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Generalized geometrical scaling for elastic hadron-hadron scattering at high energies

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We propose generalized geometrical scaling for elastic hadron-hadron scattering which has the scaling dimensional parameter $\sqrt{\sigma_t/\kappa(x)}$, $\kappa(x)$ being a function of the elasticity x characteristic of the form of the eikonal. This hypothesis presents a unified explanation of the features of the differential cross sections of pp and $\bar{p}p$ scattering in the energy region from the CERN Intersecting Storage Ring to the Fermilab Tevatron collider with the eikonal corresponding to the dipole-type form factor.

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Theoretically geometrical scaling is predicted for an elastic hadron-hadron scattering amplitude asymptotically under some generally accepted conditions, if the total cross section rises as $\ln^2(s)$ in the high-energy limit $s\to\infty$ [1]. Experimentally such behavior is observed at energies reached at the CERN Intersecting Storage Ring (ISR) [2] which are certainly not in the asymptotic range. This suggests the possibility that the observed geometrical scaling reflects an underlying general property of the hadron diffractive interaction not confined to the asymptotic range. The purpose of this work is to examine a new scaling hypothesis which is a generalization of conventional geometrical scaling (GS) [3] and to show that this generalized geometrical scaling (GGS) gives a unified explanation of the behavior of pp and $\bar{p}p$ scattering in the energy region from the ISR to the Fermilab Tevatron collider.

Neglecting spin effects, the c.m. system (c.m.s.) scattering amplitude is written in the impact parameter representation as

$$f(s,t)=ik\int_0^\infty\{1-e^{-\Omega(s,b)}\}J_0(\sqrt{-t}\,b)b\,db\ , \qquad (1)$$

where b is the impact parameter, k and s are the momentum and the squared energy, respectively, t is the squared momentum transfer, J_0 the cylindrical Bessel function of the order zero, and $\Omega(s,b)$ the eikonal. Here we neglect the real part of the scattering amplitude which will be introduced later by using the prescription given by Martin [4].

The conventional geometrical scaling requires a strong restriction of the behavior of the total (σ_t) and the elastic (σ_{el}), cross sections, i.e., stationary elasticity $x\,(=\sigma_{el}/\sigma_t)$, and cannot cope with an arbitrary variation of cross sections. If the geometrical scaling is a fundamental property of the strong interaction, we expect its existence in a more general situation. We, therefore, assume only that $\Omega(s,b)$ is factorized as [5]

$$\Omega(s,b)=w(s)g(b/r(s))\ , \qquad (2)$$

where w and r are functions depending only on s and should reproduce *both* of the total and the elastic cross sections consistently.

In the impact-parameter representation we have

$$\sigma_t=4\pi\int_0^\infty b\,db[1-e^{-w(s)g(b/r(s))}]$$
$$=4\pi r^2\int_0^\infty \beta\,d\beta[1-e^{-w(s)g(\beta)}]\ , \qquad (3)$$

$$\sigma_{el}=2\pi\int_0^\infty b\,db[1-e^{-w(s)g(b/r(s))}]^2$$
$$=2\pi r^2\int_0^\infty \beta\,d\beta[1-e^{-w(s)g(\beta)}]^2\ . \qquad (4)$$

The elasticity x is, therefore, independent of the scaling parameter r, and the interaction strength function w is uniquely determined by x. In this sense w is a function of x and we write

$$w(s)=h(x(s);g)\ . \qquad (5)$$

For a value of w fixed for given x, the scale parameter r should be chosen to give σ_t. Hence from Eq. (3) we have the equation

$$r(s)=\sqrt{\sigma_t(s)/\kappa(x(s);g)}\ , \qquad (6)$$

where

$$\kappa(x;g)\equiv 4\pi\int_0^\infty \beta\,d\beta[1-e^{-h(x;g)g(\beta)}]\ . \qquad (7)$$

This implies that only the scaling satisfying Eq. (6) works consistently. The problem is, then, whether or not there exists $g(\beta)$ such that it reproduces elastic differential cross sections.

Of the two well-discussed extreme geometrical pictures, the geometrical scaling hypothesis (GS) [3] assumes $w(s)=\text{const}$, $r(s)\propto\sqrt{\sigma_t(s)}$, while the factorized eikonal (FE) model [6] is given by $r(s)=\text{const}$.

The scaling behavior of the generalized geometrical scaling (GGS) is completely specified by the functional form of $g(\beta)$. For example, $\kappa(x;g)$ is proportional to x for the uniform disk; $g(\beta)=$const for $0\le\beta\le\beta_0$ and 0 for $\beta>\beta_0$ [7].

In Fig. 1 we show the inverse of the scaling parameter $\mu(s)\equiv 1/r(s)$ of the "dipole" case $[g(\beta)=\frac{1}{8}\beta^3K_3(\beta)$, $K_3(\beta)$ being the modified Bessel function of the order 3] [8] as well as the Gaussian case $[g(\beta)=\exp(-\beta^2)]$ for the total and elastic cross-section data of $\bar{p}p$ scattering in the ISR-Tevatron energy region. Here we have used the results of the empirical fit [9] as the experimental data and we have neglected the contribution from the real part of the scattering amplitude, which reduces the value of μ, for example, from 0.785 (0.778) to 0.779 (0.772) GeV at $\sqrt{s}=546$ (1800) GeV for $\rho=0.14$, the ratio of the real to the imaginary part of the forward scattering amplitude. We also show the curves for GS ($\mu\propto\sqrt{1/\sigma_t}$) and the uniform disk ($\sqrt{x/\sigma_t}$) for comparison. The exact FE model gives a horizontal straight line.

In order to test the validity of the present scaling assumption, we have to determine eikonals at individual energies of experiments and examine their variation. Here for simplicity we take the "dipole" form in view of the Chou-Yang geometrical picture [8]. The dipole will give qualitatively correct behavior about the features of GGS discussed below, though we may need some tuning of the form of the eikonal around the dipole in order to attain a better fit to the measurements of the differential cross section [10]. In the following we discuss pp and $\bar{p}p$ scattering at ISR and higher energies.

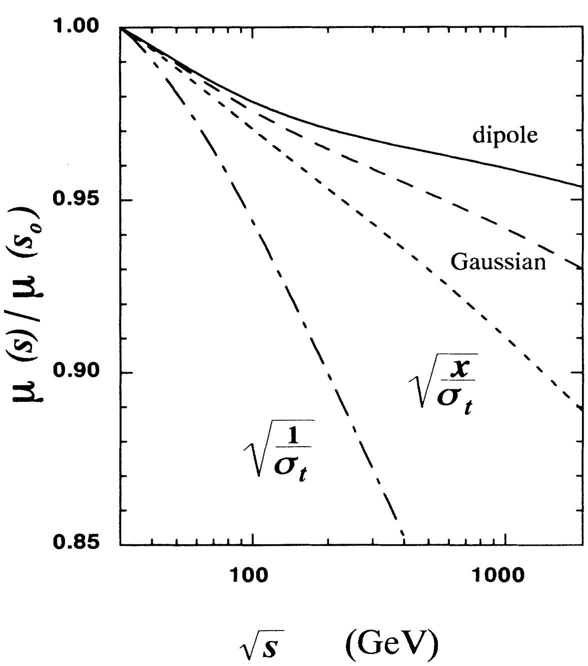

FIG. 1. The inverse of the scaling parameter $\mu(s)\equiv 1/r(s)$ is shown for the "dipole" and the Gaussian case for the total and elastic cross section data in the ISR-Tevatron energy region [9]. The curves corresponding to $\sqrt{1/\sigma_t}$ and $\sqrt{x/\sigma_t}$ are also shown. All the curves are normalized to unity at $s_0=(30\text{ GeV})^2$. The factorized eikonal (FE) implies a horizontal straight line. The value of μ for the "dipole" is 0.815 GeV at s_0.

(1) Obviously, if the elasticity $x(s)$ is independent of energy, GGS becomes the conventional GS [3]. In the ISR region we know that the elasticity is nearly constant for pp. In fact it was shown that GS holds well for pp scattering over the ISR region [2].

For $\bar{p}p$, the latest empirical fit [9] gives the elasticity weakly increasing with energy in the ISR region, which induces some deviation of μ from the curve $\sqrt{1/\sigma_t}$ as seen in Fig. 1. This suggests that GS is weakly violated. However, there are not enough data for discussing this problem further. The observed $\bar{p}p$ differential cross sections at $\sqrt{s}=53$ GeV would be better fitted in terms of the pp eikonal determined at the same energy if x and/or ρ of $\bar{p}p$ were somewhat larger than the reported ones.

Although it is not the purpose of this work to analyze the energy region of the CERN Super Protron Synchrotron (SPS) and lower where meson-nucleon data are also available [11], we comment on how GGS works for $\bar{p}p$ scattering below ISR energy. We have found the differential cross sections of $\bar{p}p$ scattering measured at $P_L=30$ and 50 GeV/c are not consistent with the *simple* application of GGS. However, the analysis by Kroll [12] suggests that GGS would work down to $P_L=50$ GeV/c ($\sqrt{s}=9.8$ GeV), if nondiffractive components were taken into account.

(2) Next we see what happens at CERN S$p\bar{p}$S and Tevatron energies. The elasticity $x(s)$ of $\bar{p}p$ scattering starts increasing markedly above ISR energies. This leads to slower decreasing of $\mu(s)$ with increasing energy as shown in Fig. 1. It is to be noted that the scaling with $\sqrt{x/\sigma_t}$ [7] holds approximately in the ISR region. Above 200 GeV the energy variation of μ becomes even slower and we enter into a quasi-FE region and some FE-type effects are then induced [13]: (i) The normalized differential cross section $(1/\sigma_t^2)d\sigma/dt$ is no longer a function of $\sigma_t t$; instead it changes with $\sigma_t t/\kappa$, but only approximately due to the energy variation of $w(s)$, and (ii) the curvature structure of the forward peak or the change of slope around $-t=0.1-0.2$ (GeV/c)^2 observed at ISR and lower energies will become weaker [7] and the second bump will rise as σ_t rises.

We have calculated the differential cross section by GGS with the "dipole" eikonal at 546 GeV. The results are shown in Fig. 2 together with the experimental data [14]. If the contribution from the real part is included by using the prescription given in Ref. [4], the predictions of GGS are in good agreement with the experiments. Here ρ is taken to be 0.14 of dispersion-relation calculation [15]. The local slopes of the forward peak are 16.2, 14.3, and 13.6 (GeV/c)$^{-2}$ at $-t=0$, 0.15, and 0.3 (GeV/c)^2, respectively, indicating still a considerable change of slope at small momentum transfers.

(3) As the energy goes up, we will have a nearly structureless forward peak without showing the apparent change of the slope at small momentum transfers. This occurs at around $x(s)=Y(s)$ where $Y(s)$ is $(1+\rho^2)\sigma_t/16\pi B$, B being the forward slope [13,16]. The results of the calculation at 1.8 TeV given in Fig. 3 fit well the measurements of the differential cross section [17], where we have included the real part with $\rho=0.14$

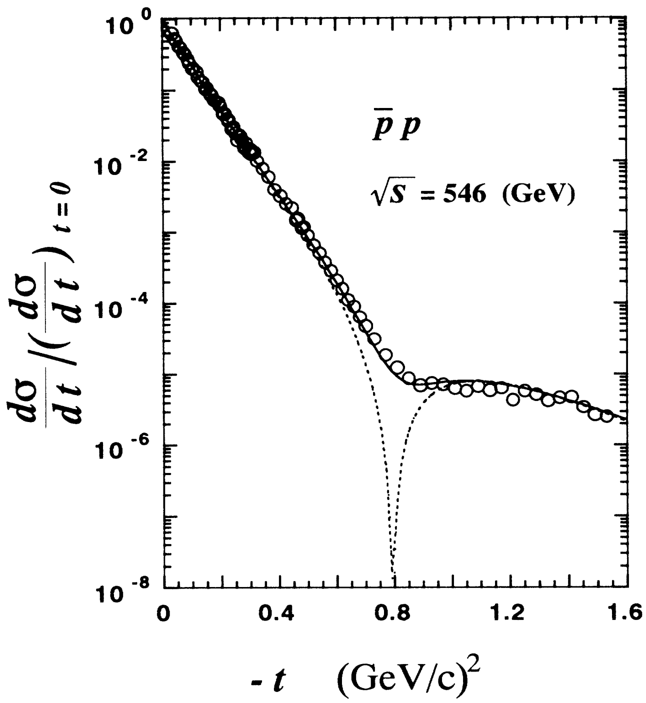

FIG. 2. The differential cross sections calculated at 546 GeV by GGS with the "dipole" eikonal together with the experimental data [14]. Here we have taken $\sigma_t = 61.4$ mb and $\sigma_{el} = 12.8$ mb of the empirical fit [9], $\rho = 0.14$ of the dispersion relation [15], and μ is 0.779 GeV from scaling. The solid curve is for the full amplitude, while the dotted one is for the imaginary part.

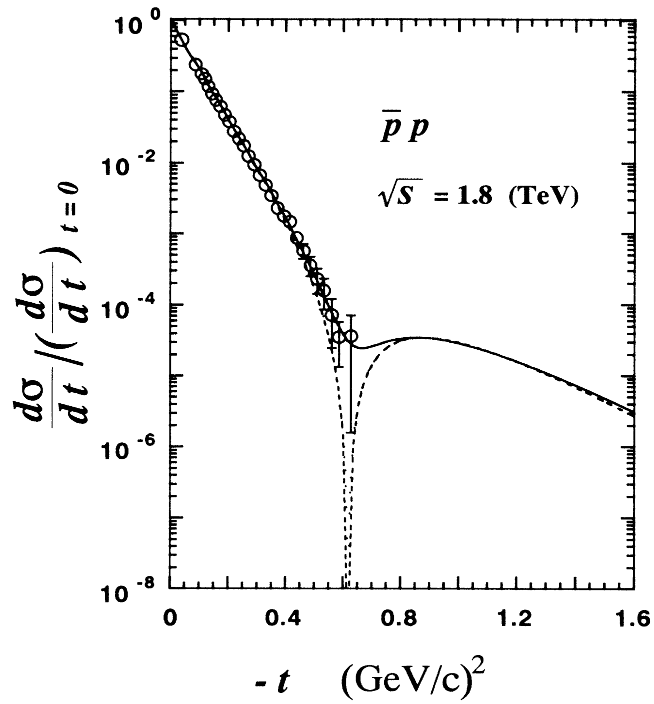

FIG. 3. The differential cross sections at 1.8 TeV calculated by GGS with the "dipole" eikonal together with the experimental data [17]. Here we have taken $\sigma_t = 74.8$ mb and $\sigma_{el} = 17.6$ mb of the empirical fit [9], $\rho = 0.14$ of the experiment [18], and μ is 0.772 GeV from scaling. The solid curve is for the full amplitude, while the dotted one is for the imaginary part.

[18]. The concave curvature structure seems to practically disappear at $-t = 0\text{–}0.3$ $(\text{GeV}/c)^2$, consistent with the experimental data. Here the calculation gives $x/Y \approx 1.04$. If we examine the slope more closely, however, it is not really constant: the calculation gives the slopes 17.3, 15.7, and 15.9 $(\text{GeV}/c)^{-2}$ at $-t = 0$, 0.15, and 0.3 $(\text{GeV}/c)^2$, respectively, which may be compared with the latest value of 16.99 ± 0.47 $(\text{GeV}/c)^{-2}$ obtained over the range $0.001 \le -t \le 0.14$ $(\text{GeV}/c)^2$ [18].

(4) Above this energy the forward peak will take a grossly convex or downward curvature structure if $Y(s) > x(s)$. Finally, if the total cross section rises indefinitely with increasing energy and the elasticity approaches a finite limit asymptotically, then the GS structure will again appear [19].

We have shown that the hypothesis of generalized geometrical scaling (GGS) explains the features of the measured differential cross sections in the ISR-Tevatron energy region reasonably well. To test this hypothesis further, we need detailed information on the eikonal, which requires accurate experimental data covering a wider t range in S$p\bar{p}$S and Tevatron energy regions as well as in energy regions reached at the CERN Large Hadron Collider and Superconducting Super Collider. The geometrical-picture analysis will give us valuable information on hadron-hadron interactions.

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