Virtual-photon bremsstrahlung in a quark-gluon plasma

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The production of dielectron pairs due to bremsstrahlung of quarks is calculated in a hot quark-gluon plasma. It is shown that multiple scattering of quarks leads to an infrared stable result for dielectron emission rates. The influence of the Landau-Pomeranchuk effect, due to destructive interference of virtual photons, is calculated and analyzed. For the soft electromagnetic signal from a QCD plasma, this effect is more important than the one due to the thermal quark mass; it furthermore reduces significantly the dielectron production rates. It is also shown that in the low invariant-mass region the contributions from quark and pion virtual bremsstrahlung are of the same order of magnitude.

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I. INTRODUCTION

When considering ultrarelativistic heavy-ion collision, one expects that strongly interacting (QCD) matter can be produced in a deconfined quark-gluon plasma (QGP) phase [1,2]. The important question here is the existence of physical observables whose behavior could confirm that a QGP has indeed momentarily been formed during the collision. Dileptons and photons have long been proposed as a signal for QGP formation [3–8]. Since these are electromagnetically interacting particles, once produced, they will escape the medium carrying information about the primordial state of matter in heavy-ion collisions. As this matter is produced with high density and temperature, new collective phenomena could play an important role here. The modification of the dilepton spectrum due to dynamical screening effects is an example of how many-body correlations could change the vacuum results [9–12].

In our present work we shall discuss the influence of the Landau-Pomeranchuk effect on low-mass dilepton pairs created through virtual bremsstrahlung in a dense QGP. This effect consists in the destructive interference of photons emitted from different parts of the trajectory of a fast charged particle moving through a dense medium.

The importance of the Landau-Pomeranchuk effect on the properties of hot QCD matter has been discussed previously in the literature. Its role in determining the radiative energy loss in a plasma has been pointed out in Ref. [13]. The suppression of bremsstrahlung from pions in a hadronic medium due to the Landau-Pomeranchuk scattering effect was discussed in our previous publication [14]. We will consider here the influence of this effect on the bremsstrahlung of virtual photons in the QGP. The reason why the case of QGP requires a special investigation is connected with the radical change in behavior of elastic particle cross sections in a QGP as compared to a pion gas. In a pion gas the scattering can be assumed to be mainly due to *s*-wave interactions, with a nearly isotropic final momentum distribution. In contrast, in a QGP, elastic cross sections are described mainly by *t*-exchange Feynman diagrams. This means that the small-angle scattering dominates and correlations between subsequent collisions must be taken into account. This problem is well known in the theory of fast charged particles penetrating through solids [15–17]. In this case the transport equation can be reduced to a Fokker-Planck equation and the main parameter appears to be not the elastic cross section itself, but the mean square of the scattering angle of the fast particle per unit path.

The motivation of our present work is contained in the observation that in a QGP the main source of low invariant-mass dielectron pairs is virtual quark bremsstrahlung [18]. Thus a detailed study of bremsstrahlung production and in particular the modification of the result due to the medium is of crucial importance when discussing the soft electromagnetic signal of QGP formation in heavy-ion collision. The results on dielectron production rates due to quark bremsstrahlung which we shall present in this paper are not very sensitive to the value of the quark mass m_q assumed in numerical calculations. In the limit of massless quarks, our results are infrared stable. These properties are in contrast with Ref. [18], where the m_q dependence of the dielectron rate is very strong and the result diverges in the limit of $m_q \to 0$. The origin for this difference is that in our approach we include the off-shell condition for photons in a quark propagator.

In addition to a kinematically consistent treatment of virtual bremsstrahlung by quarks, we explicitly include multiple scattering. We shall explain that this effect cannot be neglected as it modifies the low invariant-mass dielectrons spectrum by a several orders of magnitude.

The paper is organized as follows. In Sec. II we present the results on virtual-photon bremsstrahlung with the Landau-Pomeranchuk effect explicitly taken into account. In Sec. III we extend our discussion to dilepton production. In Sec. IV the numerical results for the dielectron production rates in a QGP are obtained. Finally, in Sec. V we present a brief summary and concluding remarks.

II. LANDAU-POMERANCHUK EFFECT

In order to discuss the role of the Landau-Pomeranchuk effect on photon production in a QGP due to quark bremsstrahlung, we shall adopt here a semiclassical approach developed in Ref. [16].

In the soft-photon approximation, the energy of photons emitted by a moving charged particle per unit photon momentum is described by the well-known expression

$$\frac{dI}{d^3k}=\frac{e_q^2\alpha}{(2\pi)^2}\left|\int_{-\infty}^{\infty}dt\, e^{i[\omega t-\mathbf{k}\cdot\mathbf{r}(t)]}\mathbf{n}\times\mathbf{v}\right|^2 , \tag{2.1}$$

where $\mathbf{r}(t)$ and $\mathbf{v}(t)$ are the coordinate and velocity of the charged particle, e_q is its charge, α is the fine-structure constant, and $\mathbf{n}=\mathbf{k}/|\mathbf{k}|$ is the unit vector directed along the photon momentum.

The result in Eq. (2.1) can also be written as

$$\begin{aligned}\frac{dI}{d^3k}=\frac{e_q^2\alpha}{(2\pi)^2}2\,\mathrm{Re}\int_0^{\infty}dt\int_0^{\infty}d\tau\, e^{-i\{\omega\tau-\mathbf{k}\cdot[\mathbf{r}(t+\tau)-\mathbf{r}(t)]\}}\\ \times\{\mathbf{v}(t+\tau)\cdot\mathbf{v}(t)\\ -[\mathbf{n}\cdot\mathbf{v}(t+\tau)][\mathbf{n}\cdot\mathbf{v}(t)]\} ,\end{aligned} \tag{2.2}$$

assuming that the particle starts to move at the initial time $t_0=0$. Equation (2.2) describes the energy emitted from a given particle trajectory $\mathbf{r}(t)$.

It is clear from Eq. (2.2) that one obtains a nonvanishing result even when the test particle is moving in the vacuum. This is because the radiation from an infinitely accelerated particle at the initial time $t_0=0$ is included in Eq. (2.2). Since in our further discussion we shall only be interested in medium effects on photon radiation, the above vacuum contribution will be subtracted from our final result [see Eq. (2.17)]. One has to note also that, as the time integration in Eq. (2.2) is carried out from zero to infinity, we explicitly assume that the lifetime and volume of the plasma are infinite. It will be shown that the multiple scattering of a charged particle in a medium decreases the probability of soft—real or virtual—photon emission. Taking into account the finiteness of the plasma volume, one would get an additional suppression for photon radiation. In the present discussion, however, we shall not include the influence of the plasma size on the photon production rates. Thus our results should be only considered as a lower bound for the suppression of dilepton and photon production in a QGP due to the medium.

In the following calculations, we shall assume a small-angle approximation. Under this approximation, a particle of energy E and rest mass m moving through the medium emits photons and scatters at very small angles of the order of $m/E\ll 1$. This approximation is known to be valid in the ultrarelativistic limit.

Approximating the longitudinal component of $\mathbf{n}$ as $n_z\approx 1-\mathbf{n}_t^2/2$ and $v_z\approx 1-m_q^2/2p_1^2-\mathbf{v}_t^2/2$, where $\mathbf{n}_t$ and $\mathbf{v}_t$ are, respectively, the transversal components of the unit vector $\mathbf{n}$ and of the particle velocity $\mathbf{v}$ (m_q is the quark mass and p_1 is the particle three-momentum with $m_q\ll p_1$), one can see that

$$\begin{aligned}\mathbf{v}_1\cdot\mathbf{v}_2-(\mathbf{n}\cdot\mathbf{v}_1)(\mathbf{n}\cdot\mathbf{v}_2)\approx n_t^2+(\mathbf{v}_{1t}\cdot\mathbf{v}_{2t})-(\mathbf{n}_t\cdot\mathbf{v}_{1t})-(\mathbf{n}_t\cdot\mathbf{v}_{2t})\\ =(\mathbf{v}_{1t}-\mathbf{n}_t)\cdot(\mathbf{v}_{2t}-\mathbf{n}_t) ,\end{aligned} \tag{2.3}$$

where terms of the order of $O(v_t^2n_t^2)$ have been neglected.

Similarly, one can write

$$\begin{aligned}\omega\tau-\mathbf{k}\cdot[\mathbf{r}(t+\tau)-\mathbf{r}(t)]&\approx\omega\tau-\omega\left[1-\frac{M^2}{2\omega^2}\right]\left[\left[1-\frac{n_t^2}{2}\right]\int_t^{t+\tau}v_z dt'+\int_t^{t+\tau}(\mathbf{n}_t\cdot\mathbf{v}_t)dt'\right]\\ &=\frac{\omega\tau}{2}\left[\frac{M^2}{\omega^2}+\frac{m_q^2}{p_1^2}\right]+\frac{\omega}{2}\int_t^{t+\tau}dt'[\mathbf{v}_t(t')-\mathbf{n}_t]^2 .\end{aligned} \tag{2.4}$$

Following Ref. [16], in order to obtain the total energy emitted, we have to average Eq. (2.2) over all possible particle trajectories. Then

$$\begin{aligned}\left\langle\frac{dI}{d^3k}\right\rangle=\frac{e_q^2\alpha}{(2\pi)^2}2\,\mathrm{Re}\int_0^{\infty}dt\int_0^{\infty}d\tau\int d^3r\, d^2v_t w_1(\mathbf{r},\mathbf{v}_t,t)\int d^3\rho\, d^2v_{1t}w_2(\mathbf{r}+\boldsymbol{\rho},\mathbf{v}_{1t},t+\tau|\mathbf{r},\mathbf{v}_t,t)\\ \times(\mathbf{v}_t-\mathbf{n}_t)\cdot(\mathbf{v}_{1t}-\mathbf{n}_t)e^{-i(\omega\tau-\mathbf{k}\cdot\boldsymbol{\rho})} ,\end{aligned} \tag{2.5}$$

where $w_1(\mathbf{r},\mathbf{v}_t,t)$ is the probability for the particle to have coordinate $\mathbf{r}$ and transversal velocity $\mathbf{v}_t$ at time t, and $w_2(\mathbf{r}+\boldsymbol{\rho},\mathbf{v}_{1t},t+\tau|\mathbf{r},\mathbf{v}_t,t)$ is the probability to have coordinate $\mathbf{r}+\boldsymbol{\rho}$ and transversal velocity $\mathbf{v}_{1t}$ at time $t+\tau$, provided that at the moment t these coordinate and velocity were equal to $\mathbf{r}$ and $\mathbf{v}_t$, respectively.

As our result for the emission rate should be translationally invariant, the correlation probability w_2 depends obviously only on $\boldsymbol{\rho}$ and τ. Moreover, one can see from Eq. (2.4) that $(\omega\tau-\mathbf{k}\cdot\boldsymbol{\rho})$ depends actually on some special combination of the $\boldsymbol{\rho}$ components:

$$\xi\equiv\int_t^{t+\tau}dt'[\mathbf{v}_t(t')-\mathbf{n}_t]^2 \ . \tag{2.6}$$

Therefore averaging over $\boldsymbol{\rho}$ is equivalent to averaging over ξ. For these reasons we will use a shorter notation $w_2(\xi,\mathbf{v}_{1t},\tau|\mathbf{v}_t)$ for w_2. We will also assume that the energy of the particle is constant along the whole trajectory. This assumption is evidently in agreement with a soft-photon approximation.

Averaging over $\mathbf{r}$ and $\mathbf{v}_t$ in Eq. (2.5) is straightforward and gives only an angular spread for the photons emitted by the charged particle. As we are interested in a spectral distribution dI/dk, we can integrate Eq. (2.5) over d^2n_t and obtain a time-independent integrand:

$$\frac{dI}{dk\,dt}=\frac{e_q^2\alpha k^2}{(2\pi)^2}2\,\mathrm{Re}\int d^2n_t\int_0^\infty d\tau\int d\xi\, d^2v_{1t}(\mathbf{n}_t-\mathbf{v}_t)\cdot(\mathbf{n}_t-\mathbf{v}_{1t})e^{-i(\omega\tau-\mathbf{k}\cdot\boldsymbol{\rho})}w_2(\xi,\mathbf{v}_{1t},\tau|\mathbf{v}_t) \ . \tag{2.7}$$

In the above and in all further equations, we shall omit the averaging sign $\langle\cdots\rangle$; however, the averaging over the trajectories is always made if necessary.

Let us turn now to the correlation probability function w_2. This function satisfies a kinetic equation which can be reduced to a Fokker-Planck equation [17]. In the small-angle approximation, one obtains [16]

$$\frac{\partial w_2}{\partial\tau}+(\mathbf{v}_{1t}-\mathbf{n}_t)^2\frac{\partial w_2}{\partial\xi}=\frac{\langle\theta_S^2\rangle}{4}\Delta_{v_{1t}}w_2 \ , \tag{2.8}$$

where

$$\Delta_{v_{1t}}=\left[\frac{\partial^2}{\partial v_{1x}^2}+\frac{\partial^2}{\partial v_{1y}^2}\right]$$

is the Laplace operator in the transversal velocity and $\langle\theta_S^2\rangle$ is the mean square of scattering angle of the fast particle per unit path:

$$\langle\theta_S^2\rangle=\left\langle nv_{12}\int d\Omega\,\theta^2\frac{d\sigma}{d\Omega}\right\rangle \ , \tag{2.9}$$

where n is the density of the medium, v_{12} is the relative velocity, $d\sigma/d\Omega$ is the elastic cross section, and θ is the angle between the initial and final particle momenta. The average $\langle\theta_S^2\rangle$ depends on both particle's momentum and the medium properties.

Equation (2.8) is to be solved under the initial condition

$$w_2(\tau=0)=\delta(\xi)\delta(\mathbf{v}_{1t}-\mathbf{v}_t) \ . \tag{2.10}$$

Introducing new variables $\boldsymbol{\theta}\equiv\mathbf{v}_t-\mathbf{n}_t$ and $\boldsymbol{\theta}_1\equiv\mathbf{v}_{1t}-\mathbf{n}_t$, one can reduce Eq. (2.7) to

$$\frac{dI}{dk\,dt}=\frac{e_q^2\alpha k^2}{2\pi^2}\mathrm{Re}\int d^2\theta\,\boldsymbol{\theta}\int_0^\infty d\tau\int d^2\theta_1\boldsymbol{\theta}_1F(\boldsymbol{\theta},\boldsymbol{\theta}_1,\tau) \ , \tag{2.11}$$

where we have defined

$$F(\boldsymbol{\theta},\boldsymbol{\theta}_1,\tau)\equiv\int_{-\infty}^{\infty}d\xi\exp\left[-i\frac{\omega\tau}{2}\left[\frac{m_q^2}{p_1^2}+\frac{M^2}{\omega^2}\right]-i\frac{\omega\xi}{2}\right]w_2(\xi,\boldsymbol{\theta}_1,\tau|\boldsymbol{\theta}) \ . \tag{2.12}$$

Combining Eqs. (2.8) and (2.9) and Eq. (2.12), we also get

$$\frac{\partial F}{\partial\tau}+i\frac{\omega}{2}\left[\frac{m_q^2}{p_1^2}+\frac{M^2}{\omega^2}+\theta_1^2\right]F=q\Delta_{\theta_1}F \ , \tag{2.13}$$

where

$$F(\tau=0)=\delta(\boldsymbol{\theta}_1-\boldsymbol{\theta}) \tag{2.14}$$

and $q\equiv\langle\theta_S^2\rangle/4$.

The solution of Eqs. (2.13) and (2.14) can be found as

$$F(\boldsymbol{\theta},\boldsymbol{\theta}_1,\tau)=\frac{1}{\pi\sinh(\sqrt{2i\omega q}\,\tau)}\left[\frac{i\omega}{8q}\right]^{1/2}\exp\left[-i\frac{\omega\tau}{2}\left[\frac{m_q^2}{p_1^2}+\frac{M^2}{\omega^2}\right]-(\theta^2+\theta_1^2)\left[\frac{i\omega}{8q}\right]^{1/2}\coth(\sqrt{2i\omega q}\,\tau)\right.$$
$$\left.+\left[\frac{i\omega}{2q}\right]^{1/2}\frac{\boldsymbol{\theta}\cdot\boldsymbol{\theta}_1}{\sinh(\sqrt{2i\omega q}\,\tau)}\right] \ . \tag{2.15}$$

Thus, from Eq. (2.11), we also have

$$\frac{dI}{dk\,dt}=-\frac{e_q^2\alpha k^2}{2\pi^2}\left[\frac{m_q^2}{p_1^2}+\frac{M^2}{\omega^2}\right]\mathrm{Re}\int_0^\infty d\tau\exp\left[-i\frac{\omega\tau}{2}\left[\frac{m_q^2}{p_1^2}+\frac{M^2}{\omega^2}\right]\right]\int d^2\theta\exp\left[-\theta^2\left[\frac{i\omega}{8q}\right]^{1/2}\tanh(\sqrt{2i\omega q}\,\tau)\right]. \quad (2.16)$$

The angular integration in the above equation can be done analytically, and after subtracting the vacuum contribution, one obtains finally the following result for the number of virtual photons emitted per unit frequency interval and per unit time by a fast charged particle moving in a medium:

$$\frac{dN^{\gamma^*}}{d\omega\,dt}=\frac{e_q^2\alpha k}{\pi\omega}\left[\frac{m_q^2}{p_1^2}+\frac{M^2}{\omega^2}\right]\int_0^\infty dx\left[\frac{1}{\tanh x}-\frac{1}{x}\right]\times e^{-2sx}\sin 2sx\ , \quad (2.17)$$

where

$$s=\frac{1}{8}\left[\frac{m_q^2}{p_1^2}+\frac{M^2}{\omega^2}\right]\left[\frac{\omega}{q}\right]^{1/2}. \quad (2.18)$$

In the limit when the virtual photon is going on shell, $M\to 0$, Eq. (2.17) coincides with the result obtained previously by Migdal [16] for real-photon production. It is interesting to note, as we show later, that the rates in Eq. (2.17) are infrared stable in the limit of massless quarks. This is because the invariant mass of a virtual photon acts as an infrared cutoff in the above equation.

Following Ref. [16], let us introduce a new function $\phi(s)$, defined as

$$\phi(s)=24s^2\int_0^\infty dx\left[\frac{1}{\tanh x}-\frac{1}{x}\right]e^{-2sx}\sin 2sx\ , \quad (2.19)$$

which determines the virtual-photon distribution in Eq. (2.17).

We have found that the above function can be approximated by

$$\phi(s)^*\sim\left[1+\left[\frac{1}{6s}\right]^2\right]^{-1/2}. \quad (2.20)$$

The results for the exact equation (2.19) and approximate equation (2.20) values of the function $\phi(s)$ are shown in Fig. 1. As seen in Fig. 1, within 10% accuracy $\phi(s)$ can be well described by Eq. (2.20). This accuracy is sufficient to accept the approximate result on the function $\phi(s)$ in our further considerations.

Using Eq. (2.10), the result for the virtual-photon emission rate can be written in a much simplified form:

$$\frac{dN^{\gamma^*}}{d\omega\,dt}=\frac{8e_q^2\alpha k}{3\pi\omega^2}q\left[\left[\frac{m_q^2}{p_1^2}+\frac{M^2}{\omega^2}\right]^2+\frac{16q}{9\omega}\right]^{-1/2}. \quad (2.21)$$

It is easy to see that the above expression remains finite in the limit of massless quarks. Thus the scattering of particles in a medium regularizes the infrared behavior of the corresponding amplitudes.

The influence of the Landau-Pomeranchuk effect on virtual-photon emission by moving quark in a medium is contained in the last term in the square brackets in Eq. (2.21). If this term is negligibly small as compared to the first one, then the effect is absent.

III. DILEPTON PRODUCTION IN A QUARK-GLUON PLASMA

In the previous section, we have derived the expression for the production of virtual photons in a QGP due to quark bremsstrahlung. The role of the Landau-Pomeranchuk effect, as seen in Eq. (2.21), is completely determined by the parameter q related to quark-quark and quark-gluon elastic scattering in a plasma. To the lowest order in the α_s, the small-angle elastic cross sections in a QGP are described by the t-channel-exchange Feynmann diagrams, which give

$$\frac{d\sigma_{ab\to ab}}{dt}=\frac{2\pi\alpha_s^2}{t^2}C_{ab}\ , \quad (3.1)$$

where

$$C_{ab}=\begin{cases}\frac{4}{9} & \text{for } qq\to qq,\\ 1 & \text{for } qg\to qg\ .\end{cases}$$

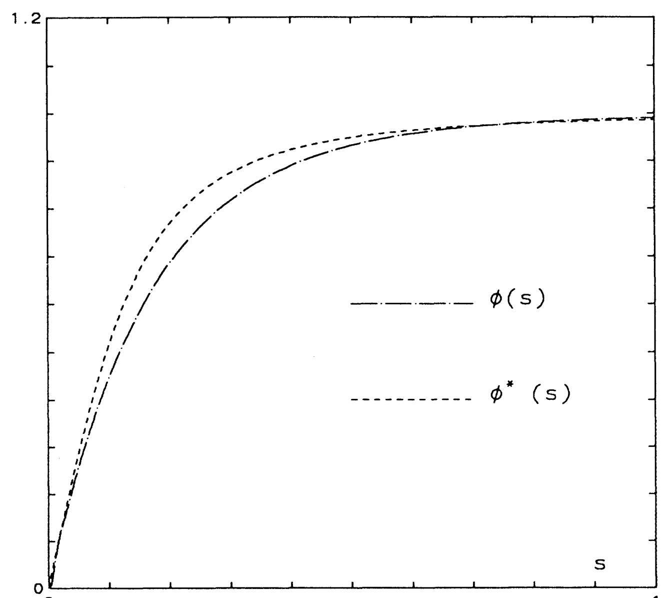

FIG. 1. Function $\phi(s)$ (see text for details): dot-dashed line, exact result [see Eq. (2.19)]; dashed line, approximate result [see Eq. (2.20)].

When integrating the cross sections in Eq. (3.1) over $d\Omega$ with a weight factor θ^2 [see Eq. (2.10)], then, in a small-angle approximation, one finds a typical logarithmically divergent integral:

$$\int_{\theta_{\min}}^{\theta_{\max}} \frac{d\theta}{\theta} = \ln\left[\frac{\theta_{\max}}{\theta_{\min}}\right] .$$

To estimate this integral, one chooses the upper limit $\theta_{\max}$ to be close to unity: This is the upper boundary for the validity of the small-angle approximation. The lower limit $\theta_{\min}$ in a Coulomb-like media is usually cut off by many-body effects such as charge screening. In the QGP a typical screening scale is a Debye mass m_D proportional to gT, with g being the coupling constant and T the temperature of the medium [19,9]. Therefore $\theta_{\max}/\theta_{\min} \sim p_1/gT$, where p_1 is the momentum of the charged particle. In a thermal system, however, temperature gives a natural scale for the particle momentum. Thus, restricting ourselves to logarithmic accuracy, we shall neglect in our further calculations the weak dependence of the above integral on the charged-particle momentum p_1 and replace $\ln(\theta_{\max}/\theta_{\min})$ with a constant factor L_C proportional to $\ln\alpha_s$.

We have to point out that multiple scattering and charge screening are not the only collective effects to be expected in a dense medium. Additionally, there is the dynamical generation of a thermal mass for fermionic degrees of freedom in the medium. According to Ref. [19], fermions in a QGP acquire a thermal mass $m_q^2 \sim 2\pi\alpha_s/3T$ induced by many-body dynamical effects. From Eq. (2.21), however, one can deduce that when considering the process of real- and virtual-photon bremsstrahlung in a plasma, the thermal-mass-generation effect is negligible compared with the multiple-scattering one. Indeed, taking the limit $M \to 0$ in Eq. (2.21), one can see that the second term in the square brackets is of the order $\alpha_s^2 \ln\alpha_s$, while the first one is only of the order of α_s^2. For this reason the Landau-Pomeranchuk effect is of crucial importance when discussing soft electromagnetic signal from a QGP. Actually, in the numerical calculation, because of the relatively large value of the coupling constant α_s at the temperature of the order of 0.2–0.4 GeV, the thermal quark mass gives approximately a 30% correction to the virtual-photon production rates. At large values of the invariant mass, both Landau-Pomeranchuk and dynamical screening effects are negligible. Because of the soft-photon approximation, the result of Eq. (2.21) cannot be applied with confidence in the kinematic range of large M.

Assuming for simplicity a Boltzmann distribution for both quarks and gluons, we obtain now, from Eqs. (2.9) and (3.1),

$$q = C\alpha_s^2 T^3 p_1^{-2} , \tag{3.2}$$

where $C = L_C d/\pi \approx 8.5 L_C$. Here d is the product of the quark-gluon degeneracy factor and the weights C_{ab} from Eq. (3.1): $d = \frac{4}{9} \times 2 \times 2_f \times 2_s \times 3_c + 2_s \times 8_c = 26\frac{2}{3}$.

Thus Eq. (3.2) together with Eq. (2.17) allow us to determine the number of soft virtual photons emitted by a charged particle moving in a QCD medium per unit path and unit frequency interval.

We now turn our attention to the dilepton production rate $dN^{e^+e^-}/d^4x\, dM$. This is related to the rate of virtual photons dN^{γ^*}/d^4x through a well-known formula [6,20]

$$\frac{dN^{e^+e^-}}{d^4x\, dM} = \frac{2\alpha}{3\pi M}\left[1 - \frac{4m_l^2}{M^2}\right]^{1/2}\left[1 + \frac{2m_l^2}{M^2}\right]\frac{dN^{\gamma^*}}{d^4x} . \tag{3.3}$$

In the following equations, we shall ignore the threshold factors since they are close to unity for dielectron production.

To obtain dN^{γ^*}/d^4x, one needs to integrate expression (2.21) over the virtual-photon frequency $d\omega$ and sum over all emitting particles in a unit volume. The difficulty here is that this procedure is out of the range of the soft-photon approximation. We have imposed the integration limits *a priori* with the help of the energy-momentum conservation laws. The procedure is similar to the case of our previous publication [14], where the Landau-Pomeranchuk effect in a dense hadronic medium has been discussed. In a QGP the only difference is that now the photon emission is a result of multiple small-angle scatterings. In this case one does not need to sum Eq. (2.21) over the scatterer's moment distribution, but instead one averages over different collisions:

$$\frac{dN^{\gamma^*}}{d^4x} = \Sigma_f 2d_q \int d^3p_1 f(\mathbf{p}_1)\frac{\int d^3p_2 f(\mathbf{p}_2)|\mathbf{v}_{12}|}{\int d^3p_2 f(\mathbf{p}_2)|\mathbf{v}_{12}|} \times \int_M^\Delta d\omega \frac{dN^{\gamma^*}}{d\omega\, dt} , \tag{3.4}$$

where $f(\mathbf{p})$ is the momentum distribution function,

$$|\mathbf{v}_{12}| = [s(s - 4m_q^2)]^{1/2}/2E_1E_2$$

is the relative velocity, E_1 and E_2 are the energies of colliding particles, $s = 2(m_q^2 + E_1E_2 - \mathbf{p}_1 \cdot \mathbf{p}_2)$, and $d_q = 2_s \times 3_c = 6$ is the quark degeneracy factor. The additional factor 2 is related to the antiquark contribution. The upper limit Δ is determined by the largest possible energy that can be radiated in a given collision.

In the following calculations, we will assume for simplicity that quarks and gluons are nearly massless and both of them are distributed according to the Boltzmann distribution function

$$f(\mathbf{p}) = (2\pi)^{-3} e^{-p/T} . \tag{3.5}$$

In this case $s = 2p_1p_2(1 - \cos\theta)$ and $|\mathbf{v}_{12}| = (1 - \cos\theta)$, where θ is the angle between $\mathbf{p}_1$ and $\mathbf{p}_2$. Combining Eqs. (3.3)–(3.5) with Eq. (2.21) and transforming the θ integration into an integration over s, one has

$$\frac{dN^{e^+e^-}}{d^4x\,dM}=\frac{2\alpha}{3\pi M}\frac{4d_q(e_u^2+e_d^2)\alpha}{3}\frac{C\alpha_s^2}{(2\pi)^3}$$
$$\times\int_0^\infty dp_1 p_1^{-2}e^{-p_1/T}\int_0^\infty dp_2 e^{-p_2/T}\int_{M^2}^{4p_1p_2}s\,ds\int_M^\Delta d\omega\frac{k}{\omega^2}\left[\left[\frac{m_q^2}{p_1^2}+\frac{M^2}{\omega^2}\right]^2+\frac{16C\alpha_s^2T^3}{9p_1^2\omega}\right]^{-1/2}, \tag{3.6}$$

where

$$\Delta=\frac{1}{2s}\{s[E+(E^2-s)^{1/2}]+M^2[E-(E^2-s)^{1/2}]\} \tag{3.7}$$

and $E=p_1+p_2$.

Two of the four integrations in the above equation can be done analytically by interchanging corresponding integrals. After this, the following result for the number of soft-dilepton pairs produced in a QGP is obtained:

$$\frac{dN^{e^+e^-}}{d^4x\,dM}=\left[\frac{2\alpha}{3\pi^2}\right]^2 d_q(e_u^2+e_d^2)\frac{C\alpha_s^2T^2}{M}\int_0^\infty dp_1p_1^{-2}\int_M^\infty d\omega\frac{k}{\omega^2}F(p_1,\omega)\left[\left[\frac{m_q^2}{p_1^2}+\frac{M^2}{\omega^2}\right]^2+\frac{16C\alpha_s^2T^3}{9p_1^2\omega}\right]^{-1/2}, \tag{3.8}$$

where

$$F(p_1,\omega)=\begin{cases} p_1(M^2+4p_1T)e^{(-1/T)(p_1+M^2/4p_1)} & \text{for } M<\omega<p_1+\dfrac{M^2}{4p_1},\\ \omega(M^2+4\omega T)e^{-\omega/T}+4[p_1^2(\omega+T)-\omega^2(p_1+T)]e^{-(p_1+\omega)/T} & \text{for } \omega>p_1+\dfrac{M^2}{4p_1}.\end{cases}$$

In the derivation of the above equation, we have neglected small terms of the order of $O(M^2/\omega^2)$ where it was possible.

The result in Eq. (3.8) is our final expression for the dilepton production rates due to quark bremsstrahlung in a QCD plasma with the Landau-Pomeranchuk effect taken explicitly into account.

IV. NUMERICAL RESULTS AND DISCUSSION

In the following section, we present a more detailed quantitative discussion of e^+e^- emission rates from a QGP. We restrict our analysis to the kinematical window of the dielectron spectrum, $dN/d^4x\,dM$, where the invariant mass M of the pair is smaller then the ρ vector-meson mass. This restriction is mainly because of the soft-photon approximation we have adopted in our calculations. Also, as it was already shown in Ref. [1,8], the hard part of the spectrum is dominated either by the Drell-Yan production or by the thermal pion or quark annihilation processes provided that the temperature of the system is high enough. Thus one cannot expect that bremsstrahlung could be of any importance here.

We shall not discuss a detailed model for QGP formation in ultrarelativistic heavy-ion collisions. We simply assume a static QGP in thermal equilibrium with a fixed temperature. Thus the expansion of the plasma and the time dependence of any thermodynamical parameters are not considered here. The lifetime of a QGP is assumed to be infinite.

The above model of thermalized QCD matter is certainly far from reality. In heavy-ion collisions, if the quark-gluon plasma is produced, it will exist in a finite spacetime volume and will undergo hydrodynamical expansion, leading to a variation of the temperature. The aim of our considerations, however, is mostly to explain the role of destructive interference of virtual photons due to the Landau-Pomeranchuk effect. For this purpose a detailed and more realistic model for a QGP is not important. We shall indicate in our conclusions the possible modifications of the results if the expansion dynamics and a finite lifetime of a QGP are taken into account.

In our numerical calculations, we have assumed that the quark mass and the strong-coupling constant are temperature-independent parameters. The constant values of $m_q=5$ MeV, $\alpha_s\sim 0.3$, and $L_C\sim 1.0$ have been assumed in our numerical calculations. Variations of the strong-coupling constant in the temperature range $0.15<T<0.3$ GeV are expected to be small, $0.2<\alpha_s<0.3$ [21].

The dependence of the quark mass on temperature is more interesting. Our results, in contrast with the results obtained in Ref. [18], are not very sensitive on the value of the quark mass. As we have already indicated in the previous sections, an infrared stable result for bremsstrahlung in the limit of massless quarks has been obtained. This result is natural for virtual-photon production as the invariant mass plays the role of the infrared cutoff. In the range of invariant mass of e^+e^- pairs, where $m_q<M$, the results are not sensitive to the value of m_q. We have found out that the replacement of the current by a thermal quark mass will not change our results more than 30%. It will not be important for $M>0.4$ GeV, either.

To begin our discussion, we show in Fig. 2 the resulting invariant-mass spectra for virtual bremsstrahlung from quarks in a QGP with temperature $T=0.15$ GeV. The invariant-mass dilepton distribution without

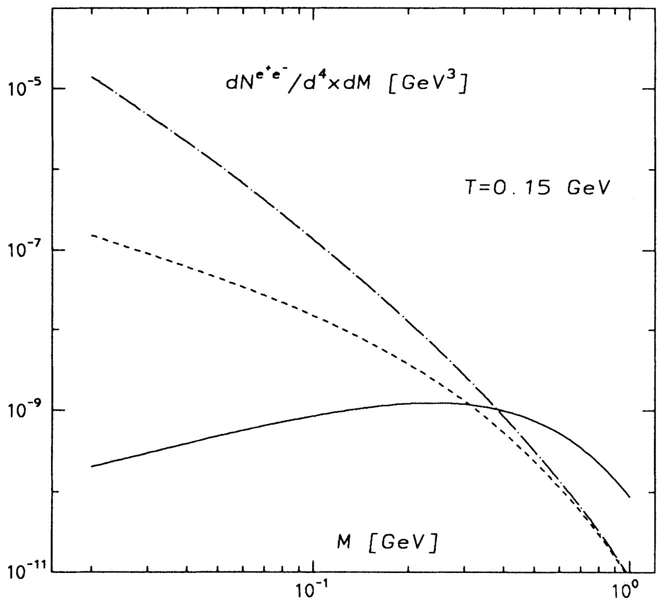

FIG. 2. Dielectron production rate in a quark-gluon plasma with fixed temperature $T = 0.15$ GeV: dot-dashed line, virtual-quark bremsstrahlung without medium effect; dashed line, virtual-quark bremsstrahlung with the Landau-Pomeranchuk effect taken explicitly into account; solid line, dielectron mass distribution due to quark-antiquark annihilation.

Landau-Pomeranchuk effect is shown by a dot-dashed line. The final result with medium effect is indicated by a dashed line. As can be seen in this figure, the Landau-Pomeranchuk effect leads to a significant reduction (up to two orders of magnitude) of dilepton production due to bremsstrahlung of virtual photons from quarks moving in a QGP. The reduction of the rates at the temperature $T = 0.15$ is particularly important in the low invariant-mass region, where the mass of the pair is lower than 0.1 GeV. With increasing M the effect is reduced, and at $M \sim 1$ GeV no decrease is seen anymore. This behavior can be understood intuitively as follows. The time necessary for e^+e^- pair production is proportional to its universe mass. Increasing the invariant mass means also decreasing the production time. Thus the amount of collisions of the test quark with a surrounding plasma constituents is decreasing as well. Consequently, the medium effect is less important. We have to stress, however, that when going beyond the soft-photon approximation the suppression will be seen for larger invariant-mass values. Our results should be valid when the values of the invariant mass of dielectron pairs is of the order of the inverse of the strong-interaction collision time. In our case it would be the part of the spectrum where the mass is less then $M \sim 0.3$–0.5 GeV.

When discussing dilepton production in a QGP, the bremsstrahlung is not the only source of e^+e^- pairs. Dielectrons can be produced by the basic quark-antiquark annihilation process. In Fig. 2 the solid line indicates the contribution due to the annihilation of quarks. As seen in Fig. 2, at the temperature $T = 0.15$ GeV the most important thermal source for dileptons in the mass window with $M < 0.4$ GeV is bremsstrahlung. Even the reduction due to the Landau-Pomeranchuk effect is not sufficient to see dileptons from quarks annihilation in a plasma.

In Fig. 3 we show the same quantities as in Fig. 2, but for a higher temperature, $T = 0.3$ GeV. There are two properties which are different in these figures. First of all, one can see that the dominance of the bremsstrahlung contribution to the overall thermal spectrum is shifted to higher invariant mass. Here it dominates over the quark annihilation contribution up to $M \sim 1$ GeV, instead of $M \sim 0.4$ GeV, as it was indicated in Fig. 2. This evidently means that with increasing temperature bremsstrahlung production increases faster than the production of e^+e^- pairs due to the quark annihilation process. Second, when comparing Fig. 2 with Fig. 3, the Landau-Pomeranchuk effect is seen to be an increasing function of temperature. This result is also indicated in Fig. 4, where the thermal spectrum $dN/d^4x\, dM$ is shown at fixed value of $M = 0.1$ GeV for different temperatures.

An increase of the medium effect on bremsstrahlung with temperature as seen in Fig. 4 could be expected. With increasing temperature the density of a medium is increasing. Consequently, the amount of collisions is also increasing, leading to a higher suppression of dilepton production due to destructive interference of virtual photons produced between the collisions.

Until now, we have only considered e^+e^- pair production in a quark-gluon plasma. To discuss the above results in the context of dilepton production as a probe of quark-gluon plasma formation [20], one needs to compare these with those obtained in the hadron gas phase. This comparison should be performed within a consistent scenario of heavy-ion collisions which admits initial QGP

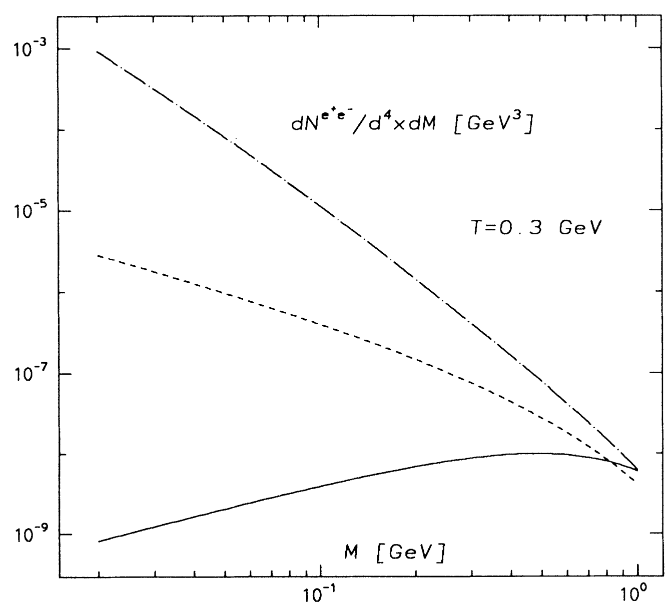

FIG. 3. As in Fig. 2, but for temperature $T = 0.3$ GeV.

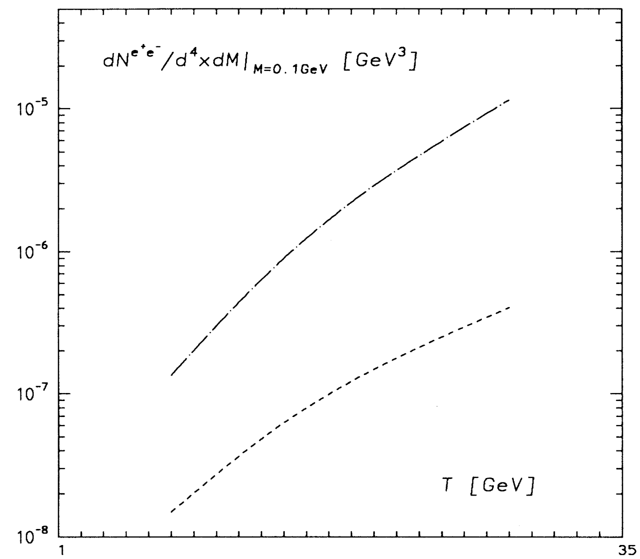

FIG. 4. Dielectron production rate $dN/d^4x\,dM$ at fixed value of invariant mass $M=0.1$ GeV as a function of temperature: dot-dashed line, virtual-quark bremsstrahlung with no medium effect; dashed line, virtual quark bremsstrahlung with the Landau-Pomeranchuk effect.

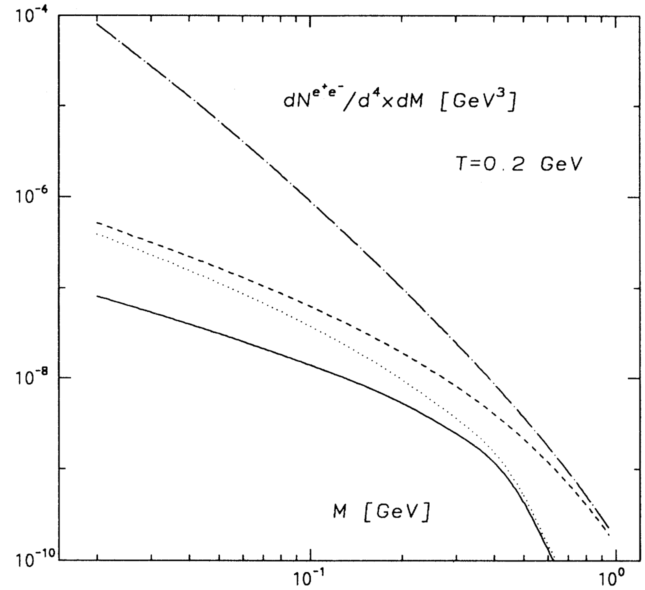

FIG. 5. Comparison of QGP and hadron gas contributions to the soft part of dielectron spectrum $dN/d^4x\,dM$ at fixed temperature $T_c=0.2$ GeV: dot-dashed, virtual-quark bremsstrahlung with no medium effect; dashed line, virtual-quark bremsstrahlung with the Landau-Pomeranchuk; dotted line, virtual-pion bremsstrahlung without medium effect; solid line, virtual-pion bremsstrahlung with the Landau-Pomeranchuk effect.

formation with subsequent hadronization. In order to get some estimate of the importance of the QGP contribution to the overall low invariant-mass dilepton spectrum, we show in Fig. 5 both quark and hadron gas contributions at a fixed temperature $T=0.2$ GeV. This type of comparison could only be valid at the critical temperature where a QGP and hadron phase coexist.

The results for virtual bremsstrahlung from pions (dotted line) and from quarks (dot-dashed line), presented in Fig. 5, shows that in the soft part of $dN/d^4x\,dM$ spectrum quark bremsstrahlung is of crucial importance. Without the Landau-Pomeranchuk effect, a pion gas and QGP contributions differ considerably. This is not the case when the Landau-Pomeranchuk effect is included. Although here the pion bremsstrahlung (solid line) is below the quark contribution (dashed line) as well, the difference is only a factor 3–6. This evidently means that medium suppression of virtual photons radiated from a charged particle is larger in a QGP than in a hadron gas. This behavior is certainly not only related to a difference in the cross sections as discussed in the Introduction. At a fixed temperature, the number of degrees of freedom and consequently the density of a QGP are higher than in a hadron gas. This naturally implies higher suppression in a QGP.

From Fig. 5 one could conclude that the QGP bremsstrahlung is dominating the $dN/d^4x\,dM$ spectrum below the ρ resonance peak. We have to point out, however, that the above results could be changed when a realistic scenario for QGP formation in heavy-ion collisions is taken into account. As the QGP is initially formed with a higher temperature than the hadron gas, their contribution will be larger than indicated in Fig. 5 by a dot-dashed line. However, the Landau-Pomeranchuk effect will be stronger in this case. Since the hadron gas contribution will increase as well when integrating over the history of the collision, we expect that even if the expansion dynamics is taken into account the contributions due to quark and pion virtual bremsstrahlung will be of the same order.

A more detailed analysis of thermal soft-dilepton production in an expending medium and comparisons with nonthermal emission are under consideration.

V. SUMMARY AND CONCLUSIONS

In this paper we have calculated the invariant-mass distribution of dileptons produced in a QGP. We have concentrated our discussion on the low invariant-mass part of the $dN/d^4x\,dM$ spectrum and applied the soft-photon approximation.

As a main source of dilepton pairs, we have considered bremsstrahlung-type emission by quarks moving in a QCD medium. We have derived the result for the dilepton spectrum which is infrared finite in the limit of massless quarks. We have then explicitly included the Landau-Pomeranchuk effect and shown that it leads to a significant reduction of the low invariant-mass dilepton spectrum.

The quantitative dependence of the Landau-Pomeranchuk effect on temperature and invariant mass has also been described. This effect was found to increase with temperature and decrease with increasing invariant mass.

Our results suggest that even with suppression from the medium bremsstrahlung is the main source of dileptons produced in a QGP at low invariant mass. Thus, when comparing the QGP with a hadron gas contribution, the bremsstrahlung, instead of the quark annihilation process, should be taken into account as a main source of pairs in the soft part of the $dN/d^4x\,dM$ spectrum.

As the Landau-Pomeranchuk effect is found to be of the order of $\alpha_s^2 \ln\alpha_s$ while the dynamical screening due to a hard thermal loop [19,9,10] is of the order of α_s^2, the former one seems to be more important when analyzing soft-virtual-photon radiation in a QGP.

Our discussion also shows that the significant increase of the rates in a QGP due to quark bremsstrahlung in the low invariant-mass region may possibly be sufficient to be a noticeable source of dileptons in heavy-ion collisions. This is because at a fixed temperature the QGP bremsstrahlung of virtual photons is found to give the dominant contribution to the thermal dielectron spectrum. At the same time, the pion bremsstrahlung and Dalitz π_0 and η decays give a very large contribution in this kinematical part of the emission rate [20]. Thus the verification of the above statement would require calculations in a more realistic scenario of the heavy-ion collision.

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