underset{\sim}{\eta}_{edf} \mathrm{Tr}[\tilde{\mathcal{E}}^a \tilde{\mathcal{E}}^e) \mathrm{Tr}(\tilde{\mathcal{E}}^b \tilde{\mathcal{E}}^d) [\mathrm{Tr}(\tilde{\mathcal{E}}^c) \mathrm{Tr}(\tilde{\mathcal{E}}^f) - \mathrm{Tr}(\tilde{\mathcal{B}}^c) \mathrm{Tr}(\tilde{\mathcal{B}}^f)] + \tfrac{\Lambda}{36} \underset{\sim}{\eta}_{abc} \underset{\sim}{\eta}_{edf} \mathrm{Tr}(\tilde{\mathcal{E}}^a \tilde{\mathcal{E}}^b \tilde{\mathcal{E}}^c) \mathrm{Tr}(\tilde{\mathcal{E}}^d \tilde{\mathcal{E}}^e \tilde{\mathcal{E}}^f) = 0, \end{aligned} \tag{11}$$

where $\mathcal{D}_a$ is a covariant derivative built from the U(2) connection defined by Eq. (5). Notice that again we have rescaled the Hamiltonian constraint with a factor $\det(E)^2$ in order to make it polynomial. This means the model is strictly equivalent to Einstein-Maxwell theory only if one considers nondegenerate triads. From now on we will concentrate on nondegenerate triads only.

It is worthwhile noticing that this is just *a rewriting* of the equations, that is, the theory remains exactly the same. Therefore, for instance, the constraint algebra and the consistency of the theory with the reality conditions [9] are automatically preserved.

A remarkable fact of this construction is that the "kinematic" constraints, the Gauss law and the diffeomorphism constraint, look *exactly* the same as those of the vacuum theory, only evaluated for a different group U(2). This will allow us to import some ideas from the vacuum theory to the unified model.

III. CONNECTION REPRESENTATION AND THE CHERN-SIMONS FORM

We now attempt to quantize the theory in the "connection representation," that is, we take a polarization in which wave functions are functionals of the connection $\Psi[\mathcal{A}]$ and $\hat{\mathcal{A}}_a \Psi[\mathcal{A}] = \mathcal{A}_a \Psi[\mathcal{A}]$, $\hat{\tilde{\mathcal{E}}}^a \Psi[\mathcal{A}] = \frac{\delta}{\delta \mathcal{A}_a} \Psi[\mathcal{A}]$. In the vacuum theory two main results have been achieved with this representation: (a) the result of Jacobson and Smolin [10] that showed that Wilson loops constructed with Ashtekar's connection were solutions to the Hamiltonian constraint, and (b) the result of Kodama [11], later extended in Ref. [4], that showed that the exponential of the Chern-Simons form constructed with Ashtekar's connection was a solution to all the constraints with a cosmological constant.

Let us examine these two results in the context of our unified model. Let us start with result (a). Consider Wilson loops

$$W_\gamma(\mathcal{A}) = \mathrm{Tr}\left[\mathrm{P} \exp\left(\oint_\gamma dy^a \mathcal{A}_a(y)\right)\right].$$

For the vacuum case these quantities solve the Hamiltonian constraint. For our model, however, they fail to be solutions. This is due to two reasons. (1) Since we rescaled our constraint by the determinant of $\tilde{E}$, this determinant should be nonvanishing for the state of interest if one wants the model to be equivalent to Einstein-Maxwell theory defined by Eqs. (1)–(4). For all the known solutions of the vacuum theory based on Wilson Loops [12], the determinants always vanish and therefore we have to exclude them. (2) Even if one considered a loop with a generic triple intersection in order to make the determinant nonvanishing, it is unlikely that finite combinations of these Wilson loops will be able to solve the constraint. This is basically due to the fact that the Maxwellian part of the Hamiltonian constraint has a term $\tilde{\mathbf{b}}^2$ which is purely multiplicative in this representation and therefore cannot be canceled by any of the other terms. So there seems to be no analogue of result (a) in our model.

We move on to result (b). Let us now consider the state

$$\Psi_\Lambda^{\rm CS}[\mathcal{A}] = \exp\left(-\frac{6}{\Lambda}{\rm Tr}\int \mathcal{A}\wedge d\wedge\mathcal{A} - \frac{2}{3}\mathcal{A}\wedge\mathcal{A}\wedge\mathcal{A}\right). \tag{12}$$

Such a state is an eigenstate of the electric field, with an eigenvalue proportional to the magnetic field:

$$\hat{\tilde{\mathcal{E}}}^a \Psi_\Lambda^{\rm CS}[\mathcal{A}] = -\tfrac{3}{\Lambda}\tilde{\eta}^{abc}\mathcal{F}_{bc}\Psi_\Lambda^{\rm CS}[\mathcal{A}] = -\tfrac{3}{\Lambda}\tilde{\mathcal{B}}^a\Psi_\Lambda^{\rm CS}[\mathcal{A}]. \tag{13}$$

With this formula, it is immediate to see that the state (with $\Lambda = 3$) is annihilated by the quantum Hamiltonian constraint of the unified model (11). The only requirement is to have a factor ordering with the $\mathcal{E}$'s to the right of the $\mathcal{B}$'s, to simply notice that all terms in (4) cancel by virtue of Eq. (13).

How is this fact to be understood in terms of the original version of Einstein-Maxwell theory? It can be readily seen that this state can be decomposed like $\Psi_\Lambda^{\rm CS}[\mathcal{A}] = \Psi_\Lambda^{\rm CS}[A]\Psi_\Lambda^{\rm CS}[\mathbf{a}]$ in terms of the usual Ashtekar variables. It is a remarkable fact that this state actually manages to solve all the constraints of the theory with a cosmological constant. This is quite easy to see. Consider the action of the Hamiltonian constraint (4) on the product $\Psi_\Lambda^{\rm CS}[A]\Psi_\Lambda^{\rm CS}[\mathbf{a}]$. The first two terms do not involve electromagnetic variables so they only act on $\Psi_\Lambda^{\rm CS}[A]$ and they annihilate it in the same way as they did for the vacuum theory [11]. The last term has both electromagnetic and gravitational variables. The gravitational part acts on $\Psi_\Lambda^{\rm CS}[A]$ and since this function is an eigenstate of the triad with eigenvalue equal to the "magnetic metric" ($\hat{\tilde{\mathcal{E}}}_i^a\Psi_\Lambda^{\rm CS}[A] = \tilde{\eta}^{abc}F_{bc}^i\Psi_\Lambda^{\rm CS}[A]$), it simply gives an overall factor. The electromagnetic part vanishes since $\hat{\tilde{\mathbf{e}}}^a\Psi_\Lambda^{\rm CS}[\mathbf{a}] = -i\tilde{\mathbf{b}}^a\Psi_\Lambda^{\rm CS}[a]$ (with $\Lambda = 3$) and this annihilates the $\tilde{\mathbf{e}}^f\tilde{\mathbf{e}}^g + \tilde{\mathbf{b}}^f\tilde{\mathbf{b}}^g$ portion of the Hamiltonian density of Maxwell theory in Eq. (4).

This result is quite robust. The only requirement needed to prove it is to assume a factor ordering in the gravitational variables with the E's to the right of the A's. By the way, this factor ordering is the one that has been used to build the loop representation [13] and is the one in which the algebra of constraints closes (formally) both in the connection [1] and loop representations [14]. The factor ordering between gravitational and electromagnetic variables can be arbitrary. As was shown, the state $\Psi_\Lambda^{\rm CS}[\mathcal{A}]$ is annihilated separately by the gravitational and electromagnetic part of the Hamiltonian constraint. This state will have important consequences in the loop representation.

The reader may be surprised by the fact that the result holds for a particular value of the cosmological constant. This is really not so, since while defining the unified model we had a freedom to rescale arbitrarily the electromagnetic variables versus the gravitational ones by a constant. By choosing this constant appropriately one can have a state for an arbitrary value of Λ. One can consider a more general state defined by the product $\Psi_\Lambda^{\rm CS}[A]\Psi_\Theta^{\rm CS}(a)$ (a different constant in the exponential of the electromagnetic part) and this also manages to solve the constraints, if $\Theta = 3$ and Λ is arbitrary. One can write this state in terms of the connection $\mathcal{A}$ but it has a more complicated and less appealing form than the state defined by (12).

What can one say about the physical relevance of this state? The Chern-Simons form is not a physically relevant state for Maxwell theory, since it is not normalizable and therefore does not belong in the Fock space. Evidently there is the possibility that a similar situation arises in our unified model. However, the measure in the inner product of the unified model, which we do not know, is potentially going to be very different from that of Maxwell theory. This is due to the fact that the unified model is invariant under diffeomorphisms and Maxwell theory is not. Therefore we cannot *a priori* rule out this state as a candidate to a physical state of the theory.

Ashtekar, Rovelli, and Smolin [15] have shown that if one formulates Maxwell theory in terms of self-dual variables the Chern-Simons form actually becomes the vacuum of the theory. This suggests the intriguing possibility of trying to rebuild our unified model in terms of a self-dual Maxwell connection. This would allow a better physical interpretation of the Chern-Simons state and would make the quantum representation more symmetric in the sense that the connection $\mathcal{A}$ would now be constructed superposing two self-dual connections.

IV. LOOP REPRESENTATION

The construction of the loop representation for this theory follows the same steps as those for the vacuum theory so we will only highlight some points. The reader interested in details of the construction of loop representations is referred to [13,16,2]. The main difference with the usual case is that the group is U(2) instead of being SU(2). This changes the form of the Mandelstam identities and therefore the kinematics of the loop representation is different. As usual we identify

$$\Psi[\gamma_1\circ\gamma_2] = \Psi[\gamma_2\circ\gamma_1]. \tag{14}$$

but there is no relation between wave functions of retraced loops, i.e., $\Psi[\gamma] \neq \Psi[\gamma^{-1}]$. In the vacuum theory one also has the Mandelstam identity, which states

$$\Psi[\gamma_1,\gamma_2] = \Psi[\gamma_1\circ\gamma_2] + \Psi[\gamma_1\circ\gamma_2^{-1}]. \tag{15}$$

That is, it allows to express any wave function of n loops as a function of $n-1$ loops. This can be used recursively to reduce all wave functions to functions of only one loop.

For our case the Mandelstam identity now reads

$$\Psi[\gamma_1,\gamma_2,\gamma_3]=\Psi[\gamma_1\circ\gamma_2,\gamma_3]+\Psi[\gamma_2\circ\gamma_3,\gamma_1]+\Psi[\gamma_3\circ\gamma_1,\gamma_2]-\Psi[\gamma_1\circ\gamma_2\circ\gamma_3]-\Psi[\gamma_1\circ\gamma_3\circ\gamma_2]. \tag{16}$$

Where we see that it only allows us to reduce a function of n loops to a function of $n-1$ and $n-2$ loops. Therefore we cannot reduce all wave functions to functions of only one loop, we need at least two loops to represent a generic wave function. Therefore the kinematics of the theory is different from the vacuum one, as one may expect from the fact that gauge group is different in both theories.

An interesting point is that one could proceed in the traditional fashion [without combining both connections into a U(2)] and construct two loop representations: one for the Ashtekar connection and another one for the Maxwell one (as was done in [17] for the 2+1 case). One would then have wave functions depending on two loops: an "electromagnetic" and a "gravitational" one. Here we also encounter two loops, but each of them carrying information about both gravitation and electromagnetism. There are other important differences in the construction of the loop representation in both cases but we will not discuss them here. It seems more natural, from the point of view of unification, to deal with loops that carry the information of both gravity and electromagnetism and that is the point of view we will adopt in this paper.

The diffeomorphism constraint still works as a generator of infinitesimal diffeomorphisms in loop space, and we can represent it explicitly in terms of the loop derivative as in the vacuum case [13] simply taking into account that it is acting on two loops. The important point here is that the space of physical states of the theory will still be represented by functionals of loops that are invariant under diffeomorphisms, i.e., they will be functionals of the link class of the loop rather than of the loop itself, exactly as in the vacuum case.

We now turn our attention to the Hamiltonian constraint. Again, it could be realized in loop space in terms of the area derivative, as one does for the vacuum theory [13]. The calculation is lengthy and at the moment we will not make use of a specific form so we will not exhibit it here. In this case, the constraint is very different from that of the vacuum theory, as expected.

One of the main achievements of the loop representation for the vacuum theory was to make possible the construction of states that solve *all* the constraints of the theory. We will see that this is the case also for the model of interest. First of all it is quite simple to see, from the structure of the Hamiltonian constraint, that wave functionals with support on smooth nonintersecting loops are *not* solutions to the constraint (although we have not written the constraint explicitly, the result is obvious from its form in the connection representation). As in the connection representation this stems from the fact that there is present a purely multiplicative term in the constraint that fails to annihilate these functionals. Moreover we should remember that the constraint was rescaled by a factor that vanishes for loops with less than a triple intersection so we may want to exclude these states altogether.

In spite of this we can construct a solution to all the constraints in the loop representation following the same reasoning that also allowed us to construct a solution in the vacuum case: since the Chern-Simons form is a solution in the connection representation, it should also be a solution in the loop representation. In general, we do not know how to compute the transform of a state into the loop representation,

$$\Psi[\gamma_1,\gamma_1]=\int d\mathcal{A}\,\Psi[\mathcal{A}]W_{\gamma_1}(\mathcal{A})W_{\gamma_2}(\mathcal{A}) \tag{17}$$

[remember that because our model is U(2) symmetric we need at least two loops to define a wave function in the loop representation], since we do not know how to evaluate the integral on the right. It turns out however that this transform is known for a state given by a Chern-Simons form taking advantage of the results of Witten and others [18,19] in Chern-Simons field theories. It is equal to a well known knot polynomial (in an arbitrary variable t related to the constant in front of the Chern-Simons form in the wave function, for our case Λ), the Kauffman bracket [18,19]. For our particular model, it is the Kauffman bracket for two loops, $K[\gamma_1,\gamma_2](\Lambda)$. The Kauffman bracket is a regular isotopic polynomial; that is, it is an invariant of framed loops (see Refs. [18,19] for a discussion of the problem of framing), and is related to the ambient isotopic Jones polynomial $J[\gamma_1,\gamma_2](\Lambda)$ (which is a true invariant of unframed loops) [19]. Summarizing, the Kauffman bracket is a solution of all the quantum constraints of the unified model. Notice that usually the Kauffman bracket and Jones polynomial are associated with SU(2) groups and with a single loop, although their generalization to other groups and more than one loop are known and have been discussed [19]. Here one should take the generalizations for a U(2) group and two loops.

This solution also sheds light on the connection between our unified loop representation based on a U(2) group and the more traditional one based on two separate SU(2) and U(1) groups. Let us rewrite Eq. (17) to make explicit the dependence on both connections,

$$\Psi(\gamma_1,\gamma_2)=\int dA\int d\mathbf{a}\,\Psi[A,a]W_{\gamma_1}(A)W_{\gamma_2}(A)W_{\gamma_1\circ\gamma_2}(\mathbf{a}). \tag{18}$$

Here we see how the intermingling of electromagnetism and gravity happens in the language of loops by noticing that both the Wilson loops built with the A and the $\mathbf{a}$ connection depend on both loops. If we now take a product state as the one we considered in Sec. III, $\Psi^{\rm CS}_{\Lambda}[A]\Psi^{\rm CS}_{\Theta}[\mathbf{a}]$, one can again evaluate the transform. The transform of the SU(2) portion gives the Kauffman brackets associated with SU(2). The transform of the U(1) portion gives the exponential of the Gauss linking number (times a function of Θ). For the particular case of $\Lambda=\Theta$ the net result can be rewritten as the Kauffman bracket associated with a U(2) symmetry.

In the loop formulation of gauge theories, it is usual to introduce charges by opening up the loops. The open path formalism describes lines of flux with charges at their ends. This has been studied for the Maxwell theory [20]. It is interesting to note what happens if one attempts to construct such a formalism for our unified model. If one opens up one of the loops in question, one not only fails to satisfy the Gauss law of the Maxwell theory (which introduces electric charges) but also one fails to satisfy the Gauss law of the Ashtekar formalism. This latter fact only occurs if one couples the theory to fermions. That is, the loop representation requires that charged objects should be fermionic.

We end by mentioning that in the vacuum theory one can perform an analysis order by order in the Jones polynomial and retrieve physical states for the theory without cosmological constant [21]. It would be interesting to try to carry out a similar analysis for the model we are considering.

V. CONCLUSIONS

We have studied the Ashtekar formulation of the Einstein-Maxwell theory. We have shown how one can rewrite the equations in terms of a single U(2) connection. The kinematic structure of the theory is quite similar to that of pure general relativity and allows the generalization of several results of that case to the combined Einstein-Maxwell theory. In particular, the loop representation is quite natural and the Jones polynomial turns out to be related to a physical state of the theory, as happens in pure general relativity. Summarizing, we can see that the Ashtekar variables and/or loop representation approach to the quantization of gravity can lead to quite appealing results when one incorporates other interactions in an unified fashion. The fact that the Jones polynomial plays a role in both the vacuum theory and the unified model points out to a possible deep role of this mathematical structure in gravitational physics yet to be understood.

ACKNOWLEDGMENTS

We wish to thank Abhay Ashtekar, Lee Smolin, and Charles Torre for useful discussions. We are also indebted to Karel Kuchař for a careful reading of the manuscript and useful discussions. This work was supported in part by Grant No. NSF PHY92-07225 and by research funds from the University of Utah. J.P. wishes to thank PEDECIBA and the Universidad de la República for hospitality and support in visits to Montevideo, where part of this work was accomplished.

[1] A. Ashtekar, Phys. Rev. Lett. **57**, 2244 (1986); Phys. Rev. D **36**, 1587 (1987).
[2] C. Rovelli and L. Smolin, Phys. Rev. Lett. **61**, 1155 (1988); Nucl. Phys. **B331**, 80 (1990).
[3] B. Brügmann, R. Gambini, and J. Pullin, Phys. Rev. Lett. **68**, 431 (1992); see also R. Gambini, B. Brügmann, and J. Pullin, in *Proceedings of the XXth International Conference on Differential Geometrical Methods in Theoretical Physics*, New York, New York, 1991, edited by S. Catto and A. Rocha (World Scientific, Singapore, 1991).
[4] B. Brügmann, R. Gambini, and J. Pullin, Nucl. Phys. **B385**, 587 (1992).
[5] See, for instance, C. Rovelli, Class. Quantum Grav. **8**, 297 (1991); **8**, 317 (1991), and references therein.
[6] P. Peldán, Phys. Rev. D **46**, R2279 (1992).
[7] A. Magnon, Class. Quantum Grav. **9**, 169 (1992).
[8] A. Ashtekar and J. Romano (private communication).
[9] A. Ashtekar, J. Romano, and R. Tate, Phys. Rev. D **40**, 2572 (1989).
[10] T. Jacobson and L. Smolin, Nucl. Phys. **B299**, 295 (1988).
[11] H. Kodama, Phys. Rev. D **42**, 2548 (1990).
[12] B. Brügmann and J. Pullin, Nucl. Phys. **B363**, 221 (1991).
[13] R. Gambini, Phys. Lett. B **255**, 180 (1991); B. Brügmann and J. Pullin, Nucl. Phys. **B390**, 399 (1993).
[14] R. Gambini, A. Garat, and J. Pullin (in preparation).
[15] A. Ashtekar, C. Rovelli, and L. Smolin, J. Geom. Phys. **8**, 7 (1992).
[16] R. Gambini and A. Trias, Nucl. Phys. **B278**, 436 (1986).
[17] C. Nayak, Gen. Relativ. Gravit. **23**, 931 (1991).
[18] E. Witten, Commun. Math. Phys. **121**, 351 (1989); L. Smolin, Mod. Phys. Lett. A **4**, 1091 (1989); see also B. Brügmann, R. Gambini, and J. Pullin, Nucl. Phys. **B385**, 587 (1992), for a discussion of intersecting loops.
[19] E. Guadagnini, M. Martellini, and M. Mintchev, Nucl. Phys. **B330**, 575 (1990); E. Guadagnini, Int. J. Mod. Phys. A **7**, 877 (1992).
[20] H. Fort and R. Gambini, Phys. Rev. D **44**, 1957 (1991).
[21] B. Brügmann, R. Gambini, and J. Pullin, Gen. Relativ. Gravit. **25**, 1 (1993).