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Mass formulas for stationary Einstein-Yang-Mills black holes and a simple proof of two staticity theorems

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We derive two new integral mass formulas for stationary black holes in Einstein-Yang-Mills theory. From these we derive a formula for $\mathcal{J}\Omega - QV$, from which it follows immediately that any stationary, nonrotating, uncharged black hole is static and has a vanishing electric field on the static slices. In the Einstein-Maxwell case, we have, in addition, the "generalized Smarr mass formula," for which we provide a new, simple derivation. When combined with the other two formulas, we obtain a simple proof that nonrotating Einstein-Maxwell black holes must be static and have a vanishing magnetic field on the static slices. Our mass formulas also can be generalized to cases with other types of matter fields, and we describe the nature of these generalizations.

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In a recent paper by the authors [1], it was shown, among other things, that a solution of the Einstein-Yang-Mills (EYM) equations describing a stationary black hole with a bifurcate Killing horizon and satisfying $VQ = \mathcal{J}\Omega = 0$ is necessarily static, and has a vanishing electric field on the static slices. A stronger result was obtained in the Einstein-Maxwell case: It was proven that a solution of the Einstein-Maxwell equations describing a stationary black hole with a bifurcate Killing horizon and satisfying $\mathcal{J}\Omega = 0$ is necessarily static, and has a vanishing magnetic field on the static slices. These staticity theorems were obtained by deriving a generalized first law of black hole mechanics, using it to infer extremal properties of stationary black hole solutions, and then showing that these extremal properties could be violated unless the black hole is static. The theorems do not require the stationary Killing field to be globally timelike in the exterior region; i.e., "ergoregions" are permitted. Thus, in particular, the Einstein-Maxwell staticity theorem closed a gap in the black hole uniqueness theorems which had been open for nearly two decades (see [2]).

The purpose of this paper is to derive some new "mass formulas" relating the asymptotically defined attributes of a stationary black hole in EYM theory, and to use them to give a simple proof of the above staticity theorems. For definiteness, we will restrict our considerations to SU(2)-EYM theory, although all of our equations and results also apply straightforwardly to Einstein-Maxwell theory. More generally, mass formulas analogous to the ones we derive will exist for many other theories, and we will explain the conditions under which such generalizations can be obtained.

We first briefly review the "3+1" formulation of the EYM equations given in [1]. Initial data in EYM theory consists of the specification of the fields $(h_{ab}, \pi^{ab}, A_a{}^\Lambda, E^a{}_\Lambda)$ on a three-dimensional manifold Σ. Here h_{ab} is a Riemannian metric on Σ, $A_a{}^\Lambda$ is the gauge field component tangent to Σ, π^{ab} is the canonically conjugate momentum to h_{ab}, and $E^a{}_\Lambda$ is the electric field (viewed as a density of weight $\frac{1}{2}$), which is (up to the numerical factor of $\frac{1}{4}$) the momentum canonically conjugate to $A_a{}^\Lambda$. Here, and throughout this paper, lower case greek indices are used to denote spacetime tensors, and latin indices are used to denote tensors in the hypersurface Σ. The projection of a spacetime tensor into Σ is denoted by replacing the greek indices by latin indices; e.g., the projection of the spacetime vector t^μ into Σ is denoted as t^a. Capital greek indices are used for the Lie algebra of the Yang-Mills field.

Constraints are present in Einstein-Yang-Mills theory. On a hypersurface Σ on which $\pi_a{}^a=0$, the allowed initial data are restricted to those that at each point $x\in\Sigma$ satisfy

$$0=\sqrt{h}\,\mathcal{D}_a(E^a{}_\Lambda/\sqrt{h})=\sqrt{h}\,D_a(E^a{}_\Lambda/\sqrt{h})+c_{\Lambda\Gamma}{}^\Delta A_a{}^\Gamma E^a{}_\Delta, \tag{1}$$

$$0=-R+\frac{1}{h}\pi_{ab}\pi^{ab}+\frac{2}{h}E_a{}^\Lambda E^a{}_\Lambda+F_{ab}{}^\Lambda F_\Lambda{}^{ab}\ , \tag{2}$$

$$0=\sqrt{h}\,D_b(\pi_a{}^b/\sqrt{h})-2F_{ab}{}^\Lambda E^b{}_\Lambda\ , \tag{3}$$

where D_a is the derivative operator on Σ compatible with the metric h_{ab}, $\mathcal{D}_a$ denotes the (metric compatible) gauge-covariant derivative operator, and R denotes the scalar curvature of h_{ab}.

We shall be concerned in this paper with spacetimes representing a stationary black hole with a bifurcate Killing horizon. As discussed in [3], this should encompass all stationary black hole solutions in EYM theory except for the "degenerate" solutions which have vanishing surface gravity. Recall that a stationary black hole with a bifurcate Killing horizon automatically possesses a Killing field t^μ, which approaches a time translation in the asymptotic region, and a Killing field χ^μ, which vanishes on the bifurcation surface S. If χ^μ fails to coincide with t^μ, then the spacetime also possesses an axial Killing field ϕ^μ such that

$$\chi^\mu=t^\mu+\Omega\phi^\mu\ , \tag{4}$$

where the constant Ω is known as the angular velocity of the horizon. It has recently been proven [4] that any stationary black hole with a bifurcate Killing horizon admits an asymptotically flat maximal ($\pi_a{}^a=0$) hypersurface which is asymptotically orthogonal to t^μ, and whose boundary is the bifurcation surface S of the horizon. We choose Σ to be such a hypersurface.

Now, consider the evolution equations for the initial data which are induced on Σ. Choose the lapse and shift functions, $N^\mu=(N,N^a)$, to coincide with a Killing field in the spacetime. Then, the EYM evolution equations yield the following relations, obtained by setting $\pi_a{}^a=0$ in the equations given in [1]:

$$0=\dot{\pi}^{ab}=-\{\sqrt{h}\,Na^{ab}+\sqrt{h}\,[h^{ab}D^cD_c(N)-D^aD^b(N)]-\pounds_{N^i}\pi^{ab}\}\ , \tag{5}$$

$$0=\dot{h}_{ab}=\frac{N}{\sqrt{h}}2\pi_{ab}+\pounds_{N^i}h_{ab}\ , \tag{6}$$

$$0=\dot{E}^a{}_\Lambda=-[\sqrt{h}\,\mathcal{D}_b(NF^{ab}_\Lambda)+Nc_{\Lambda\Gamma}{}^\Delta A_0{}^\Gamma E^a{}_\Delta-\pounds_{N^i}E^a{}_\Lambda]\ , \tag{7}$$

$$0=\dot{A}_a{}^\Lambda=NE_a{}^\Lambda/\sqrt{h}+\mathcal{D}_a(NA_0{}^\Lambda)+\pounds_{N^i}A_a{}^\Lambda\ , \tag{8}$$

with

$$a^{ab}\equiv\frac{2}{h}(E^a{}_\Lambda E^{b\Lambda}-\tfrac{1}{2}h^{ab}E_c{}^\Lambda E^c{}_\Lambda)+2(F^{ac}_\Lambda F_c{}^{b\Lambda}+\tfrac{1}{4}h^{ab}F^{cd}_\Lambda F_{cd}{}^\Lambda)+(R^{ab}-\tfrac{1}{2}h^{ab}R)+\frac{1}{h}(2\pi^a{}_c\pi^{bc}-\tfrac{1}{2}h^{ab}\pi^{cd}\pi_{cd})\ , \tag{9}$$

where

$$\pounds_{N^i}h_{ab}=2D_{(a}N_{b)}\ , \tag{10}$$

$$\pounds_{N^i}A_a{}^\Lambda=N^bD_b\,A_a{}^\Lambda+A_b{}^\Lambda D_aN^b \tag{11}$$

and

$$\pounds_{N^i}\pi^{ab}=\sqrt{h}\,N^cD_c(\pi^{ab}/\sqrt{h})-2\pi^{c(a}D_cN^{b)}+\pi^{ab}D_cN^c\ , \tag{12}$$

$$\pounds_{N^i}E^a{}_\Lambda=\sqrt{h}\,N^cD_c(E^a{}_\Lambda/\sqrt{h})-E^c{}_\Lambda D_cN^a+E^a{}_\Lambda D_cN^c\ . \tag{13}$$

We begin by deriving a simple equation satisfied by the lapse function $N=-k^\mu n_\mu$ associated with any Killing field k^μ, where n^μ denotes the unit normal to the maximal hypersurface Σ. Contracting (5) with h^{ab}, we find

$$D_cD^c(N)=-\tfrac{1}{2}Na^b{}_b-D_a(N_b)\pi^{ab}/\sqrt{h}\ . \tag{14}$$

From Eqs. (6) and (10) we find

$$D_a(N_b)\pi^{ab}=-N\pi_{ab}\pi^{ab}/\sqrt{h}\ . \tag{15}$$

Substituting in (14) and using the constraint (2) in the expression for $a^b{}_b$ we obtain

$$D_cD^c(N)=\rho N\ , \tag{16}$$

where

$$\rho=\frac{1}{h}\pi_{ab}\pi^{ab}+\frac{1}{h}E_a{}^\Lambda E^a{}_\Lambda+\tfrac{1}{2}F_{ab}{}^\Lambda F^{ab}_\Lambda\ , \tag{17}$$

so that ρ is non-negative. Note that the derivation of Eq. (16) used only the "Einstein portion" of the EYM equations, and, thus, it is easily generalized to any other Einstein-matter system (even if the full system is not derivable from a Hamiltonian). Indeed, a generalization of our derivation shows that in any spacetime foliated by maximal hypersurfaces, the lapse function N of this foliation satisfies Eq. (16) with ρ replaced by

$$\rho=\frac{1}{h}\pi_{ab}\pi^{ab}+R_{\mu\nu}n^\mu n^\nu\ . \tag{18}$$

(This result also could be derived from the Raychaudhuri equation for nongeodesic timelike congruences; see Eq. (4.26) of [2].) Thus, in particular, for a stationary black hole with a bifurcate horizon in any Einstein-matter system, Eq. (16) holds with ρ given by (18). Note that ρ will be non-negative provided only that the matter satisfies the strong energy condition. When ρ is non-negative, the maximum principle can be usefully applied to Eq. (16), and solutions to Eq. (16) are uniquely determined by their boundary value at S and their asymptotic value at infinity.

As our first application of Eq. (16), we choose $N^\mu=\phi^\mu$, where ϕ^μ is the axial Killing field, so $N=-n_\mu\phi^\mu$. The boundary conditions ($N_{|S}=0$ and $N_{|\infty}=0$) yield the unique solution $N=0$ on Σ. Thus, we find that ϕ^μ is tangent to Σ. This result also could be proven by a generalization of known uniqueness results on maximal foliations (see Theorem 5.5 of [5]), since if ϕ^μ failed to be everywhere tangent to Σ, we could obtain a new maximal hypersurface asymptotic to Σ by applying a rotation to Σ.

Next we apply Eq. (16) to the stationary Killing field t^μ. We write λ for the lapse function N in this case, i.e., we define

$$\lambda=-n_\mu t^\mu \ . \tag{19}$$

Since λ satisfies the boundary conditions $\lambda_{|S}=0$ and $\lambda_{|\infty}=1$, the maximum principle implies that λ is strictly positive on Σ outside of S. (Also $\lambda<1$ throughout Σ.) Integrating Eq. (16) over Σ, we obtain

$$\int_\infty dS^a D_a\lambda-\int_S dS^a D_a\lambda=\int_\Sigma \lambda\rho \ , \tag{20}$$

where, here and below, all volume integrals over Σ are taken with respect to the natural volume element determined by h_{ab}, and our convention on the unit normal to S is that it point "radially outward," i.e., into Σ. The surface integral at infinity is simply $4\pi M$. The surface integral at S is just κA, where κ denotes the surface gravity of χ^μ on S, and A is the area of S. Therefore, we obtain

$$4\pi M-\kappa A=\int_\Sigma \lambda\rho \ . \tag{21}$$

Equation (21) is our first "mass formula" for black holes. It should be emphasized that this formula applies to an arbitrary stationary black hole with a bifurcate Killing horizon, with ρ given by Eq. (18), which takes the explicit form (17) in the EYM case. Both ρ and λ are non-negative whenever Einstein's equation holds with matter satisfying the strong energy condition. Hence, it follows immediately that for all such black holes we have

$$4\pi M\geq \kappa A \ . \tag{22}$$

This inequality was recently derived by Visser [6] [using Eq. (23) below] for the case of nonrotating black holes. Our derivation shows that Eq. (22) remains valid for all stationary black holes, provided only that the matter present in the exterior region satisfies the strong energy condition. In particular, if there exist any "colored excitations" of the Kerr-Newman black holes (as we conjectured in [1]), they must satisfy Eq. (22).

To derive our second mass formula, we start with the well-known integral mass formula of Bardeen, Carter, and Hawking [7]:

$$M-\frac{\kappa A}{4\pi}-2\Omega J_H=2\int_\Sigma (T_{\mu\nu}-\tfrac{1}{2}Tg_{\mu\nu})t^\mu n^\nu \ , \tag{23}$$

where $T_{\mu\nu}$ is the energy-momentum tensor of matter, and J_H is the "angular momentum of the black hole," defined by

$$J_H=\frac{1}{16\pi}\int_S \varepsilon_{\mu\nu\sigma\rho}\nabla^\sigma\phi^\rho \ . \tag{24}$$

Equation (23) is obtained by starting with the Komar formula for the mass of a stationary spacetime and converting this surface integral at infinity to a volume integral over a hypersurface Σ passing through S (see, e.g., [8]). It holds for any stationary black hole with a bifurcate Killing horizon satisfying Einstein's equation with arbitrary matter. Note that it is not necessary for the validity of Eq. (23) that Σ be a maximal hypersurface.

We restrict attention, now, to the case where matter is a Yang-Mills field. Then we can write Eq. (23) more explicitly as

$$M-\frac{\kappa A}{4\pi}-2\Omega J_H=\frac{1}{4\pi}\int_\Sigma \{\lambda[(1/h)E_a{}^\Lambda E^a{}_\Lambda+\tfrac{1}{2}F_{ab}{}^\Lambda F^{ab}_\Lambda]+2t^aE^b{}_\Lambda F_{ab}{}^\Lambda/\sqrt{h}\} \ . \tag{25}$$

A more useful form of Eq. (25) can be obtained by relating J_H to the canonical angular momentum in EYM theory, defined by [1]

$$\mathcal{J}_\infty=-\frac{1}{16\pi}\int_\infty (2\phi_b\pi^{ab}+4\phi^bA_b{}^\Lambda E^a{}_\Lambda)/\sqrt{h}\ dS_a \ . \tag{26}$$

Converting this surface integral to a volume integral over Σ and using the constraint equations as in the derivation Eq. (53) of [1], we find

$$\mathcal{J}_\infty=-\frac{1}{16\pi}\int_\Sigma (\pi^{ab}\pounds_{\phi^i}h_{ab}+4E^a{}_\Lambda\pounds_{\phi^i}A_a{}^\Lambda)/\sqrt{h}+\mathcal{J}_H \ , \tag{27}$$

where $\mathcal{J}_H$ is defined by

$$\mathcal{J}_H=-\frac{1}{16\pi}\int_S (2\phi_b\pi^{ab}+4\phi^bA_b{}^\Lambda E^a{}_\Lambda)/\sqrt{h}\ dS_a \ . \tag{28}$$

The integral over Σ in Eq. (27) vanishes because the axial Killing field ϕ^μ is equal to its tangential projection ϕ^i. Thus, we obtain

$$\mathcal{J}_\infty=\mathcal{J}_H \ . \tag{29}$$

Furthermore, using the fact that

$$\nabla^\mu\phi^\nu=D^\mu\phi^\nu-2\phi_\rho n^{[\mu}K^{\nu]\rho} \ , \tag{30}$$

where $K^{\nu\rho}$ is the extrinsic curvature of Σ, we see that the first term in the formula (28) for $\mathcal{J}_H$ is just J_H. The second term in Eq. (28) can be computed by noting that on S, t^a and ϕ^a coincide (up to the constant Ω) as a result of Eq. (4), and the fact that χ^b vanishes on S. Therefore we can write

$$4\pi\Omega(J_H-\mathcal{J}_\infty)=-\int_S dS_a t^b A_b{}^\Lambda E^a{}_\Lambda/\sqrt{h} \ . \tag{31}$$

We now convert this surface integral into a volume integral over Σ, using the constraint Eq. (1). When we do so, we get no contribution from the boundary at infinity on account of the asymptotic falloff behavior of $A_b{}^\Lambda$ and $E^a{}_\Lambda$. We obtain

$$4\pi\Omega(J_H-\mathcal{J}_\infty)=\int_\Sigma[t^bE^a{}_\Lambda F_{ab}{}^\Lambda/\sqrt{h}+\pounds_t(A_a{}^\Lambda)E^a{}_\Lambda/\sqrt{h}\,] \ . \tag{32}$$

We now use Eq. (8) to substitute for $\pounds_t(A_a{}^\Lambda)$ and again use the constraint Eq. (1). We obtain

$$4\pi\Omega(J_H-\mathcal{J}_\infty)=-\int_\Sigma(t^aE^b{}_\Lambda F_{ab}{}^\Lambda/\sqrt{h}+\lambda E^a{}_\Lambda E_a{}^\Lambda/h)-\int_\infty dS_a\lambda E^a{}_\Lambda A_0{}^\Lambda/\sqrt{h} \ , \tag{33}$$

where we used the fact that there is no surface integral contribution from S in this equation since $\lambda=0$ on S. As shown in [1], for SU(2)-Yang-Mills theory (or Maxwell theory) the surface integral at infinity in this equation yields simply $4\pi VQ$, where V is the asymptotic magnitude of $A_0{}^\Lambda$ and Q is the Yang-Mills electric charge at infinity. Therefore, we obtain,

$$4\pi[VQ-\Omega(\mathcal{J}_\infty-J_H)]=-\int_\Sigma(t^aE^b{}_\Lambda F_{ab}{}^\Lambda/\sqrt{h}+\lambda E^a{}_\Lambda E_a{}^\Lambda/h) \ . \tag{34}$$

Our desired second mass formula is obtained by using this equation to eliminate J_H from Eq. (25). We obtain

$$4\pi M-\kappa A+8\pi(VQ-\Omega\mathcal{J}_\infty)=\int_\Sigma\lambda[\tfrac{1}{2}F_{ab}{}^\Lambda F^{ab}_\Lambda-(1/h)E_a{}^\Lambda E^a{}_\Lambda] \ . \tag{35}$$

Note that Eq. (35) holds for an arbitrary hypersurface Σ which is asymptotically orthogonal to t^μ and has S as its inner boundary; i.e., in this formula it is not necessary that Σ be a maximal hypersurface.

As previously mentioned, our starting point, Eq. (23), in the derivation of Eq. (35) holds for an arbitrary Einstein-matter system. Furthermore, the notion of $\mathcal{J}_\infty$ is well defined for any Einstein-matter system derivable from a Hamiltonian. However, considerable use was made of the explicit form of the Yang-Mills field equations in deriving Eq. (34). Thus, it is not clear that Eq. (35) would have a close analogue for other Einstein-matter systems derivable from a Hamiltonian.

We now subtract Eq. (35) from Eq. (21), using Eq. (17). We obtain

$$8\pi(\Omega\mathcal{J}_\infty-VQ)=\int_\Sigma\lambda(\pi_{ab}\pi^{ab}+2E^a{}_\Lambda E_a{}^\Lambda)/h \ . \tag{36}$$

By inspection of Eqs. (36), (21), and (17), we see that any stationary black hole with a bifurcate Killing horizon in EYM theory satisfies

$$M-\frac{\kappa A}{4\pi}\geq\Omega\mathcal{J}_\infty-VQ\geq 0 \ . \tag{37}$$

Furthermore, we obtain directly from Eq. (36) the following theorem, which corresponds to Theorem 3.4 of [1].

Theorem 1. A solution of the EYM equations describing a stationary black hole with a bifurcate Killing horizon that has $\Omega\mathcal{J}_\infty-VQ=0$ is static and has a vanishing electric field on the static slices.

Proof. Since the strong energy condition is satisfied by the Yang-Mills field, Theorem 4.2 of [4] establishes that the exterior region of the black hole can be foliated by maximal hypersurfaces with a boundary S, which are asymptotically orthogonal to the timelike Killing field t^μ. Applying Eq. (36) to these hypersurfaces, we obtain $\pi^{ab}=0$ and $E^a_\Lambda=0$. It then follows directly that λn^μ is a Killing field (see [1]), and, indeed, that $t^\mu=\lambda n^\mu$.□

In our discussion thus far, we have not made use of the first law of black hole mechanics for EYM black holes (see Theorem 2.2 of [1]), which states that the changes in M, Q, $\mathcal{J}_\infty$, and A induced by an arbitrary asymptotically flat perturbation satisfying the linearized EYM equations are related by

$$\delta M+V\delta Q-\Omega\delta\mathcal{J}_\infty=\frac{1}{8\pi}\kappa\delta A \ . \tag{38}$$

As emphasized in [9], a formula of this type will exist in any Einstein-matter theory having a Hamiltonian formulation. For EYM theory, it does not appear possible to derive an integral mass formula directly from Eq. (38). However, in Einstein-Maxwell theory with a trivial U(1) bundle (i.e., with a vanishing magnetic charge), an integral formula can be derived from the first law using the additional fact that the theory is invariant under the scaling transformation $g_{\mu\nu}\to\alpha^2g_{\mu\nu}$, $A_\mu\to\alpha A_\mu$, where α is a constant. Under this transformation, a solution of the Einstein-Maxwell equations is taken into a new solution, with $M\to\alpha M$, $V\to V$, $Q\to\alpha Q$, $\Omega\to\alpha^{-1}\Omega$, $\mathcal{J}_\infty\to\alpha^2\mathcal{J}_\infty$, $\kappa\to\alpha^{-1}\kappa$, and $A\to\alpha^2A$. Substituting the linearized perturbation associated with this scale transformation into Eq. (38) we obtain the mass formula

$$M+VQ-2\Omega\mathcal{J}_\infty=\frac{1}{4\pi}\kappa A \ , \tag{39}$$

which is valid in the Einstein-Maxwell case with vanishing magnetic charge. (Note that a similar formula, valid for the case of nonvanishing magnetic charge, can be derived from Eq. (39) by using the fact that the magnetic charge always can be set to zero by a duality transformation.) Equation (39) is equivalent to the "generalized Smarr formula" derived by Carter (see Eq. (6.323) of [10] and note that Carter's Φ_H corresponds to $-V$). This mass formula is characterized by the fact that it involves only "surface terms." As our derivation makes clear, a similar formula can be obtained for any scale-invariant Einstein-matter system which has a Hamiltonian formulation.

By combining Eqs. (35), (21), and (39), we can solve for $\Omega\mathcal{J}_\infty$ and VQ separately in Einstein-Maxwell theory. We obtain

$$\Omega\mathcal{J}_\infty = \frac{1}{8\pi}\int_\Sigma \lambda(\pi_{ab}\pi^{ab}/h + F^{ab}F_{ab}) \quad (40)$$

and

$$VQ = -\frac{1}{4\pi}\int_\Sigma \lambda(E^a E_a/h - \tfrac{1}{2}F^{ab}F_{ab}) \; . \quad (41)$$

The latter equation also can be derived using only Maxwell's equations.

By inspection of Eq. (40), we see that any stationary black hole with a bifurcate Killing horizon in Einstein-Maxwell theory satisfies

$$\Omega\mathcal{J}_\infty \geq 0 \; . \quad (42)$$

By the same proof as in Theorem 1 above, we obtain the following theorem (previously proven in the discussion following Theorem 3.4 of [1]).

Theorem 2. A solution of the Einstein-Maxwell equations (with vanishing magnetic charge) describing a stationary black hole with a bifurcate Killing horizon that has $\Omega\mathcal{J}_\infty = 0$ is static and has a vanishing magnetic field on the static slices.

Thus, our mass formulas have enabled us to give an elementary proof of the staticity theorems of [1].

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