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Neutrino masses and mixing angles in a predictive theory of fermion masses

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A framework for predicting charged fermion masses in supersymmetric grand unified theories is extended to make predictions in the neutrino sector. Eight new predictions are made: the two neutrino mass ratios and the three mixing angles and three phases of the weak leptonic mixing matrix. There are three versions of the theory which are relevant for producing MSW neutrino oscillations in the Sun. One of these is preferred by the combined solar neutrino observations. Another will be probed significantly by the searches for $\nu_\mu \nu_\tau$ oscillations at the NOMAD, CHORUS, and P803 experiments. In this second version ν_τ could be a significant component of the dark matter in the Universe.

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The solar neutrino problem and the several existing and upcoming neutrino experiments have caused interest in neutrino masses and mixings to escalate in the last few years. Theoretical advances on this subject are very difficult to come by since the subject of neutrino masses is, in general, coupled to the problem of quark and charged lepton masses on which very little progress has been made.

Recently, we proposed a predictive framework, based on supersymmetric grand unified theories (GUT's), in which the 14 parameters of the quark and charged lepton mass matrices, plus the ratio of the Higgs vacuum expectation values (VEV's), can be obtained in terms of just 8 input parameters, thus leading to 6 predictions [1]. The consequences of these predictions will be tested in planned experiments [2]. Our framework has the virtue that it is a consequence of a large class of GUT's. In this paper we wish to study a subset of models within this class which are very predictive in the neutrino sector. We will show how the addition of one more input parameter, for a total of 9 inputs, allows us to account for 23 parameters, resulting in 14 predictions. In particular we will predict the 2 neutrino mass ratios, 3 mixing angles, and 3 phases of the lepton sector.

We begin with a rapid overview of our framework. A key ingredient is the Georgi-Jarlskog texture for fermion mass matrices at the GUT scale [3]:

$$M_D = \begin{pmatrix} 0 & F & 0 \\ F & E & 0 \\ 0 & 0 & D \end{pmatrix} \frac{v}{\sqrt{2}} \cos\beta ,$$
$$M_E = \begin{pmatrix} 0 & F & 0 \\ F & -3E & 0 \\ 0 & 0 & D \end{pmatrix} \frac{v}{\sqrt{2}} \cos\beta , \qquad (1)$$
$$M_U = \begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix} \frac{v}{\sqrt{2}} \sin\beta$$

where $\tan\beta = v_2/v_1$ is the ratio of electroweak breaking VEV's. The factor of 3 difference in the 22 elements of M_D and M_E is of crucial importance. It arises naturally as a consequence of the breaking of the Pati-Salam SU(4) via a VEV pointing parallel to the hypercharge generator:

$$Y_{15} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{pmatrix} . \qquad (2)$$

Three examples where this happens are when the 22 entries are generated by Higgs doublets which lie in (a) a $\overline{\mathbf{45}}$ of SU(5), (b) a $\overline{\mathbf{126}}$ of SO(10), (c) a higher dimension operator, for example $\mathbf{45} \times \mathbf{45} \times \mathbf{10}$ of SO(10).

For simplicity and definiteness we will focus on case (b) in this paper and thus study theories where the M_D and M_E arise from the following matrix of Yukawa couplings[1]:

$$\mathbf{16} \begin{pmatrix} 0 & f\,\mathbf{10}^d & 0 \\ f\,\mathbf{10}^d & e\,\overline{\mathbf{126}}^{\,d} & 0 \\ 0 & 0 & d\,\mathbf{10}^d \end{pmatrix} \mathbf{16} . \qquad (3)$$

Here d, e, and f are Yukawa couplings and $\mathbf{10}^d$ and $\overline{\mathbf{126}}^{\,d}$ are scalar fields getting SU(2)-breaking VEV's that contribute to M_D and M_E.

The entries A, B, C of the up matrix can each arise from couplings to $\mathbf{10}$ or $\overline{\mathbf{126}}$ scalar mesons.[2] There may be several such scalar mesons, distinguished by discrete symmetries necessary to ensure the texture structure of Eq. (1). Each $\overline{\mathbf{126}}$ contains an SU(5)-preserving VEV that can contribute to the Majorana mass matrix M_{NN} of the

[1]Many of our conclusions are probably more general and valid even if fermion masses come from higher dimension operators.

[2]The $\mathbf{120}$ scalar meson would lead to antisymmetric contributions to the mass matrix, and is therefore not needed.

SU(5) singlet right-handed neutrinos. To proceed further and make predictions for neutrino masses, we need to introduce some hypothesis that limits the number of parameters in the theory. We already know that we will need to introduce at least one additional parameter into the theory, namely the overall scale of the right-handed neutrino masses. To maximize predictive power we will seek theories where this is the *only* additional parameter. This has some implications for both the Yukawa couplings and the VEV's.

(I) There are no new Yukawa couplings, i.e., the Yukawa couplings giving rise to M_{NN} are the same as those that give rise to A,B,C,D,E,F.

(II) All the entries in the M_{NN} matrix must be generated from the VEV of only *one* of the $\overline{\mathbf{126}}$ multiplets.

(III) Each fermion mass matrix element is generated by the VEV of only *one* of the **10** or $\overline{\mathbf{126}}$ multiplets.

It is now easy to see that these minimality hypotheses lead us to a theory in which M_U originates in the following Yukawa couplings:

$$16\begin{bmatrix} 0 & c\,\overline{\mathbf{126}}^{\,uN} & 0 \\ c\,\overline{\mathbf{126}}^{\,uN} & 0 & bX^u \\ 0 & bX^u & a\,\overline{\mathbf{126}}^{\,uN} \end{bmatrix}16 \tag{4}$$

where (a) $\overline{\mathbf{126}}^{\,uN}$ gets an electroweak breaking VEV giving rise to Dirac masses for up quarks and neutrinos, (b) $\overline{\mathbf{126}}^{\,uN}$ also gets an SU(5)-preserving VEV contributing to M_{NN}, and (c) X^u can be either $\mathbf{10}^u$ or $\overline{\mathbf{126}}^{\,u}$. In either case it only gets a VEV in an electroweak breaking direction and gives Dirac masses to up quarks and neutrinos.

It is easy to see that if we try to deviate away from Eqs. (3) and (4) or from (a),(b),(c) we would either unnecessarily increase the number of parameters, or we would end up with a light right-handed neutrino that would become the Dirac partner to ν_e, ν_μ, or ν_τ, with a mass which is much larger than present laboratory limits. It is straightforward to find a set of symmetries which guarantees the textures of Eqs. (3) and (4). Note that X^u cannot be $\overline{\mathbf{126}}^{\,uN}$; if it were, wave function mixing would induce a nonzero 22 entry in the matrix.

The neutrino masses which follow from Eq. (4) are

$$M_{\nu N}=\begin{bmatrix} 0 & -3C & 0 \\ -3C & 0 & -3\kappa B \\ 0 & -3\kappa B & -3A \end{bmatrix}\frac{v}{\sqrt{2}}\sin\beta\,, \qquad M_{NN}=\begin{bmatrix} 0 & C & 0 \\ C & 0 & 0 \\ 0 & 0 & A \end{bmatrix}V\,, \tag{5}$$

where $\kappa=1$ if $X^u=\overline{\mathbf{126}}^{\,u}$ (case I) and $\kappa=-\frac{1}{3}$ if $X^u=\mathbf{10}^u$ (case II). V, the superheavy singlet VEV, is the one additional free parameter which occurs in the neutrino mass matrix.

There are two important ingredients we left out of our discussion so far.

(1) All the quantities involved in the mass matrices are complex. This appears to limit the predictive power; however, all but one of the phases can be eliminated by rephasing the fields.

(2) The mass matrices that we measure at the weak scale are not the same as those at the GUT scale [Eqs. (1) and (5)]; the two are connected via the renormalization group (RG).

In our previous paper [1], we analytically solved the RG equations for quark and charged lepton masses. We shall restrict ourselves in this paper to the same approximations used there. We use one loop RG equations, neglecting all Yukawa couplings except for the top, λ_t. This is a good approximation only for sufficiently small $\tan\beta$. The effects of large $\tan\beta$ will be studied in a forthcoming paper [4].

The neutrino mass derives from effective dimension-5 operators, involving lepton doublets L_i and Higgs doublet H, of the form

$$\frac{M_{\nu\nu}^{ij}}{2}L_iL_j\left[\frac{H}{v\,\sin\beta/\sqrt{2}}\right]^2,$$

where

$$M_{\nu\nu}=M_{\nu N}M_{NN}^{-1}M_{\nu N}^T\,.$$

$M_{\nu\nu}$ gets rescaled by an overall factor, as a result of RG running from M_G to M_W. These ingredients result in the following mass matrices at the weak scale:

$$M_E=\eta_e\begin{bmatrix} 0 & Fe^{i\phi} & 0 \\ Fe^{-i\phi} & -3E & 0 \\ 0 & 0 & D \end{bmatrix}\frac{v}{\sqrt{2}}\cos\beta\,, \qquad M_{\nu\nu}=\eta_\nu\frac{9Av^2}{2V}\begin{bmatrix} 0 & C/A & 0 \\ C/A & \kappa^2B^2/A^2 & 2\kappa B/A \\ 0 & 2\kappa B/A & 1 \end{bmatrix}\sin^2\beta\,, \tag{6}$$

where η_e and η_ν take into account the RG scaling from M_G to M_W. Note that, apart from the overall scale, the neutrino mass matrix depends only on κ, B/A, and C/A. As always A, B, and C refer to parameters renormalized at the GUT scale.

Harvey, Ramond, and Reiss [5] have discussed neutrino mass matrices in SO(10) GUT's which incorporate the Georgi-Jarlskog ansatz. However, they did not make any predictions for the neutrino masses. This is because, even though our ansatz is a special limit of theirs, we rely on the additional hypothesis that the VEV of only one multiplet contributes to any one matrix element (item III above). It is this additional hypothesis which gives our ansatz its strong predictive power with all neutrino masses and mixing angles determined, up to one overall scale, in terms of parameters fixed in the charged fermion sector of the theory.

The mass matrices in the lepton sector, Eq. (6), can be diagonalized by bilinear transformations of the form

$$M_E^{\rm diag}=V_e^LM_EV_e^{R\dagger}\,, \qquad M_{\nu\nu}^{\rm diag}=V_\nu M_{\nu\nu}V_\nu^T\,. \tag{7}$$

The leptonic Cabibbo-Kobayashi-Maskawa (CKM) matrix is

$$V'=V_\mu V_e^{L\dagger}\,. \tag{8}$$

A simple diagonalization of (6) leads directly to V_e^L, V_ν, and V' in the form

$$V_e^L=\begin{pmatrix} c_1' & -s_1' & 0 \\ s_1' & c_1' & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & & \\ & e^{i\phi} & \\ & & e^{i\phi} \end{pmatrix},$$

$$V_\nu=\begin{pmatrix} c_2' & s_2' & 0 \\ -s_2' & c_2' & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3' & s_3' \\ 0 & -s_3' & c_3' \end{pmatrix}, \qquad (9)$$

$$V'=\begin{pmatrix} c_1'c_2'-s_1's_2'e^{-i\phi} & s_1'+c_1's_2'e^{-i\phi} & s_2's_3' \\ -c_1's_2'-s_1'e^{-i\phi} & -s_1's_2'+c_1'c_2'c_3'e^{-i\phi} & s_3' \\ s_1's_3' & -c_1's_3' & c_3'e^{i\phi} \end{pmatrix}.$$

The angles are given by

$$s_1'=-\frac{F}{3E}, \quad s_2'=\frac{C/A}{3\kappa^2B^2/A^2}>0, \quad s_3'=-2\kappa B/A, \qquad (10)$$

and the *CP*-violating angle ϕ is determined in the previous paper [1].

In general V' contains three independent phases. However, for our case all three are related to the phase ϕ which is identical to that occurring in the quark sector, and is determined to be $\cos\phi=0.38^{+0.21}_{-0.14}$. Diagonalization of the quark mass matrices leads to a KM matrix V which is the same function of angles θ_i, ϕ as V' is of θ_i' and ϕ: $V'(\theta_i',\phi)=V(\theta_i,\phi)$. The relations between the mixing angles in the quark and lepton sectors just involve simple group theory numerical factors

$$s_1'=-\frac{1}{3}s_1, \quad s_2'=\frac{1}{3\kappa^2}s_2, \quad s_3'=2\kappa\eta_3 s_3, \qquad (11)$$

except for $\eta_3=\eta_c V_{cb}^2 m_t/m_c$ which comes from the effect of the large top Yukawa coupling on the RG scaling of $V_{cb}=s_3$. Here η_c is the QCD renormalization enhancement factor of the charm mass between m_t and m_c. While the mixing angle from the D/E sector is most accurately determined by

$$s_1'=-\left[\frac{m_e}{m_\mu}\right]^{1/2}, \qquad (12)$$

the angles in the U/ν sector must be determined from quark physics via

$$s_2=\left[\frac{m_u}{m_c}\right]^{1/2}, \quad s_3=V_{cb}. \qquad (13)$$

We now use the known numerical inputs from the $U/D/E$ sectors to make precise numerical predictions of the neutrino masses. The neutrino mass ratios are

$$\frac{m_{\nu_\tau}}{m_{\nu_\mu}}=\frac{1}{3\kappa^2}\left[\frac{B}{A}\right]^{-2}=\frac{1}{3\kappa^2}(\eta_3V_{cb})^{-2}=\begin{cases}208\pm42 & (\mathrm{I}),\\ 1870\pm370 & (\mathrm{II}),\end{cases} \qquad (14)$$

and

$$\frac{m_{\nu_\mu}}{m_{\nu_e}}=\left[\frac{C/A}{3\kappa^2B^2/A^2}\right]^{-2}=9\kappa^4\left[\frac{m_u}{m_c}\right]^{-1}=\begin{cases}3100\pm1000 & (\mathrm{I}),\\ 38\pm12 & (\mathrm{II}),\end{cases} \qquad (15)$$

where the ranges obtained correspond to ranges of the input parameters $(V_{cb},m_b,m_c,\alpha_s,m_u/m_d)$ which we have found to be consistent with the quark mass and mixing predictions of our scheme [1]. We cannot predict the overall mass scale. We find

$$m_{\nu_\tau}=\eta_\nu\frac{9Av^2}{2V}\sin^2\beta=0.8\ \mathrm{eV}\left[\frac{m_t}{170\ \mathrm{GeV}}\right]\left[\frac{10^{14}\ \mathrm{GeV}}{V}\right]$$

where we have ignored electroweak contributions to η_ν.

The most useful form for the prediction of the three mixing angles is in terms of the predictions for neutrino oscillations from flavor i to flavor j. For sufficiently large Δm_{ij}^2 the oscillation probabilities in our model can be approximated by the familiar 2×2 case: $P_{ij}=\frac{1}{2}\sin^2 2\theta_{ij}$. We find

$$\theta_{\mu\tau}\simeq V'_{\nu_\mu\tau}=s_3',$$

$$\theta_{e\mu}\simeq|V'_{\nu_e\mu}|=\left[\frac{m_e}{m_\mu}+\frac{m_{\nu_e}}{m_{\nu_\mu}}-2\left[\frac{m_e m_{\nu_e}}{m_\mu m_{\nu_\mu}}\right]^{1/2}\cos\phi\right]^{1/2}$$

and

$$\theta_{e\tau}\simeq V'_{\nu_\tau e}=s_1's_3'.$$

Using Eqs. (11)–(13), we obtain

$$\sin^2 2\theta_{\mu\tau}=\begin{cases}(2.6\pm0.5)10^{-2} & (\mathrm{I}),\\ (2.9\pm0.6)10^{-3} & (\mathrm{II}),\end{cases} \qquad (16)$$

$$\sin^2 2\theta_{e\mu}=\begin{cases}(1.7\pm0.2)10^{-2} & (\mathrm{I}),\\ (9.0\pm4.3)10^{-2} & (\mathrm{II}),\end{cases} \qquad (17)$$

$$\sin^2 2\theta_{e\tau}=\begin{cases}(1.3\pm0.3)10^{-6} & (\mathrm{I}),\\ (1.4\pm0.3)10^{-7} & (\mathrm{II}).\end{cases} \qquad (18)$$

The best hope for an experimental laboratory test of these numbers is provided by $\nu_\mu\nu_\tau$ oscillations. Present limits and the reaches of proposed experiments are shown in the $\Delta m^2-\sin^2 2\theta$ plot in Fig. 1, together with our two predictions as vertical lines.

In model I $\theta_{\mu\tau}$ is sufficiently large that the Fermilab

E531 results imply that $m_{\nu_\tau} \leq 2.5$ eV. This means that it is unlikely that planned neutrino oscillation experiments [6–8] will be able to detect the neutrino masses of this model. Given the upper bound on m_{ν_τ} we have, using Eq. (14), $m_{\nu_\mu} < 1.5 \times 10^{-2}$ eV or $m^2_{\nu_\mu} < 2.3 \times 10^{-4}$ (eV)^2. Now given this upper bound on m_{ν_μ} and our value for $\theta_{e\mu}$, Eq. (17) (see vertical line labeled I in Fig. 2), we find a possible resolution of the Cl, Kamiokande, and gallium [10–13] solar neutrino experiments by Mikheyev-Smirnov-Wölfenstein (MSW) oscillations, at the 90% confidence level. Our value of $\theta_{e\mu}$ implies that, as the error bars on the Ga experiments [12,13] are decreased, a low number of about 50 ± 10 solar neutrino units (SNU) will result [9]. To test this region of parameter space in the laboratory would require a long base line $\nu_\mu \nu_\tau$ oscillation search with sensitivity to smaller mixing angles than the present proposals. We note that the neutrinos in this solution are all too light to be a significant component of the dark matter.

In model II $\theta_{\mu\tau}$ is just beyond the E531 limits. This is very exciting because it means that the upcoming $\nu_\mu \nu_\tau$ oscillation searches will probe a large range of Δm^2 in this model [6–8]. In particular, if ν_τ makes a significant contribution to the dark matter in the Universe, then ~ 50 events will be seen and $\sin^2 2\theta_{\mu\tau}$ will be determined to be within 15% of 3×10^{-3}. We can still obtain an upper limit on m_{ν_τ} if we demand that ν_τ does not overclose the Universe. We have

$$m_{\nu_\tau} \leq 93 \text{ eV}(\Omega_\nu h^2)$$

with the Hubble constant $H_0 = 100h$ km/s Mpc and $\frac{1}{2} \leq h \leq 1$. For $h = \frac{1}{2}$ and $\Omega_\nu = 1$, we have $m_{\nu_\tau} < 23$ eV. This implies, using Eq. (14), $m_{\nu_\mu} < 1.5 \times 10^{-2}$ eV or $m^2_{\nu_\mu} < 2.4 \times 10^{-4}$ eV^2.

Now consider possible MSW oscillations. We find two possible solutions to the solar neutrino problem consistent with the "cosmological" upper bound on m_{ν_μ} and our value for $\theta_{e\mu}$, Eq. (17) (depicted as the vertical line labeled II in Fig. 2). In fact, if a signal is seen in $\nu_\mu \nu_\tau$ oscillations at CERN or at Fermilab which is consistent with our value of $\theta_{\mu\tau}$, then we predict values of $\theta_{e\mu}$ and m_{ν_μ} which are significant for the Cl, Kamiokande, and gallium solar neutrino experiments. This is our upper model II solution. In this case the Ga counting rate is about 110 SNU [9]. Note, this region is not consistent with combined fits to the present observations of solar neutrinos at 90% C.L. [10–13]. Nevertheless, perhaps it is too early to rule it out.[3] It is an interesting solution since, in addition to having a τ neutrino with mass of order 20 eV and thus a significant component of the dark matter,[4] it also provides a possible solution to the supernova shock reheating problem. The values of m_{ν_τ} and $\theta_{e\tau}$ are just in the range required by Fuller *et al.* [15] to allow for $\nu_\tau \nu_e$ oscillations in the supernova. This results in higher ν_e energies behind the shock, thus producing an increase in the heating rate.

If no signal is seen in $\nu_\mu \nu_\tau$ oscillations at CERN or at Fermilab, then the neutrino mass limits are sufficiently suppressed that there is no hope that $\nu_\mu \nu_e$ oscillations could be found at subsequent experiments such as the long baseline proposal at Fermilab (P822) [16]. Even if a signal were seen in $\nu_\mu \nu_\tau$ oscillations at CERN, a subsequent signal could only be seen at experiments such as proposed in Fermilab P822 if the ν_τ mass were above the cosmological limit of ~ 23 eV, a limit which we would expect to apply to ν_τ in this theory.

A third possible MSW solution to the solar neutrino problem is seen in Fig. 2 as the lower model II solution. It is not favored by GALLEX, but is consistent with chlorine and Kamiokande.

In conclusion, we have a very predictive model for neu-

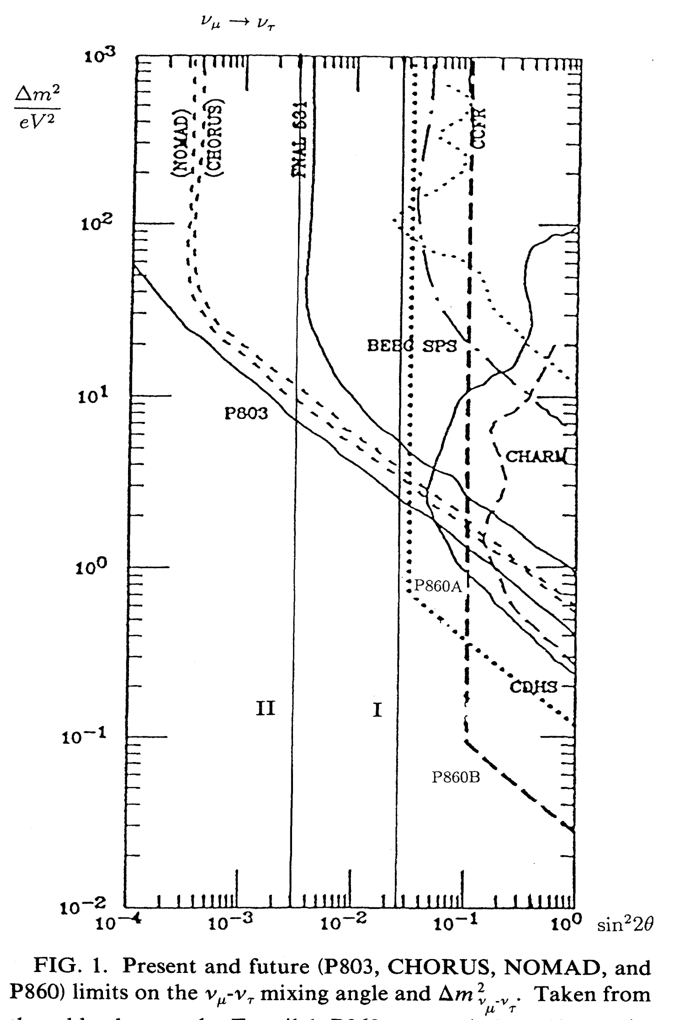

FIG. 1. Present and future (P803, CHORUS, NOMAD, and P860) limits on the ν_μ-ν_τ mixing angle and $\Delta m^2_{\nu_\mu\text{-}\nu_\tau}$. Taken from the addendum to the Fermilab P860 proposal, June 1992. The vertical lines labeled I and II are our predictions for $\kappa = 1$ and $\kappa = -\frac{1}{3}$, respectively.

[3]It might, in fact, be consistent with the solar model with a 5% higher core temperature [9].

[4]This could lead to a mixture of cold dark matter, some type of neutralino, and hot dark matter which seems to be preferred by recent Cosmic Background Explorer (COBE) data [14].

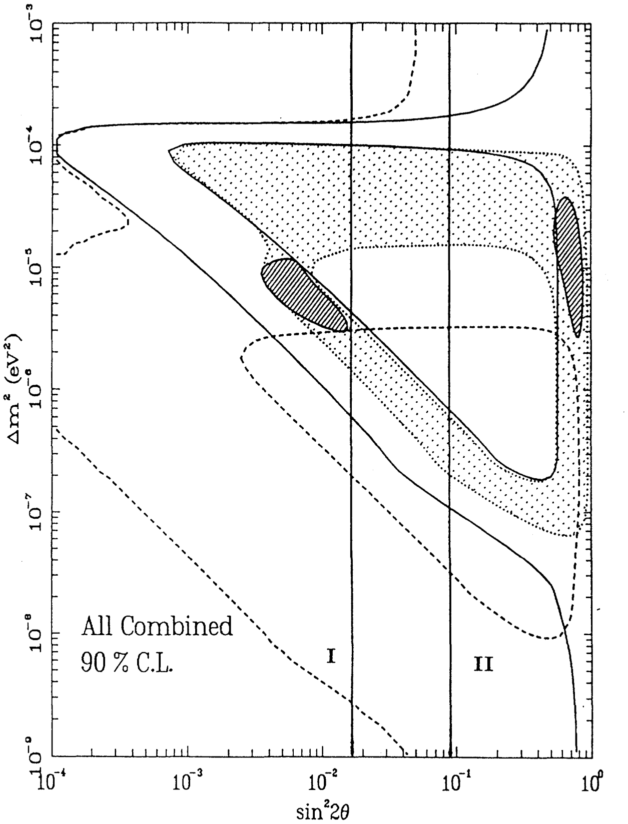

FIG. 2. The region in ν_e-ν_μ mixing and $\Delta m^2_{\nu_e\text{-}\nu_\mu}$ relevant for the MSW solution to the solar neutrino problem. The 90% C.L. regions are shown for the Homestake (dotted region), Kamiokande II+III (solid line), and GALLEX (dashed line) experiments. The shaded region is the combined fit of these three experiments (90% C.L.). This figure is reproduced from Ref. [9]. The vertical lines labeled I and II are our predictions for $\kappa=1$ and $\kappa=-\frac{1}{3}$, respectively.

trino masses and mixing angles. In terms of just one arbitrary parameter, the overall scale of the neutrino masses, we predict all 9 observable quantities of the neutrino sector. In version I of our model, we find the value of $\theta_{e\mu}$ and the allowed values for m_{ν_μ} lead to $\nu_e \nu_\mu$ resonant MSW neutrino oscillations which seem to be favored by present experiments to solve the solar neutrino problem. In this case we predict that with greater statistics the GALLEX and SAGE experiments will settle on a result of around 50 SNU. On the other hand, in version II of our model a large region of parameter space will be probed by the NOMAD and CHORUS experiments for $\nu_\mu \nu_\tau$ oscillations under construction at CERN or by P803 proposed at Fermilab. For a sufficiently large ν_τ mass, including values for which the ν_τ contributes a significant amount of dark matter to the Universe, these experiments will make a precision test of our theory, measuring $\sin^2 2\theta_{\mu\tau}$ to 15% accuracy. This model is also relevant for solar neutrino experiments, but is in a region of the $\Delta m^2 - \sin^2 2\theta_{e\mu}$ which is not consistent with the present data at 90% C.L. Our three possible MSW solutions to the solar neutrino problem are given in Table I, along with some of their properties.

TABLE I. The three possibilities for neutrino masses in the case where the solar neutrino problem is solved via MSW. The ν_μ-ν_τ oscillation lengths are for $E_{\nu_\mu}=20$ GeV.

Model	I	II_{upper}	II_{lower}
m_{ν_μ}	1.7×10^{-3} eV	10^{-2} eV	6×10^{-4} eV
m_{ν_τ}	0.4 eV	20 eV	1 eV
SAGE/GALLEX			
Counting rate	$\sim$50 SNU	$\simeq$110 SNU	$\leq$10 SNU
ν_μ-ν_τ osc.			
$L_{\nu_\mu\nu_\tau}$	100 km	44 m	16 km
$\sin^2 2\theta_{\mu\tau}$	0.026	0.003	0.003

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